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by

Manfred Stadler

Faculty of Economics and Social Sciences
www.wiwi.uni-tuebingen.de

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Abstract

As is well-known from the literature on oligopolistic competition with incomplete information, firms have an incentive to share private demand information. However, by assuming verifiability of demand data, these models ignore the possibility of strategic misinformation. We show that if firms can send misleading demand information, they will do so. Furthermore, we derive a costly signaling mechanism implementing demand revelation, even without verifiability. For the case of a gamma distribution of the firms' demand variables, we prove that the expected gross gains from information revelation exceed the expected cost of signaling if the skewness of the distribution is sufficiently large and the products are sufficiently differentiated.

Keywords: Information sharing; simultaneous signaling; demand uncertainty

JEL Classification: C73, D82, L13

* University of Tübingen, Department of Economics, Mohlstraße 36, D-72074 Tübingen, Germany.
e-mail: manfred.stadler@uni-tuebingen.de.

1 Introduction

The strategic interaction of firms is typically characterized by problems of asymmetric information. Firms are usually better or at least earlier informed about their own cost and demand parameters than about those of their rivals. Therefore, it is an important issue in the industrial organization literature to study the incentives of firms to exchange their private information (see, e.g. Vives 1999, ch. 8). The literature on information sharing in oligopoly is large. However, most models deal with Cournot competition in homogeneous markets. Only few papers deal with price competition in heterogeneous markets, even if this mode of competition seems to be relevant in most industries. Gal-Or (1986) and Sakai (1991) study the expected gains of exchanging cost information, while Vives (1984) and Sakai (1986) analyze the exchange of demand information. The authors derive the well known result that firms ex ante have no incentive to share private value cost information but indeed have an incentive to share private value demand information. These results, summarized in the rather general duopoly model of Raith (1996), show that the decisive factors for information sharing include not only the mode of competition (prices or quantities) but also the kind of private information (common value or private value) and the relation of the products (substitutes or complements). In some cases, firms ex ante expect to benefit from sharing information, in other cases the reverse is true.

The cited models have in common that they ignore the problem of strategically misinforming competitors by assuming that firms can agree to reveal information as soon as it becomes privately known. As an alternative, they assume an outside institution, such as a trade association, which transfers true information. The role of this association is to ensure that no firm can deviate from its precommitment to disclose the true information. Such an institution may exist in a regulated environment but not in a competitive market. Without precommitment, however, the decision to disclose depends on the realization of the private information. For a large subset of realizations, firms will prefer not to disclose. Even if firms are willing to report about their private information, the question arises whether they have an incentive to disclose the true value or whether they find it in their best interest to mislead the rivals. So far, this crucial question has only been addressed by Ziv (1993) in the context of private value cost information in a homogeneous market with quantity competition. By introducing a costly two-sided signaling mechanism, Ziv (1993) solves the precommitment problem by showing that, if it is not too costly, the firms' optimal strategy is to truthfully disclose their private information by sending a message. Of course, competitors could send no message at all, but if this strategy is interpreted as the worst possible case, such behavior will certainly be dominated by sending the true signal.

The intention of the present paper is twofold: First, it complements Ziv's results by analyzing the information revelation strategies of firms in case of private demand information instead of private cost information. Thereby, we generalize the model by accounting for heterogeneous markets and by allowing not only for quantity but also for price competition. These extensions prove to be essential since the information revelation behavior in the sequentially rational equilibrium decisively depends on the degree of product differentiation. Secondly, we provide a parametric solution for the condition that firms agree to implement the proposed revelation mechanism. We show that, depending on mean and variance of the distribution of the demand variable, the expected gross gains from information revelation exceed the expected cost of signaling only if the products are sufficiently heterogeneous.

The remainder of the paper is structured as follows: Section 2 introduces the model and determines the expected gains from information sharing. Section 3 proves that, without a signaling mechanism implementing truth telling, firms have no incentive to reveal their private demand information. Section 4 derives a costly signaling mechanism implementing a truthful exchange of private demand information if the expected gross gains from information revelation exceed the expected cost of signaling. Section 4 derives the condition under which risk-neutral firms should agree to implement the proposed signaling device and gives a parametric example by assuming gamma distributed demand variables. Section 5 concludes.

2 The Model

We consider a simultaneous-move game between two firms producing differentiated products. Each strategy $s_i, i = 1, 2$ belongs to the positive real line and the profit functions $\pi^i(s_i, s_j)$ are twice continuously differentiable. If the decision variables are prices, the game is one of price competition, if they are quantities, it is one of quantity competition. To keep the model analytically tractable, we rely on the quadratic profit functions

$$\pi^i(s_i, s_j) = s_i(a_i - s_i + ks_j) \ , \quad i, j = 1, 2, i \neq j \ ,$$

where the demand variables a_i are independent and identically distributed on the support $[\underline{a}, \overline{a}]$. To avoid negative quantities or prices for $k < 0$, we have to assume that $2\underline{a} + k\overline{a} > 0$. The parameter k measures the degree of product differentiation. If $k = 0$, the price of each variety depends only on the quantity of the variety produced and vice versa. In the case of quantity competition, the products of the two firms are substitutes (complements) according to whether $k < 0$ ($k > 0$). In the case of price competition, they are substitutes (complements) for $k > 0$ ($k < 0$).

The game consists of three stages. In the first stage, each firm i receives private information about the realization of its demand parameter a_i. In the second stage, each firm can send a message $\hat{a}_i \in [\underline{a}, \overline{a}]$ about this parameter. After receiving the other firm's message, firms simultaneously compete in quantities or prices in the third stage of the game. The payoffs are the resulting profits net of the message cost.

As a benchmark we will first derive the ex ante expected profits of the firms in the two cases of truthful information sharing and information concealing.[1]

2.1 Equilibrium with Private Demand Information

If firms conceal their private information, the game is one of static quantity or price competition with asymmetric demand information and the expected profit of firm i is

$$E^i\pi^i = s_i(a_i - s_i + kE^i(s_j)) \ , \quad i, j = 1, 2, i \neq j \ .$$

Since the firms' demand variables are assumed to be identically distributed, it is easy to solve for the firms' strategies in the Bayesian equilibrium, given their private demand information. The first-order conditions lead to the equilibrium strategies

$$s_i = \frac{(2-k)a_i + kE(a)}{2(2-k)} \ , \quad i = 1, 2 \ .$$

[1] As Raith (1996) has shown, it is never optimal for a firm to only partially share information by sending a signal with intermediate noise to the rivals.

The resulting expected profits are

$$E_p^i \pi^i = \frac{[(2-k)a_i + kE(a)]^2}{4(2-k)^2} \ , \quad i = 1, 2 \ ,$$

where the subscript p denotes the case of private information. The ex ante expected profits, before firms receive their own demand information, are

$$EE_p\pi = \frac{1}{(2-k)^2} \, (E(a))^2 \ + \ \frac{1}{4} \, V(a) \ , \tag{1}$$

where $V(a) = E(a^2) - (E(a))^2$ denotes the variance of the demand distribution.

2.2 Equilibrium with Truthful Information Sharing

If firms truthfully share demand information, then their Nash-equilibrium strategies in the third stage are

$$s_i = \frac{2a_i + ka_j}{4 - k^2} \ , \quad i, j = 1, 2, i \neq j \ ,$$

and yield the profits

$$\pi_s^i = \left(\frac{2a_i + ka_j}{4 - k^2} \right)^2 \ , \quad i, j = 1, 2, i \neq j \ ,$$

where the subscript s denotes the case of truthful information sharing. If the disclosure of information is anticipated, the ex ante expected profits are

$$E\pi_s = \frac{1}{(2-k)^2} \, (E(a))^2 \ + \ \frac{4 + k^2}{(4 - k^2)^2} \, V(a) \ . \tag{2}$$

The difference $\Delta \equiv E\pi_s - EE_p\pi$ indicates whether truthful information sharing increases the firms' expected profits at a point in time where their own demand parameters are still unknown. If firms are risk-neutral they prefer truthful information sharing to information concealing if $\delta > 0$.

Proposition 1: Before firms have recognized the realization of their demand parameters, they strictly prefer truthful information sharing to information concealing, independent of whether the products are substitutes or complements.

Proof: From (1) and (2), the difference between the ex ante expected profits is

$$\Delta = \frac{(12-k^2)k^2}{4(4-k^2)^2}\ V(a) \quad > \quad 0 \ \ \forall \ \ |k| \in (0,1] \ . \tag{3}$$

Of course, if the decision variables s_i are strategically independent, i.e. $k = 0$, there is no gain from information sharing. If they are either strategic substitutes or complements, however, firms strictly prefer a truthful information transfer to a concealment of information. The expected gains are increasing in variance $V(a)$ and in the substitution parameter $|k|$. The maximum expected gains can be realized for the values $k = 1$ (including the case of price competition for market shares) and $k = -1$ (including the case of Cournot competition).

3 The Truth-Telling Problem

The preceding analysis of the expected gains from information sharing holds at a point in time where firms do not yet know the realization of their own demand variables. Once they receive private demand information, however, their incentives to share information disappear.

To demonstrate this effect, we now assume that each firm, after having received its private demand information, can send a message $\hat{a}_i$ about a_i, whereby it is not confirmed to the truth.

The maximization of the expected profit

$$E^i\pi^i = s_i(a_i - s_i + kE^i(s_j)) \ , \quad i,j = 1,2, i \neq j \ ,$$

leads the reaction functions

$$s_i = (a_i + kE^i(s_j))/2 \ , \quad i,j = 1,2, \ i \neq j \ .$$

From the viewpoint of the other firm, which does not know the realization of the rival's demand parameter, the expected strategy of the rival is

$$E^i(s_j) = (E^i(a_j|\hat{a}_j) + kE^j(s_i|\hat{a}_i))/2 \ , \quad i,j = 1,2, \ i \neq j \ .$$

Inserting these expressions into the reaction functions gives

$$s_i = (2a_i + kE^i(a_j|\hat{a}_j) + k^2 E^j(s_i|\hat{a}_i))/4 \; , \quad i,j = 1,2, \; i \neq j \; . \tag{4}$$

Since the profit function is quadratic, we can use Radner's (1962) approach and assume the linear solution equations

$$s_i = \xi_1 a_i + \xi_2 E^j(a_i|\hat{a}_i) + \xi_3 E^i(a_j|\hat{a}_j)$$

implying that

$$E^j(s_i|\hat{a}_i) = (\xi_1 + \xi_2)E^j(a_i|\hat{a}_i) + \xi_3 E^i(a_j|\hat{a}_j) \; .$$

Using (4), we equate the coefficients to find $\xi_1 = 1/2$, $\xi_2 = k^2/(8-2k^2)$, and $\xi_3 = k/(4-k^2)$ and, hence, derive the solution equations

$$s_i = \frac{(4-k^2)a_i + k^2 E^j(a_i|\hat{a}_i) + 2kE^i(a_j|\hat{a}_j)}{2(4-k^2)} \; , \quad i,j = 1,2, \; i \neq j \; .$$

The expected profits are therefore

$$E^i\pi^i = \frac{[(4-k^2)a_i + k^2 E^j(a_i|\hat{a}_i) + 2kE^i(a_j|\hat{a}_j)]^2}{4(4-k^2)^2} \; , \quad i,j = 1,2, \; i \neq j \; . \tag{5}$$

Proposition 2: Without a costly signaling mechanism, firms have no incentive to send true messages about their private demand information. Instead, each firms has an incentive to signal the highest possible value of its demand parameter.

Proof: It is clear that $E^j(a_i|\hat{a}_i) \in [\underline{a}, \overline{a}], i,j = 1,2, i \neq j$. Hence, the derivative

$$\frac{dE^i\pi^i}{dE^j(a_i|\hat{a}_i)} = \frac{(4-k^2)k^2 a_i + 2k^3 E^i(a_j|\hat{a}_j) + k^4 E^j(a_i|\hat{a}_i)}{2(4-k^2)}$$

is positive for all $a_1, a_2, E^1(a_2|\hat{a}_2), E^2(a_1|\hat{a}_1)$, given our assumption that $2\underline{a} + k\overline{a} > 0$. This implies that each firm has an incentive to signal the highest possible value of its demand parameter. Since these signals are certainly not credible, rivals will not react on them. What is needed for a truth-telling perfect Bayesian equilibrium is a mechanism design that implements revelation of the private information.

4 A Simultaneous Signaling Mechanism Implementing Truth-Telling

In this section, we derive a signaling mechanism that indeed implements truth-telling behavior of both firms. The mechanism involves consistent beliefs of each firm about the rival's behavior. In the resulting sequentially rational equilibrium, truth-telling is not an ad hoc assumption as in the predecessor models of information sharing but a consequence of an incentive-compatible device, whereby it is in the best interest of the firms to send true messages.

In the first stage of the game, each firm receives private demand information. When sending a signal in the second stage, each firm has to take into account the implications in the third stage. It is clear from Proposition 2 that each firm has an incentive to convince the rival that its demand parameter a_i has the highest possible realization $\overline{a}$. This implies that in order to induce the firm to send the truthful lower message, it must incur other cost when announcing high demand. Adopting the procedure suggested by Ziv (1993), we therefore introduce a costly signaling mechanism for firms to send a message about the realization of their demand parameter.

We therefore define the cost function $f(\hat{a}_i)$ indicating the amount of money that firm i has to pay when sending the message $\hat{a}_i$. This spending can be observed in dissipative advertising and charity expenses, among others.[2] The introduction of a costly signaling device defines a signaling game where the cost $f(\hat{a}_i)$ of sending the message is the signaling cost. Of course, a firm does not have to send an explicit message at all. But if this strategy is interpreted as the worst possible case, such behavior will be dominated by sending the true signal.

By accounting for the signal cost $f(\hat{a}_i)$ when sending the message $\hat{a}_i$ the expected net profits (5) extend to

$$E^i_f \pi^i = \frac{[(4-k^2)a_i + k^2 E^j(a_i|\hat{a}_i) + 2kE^i(a_j|\hat{a}_j)]^2}{4(4-k^2)^2} - f(\hat{a}_i) \ , \quad i,j = 1,2, i \neq j \ . \tag{6}$$

We characterize a separating perfect Bayesian equilibrium in which the costly message serves as a signal of the demand parameter. Let $E^j(a_i|\hat{a}_i)$ be the belief of firm j

[2] As an alternative, rather than burning money, rivals may exchange transfer payments, thereby reducing net signaling cost.

that relates firm i's message $\hat{a}_i$ to its demand parameter a_i. Thus, if firm i sends the message $\hat{a}_i$ (or the signal $f(\hat{a}_i)$, respectively), then it is inferred to have the demand parameter $E^j(a_i|\hat{a}_i) \in [\underline{a}, \overline{a}]$. A perfect Bayesian equilibrium requires that for each firm i the expected profit (6) is maximized with respect to the message $\hat{a}_i$. In addition to these incentive compatibility constraints, firms' beliefs must be consistent with the equilibrium play, that is, $E^j(a_i|\hat{a}_i) = a_i$. It its straightforward to derive the incentive-compatible signal-cost function in the perfect Bayesian equilibrium.

Proposition 3: In the separating perfect Bayesian equilibrium the signal-cost functions are determined by

$$f(\hat{a}_i) = \frac{[k^2(\hat{a}_i^2 - \underline{a}^2) + k^3(\hat{a}_i - \underline{a})E(a)]}{(4-k^2)^2} \quad > \quad 0 \; i = 1,2 \;,\; \forall \;\; |k| \in (0,1].$$

Proof: The expected profits (6) are maximized if the first-order conditions hold, i.e.

$$\frac{dE^i\pi_f^i}{d\hat{a}_i} = \frac{(4-k^2)k^2 a_i + 2k^3 E^i(a_j|\hat{a}_j) + k^4 E^j(a_i|\hat{a}_i)}{2(4-k^2)^2} \cdot \frac{dE^j(a_i|\hat{a}_i)}{d\hat{a}_i} - \frac{df(\hat{a}_i)}{d\hat{a}_i} = 0 \;.$$

In order to induce consistent beliefs, these conditions must be fulfilled at $\hat{a}_i = a_i$, implying $E^j(a_i|\hat{a}_i) = a_i$ and, hence, $\frac{dE^j(a_i|\hat{a}_i)}{d\hat{a}_i} = 1$. Since the information transfer is assumed to be simultaneous, firm i does not know a_j when sending its signal, so that $E^i(a_j|\hat{a}_j) = E^i(a_j) = E(a)$. Using these expressions, we can solve for the marginal signal-cost functions

$$\frac{df(\hat{a}_i)}{d\hat{a}_i} = \frac{2k^2\hat{a}_i + k^3 E(a)}{(4-k^2)^2} \;, i = 1,2 \;.$$

Integrating and inserting the initial conditions $f(\underline{a}) = 0$ leads to the equilibrium signal-cost functions

$$f(\hat{a}_i) = \frac{[k^2(\hat{a}_i^2 - \underline{a}^2) + k^3(\hat{a}_i - \underline{a})E(a)]}{(4-k^2)^2} \;, \quad i = 1,2 \;, \tag{7}$$

which are positive for all $|k| \in (0,1]$, given our assumption that $2\underline{a} + k\overline{a} > 0$. Thus, each firm invests a positive amount of money as long as it receives a demand realization higher than the worst one, i.e. $a_i > \underline{a}$.

If firms agree to implement the derived signaling mechanism, information sharing results as a feature of the equilibrium strategy. Each firm reveals its private demand information because this behavior maximizes its expected profit. Consequently, the firms' messages are credible and taken into account when setting quantities or prices in the last stage. A crucial condition for the firms to implement the signaling device is, however, that the expected signal costs do not overcompensate the expected gross gains from a truthful information exchange.

5 Conditions for Implementing the Signaling Device

Whether firms will reach an agreement to implement the proposed signaling device depends on whether the expected gross gains from information sharing exceed the expected signaling cost. Having solved for the signaling-cost functions, it is straightforward to derive this net gain.

Proposition 4: The expected gross gains from information sharing exceed the expected signaling cost if and only if

$$\frac{(12-k^2)k^2V(a) - 4k^2((E(a^2)-\underline{a}^2) + k(E(a))^2 - \underline{a}E(a))}{4(4-k^2)^2} \ > \ 0 \ . \tag{8}$$

Proof: In the separating perfect Bayesian equilibrium the expected value of the signaling cost as given in (7) is

$$Ef(a) = \frac{[k^2(E(a^2)-\underline{a}^2) + k^3(E(a)-\underline{a})E(a)]}{(4-k^2)^2} \ .$$

Using the expression for Δ in (3), we derive the expected net gains from signaling true demand information as

$$\Delta - Ef(a) = \frac{(12-k^2)k^2V(a) - 4k^2((E(a^2)-\underline{a}^2) + k(E(a))^2 - \underline{a}E(a))}{4(4-k^2)^2} \ .$$

Thus, whether truthful information signaling will occur or not, decisively depends on the degree of product differentiation as well as on the first two moments of the distribution function of the demand variables.

In order to make sharper predictions we present a parametric example for $k \in (0,1]$, including the important case of price competition in heterogeneous markets. Therefore, we assume that $\underline{a} = 0$, $\overline{a} \to \infty$ and use the rather general gamma distribution.[3] The gamma distribution seems to be an appropriate example for the description of demand uncertainty since it is nonnegative and is skewed to the right, i.e. to the realization of high values of the demand variables. Its probability density function is given by

$$g(a) = \frac{n^m}{\Gamma(m)} a^{m-1} e^{-na} \ , \quad \Gamma(m) = \int_0^\infty x^{(m-1)} e^{-x} dx$$

on the support $[0, \infty]$ with the shape parameter $m > 0$ and the inverse scale parameter $n > 0$. The gamma distribution has mean $E(a) = m/n$, variance $V(a) = m/n^2$, and skewness $S(a) = 2/\sqrt{m}$.

Proposition 5: In case of the gamma distribution, the expected gross gains from true signaling exceed the expected cost of signaling only if the skewness of the distribution is sufficiently large (small values of m) and the products are sufficiently differentiated (small values of k).

Proof: Inserting mean and variance as well as the limits of the support into (8) yields the inequality

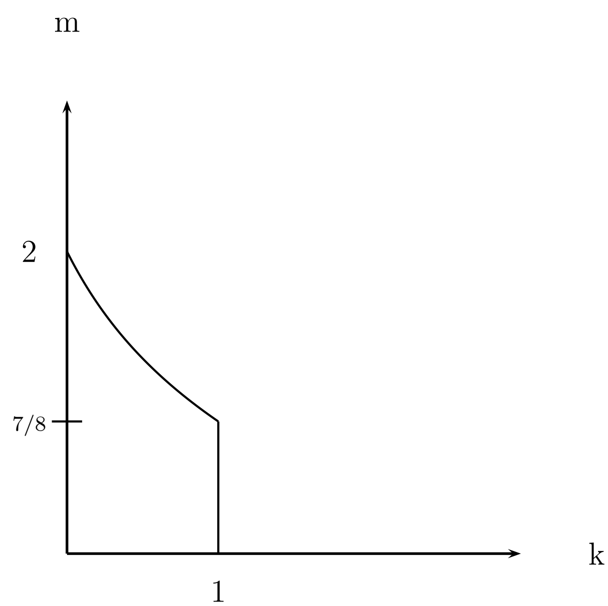
$$m < \frac{8 - k^2}{4(1+k)} \ . \tag{9}$$

Figure 1 illustrates the region in the parameter space (k, m) where the condition for expected profitability of the information sharing device by costly signaling is fulfilled.

Since k is restricted to the interval $(0,1]$ in this example, the inequality (9) is always fulfilled if $m \leq 7/8$, but it is never fulfilled if $m > 2$. For all intermediate values of m the substituiton parameter k has to be sufficiently small. As an example, in the case of $m = 1$, where the gamma distribution degenerates to the exponential distribution with p.d.f. $g(a) = ne^{-na}$, $k < 2(\sqrt{2} - 1) \approx 0.83$ must hold for (9) to be fulfilled.

[3] Ziv (1993) has demonstrated his results by using a binary distribution of the unit cost of production. A discrete probability function, however, is not an appropriate example for a mechanism relying on the revelation of information about continuous random variables.

Figure 1: Parameter range for information signaling



Thus, in the standard model of price competition with substitute products - as well as in the model of quantity competition with complement products -, firms will agree to implement the proposed signaling device and will reveal their private demand information by sending true messages, if the products are sufficiently differentiated.

6 Conclusion

The revelation of private demand information rises the expected profits of the competitors in a market, and consequently firms are interested in sharing this private information. This information exchange, however, cannot be done without a truth-telling mechanism, since firms could realize even higher gains by claiming to be larger than they really are. There are at least three channels through which firms can infer the private demand information of the rivals. The first is the existence of an outside institution, such as a trade association, which is able to transfer true information. However, even if such an institution may exist in a regulated environment it will hardly emerge in a competitive market. The second channel is an intertemporal transmission of private demand information. Of course, in a repeated game of competition with constant demand parameters, firms can simultaneously signal their private information by their strategic price and quantity decisions in previous periods. This paper presented a third channel by introducing a two-sided signaling mechanism implementing information revelation even in a one-period (but multi-stage) game. Due to the cost of signaling, firms will agree to apply this mechanism only if the expected gains from information sharing exceed the expected cost of signaling. We proved that this condition is fulfilled if the products are sufficiently differentiated. Thus, even in the absence of an outside institution and even in a one-period context, firms can agree to implement a signaling device by which a truthful exchange of their private demand information is possible.

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