Theory and Practice of Cost Functions

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The cost function is the backbone of any archaeological least-cost analysis. In this paper, first the general properties of cost functions are outlined, for example all costs must be positive. It is shown that some of the functions used in archaeological least-cost path (LCP) studies do not fulfil these requirements. Nearly all LCP studies are slope-based. Therefore several slope-dependent cost functions for walkers and vehicles are discussed. Slope is often combined with some other factor, so the different methods for combining cost components are compared. The theoretical concepts presented are applied to reconstruct part of an ancient trail. The study area is a hilly terrain in Rhineland, Germany, with many small rivers and creeks.

Keywords: least-cost paths, cost function.

1. Introduction

In prehistory, paths are often the result of centuries of experience with movement within a certain landscape. According to the implicit model of least-cost path analysis, the routes are improved gradually so that finally optimal paths result. With the increasing availability of GIS software, the number of archaeological studies trying to reconstruct prehistoric routes by least-cost path analysis (LCP) grows (e.g. FIZ *et al*., 2008; ZAKŠEK *et al*., 2008). Though pressing a few buttons in GIS software is quite easy, the choice of the appropriate algorithm for the least-cost analysis is quite a complex issue.

The backbone of any archaeological least-cost analysis is the cost function. Other issues including the number of neighbours considered in the raster grid, the accuracy and resolution of the elevation data or the method dealing with anisotropic data like slope are also important (HERZOG *et al*., 2011). However, the focus of this publication is on the choice of the cost function.

Nearly all LCP studies are slope-based, and slope is often combined with some other component. Therefore several slope-dependent cost functions for walkers and vehicles are discussed.

Whatever cost function is chosen, it has to be kept in mind that the time and energy needed for a walk varies depending on weather, sex and fitness of the walker etc. but the LCP algorithm produces only one result.

2. General properties of cost functions

The cost of traveling from location A to B at some distance always involves some positive costs in terms of time or energy required, i.e. CostDist(A,B) > 0, for all B≠A (WORBOYS *et al*., 2004, 124; ERICSON *et al*. 1980). LCP software is typically based on Dijkstra's algorithm (e.g. WORBOYS *et al*., 2004, 215–216), which requires weighted graphs with positive weights. Only a cost function with positive values ensures that the weights in the graph are positive.

In addition, the triangle inequality holds for the cost distance function, i.e. the LCPs created on the basis of a valid cost function. The triangle inequality implies that the costs of the direct LCP from A to B are smaller or equal to the sum total of the costs for the two LCPs from A to C and C to B, independently of the location of C. This means that a detour from the LCP will never save costs.

Multiplication of the cost function values by a constant factor does not change the result of the LCP analysis. Such a multiplication is necessary when converting from one unit of measurement to another, e.g. from kilocalorie to joule. It is obvious that the path connection which requires the minimum amount of kilocalories is also optimal with respect to joule consumption.

But a transformation by a monotonically increasing function does make a difference: If crossing a barrier is n times as costly as walking in the surrounding area, a detour with a length below n cuts costs, if the barrier is avoided by this detour. In this example, different values of n will bring about different LCP results.

In general, only ratio-scale variables are appropriate for LCP analysis (e.g. CONOLLY *et al*., 2006: 255). If the analysis is based in part on intangibles, like the attraction to a center of worship, it is difficult to find an adequate cost model.

Cost functions are expected to be continuous, i.e. small differences in the environment will result in small differences in cost estimates. For example, slight changes in slope should not produce sudden jumps in costs. An exception is the crossing of streams: Whereas small creeks can often be traversed easily by jumping to the other side, with increasing breadth, a point is reached where jumping is no longer possible but significantly more effort is necessary to cross the stream.

Classifying can convert a continuous cost function into a non-continuous one (e.g. ZAKŠEK *et al*., 2008); though calculations might become easier this way, the classifying approach is counter-intuitive and with modern computers such a simplification is no longer necessary. However, classifying a cost function into a large amount of small classes reduces the implementation effort of a least-cost algorithm if the resulting program is to support many different cost functions. The errors introduced this way are negligible.

Isotropic costs are independent of the direction of movement, typical isotropic costs are those based on vegetation, soil properties, lakes, barriers like large rivers without fords, taboos, social attraction or visibility aspects. Anisotropic costs are dependent on the direction of movement. Slope is anisotropic, and slope is so important that sometimes the term anisotropic costs actually refers to costs based on slope (CONOLLY *et al*., 2006: 253). With anisotropic costs, CostDist(A,B) ≠ CostDist(B,A), where CostDist is the cost of the LCP connecting A and B. The cost function of a path taken in both directions becomes symmetric:

$$\text{Bicost}(A,B) = \text{CostDist}(A,B) + \text{CostDist}(B,A)$$

This assumption is only valid if similar conditions prevailed on the route in both directions. For example, when coal was transported one way and the empty wagons returned on the same route, the direction of movement is important.

3. Slope-dependent cost functions

As mentioned above, slope is considered an important component in most LCP studies. For walkers, vehicles and pack animals, different slope-dependent cost functions are appropriate.

3.1. The Tobler cost function

The Tobler hiking function seems to be the most popular cost function in archaeological LCP calculations, and has been applied in many studies. The velocity of walking is given by

$$V(s) = 6\, e^{-3.5\,|S+0.05|}$$

where s is the slope (calculated by vertical change divided by horizontal change). It is easy to calculate the time (minutes) required for walking a certain distance from the velocity of walking, and the time can be considered as cost.

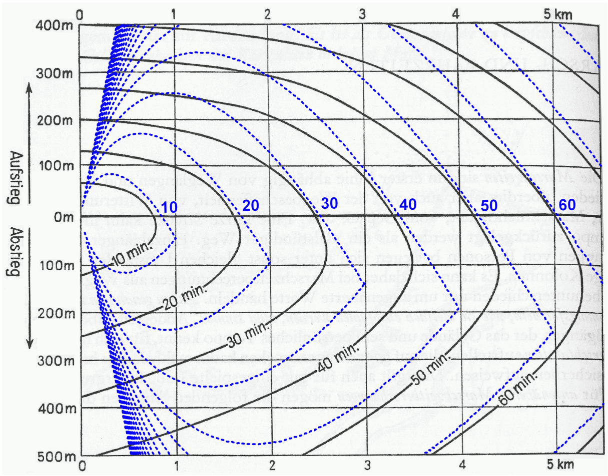

Figure 1: *The diagram published by IMHOF (1950, 218) and the Tobler isolines (dotted blue).*

According to Tobler (1993), the hiking function was estimated from empirical data published by IMHOF (1950: 217–220). Imhof's text includes only very few numbers, but displays a diagram which visualizes his cost curve. He does not give any information on the source of the data for the diagram. The Imhof diagram displays the time required for walking to any point which is up to 5 km in map distance from the origin, and involves up to 500 m of descent or up to 400 m in ascent. The diagram consists of the corresponding isolines in 10 minute intervals.

The Imhof diagram was compared with data generated on the basis of Tobler's cost function (Figure 1). Only points reachable via gradients that are not steeper than 100% (45°) were included in the calculation. Figure 1 shows that the pointed nature of Tobler's cost function is also visible in the isolines. However, the fit of the cost function to the Imhof diagram is not that perfect.

3.2. Backpacker's equations

WHEATLEY *et al*. (2002, 154) suggest starting from the 'backpackers equations' such as that proposed by ERICSON *et al*. (1980):

$$\Delta\_D + 3.168*\Delta\_H\_up + 1.2*|\Delta\_H\_down|$$

where Δ_D is the horizontal distance covered, Δ_H_up is positive and Δ_H_down negative height change.

The minimum of this cost function is at a slope of 0%, i.e. walking on level ground. The cost formula does not take into account that many investigators found that the lowest energy expenditure is on a 10% (5.7°) downslope gradient (e.g. MINETTI *et al*., 1993).

The formula implemented by the GRASS GIS r.walk procedure (version 6.5) is a piecewise linear cost function based on a rule of thumb published by LANGMUIR (2004, 40) and calculates time in seconds:

a*Δ_D + b*Δ_H_up + c*Δ_H_gd + d*Δ_H_sd

where Δ_D and Δ_H_up are defined as above, Δ_H_gd and Δ_H_sd are gentle and steep downhill differences. All distance measurements are in meters. The default values for the multipliers are: a=0.72, b=6.0, c=1.9998, d=-1.9998. Note that the downhill Δ_H values are negative as they reflect a negative change in height. The downhill default slope value threshold is at 21.25%, i.e. walking downhill for slopes up to 21.25% (12°) is considered favorable, for steeper descents the costs increase. The default r.walk cost function is not continuous: According to the formula, walking a horizontal distance of 1 km on a downslope gradient of 21% requires 6 minutes whereas the estimate for descending the same distance at a slightly steeper slope of 21.5% is 19.2 minutes. To create a continuous function, the constant c should be set to 0 or, alternatively, c and d should be identical.

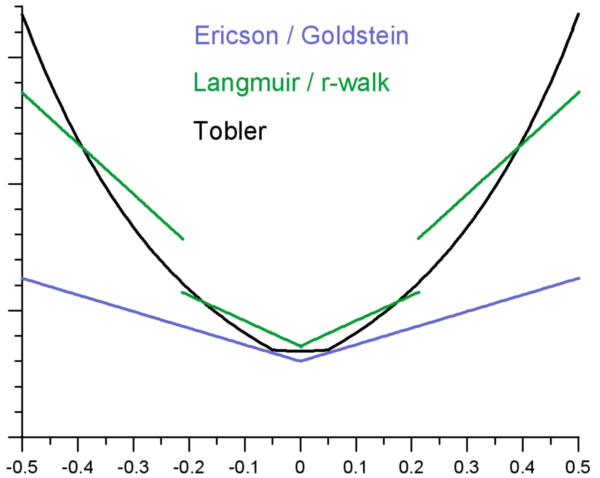

Figure 2: *Three slope-dependent bidirectional cost functions estimating time, depicted for mathematical slope in the range of –0.5 to +0.5 (i.e. –50% to +50%).*

Figure 2 compares the Tobler cost function and the two backpackers equations for return paths. The time estimates from the Ericson/Goldstein equation for steep slopes are significantly lower than with the other two cost functions.

3.3. Expert cost functions for walkers

Physiologists are experts in measuring energy expenditure in humans and animals, and for this reason, cost functions presented by physiologists are often considered more appropriate than a backpacker´s rule of thumb. However, the function for walkers suggested by MINETTI *et al*. (2002) is negative for steep downhill slopes (Figure 3). This problem can be avoided by applying the quartic polynomial cost function proposed by LLOBERA *et al*. (2007). But with the estimates derived from the Llobera and Sluckin cost curve, five out of 13 values are not within the range of the mean plus or minus the standard deviation given in the MINETTI *et al*. (2002) paper. Another disadvantage of both the quartic and the quintic polynomials is that their minima are at downslope gradients of 17.7% (10°) and 15.25% (8.7°) respectively, whereas experiments suggest a minimum at about 10%.

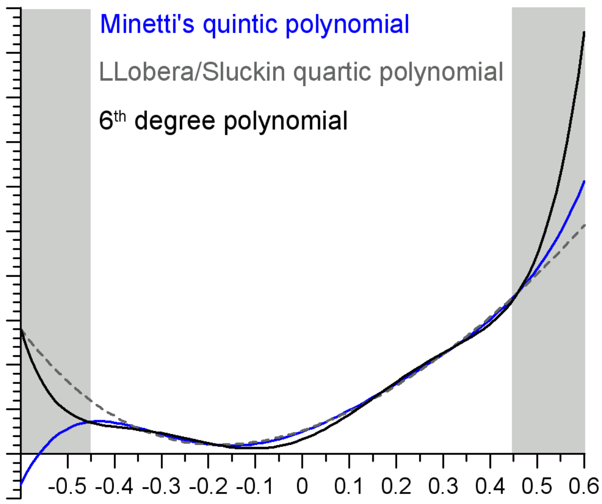

Figure 3: *Three cost functions estimating energy expenditure. Measurement data is available for the mathematical slope in the range of –0.45 to +0.45 (i.e. –45% to +45%).*

An alternative is to use a sixth degree polynomial approximation to the energy expenditure values found by MINETTI *et al*. (2002):

$$1337.8\ s^6 + 278.19\ s^5 - 517.39\ s^4 - 78.199\ s^3 + 93.419\ s^2 + 19.825\ s + 1.64$$

where s is the mathematical slope. This cost function shows none of the disadvantages discussed above, the minimum of the curve is at a downhill gradient of about 10.5% (6°).

3.4. Cost functions for vehicles

With increased production and trade, the need for transporting heavy and bulky goods grows. Humans or pack animals have only a limited capacity, and it is for this reason that wheeled transport plays a more and more important role after the invention of wheeled vehicles which first appeared in the 4th millennium BC in the area between the Rhine and the Tigris (BURMEISTER, 2004). Ever since their invention, wagons and carts have been used for ritual purposes and for prestige, so least-cost approaches in the context of wagon routes might not be appropriate in some situations.

MINETTI (1995) notes that the locomotion of biological systems is less efficient than that of vehicles, because animals and humans spend some metabolic energy for braking the motion. MINETTI *et al*. (1993) compare a walking human to the movement of a rimless spoked wheel. They found that humans do no longer accelerate if the downhill slope exceeds 15%, which is not true for vehicles.

Cost functions for wheeled vehicles do not agree with those for walkers due to differences in critical slope. The critical slope is the transition when it becomes more efficient to use switchbacks instead of the direct uphill or downhill route (LLOBERA *et al*., 2007). MINETTI (1995) analyzed the metabolic expenditure of walking different gradients and found that the critical slope for pedestrians is at about 25% for both descending and ascending paths; this value is a compromise between speed and energy optimization. This is a broad minimum, most of the gradients within the range of 15 to 40% are still fairly efficient.

In general, the critical slope for vehicles is less than that of walkers. Roman roads may serve as an example for wheeled traffic: GREWE (2004, 30) points out that for Roman roads the steep slopes were avoided in order to allow horse or oxen-drawn vehicles to proceed. According to Grewe, the slopes of Roman roads in the Rhineland normally do not exceed 8%, but at some exceptional locations 16% to 20% were recorded.

LLOBERA *et al*. (2007) calculate the critical slope of simple quadratic cost functions before addressing more complex problems. Based on their results, a symmetric quadratic cost function can be easily constructed for a given critical slope š:

$$\mathrm{Cost}(s) = 1 + (s / \check{s})^2$$

where š and s are percent slope values (or both are mathematical slope values). Typically, š is in the range of 8 to 16.

4. Combining Costs

In general, costs for traversing a landscape consist of two components: Anisotropic cost (slope), and isotropic cost. Different approaches for combining slope and other components have been suggested in archaeological LCP studies.

4.1. Adding costs

An example of cost addition was presented by FIZ *et al.* (2008). They add the slope friction and a wetness avoidance cost component after dividing each friction component by its maximum value. The value of each weighted cost component depends on the maximum cost value, and this is typically disliked by statisticians. This disadvantage could be avoided by choosing some other method of weighting which is robust in the presence of outliers. But another problem cannot be fixed that easily: Changing the extent of the study area may also change the cost function, for example if steeper slopes are found in the wider area included later.

4.2. Multiplying costs

The study by Zakšek *et al*. (2008) is an example for cost multiplication. The authors multiply the slope by the visibility costs. Multiplication does not require any weighting. In some physiological studies multipliers were determined in order to account for certain features preventing fast progress, for example the terrain factor for loose sand is about 2. However, some experiments suggest that the terrain factor on steep slopes is higher than on level ground. So multiplying is simple and more realistic than adding but does not provide a perfect model.

5. Example: The Heerweg

It is often difficult to assess the plausibility of archaeological route reconstructions based on the location of settlements because no evidence is available to check the LCP results. The ancient route Heerweg (NICKE, 2001: 85–88) can be traced on historic maps created between 1840 and 1844 and therefore serves as a test case for evaluating the reliability of route reconstruction using LCP techniques.

The focus of the study is on the western part of the Heerweg which connects the Cologne district of Mülheim with Lüdenscheid in North-Rhine-Westphalia, Germany. According to the description of Nicke, the route starts close to the river Rhine in Cologne-Mülheim (47 m asl, no. 1 in Figures 4 and 6); the first five kilometers are on fairly level ground, but then the route reaches a hilly region and becomes steeper. The first intermediate stop is at Bechen (248 m asl, no. 2 in Figures 4 and 6), though an alternative shorter route is also described by Nicke. The place name Wipperfürth (272 m asl, no. 3 in Figures 4–6) alludes to the ford which is close to this town. Two bridges crossing the river Wupper can be found near Wipperfürth on the map published in 1840. Both bridges were most probably constructed at ancient ford locations According to Nicke, various routes were used over time by the Heerweg after crossing the river Wupper. East of Wipperfürth, the Heerweg passes Halver (419 m asl, no. 4 in Figures 4 and 6), and the final stop considered in this study is Lüdenscheid (423 m asl, no. 5 in Figures 4 and 6). Unfortunately, the route description of Nicke lacks detail for the section between Halver and Lüdenscheid, which is why several possible routes were recorded on the basis of the historic maps.

Another feature helps to identify the correct route location: place names like “Obern Herweg” or “Dieves Herwege”. Settlements with such names are marked by black triangles in Figures 4 and 6. The largest distance between a Heerweg place name and one of the routes described by Nicke is Küppersherweg, which is about 1.6 km south of a Heerweg route.

When the Heerweg starts to climb towards Bechen, it reaches the hilly region. In the study area, which includes parts of the fairly flat Rhine valley, only 25% of

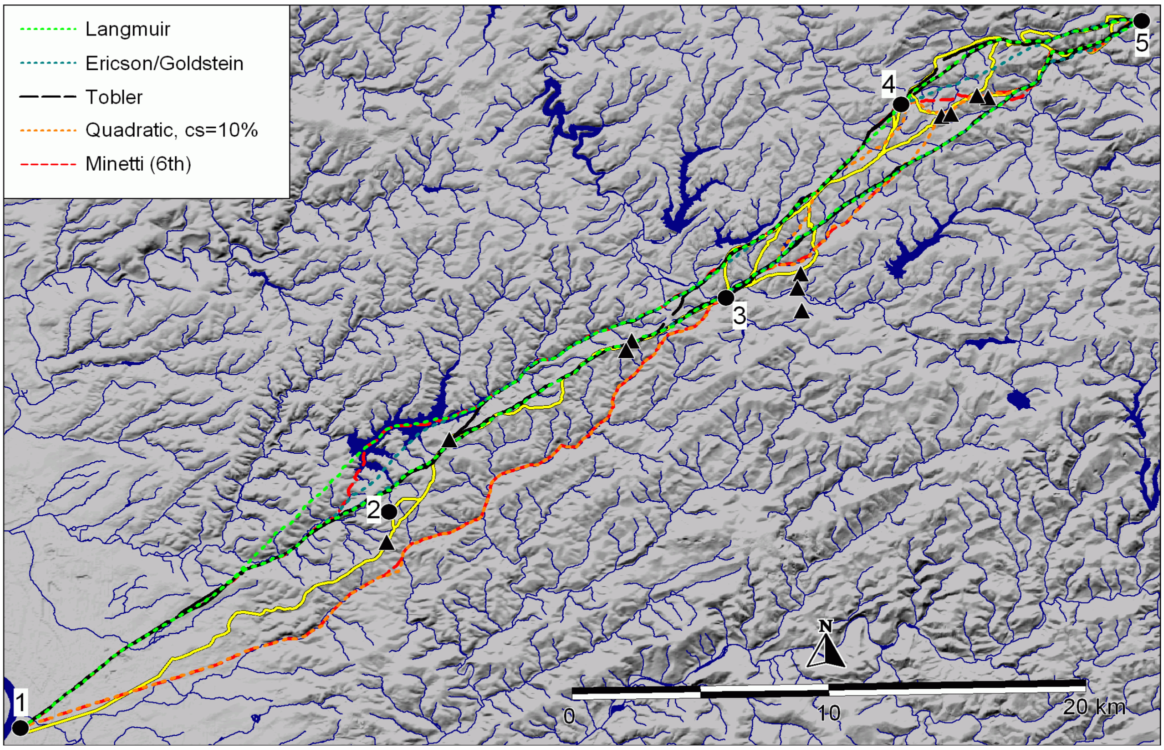

Figure 4: *The Heerweg (yellow) and LCPs based on slope costs only.*

the slopes are below 6% (3.4°), the median slope is 11.3% (6.5°), and 25% of the slopes exceed 17.9% (10.2°). Due to the prevailing western winds, precipitation in the hilly region is significantly higher than in the Cologne area. The climatic and geologic conditions allow growing crops in only very few areas, whereas most farms focus on stock breeding. Due to these natural factors, the landscape remained nearly unoccupied for a long time. During the 11th to 13th century population growth in the area west of the river Rhine forced people to leave their home villages and to move east. This is reflected in the dates when the towns along the Heerweg were first mentioned in historic sources (NICKE, 2001, 200–206): Archbishop Friedrich I referred to Wipperfürth in a deed dating from 1131 BC. Wipperfürth is not only at the intersection of two ancient roads (in addition to the Heerweg, the Eisenstraße from Kreuztal to Lennep), but also the end point of another old route named Polizeiweg by Nicke. The initial layout of Halver is a village built around a church, and the town has preserved much of this character until today. The village was first mentioned in connection with the abbey of Werden near Essen around 900 BC. Halver is also an intermediate stop of two other ancient routes, the Zeitstraße connecting Bonn with Dortmund and the Hileweg leading from Essen to Limburg. In a deed dated in 1067 BC, the church of Lüdenscheid was first mentioned.

Even today, the study area considered is not as densely occupied as the rest of the Rhineland. Nevertheless, the landscape underwent some changes by modern construction and mining activities. Some of these are already recorded on the historic maps of 1840–1844. For example, about 6 km after leaving Mülheim, pits and artificial lakes are depicted in the area around the Heerweg. The digital elevation model supplied by the ordnance survey of North-Rhine Westphalia with a resolution of 50 m also shows some large pits resulting from quarrying activities or from extracting other bulk material like brick earth. However, it is not within the scope of this study to reconstruct the relief of the landscape of medieval times or earlier.

The modern water layer includes many lakes resulting from the construction of a dam for creating water reservoirs (Figure 4). A polyline water layer is also available, which includes lines within modern water reservoirs. These lines are a smoothed representation of the streams depicted in the historic maps. Thus, for the route calculations including the streams these lines were used instead of the lakes. Moreover, two modern canals were deleted from the layer.

5.1. Slope-dependent route calculations

The first LCP calculations focused on slope only (Figure 4). None of the initial routes passes Halver. Therefore it was decided to calculate both the direct route from Mülheim to Lüdenscheid, as well as the route with a forced intermediate stop at Halver. The calculations are based on the assumption that the same route was used in both directions. The software used in this study was created by the author because none of the GIS LCP procedures currently available offers all the features required. The software (HERZOG *et al.*, 2011) models anisotropic movements in a similar way as the r.walk approach of GRASS GIS. Moreover, for each step, 48 neighboring cells were considered, and long moves were subdivided to ensure that the long moves take the costs of crossing thin barriers into account. The Dijkstra algorithm including backlinks was implemented.

Figure 4 shows that some of the LCPs based on different cost functions agree in some parts, but in general, several LCP corridors are found. The direct Mülheim to Lüdenscheid LCP created on the basis of the sixth degree polynomial cost function runs south of the historic route and uses the valley of the Sülz creek. After passing Wipperfürth the LCP follows another creek valley, and after 4 km it runs south of the historic route again. Only on the last 6 km, the calculated route and one of the historic alternatives agree quite well. The LCP created with the same cost function but the intermediate stop at Halver runs north of the historic route most of the time, also following creek valleys like the Dhünn (nowadays a water reservoir). After crossing the Wupper north of Wipperfürth, it coincides with the historic route for a few kilometers, and on the final kilometers before reaching Lüdenscheid the LCP agrees well with one of the historic alternatives.

Both the direct and the LCP through Halver based on the quadratic cost function with a critical slope of 10% follow the same route until Wipperfürth, which coincides with the southern route found with the sixth degree polynomial cost function. The LCP with a critical slope of 10% from Halver to Lüdenscheid agrees quite well with the most southern historic alternative route.

The first part of the direct Tobler LCP uses the same route as the northern LCP based on the sixth degree polynomial, coincides with the historic route on 7 km before Wipperfürth, but the agreement on the final stretch of the road to Lüdenscheid is not as good as that of the two other LCPs discussed above. The Tobler LCP with intermediate stop at Halver is quite similar to the LCP based on the sixth degree polynomial between Mülheim and Halver, except that the Dhünn valley is avoided but a more direct connection is taken. The LCP between Halver and Lüdenscheid takes the northern historic alternative. However, the southern historic alternative appears to be the more probable route, since four place names referring to the Heerweg can be found along this route.

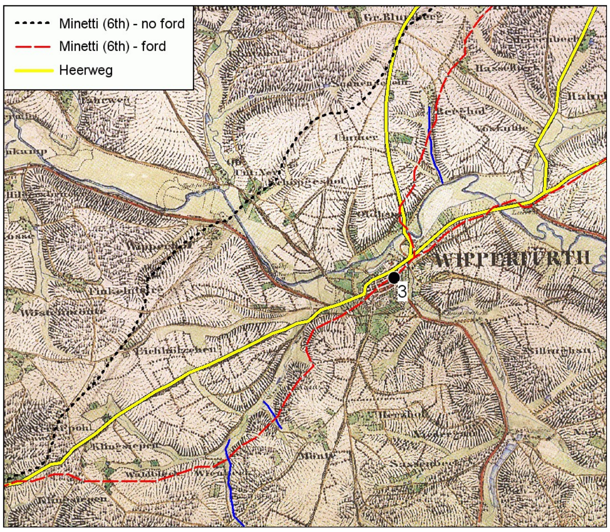

Figure 5: *Map from 1840 with digitized ancient routes (Heerweg); LCPs based on models including multipliers for streams, with and without fords.*

The LCPs calculated with the Ericson and Goldstein cost function coincide with the Tobler LCPs with two exceptions: The Ericson and Goldstein LCP from Mülheim to Halver runs between the Tobler and the sixth degree polynomial LCPs in the area of the Dhünn valley, and the Halver to Lüdenscheid LCP is straighter than the other LCPs, between the northern and the southern historic alternatives. The agreement between the Tobler and the Langmuir LCPs is quite high as well. Differences can be found on the route from Mülheim to Halver north of Bechen, where the Langmuir LCP follows the Dhünn valley and joins part of the LCP based on the sixth degree polynomial.

The agreement between the routes calculated on the basis of slope only and the historic road is not overly convincing. According to NICKE (2001, 13) hardly any construction work was employed for the medieval routes in the area considered in this study. Therefore, these routes could not follow the wet creek valleys. So the LCPs running through the creek valleys are not plausible. In modern times, road construction work started to enable transport of large amounts of heavy material, and nowadays roads run through the creek valleys. For example the historic map of 1893 already shows a road which coincides quite well with the two LCPs running through the Sülz valley.

To reconstruct the historic routes, it seemed plausible to combine slope costs with the costs of moving in the wet areas surrounding the creeks.

5.2. Combining slope and costs for water streams

As shown above, combination of costs by multiplication is a simple and intuitive choice. Buffers with a radius of 50 m were created for each stream polyline and the multipliers for the cells within these buffers were set to 5.

The black dotted line in Figure 5 shows the main problem incurred with this approach: The LCP based on the sixth degree cost function does not run through Wipperfürth but crosses the river Wupper at another location where no ford or bridge can be found. The river Wupper is the only major stream the Heerweg has to cross on the stretch between Mülheim and Lüdenscheid. Therefore, the buffer of the river was assigned a multiplier of 20, whereas in the areas of the two bridges on the historic map, fords were modeled with a multiplier of 5. This way the LCPs can be forced to cross the Wupper at the fords. Figure 5 shows also that some of the creeks depicted on the 1840 map are not included in the modern water layer (blue lines). For this reason, the LCPs deviate in some parts from the historic routes.

Figure 6 shows the LCPs based on the combined slope and water stream costs. Considering the western part of the Heerweg from Mülheim to Wipperfürth, all LCPs agree quite well with the historic route. The LCPs based on the sixth degree polynomial and the parabola with a critical slope of 10% run on the same route most of the time and come closest to the route described by Nicke. These two routes run via Bechen whereas the Tobler, the Ericson/Goldstein and the Langmuir LCPs take a more direct route. Between Wipperfürth and Halver the agreement between the LCPs and the historic routes is not very accurate. As pointed out above, this might be due to creeks missing in the water layer. Between Halver and Lüdenscheid, the LCPs come closer to the historic routes but the fit is by no means perfect.

Conclusions and future work

The Heerweg example shows that a good model fit for the first part of a route does not necessarily mean that the model will hold for the second part as well. NICKE (2001, 13–19) lists some other factors which could be included in the model: ridgelines were preferred and routes along contour lines were avoided because they required construction work. If ridgelines had too many ups and downs, a lower route preferably with south aspect – or if this was not available – east aspect was chosen. Though water streams were avoided most of the time, pack animals and the people needed fresh water at some time during the day, so that routes crossing a creek close to the source were selected.

A better result is to be expected after updating the water layer and including the additional aspects listed above. Some problems may also arise from the 50 m resolution of the DEM. For example, some of the creek valleys are deeply incised and this will be smoothed by a low resolution DEM. As mentioned above, changes in the landscape since medieval times may also account for deviations between the LCPs and the true historic routes.

The Heerweg example and several other from the hilly region east of the river Rhine show that in areas of obvious natural paths the choice of the slope-dependent cost function is of minor importance in a moderately undulating landscape, but that other cost components, namely the costs of crossing streams, often play a significant role.

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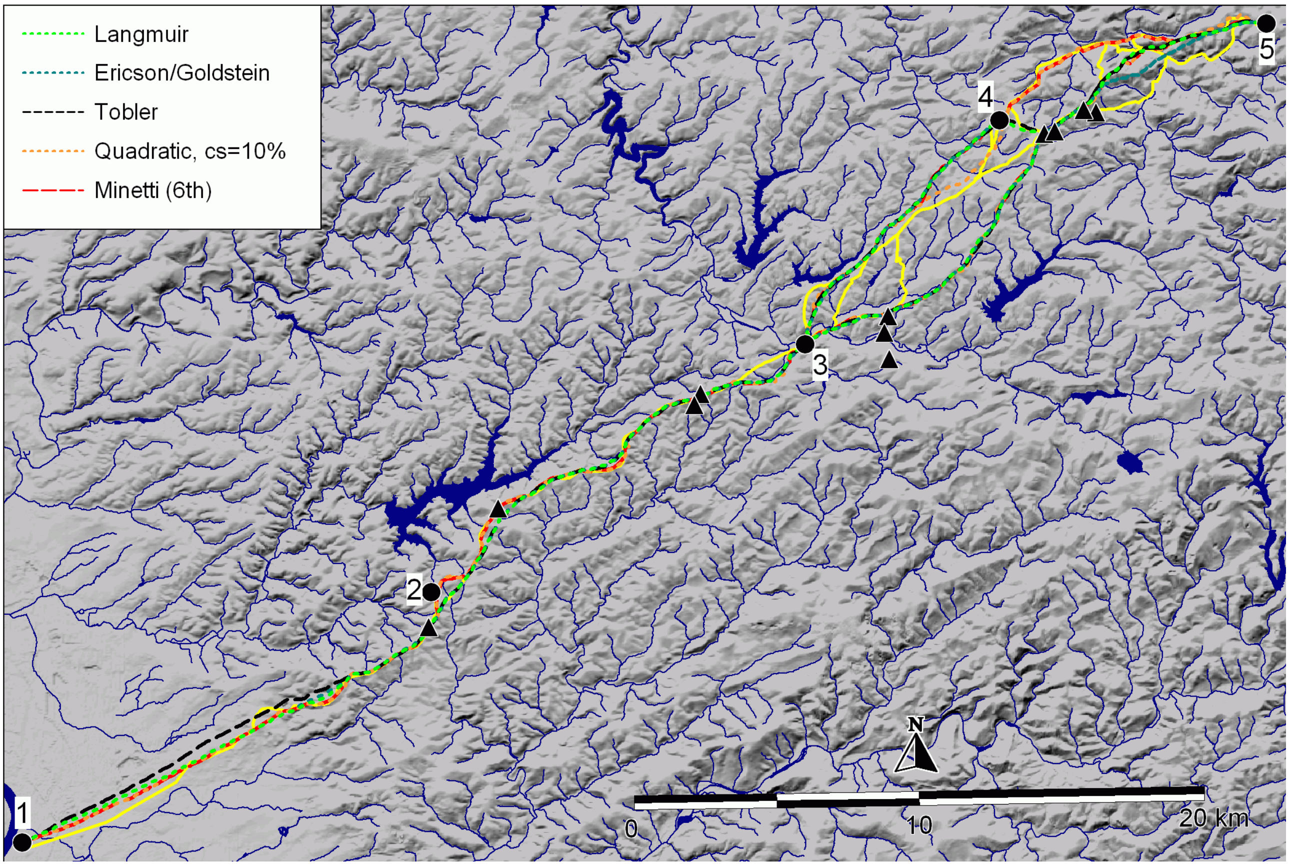

Figure 6: *The Heerweg (yellow) and LCPs based on both slope and stream costs.*

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