

NOTE

Intensity modulated photon beams subject to a minimal surface smoothing constraint

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Abstract. A method to smooth intensity-modulated photon beams is presented which can be applied in conjunction with any optimization algorithm. The method employs an additional soft constraint to minimize the area of the surface defined by the photon fluence.

1. Introduction

The optimization of intensity modulated radiotherapy affords a great degree of flexibility in the solutions which can be exploited to yield potentially more expedient fluence profiles with respect to practical considerations (Webb *et al* 1998). Smoother profiles can be applied faster with dynamic multileaf collimators (dMLCs) and can be translated more readily into leaf trajectories if MLC constraints are neglected during the optimization, although smoothing does not supplant these constraints. For compensators, the influence of scatter, beam divergence and penumbra is reduced. With more homogeneous fluences the impact of patient geometry uncertainties caused by setup errors or physiological changes should be reduced, yet this is difficult to assess. Although smoothness is not *per se* a necessary condition of fluence profiles, it is a trait which renders a treatment plan easier to execute and verify.

This note shows how a smoothing constraint can be included in any intensity modulation optimization algorithm by means of a global soft constraint.

2. Method and materials

Let $F(\Phi)$ be the objective function defined for the fluence Φ and $G_{nt}(\Phi)$ be the normal tissue constraints. Consider the optimization problem

$$\text{minimize } F(\Phi) \text{ subject to } G_{nt}(\Phi) < 0, \quad \Phi = \otimes(\phi_{1,1}^k, \dots, \phi_{n,m}^k) \quad (1)$$

for the finite-size pencil beam fluences $\phi_{i,j}^k$ which are arranged on some regular grid for each beam k . We drop the index k in the following, keeping in mind that all equations apply to fluence elements within one beam only.

The function $\phi(u, v)$ with u, v being continuous variables in the directions of i, j defines a surface on the cross-section of the beam. To introduce the smoothing constraint, we demand that $\phi(u, v)$ be a minimal surface. Intuitively, the fluence profile becomes a rubber membrane with given tension. Fluence elements which stick out of the surface (on either side) are pushed

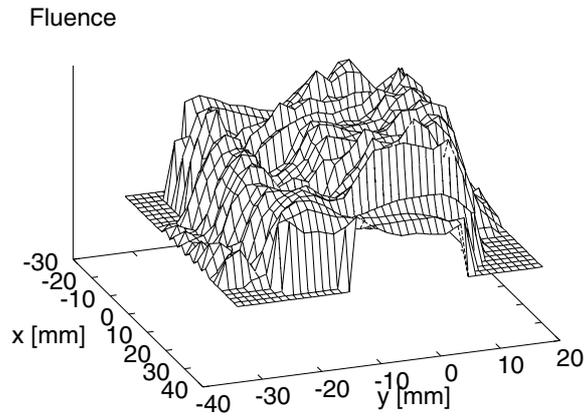


Figure 1. A posterior oblique field without smoothing constraint. The fluence models the dose requirements to the boost and PTV.

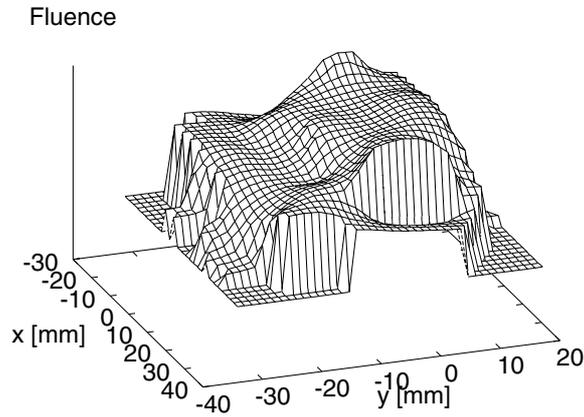


Figure 2. The same field as figure 1, including smoothing.

back in line against the ‘force’ of the objective function[†]. Given the projecting fluence was redundant for the dose distribution, it would realign with its neighbours.

A surface is minimal if and only if the mean curvature vanishes identically, in our case

$$C(\Phi) = \frac{\partial_{uu}\phi\partial_{vv}\phi - (\partial_{uv}\phi)^2}{(1 + (\partial_u\phi)^2 + (\partial_v\phi)^2)^2} \equiv 0. \quad (2)$$

The curvature can be added to the objective function to yield

$$\text{minimize } F(\Phi) + \lambda C^2(\Phi) \text{ subject to } G_v(\Phi) < 0 \quad (3)$$

with a penalty parameter λ . In this formulation, the smoothing constraint competes with the objective function only, so that the tension can be adjusted such that the loss in, say, tumour control can be tolerated.

For the purposes of treatment optimization, it is sufficient to apply this constraint in a one-dimensional form for computational efficiency. In case of application by dMLCs, only the leaf direction may be subject to smoothing. To second order in the pencil beam size, the

[†] If the objective function is interpreted as a potential energy function to be minimized, the derivative with respect to a ray weight is indeed equivalent to a force.

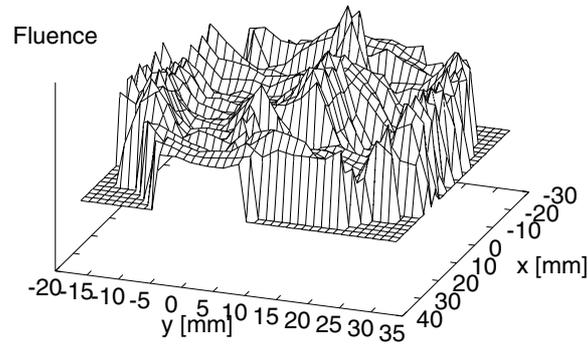


Figure 3. A frontal field, the fluence gradients correspond to the outline of the myelon and the boost volume.

discrete, one-dimensional form of equation (2) reads:

$$C(\phi_{i,j}) = \frac{(2\phi_{i,j} - \phi_{i-1,j} - \phi_{i+1,j})}{(1 + 1/4(\phi_{i-1,j} - \phi_{i+1,j})^2)}. \quad (4)$$

Notice that the derivative of F with respect to $\phi_{i,j}$ includes three additional terms, this being the reason why the two-dimensional form is computationally more expensive than the one-dimensional form applied in two directions.

3. Results

Of course, the influence of the smoothing constraint depends on λ and the current case. If λ is adjusted such that

$$\nabla_{\phi} F + c\lambda \nabla_{\phi} C = 0 \quad (5)$$

in the minimum for some given constant, $c \approx 10 \dots 100$, the loss in equivalent tumour dose can be standardized to a few cGy. With λC being several orders of magnitude smaller than F , the non-convexity of C can hardly give rise to local minima of the objective function. We did not observe any local minimum when adjusting the parameter c down to as small as 0.1.

Figures 1–4 show two fluence profiles for a lung tumour treated with the simultaneous boost technique, where the boost is irradiated to 150% of the dose to the PTV. The size of the pencil beams in figures 1–4 is 5 mm at the isocentre level, which corresponds to 2.5 mm in the figures since the fluence profiles are defined at half the isocentre distance. The loss in equivalent dose to the boost was 1.5% which was mainly due to a reduction in hot spots (figure 5). The differential DVH is displayed for the boost and the PTV only since the change to the other volumes of interest is indiscernibly small.

Remarkably, in our implementation of the optimization algorithm, the smoothing constraint reduces computation time by a factor 0.9. This is due to the fact that the curvature around the minimum is increased and hence the algorithm is less prone to zig-zagging.

4. Discussion

The inclusion of physical and practical constraints in the optimization of radiotherapy is of paramount importance. Although the smoothness of fluence profiles is not a genuine condition, it is of practical relevance to the reliability of treatment delivery and the simplicity of quality

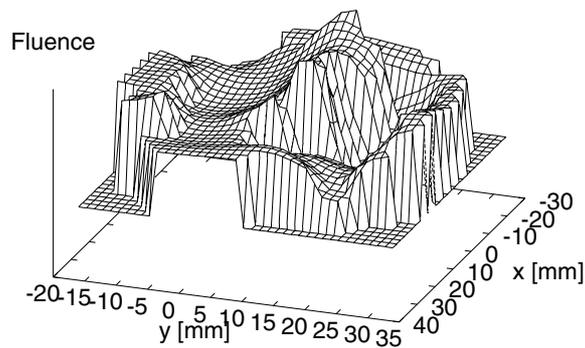


Figure 4. The same field as figure 3. Although it appears much smoother in some regions, the essential features are preserved.

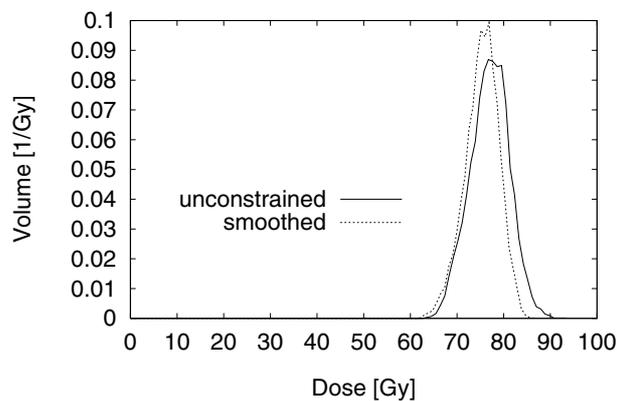


Figure 5. The differential DVH of the central boost volume. The smoothing constraint mainly affects hot spots within the target volume, yet did not decrease the minimum dose.

assurance. The minimum curvature constraint is designed to take into account the relative importance of fluence elements and is superior to projection operators in this respect. However, it does not in any way secure the feasibility of dynamical MLC delivery (as projections would), although the smoothed profiles may result in shorter treatment times with all their advantages. The theoretical decrease in tumour control can be kept at small levels, whereas in practice there might be a tendency to improve tumour control.

Acknowledgment

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Reference

Webb S, Convery DJ and Evans PM 1998 Inverse planning with constraints to generate smoothed intensity-modulated beams *Phys. Med. Biol.* **43** 2785–94