Constrained Correspondence Analysis for Seriation of Sagalassos tablewares

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Abstract. Reconstructing the time dimension is a crucial aspect of archaeological activity. Various methods are available based on a variety of scientific concepts. Often, correspondence analysis is used to obtain a seriation. However, the seriation solution does not produce explicit time frames, only a relative ordering. Furthermore, correspondence analysis ignores additional absolute dating information that may be available for some deposits. Also, correspondence analysis does not use criteria that logically restrict the order of the seriation. In this paper, we propose a constrained form of correspondence analysis that takes such restrictions into account. Using these constraints we are able to assign explicit dates to a seriated solution. We present a case-study that takes the "classical" methodology as a starting-point, and tests the established chronological framework of the Roman to early Byzantine tableware produced at Sagalassos against the application of constrained correspondence analysis. Interestingly, the results prove to be largely compatible.

Keywords: Seriation, Correspondence analysis, Constraints, Explicit dating, Sagalassos.

1 Sagalassos and its tableware

Archaeological excavation and survey projects typically produce an abundance of small finds. These mostly fragmented objects are in essence mute. The information which they contain can only be made intelligible when they are integrated into a methodological framework. The concept of this framework intrinsically determines the interpretation of the material, and the working of archaeology as a discipline. From its early days onwards, classical archaeology has been mostly object-oriented (Whitley, 2001). Artefacts, be it pottery or sculpture, have been classified, categorized, and ordered in an evolutionary sequence, and are typically published as a catalogue of finds, arranged in chronological order (Allison, 1997). The rationale behind this type of work may be self-evident, and even essential in its first stages. The results, however, are mostly restrictive in preventing the integration of the artefacts into their lost, diverse and multi-levelled contexts of production and use (Hodder, 1999: 66-79; Kingery, 1996).

The deficiencies of the traditional typo-chronological approach may even be more outspoken when studying material from ongoing excavation and survey work, as at ancient Sagalassos. Here, each summer, similar series of objects are found in a variety of contexts, but each time slight variations in assemblage composition are noted. The attested variations may result from different formation processes, different activity patterns or use of the material, and/or from short-term chronological evolutions. Each of these three processes is the focus of recent research efforts at Sagalassos, by mapping in detail erosional events in collaboration with geomorphologists, introducing concepts of contextual archaeology in the fieldwork and subsequent data-processing, and by constantly trying to improve the quality of the chronological framework of the material. This paper only deals with the last of these aspects, the definition of the chronological framework of the locally produced tableware.

The archaeological site of Sagalassos is located in southwestern Turkey. The town was formerly known as the metropolis of its ancient region of Pisidia, a region characterized by the Taurus mountains and a series of lakes. The urban site was laid out on various platforms at an altitude between 1400 and 1600m. Its origins have not yet been documented by the ongoing excavations, which have mainly highlighted important phases of expansion and urban lay-out around the beginning of our era, the earlier second century AD and the early fifth century AD. After having suffered from a major earthquake in 518 AD, the town still tried to recover, but a cocktail of epidemics, water shortages, a general lack of security and stability, a failing economy and finally another devastating earthquake around the middle of the seventh century AD, forced the inhabitants of Sagalassos to abandon their town and re-settle in the valley (Waelkens, 1993; Waelkens and Poblome, 1993; 1995; 1997; Waelkens and Loots, 2000).

In antiquity, Sagalassos may not have been much more than a provincial primus inter pares. Yet, within the context of classical archaeology in Asia Minor, the systematic interdisciplinary approach towards the reconstruction of the palaeo-ecological constraints and potential of the town and its territory, and the economic exploitation of the available resources has drastically improved our understanding of the functioning of the settlement and the everyday life of its inhabitants (Waelkens et al., 1997). One of the focal points of research is the evaluation of the importance of artisanal production to the economic network of the ancient site and its territory. A wide range of mineral resources was exploited, supplying the local building industry, local blacksmiths, a local glass workshop and a prolific mass producing pottery production centre. Also agricultural products were exploited, for instance, supplying bone-workers and the local guild of textile dyers. In this way, the town may be considered the regional pole of attraction of interdependent exchange patterns (of people, goods and ideas) and exchange mechanisms (reciprocity, redistribution and trade).

The discovery in 1987 of a potters’ quarter at Sagalassos was unexpected. Over about six hectares, located to the east of the ancient town, dumps of misfired ceramics are still
noticeable at the surface. Architectural ceramics, figurines and oil lamps, cooking and storage vessels, as well as a series of tableware were produced locally with six different clay fabrics. In economic terms, the newly discovered tableware, or Sagalassos red slip ware, can be considered the most important feature of this production centre. After a late Hellenistic antecedent, mass-production of this new type of eastern sigillata started during the early Augustan period and lasted into the first half of the seventh century AD. The ware was traded intensively in Anatolia, and could already be identified at a series of sites in the eastern Mediterranean and beyond, as far away as Italy and Nubia, and shows an interesting connection with ancient Egypt (Poblome, 1996; Poblome et al., 2001).

The basic chronological evolution of the Sagalassos wares has been reconstructed, using traditional archaeological weaponry (Poblome, 1999). The first step comprised a thorough stratigraphical analysis. In a next step, the shortlisted stratigraphical units were quantified by count and weight. Earlier experiments with estimated rim equivalents, and minimum number of individuals were stopped because of an overload of material to be processed. From the quantified data, ceramic assemblages were reconstructed. Next the assemblages were seriated, using both stratigraphical criteria and changing patterns of popularity of types and variants. In a final step, the relative chronology was dated using internal criteria, such as other datable objects, and external criteria or comparanda and exported material. In this way, nine main phases have been distinguished in the development of the local tableware, between early imperial and early Byzantine times. Each year, new ceramic assemblages need to be dated, however. Although the majority of these assemblages can fairly easily be attributed to one or other main phase, it has proven difficult to maintain a fixed chronological order within each phase, with new assemblages at times upsetting a previously reconstructed model of evolution.

In this context, the idea of improving the chronological framework of the Sagalassos tableware with constrained correspondence analysis was born. The application of this newly developed technique is not only beneficial to narrowing down the chronological position of newly discovered ceramic assemblages, but also provided a boost for the already existing chronological sequence.

2 Constrained Correspondence Analysis

The data set contains a total of 26,166 sherds from a selection of 27 relevant assemblages. The sherds belong to one of the 85 types and variants of Sagalassos red slip ware, representing the entire evolution from early imperial into early Byzantine times (Poblome, 1999). Only frequency counts were taken into account, not the weight of the sherds. In this manner, a two-way contingency table is obtained of vessel type/variant by assemblage. The basic idea behind seriation is that over time a given vessel type is introduced, becomes common, and finally is no longer used and production stops. Therefore, the basic assumption is that the distribution of sherds is single peaked. In archaeology, such single peaked distributions are often displayed in so-called battleship graphs (Ford, 1962). However, the main problem is that we do not know the temporal ordering of the assemblages a priori. Correspondence analysis (see, for example, Greenacre, 1984; Gifi, 1990) is a popular technique for seriation, thereby searching for the unknown temporal ordering of the assemblages. Note that correspondence analysis does not give explicit dates but only a relative ordering of the assemblages.

Often, however, the archaeologist has more information available on the chronological position of the ceramic assemblages. For example, stratigraphical superposition may indicate that one assemblage must be younger than another. Exceptionally, absolute dating criteria may be available for some assemblages. In this case, a ceramic assemblage could be identified with the construction fill of the Neon-Library of Sagalassos, the date of which has been very narrowly defined by a set of seven foundation inscriptions to shortly after 120 AD (Devijver, 1993).

This additional information is not used in traditional correspondence analysis. Therefore, we propose an adaptation, constrained correspondence analysis, that can handle these type of restrictions. In this paper, we discuss four types of additional information that will be used by correspondence analysis (Poblome, 1999, for more details on the assemblages):

1. For one assemblage, the exact date is known (assemblage 4), that is, 100 AD. Assemblage 4 is associated with the aforementioned Library. We have also fixed assemblages 1 to 1 AD, 22 to 410 AD, and 27 to 650.
2. Some assemblages necessarily have the same date. Assemblages 6 and 7 and assemblages 24 and 25 represent the stratigraphical continuation of one another.
3. Some assemblages are necessarily ordered in time based on stratigraphical superposition. Assemblage 2 is older than 7 (site NoN), 5 is older than 12, 13, 12 is older than 11 (site L), 3 is older than 5, 5 is older than 17, 17 is older than 19 (site LW), 6 is older than 13 (site EoN), 22 is older than 23 (site H), and 26 is older than 17 (site B3).
4. Some assemblages must be older (or younger) than an explicit date (assemblage 1, 22, 21, 24, 25 and 27). Assemblage 22 was associated with the first use of the late Roman fortification wall, assemblages 1 and 27 with the clear-up of a devastating earthquake in 518 AD, and assemblages 21 and 24 and 25 with the beginning and the end of the mass production of tableware at Sagalassos.

We will show that these constraints can be reformulated as either equality or inequality constraints. In addition, our method of constrained correspondence analysis provides explicit dating for all the assemblages, also for those of which the dates were not known a priori.

2.1 Applying equality and inequality constraints

In correspondence analysis, usually two sets of coordinates are fitted: the coordinates $r_i$ for the assemblages, and the coordinates $c_i$ for the type of ceramics. Here, we are only interested in the coordinates $r_i$ for the assemblages, since these are used for seriation. In classical correspondence analysis, all coordinates $r_i$ are optimally estimated. Below we consider how the four types of equality and inequality constraints can be handled in constrained correspondence analysis.

The first type of constraint is the exact dating constraint. To make sure that the specified assemblages (1, 4, 22, and 27) are restricted to their specific date (1, 100, 410, and 650), the
Consider the current data, where we have explicit dates for some assemblages. That is, we know
explicit dates, that is, we know
must be older (or younger) than a specific date. This type of
restrictions are set and these can be reparametrized as
coordinates of the remaining assemblages, no equality
holds for assemblages 24 and 25. These equality restrictions can be written as
or, equivalently,
In a similar way, an explicit date inequality can be obtained for assemblage 24 that must be younger than
for assemblage 24 that must be younger than
or, equivalently,
In the previous section, we indicated that assemblages 21, 23, 24, 25, and 26 must be younger than
and assemblage 21 older than 518 AD. These restrictions lead to the following inequalities:
For the last type of inequality constraint, we wish to consider inequality restrictions for some of the assemblages on
explicit dates, that is, we know a priori that some assemblages must be older (or younger) than a specific date. This type of
inequality restriction can be made explicit because explicit date information is available through the restrictions in (1).
Consider the current data, where we have explicit dates for
(r22 = 518 AD) and
(r27 = 650 AD). We also know that
assemble 22 must be dated before 518 AD, even though
is unknown at this stage. The patterned area in Figure 1 shows schematically where
may be located. This restriction can be written as
or, equivalently,
Figure 1. Schematic view of the restriction on assemblage 22 that it must be older than the explicit date of 518 AD. The patterned area indicates where r22 can be located.
2.2 Fitting constrained correspondence analysis
Groen and Poblome (in press) show that the equality and inequality constraints discussed above can be imposed by applying an alternating least squares algorithm where in each
iteration the coordinates for the assemblages are updated while keeping the coordinates for the pottery types fixed, followed by an update of the pottery types where the assemblages are kept fixed. This leads to a standard regression problem for updating the pottery types. For the update of the assemblages, restricted by equality and inequality constraints, we transform the problem to a nonnegative least-squares problem for which a direct solution exists (Lawson and Hanson, 1974). The linear constraints are fitted using results from Böckenholt and Takane (1994). The algorithm for constrained correspondence analysis has been programmed in a prototype matrix language, that is, in MatLab. For more computational results, we refer to Groenen and Poblome (in press).

2.3 Reconstructing the dates

For assemblage 16, this procedure is illustrated by a dashed line. Figure 2 clarifies this procedure using the constrained correspondence analysis solution. On the vertical axis, we see all coordinates and the dates that were restricted by (1). This link allows us also to map the assemblages with unknown dates (with or without inequality restrictions) linearly onto the date axis. Figure 2 clarifies this procedure using the constrained correspondence analysis solution. On the vertical axis, we see all coordinates obtained by constrained correspondence analysis. Those coordinates that are linearly constrained to the date axis (thus assemblages 1, 4, 22, and 27) are represented by solid circles. To reconstruct the dates for those assemblages with unknown a priori dates, we simply project such a point (like assemblage 16) onto the line connecting the points with known dates. Then, the horizontal coordinate is the reconstructed date. For assemblage 16, this procedure is illustrated by a dashed line. This line connects to the date axis at 306, which is the reconstructed date for assemblage 16.

3 The need for interpretation

The results of the constrained correspondence analysis solution are implicitly shown in Figure 2 and explicitly in Table 1. The total Chi-square in the data is 64032 of which 20563 (32.11%) is reconstructed by the constrained correspondence analysis solution, which is quite reasonable. Unconstrained correspondence analysis yields a reconstructed Chi-square of 20904 (32.64%), which is only .5% better than constrained correspondence analysis. Therefore, we conclude that imposing the restrictions from Section 2.4 hardly reduces the fit and may thus be imposed.

**Table 1.** Seriation results obtained constrained correspondence analysis. The exact equality and inequality constraints are described.

<table>
<thead>
<tr>
<th>Reconstructed</th>
<th>Assemblage</th>
<th>Phase</th>
<th>Date</th>
<th>$r_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>nr</td>
<td>Label</td>
<td>Nr</td>
<td>Date</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>TSW2</td>
<td>1</td>
<td>0-50</td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>NoN 5-8</td>
<td>1</td>
<td>0-50</td>
</tr>
<tr>
<td>73</td>
<td>3</td>
<td>L 10-16N</td>
<td>1</td>
<td>0-50</td>
</tr>
<tr>
<td>86</td>
<td>8</td>
<td>LW 18-20C</td>
<td>3</td>
<td>100-150</td>
</tr>
<tr>
<td>91</td>
<td>10</td>
<td>RB-R3, B</td>
<td>3</td>
<td>100-150</td>
</tr>
<tr>
<td>98</td>
<td>7</td>
<td>NoN 2-4</td>
<td>3</td>
<td>100-150</td>
</tr>
<tr>
<td>98</td>
<td>6</td>
<td>EoN 11-18</td>
<td>3</td>
<td>100-150</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>L 9-18S</td>
<td>2</td>
<td>50-100</td>
</tr>
<tr>
<td>112</td>
<td>5</td>
<td>L 8-9N</td>
<td>3</td>
<td>100-150</td>
</tr>
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<td>120</td>
<td>12</td>
<td>L 5-7N</td>
<td>4</td>
<td>150-200</td>
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<tr>
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<td>9</td>
<td>RB-R3, A</td>
<td>3</td>
<td>100-150</td>
</tr>
<tr>
<td>128</td>
<td>13</td>
<td>EoN 4-8</td>
<td>4</td>
<td>150-200</td>
</tr>
<tr>
<td>141</td>
<td>11</td>
<td>L 3-4N</td>
<td>4</td>
<td>150-200</td>
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<td>14</td>
<td>Kiln 5</td>
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<td>181</td>
<td>15</td>
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<td>5</td>
<td>200-300</td>
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<tr>
<td>225</td>
<td>17</td>
<td>LW 16-17C</td>
<td>6</td>
<td>300-350</td>
</tr>
<tr>
<td>306</td>
<td>16</td>
<td>Lib</td>
<td>6</td>
<td>300-350</td>
</tr>
<tr>
<td>410</td>
<td>22</td>
<td>H Floor</td>
<td>7</td>
<td>350-450</td>
</tr>
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<td>419</td>
<td>19</td>
<td>LW 9-14C</td>
<td>7</td>
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<td>18</td>
<td>LE 4-6</td>
<td>7</td>
<td>350-450</td>
</tr>
<tr>
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<td>23</td>
<td>H Fill</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>512</td>
<td>26</td>
<td>B3 D1 pre</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>518</td>
<td>21</td>
<td>WDT</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>518</td>
<td>25</td>
<td>B3 D1 post</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>518</td>
<td>24</td>
<td>Inn Corr S, 7</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>532</td>
<td>20</td>
<td>Nymph</td>
<td>8</td>
<td>450-575</td>
</tr>
<tr>
<td>650</td>
<td>27</td>
<td>LA</td>
<td>9</td>
<td>575-650</td>
</tr>
</tbody>
</table>

The distribution per type of pottery is shown in a so called battleships graph in Figure 3. Ideally, in each column there should be a single fat belly and narrow tails, indicating single peakedness of the conditional distributions. Generally, this form seems to hold, but for certain types there seems to be more than one peak. returning to the solution in Table 1, we see that the all constraints (necessarily) hold. Note that the
hypothized phases generally are reconstructed, but that there are some anomalies. For example, many assemblages of Phase 3 are hypothetically dated between 100 and 150, whereas our method dates them just before 100.

In general, the logical archaeological sequence of Sagalassos red slip ware as proposed by Poblome (1999) is maintained. The three main stages in the development of the local tableware, imperial (phases 1-5), late Roman (phases 6-7) and early Byzantine (phases 8-9), are easily recognizable in the seriated result. The relative distance between the late Roman and early Byzantine assemblages is somewhat larger than between the imperial and late Roman assemblages. This is a result of the fact that more new types were being introduced in the early Byzantine stage compared to the late Roman one, providing a clearer typological separation. As such, however, no major anomalies are noticeable, with each assemblage remaining within the limits of its stage.

![Figure 3. Battleship representation of the solution obtained by constrained correspondence analysis.](image)

Obviously, the dates proposals resulting from the constrained correspondence analysis are indicative, and to be considered at the same level as any other indication in reconstructing the relative and absolute chronology of Sagalassos red slip ware. Slight differences are, for instance, to be noted between the archaeological and the seriated dates of mainly the imperial assemblages. This may be explained by the overall similarity in the typological composition of the contemporary assemblages. In phases 1 to 5, a rather restricted series of types and variants dominates the assemblages. The dates are mostly assigned by comparing the proportional presence of those types, rather than their presence or absence.

Moreover, morphological changes have been attested for some of these types, which imply a chronological order. As these morphological changes occur within the same typological concept of the types and variants, it has not been possible to add these specifications to the constrained seriation. Mainly the assemblages of the second half of the first century and those of the second century are intrinsically very comparable, whereas the differences between these contexts and the outlying phases 1 and 5 are more marked. It is interesting to note that this tendency is also reflected in the seriated solution, without having imposed any prior distance between any of these. In this way, the result provides a good criterion for interpretation as the expected distance between the assemblages is largely respected. Clearly, any difference can be explained by the nature if the archaeological evidence, implying that the interpretation of the material remains the most crucial step. Assemblage 18, for instance, is slightly out of position and this may be the result of the fact that this is a fairly small assemblage found mainly in one room of a house which was destroyed by fire. As a consequence this assemblage is less varied in composition compared to, for instance, assemblages 21, 24 and 25 which result from a much more extensive cleaning operation with material mixed from different origins. The compositional restrictions of assemblage 18 prevent it from fitting into a larger sequence of types, which may make it more difficult to find a suitable chronological solution. This, of course, does not reduce the potential of this assemblage for other, more contextually oriented analyses. In this way, it is clear that any dating technique has to be combined with studying formation processes and linking these processes to weathering patterns of the ceramic material, and performing contextual analyses of the material evidence.

Ideally, contextual dates should be defined (Evans, 1995), which are anchored to the specific nature of the archaeological deposits (e.g. floor deposit, destruction layer) and other datable finds categories (e.g. coins, glass, oil lamps). This reflexive exercise may help to improve the definition and quality of the applied chronological criteria, isolate residual and intrusive material, which is either older material incorporated into a younger deposit or younger material which entered an assemblage at a later stage, and highlight existing loopholes in the chronology. A notable side-effect may be an increased efficiency in processing the data, typically impeding work of large-scale projects, such as the Sagalassos project.

References


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