Intrafirm Conflicts and Interfirm Competition

by

Werner Güth, Kerstin Pull & Manfred Stadler
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Abstract

We study strategic interfirm competition allowing for internal conflicts in each seller firm. Intrafirm conflicts are captured by a multi-agent framework with principals implementing a revenue sharing scheme. For a given number of agents, interfirm competition leads to a higher revenue share for the agents, higher equilibrium effort levels and higher agent utility, but lower profits for the firms. The winners from antitrust policy are thus not only the consumers but also the agents employed by the competing firms.

Keywords: agency theory, strategic interfirm competition, revenue sharing

JEL Classification: C72, L22, M52
1 Introduction

Agency theory has been developed to overcome the limitations of the monolithic firm model of classical microeconomics by allowing for interpersonal strategic conflicts within firms. While it has undoubtedly enhanced our understanding of many aspects of intrafirm organization like problems of moral hazard and adverse selection (see, e.g., Alchian, Demsetz 1972, or Holmström 1982), its basic flaw is its exclusive focus on intrafirm conflicts without considering how these are embedded in interfirm competition. Our focus is on the coexistence of intra- and interfirm competition. While Berninghaus et al. (2007) introduce hiring competition on the labor market via a principal-agent firm organization, we analyze intrafirm conflicts in an environment where firms compete on the product market (sales competition). We ask: What does sales competition imply for intrafirm conflicts? And specifically: How does sales competition affect agents’ effort levels, agent utility, and total welfare? To answer such questions we analyze the interaction of intra- and interfirm conflicts within an analytically tractable, deterministic model.

Concerning intrafirm conflicts, principals implement a revenue sharing plan granting an individual agent a compensation based on relative performance. This compensation mechanism is not only plausible (as compared to, e.g., fixed-prize tournaments) but is also empirical relevant: Rent-sharing, either in its explicit form through revenue-, gain- or profit sharing plans, or - even more prominent - rent-sharing in its implicit form with high profit firms paying high wages is widely spread in the organizational practice. Further, when revenue-sharing plans are in place, we observe that an individual agent’s portion of the overall revenue share varies according to his position and performance. For instance, in the well-known Japanese bonus payment tournaments as studied e.g. by Kräkel (2003), the total wage sum is determined by a process of collective bargaining (and will most certainly depend on the firm’s profitability), and an individual worker’s share of that collectively determined wage sum is set according to the worker’s relative performance as measured in the evaluation process (“satei”) (see Endo 1994). We analyze such a compensating scheme without asking whether this scheme is optimal.2

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1 Durable monopolies, for instance, allow for intrafirm strategic conflicts in the sense of intertemporal price competition (the monopolistic seller can serve demand earlier or later and thus engages in price competition which approaches homogenous price competition when customers become very patient (see Coase 1972 and Güth, Ritzberger 1998)).

2 Güth, Levinsky, Pull, Weisel (2010), for instance, have shown both, theoretically and experimentally, that variable prize tournament incentives (the wage sum depends on firm profitability, i.e. on collective effort) are cost minimizing as compared to piece rates and fixed prize tournaments.
Concerning *interfirm competition* our analysis complements the wide class of strategic competition models. In the Industrial Organization literature, the main focus has been on the strategic implications of managerial incentives on price and quantity competition (see, e.g., Vickers 1985, Fershtman 1985, Fershtman, Judd 1987, 2006, Sklivas 1987, Hermelín 1992, Cailland, Jullien, Picard 1995, Schmidt 1997, Jansen, van Lier, van Witteloostuijn 2007). In these models, the delegation problem is reduced to the incentive schemes for managers implemented by the owners of the firms. However, intrafirm conflicts between the workers within a single firm are usually excluded from the analysis.

Concerning the combination of intrafirm conflicts and interfirm competition on product markets, our work relates to that of Raith (2003) as well as to that of Lin (2008). Raith (2003) analyzes compensation systems based on piece rates and focuses on the product market structure being endogenously determined by firms’ entry and exit decisions. Lin (2008) studies the effect of the product market price on fixed prize tournament incentives but largely neglects the interaction between intrafirm conflicts and interfirm competition by assuming that the product price does not depend on agent effort.

The remainder of the paper is structured as follows: Section 2 presents the basic model of a monopolistic seller firm with several agents. Section 3 adds a second seller to analyze the effects of interfirm competition with a given number of agents. Section 4 derives the full incentive scheme by endogenously determining the number of agents to be employed by the rivals when respecting the agents’ participation constraints. Section 5 concludes.

## 2 The Monopolistic Seller Firm

We begin our analysis by limiting strategic interaction to conflicts within a single firm. Consider a product market with a linear demand function that can be normalized (via appropriate choices of monetary and sales units) to

\[ D(p) = 1 - p. \]

If the seller firm produces \( q \) units of the good, market clearing requires the price

\[ p = 1 - q. \]
The firm employs $n$ agents whose effort choices $e_k \geq 0; k = 1, ..., n$, determine the firm’s output level $q$ via 

$$q = \sum_{k=1}^{n} e_k .$$

In order to be able to concentrate on the interaction effects between the agents of the firm and for reasons of model tractability, we abstract from any informational asymmetry between principal and agents concerning an agent’s effort choice and instead assume each agent’s output to be (i) verifiable and (ii) a deterministic function of individual effort, i.e. we assume individual agent output to be a perfect signal of individual agent effort. Effort costs are assumed to be private, but commonly known. All agents have the same quadratic effort cost function 

$$c(e_k) = e_k^2/2 .$$

To start with, assume that all agents are already employed so that we can neglect additional fixed wages and agents’ participation constraints. Later on we will show that participation constraints can be used to endogenously determine the number of agents employed by the firm.

The principal of the firm determines the revenue share $s \in [0, 1]$, offered to the agents as a whole. Agents are assumed to be identical and to distribute their overall revenue share $spq$ proportionally according to each agent’s individual contribution $e_k/q$, as suggested by equity theory (see, e.g., Homans 1961). This means that each agent realizes the net utility 

$$U(e_k) = se_k p - e_k^2/2 = se_k (1 - q) - e_k^2/2 .$$

Maximization of utility $U$ with respect to efforts $e_k$ leads to the first-order conditions 

$$s(1 - q - e_k) - e_k = 0$$

for $k = 1, ..., n$. The second-order conditions are obviously fulfilled. The unique solution to this equation system is the symmetric effort 

$$e^*(s, n) = \frac{s}{1 + (n + 1)s} .$$

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3Due to the nonlinearity of the profit functions, our model would not be analytically solvable any more if we added a random variable.

4Contestant heterogeneity, e.g. in the sense of agents differing in their (marginal) costs of effort, might question the fairness of such reward schemes which may, in turn, crowd in other regarding concerns like inequity aversion concerning agents’ earnings (see Bolton, Ockenfels 2000 and Fehr, Schmidt 1999). Here, we abstract from contestant heterogeneity for the sake of analytical tractability.
The agents’ equilibrium effort depends positively on the revenue share \( s \) but negatively on the number \( n \) of agents employed by the firm. Anticipating the agents’ effort decisions, the principal’s profit function is

\[
\pi(s, n) = (1 - s)ne^*(1 - ne^*) = \frac{ns(1 - s^2)}{[1 + (n + 1)s]^2}.
\]

The first-order condition for maximizing profit \( \pi \) with respect to the revenue share \( s \) leads to the cubic equation

\[
(n + 1)s^3 + 3s^2 + (n + 1)s - 1 = 0,
\]

implying a negative relationship between the number \( n \) of agents and the revenue share \( s \) offered by the principal. This cubic equation has the single real solution

\[
s^*(n) = u + v - 1/(n + 1),
\]

where \( u = (D^{1/2} - b)^{1/3}, v = (-D^{1/2} - b)^{1/3}, D = a^3 + b^2, a = [(n+1)^2/3 - 1]/(n+1)^2 \) and \( b = -[(n+1)^2 - 1]/(n+1)^3 \).

Given the equilibrium revenue share \( s^* \), it is straightforward to calculate the equilibrium effort \( e^* \) of the agents and, hence, the firm’s profit \( \pi^* \) and the agents’ utility \( U^* \). The welfare consisting of the firm’s surplus (principal and agents) as well as the consumer rent can be calculated as

\[
W^* = (1 + p^*)q^*/2 - ne^{*2}/2.
\]

Table 1 presents the details of the solution for various numbers \( n = 1, 2, \ldots \) of agents employed by the monopolistic seller.\(^5\) As can be seen, the equilibrium revenue share \( s^* \) as well as the equilibrium effort level \( e^* \) decrease in the number of agents. The profit \( \pi^* \) increases in the number of agents and approaches the value \( 1/4 \) of the traditional monopoly model (in the case of zero production cost) when \( n \) tends to infinity. Welfare \( W^* \) also increases and approaches the value \( 3/8 \) of the traditional monopoly model for \( n \to \infty \). The reason is that the equilibrium effort of each agent becomes negligible and that marginal effort costs converge to zero when the number of agents approaches infinity.

\(^5\)A similar table for heterogeneous agents, e.g. \( n_l \) low-cost agents and \( n_h \) high-cost agents with \( n_l, n_h \geq 1 \) and \( n = n_l + n_h \geq 2 \), would be less illustrative but could be derived numerically. Our focus, however, is on the number \( n \) of agents and its limit cases \( n = 1 \) and \( n \to \infty \).
<table>
<thead>
<tr>
<th>$n$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
<th>$n = 5$</th>
<th>$n = 6$</th>
<th>$n \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*$</td>
<td>0.317</td>
<td>0.250</td>
<td>0.208</td>
<td>0.176</td>
<td>0.152</td>
<td>0.133</td>
<td>0.000</td>
</tr>
<tr>
<td>$e^*$</td>
<td>0.194</td>
<td>0.144</td>
<td>0.114</td>
<td>0.094</td>
<td>0.079</td>
<td>0.069</td>
<td>0.000</td>
</tr>
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<td>0.288</td>
<td>0.341</td>
<td>0.374</td>
<td>0.395</td>
<td>0.414</td>
<td>...</td>
</tr>
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<td>$p^*$</td>
<td>0.806</td>
<td>0.712</td>
<td>0.659</td>
<td>0.626</td>
<td>0.605</td>
<td>0.586</td>
<td>...</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.107</td>
<td>0.153</td>
<td>0.178</td>
<td>0.193</td>
<td>0.203</td>
<td>0.210</td>
<td>...</td>
</tr>
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<td>$U^*$</td>
<td>0.031</td>
<td>0.015</td>
<td>0.009</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
<td>...</td>
</tr>
<tr>
<td>$W^*$</td>
<td>0.156</td>
<td>0.226</td>
<td>0.263</td>
<td>0.286</td>
<td>0.301</td>
<td>0.314</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 1: Numerical results for various numbers $n$ of agents employed by a monopolistic seller.

Allowing for non-negative fixed wages in addition to the revenue shares and assuming non-binding participation constraints would change neither $e^*(s)$ nor $s^*$. The firm would like to hire as many workers as possible to reduce the revenue share $s^*$. An endogenous number of agents results, however, if a participation constraint $U \geq \bar{U} > 0$ is taken into account. The higher the reservation level $\bar{U}$, the less agents will be willing to accept the labor contract - although the agents’ revenue share increases when $n$ becomes smaller.

In the next step we introduce a second seller firm to analyze the effects of strategic interfirm competition on the equilibrium incentive scheme (section 3) and on the number of agents hired by the rivals (section 4).

### 3 Seller Competition with a Given Number of Agents

When several firms compete in serving demand, the assumption that principals share revenues with their agents implies that interfirm competition involves both hierarchy levels, principals and agents. To demonstrate the effects of such intrafirm as well as interfirm competition, we restrict ourselves to the case of two competing firms $i = 1, 2$ on a homogeneous market with $n$ agents each and firm specific sales amounting to $q_i = \sum_{k=1}^{n} e_{i,k}$. As in the case of a monopolistic seller firm, we start by assuming an exogenously given equal number $n$ of employees in each firm. Later on we will endogenously determine the number of agents employed and justify firms’ symmetry.

The inverse demand function can now be written as

$$p = 1 - q_1 - q_2.$$
Each of the agents, $k = 1, ..., n$, employed by firm $i = 1, 2$, realizes net utility

$$U_{i,k}(e_{i,k}) = s_i e_{i,k} - e_{i,k}^2/2 = s_i e_{i,k}(1 - q_1 - q_2) - e_{i,k}^2/2.$$  

Maximization with respect to the efforts $e_{i,k}$ yields the agents’ first-order conditions

$$s_i(1 - q_1 - q_2 - e_{i,k}) - e_{i,k} = 0; \quad i = 1, 2; \quad k = 1, ... n,$$

whose symmetric solution is

$$e^*_i(s_1, s_2) = \frac{s_i(1 + s_j)}{1 + (n + 1)(s_i + s_j) + (2n + 1)s_i s_j}, \quad i = 1, 2, i \neq j.$$  

Anticipating these equilibrium efforts, the profit functions of the two principals are

$$\pi_i(s_1, s_2) = (1 - s_i)n e^*_i(1 - n e^*_i - n e^*_j), \quad i = 1, 2, i \neq j.$$  

The first-order condition for maximizing the profits $\pi_i(s_1, s_2)$ with respect to $s_i$ and the obvious symmetry of the solution leads to the quadratic equation\(^6\)

$$(2n + 1)s^{*2} + 2s^* - 1 = 0,$$  

implying again a negative relationship between the number $n$ of agents in each firm and the revenue shares $s^*$ offered by the principals. The quadratic equation has the solution

$$s^* = \frac{\sqrt{2n + 2} - 1}{2n + 1},$$

implying the symmetric effort levels

$$e^*(s^*) = \frac{s^*}{1 + (2n + 1)s^*} = \frac{\sqrt{2n + 2} - 1}{(2n + 1)\sqrt{2n + 2}}.$$

\(^6\)One might wonder why the equation in the duopoly case is quadratic whereas the corresponding equation (1) in the monopoly case is cubic. To understand the difference in the degrees of the polynomials, consider the more general inverse demand function

$$p_i = 1 - q_i - dq_j; \quad i, j = 1, 2,$$

where $d \in [0, 1]$ measures the heterogeneity of the market. In this generalized setting, the solution is the fourth-order equation

$$[(1 - d^2)n^2 + 2n + 1]s^4 + [4(n + 1)]s^3 + [(1 - d^2)n^2 + 2n + 4]s^2 - 1 = 0.$$

In case of $d = 0$, the market is segmented into two independent monopoly markets. By factoring out $[1 + (n + 1)s] > 0$, the solution equation degenerates to (1). In case of $d = 1$, the market is homogeneous. By factoring out $(1 + s)^2 > 0$, the solution simplifies to (2).
for all $2n$ agents employed by the two competing firms. The profits of the principals are thus

$$\pi^* = (1 - s^*)n e^*(1 - 2ne^*) = \frac{n s^*(1 - s^2)}{[1 + (2n + 1)s^*]^2}.$$ 

The welfare in the homogeneous market adds up to

$$W^* = (1 + p^*)q^* - ne^{*2},$$

where $q^*_1 = q^*_2 = q^* = (1 - p^*)/2$ is a single firm’s output. Table 2 illustrates how the solution $(s^*, e^*, q^*, p^*, \pi^*, U^*, W^*)$ depends on the number $n$ of agents employed by each seller. Again, the equilibrium revenue share $s^*$ decreases in the number $n$ of agents as do equilibrium effort levels $e^*$. The agents’ utility levels are decreasing throughout whereas the profits follow an inverted U-shape. For small numbers of agents, profits increase (as it was already derived in the monopoly case), but for larger numbers, the counteracting interfirm competition effect dominates and profits decrease. Interestingly, profits do not converge to the level of $1/9$ of the Cournot-duopoly (in the case of zero production cost) but rather to zero for $n$ approaching infinity. The reason is that our model does not only feature the competition of principals but also that of agents. It is the interfirm competition on both hierarchy levels\(^7\) which drives the price down to zero when $n$ tends to infinity.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
<th>$n = 5$</th>
<th>$n = 6$</th>
<th>...</th>
<th>$n \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*$</td>
<td>0.333</td>
<td>0.290</td>
<td>0.261</td>
<td>0.240</td>
<td>0.224</td>
<td>0.211</td>
<td>...</td>
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<td>$e^*$</td>
<td>0.167</td>
<td>0.118</td>
<td>0.092</td>
<td>0.076</td>
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<td>0.056</td>
<td>...</td>
<td>0.000</td>
</tr>
<tr>
<td>$q^*$</td>
<td>0.167</td>
<td>0.237</td>
<td>0.276</td>
<td>0.304</td>
<td>0.325</td>
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<td>0.500</td>
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<td>$p^*$</td>
<td>0.667</td>
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</tr>
<tr>
<td>$\pi^*$</td>
<td>0.074</td>
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<td>0.091</td>
<td>0.090</td>
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</tr>
<tr>
<td>$U^*$</td>
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<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
<td>0.002</td>
<td>...</td>
<td>0.000</td>
</tr>
<tr>
<td>$W^*$</td>
<td>0.250</td>
<td>0.334</td>
<td>0.374</td>
<td>0.400</td>
<td>0.418</td>
<td>0.427</td>
<td>...</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 2: Numerical results for various numbers $n$ of agents, employed by each of the two firms.

In order to assess the effects of selling competition on intrafirm conflicts we compare the monopoly case with $2n$ agents (section 2) to the duopoly case with each firm employing $n$ agents. Obviously, introducing selling competition results in a

\(^7\)In case of employment contracts offering piece rates instead of revenue shares agents would not compete but rather face isolated optimization tasks.
higher equilibrium revenue share $s^*$, higher equilibrium effort levels $e^*$, and a higher aggregate output. Consequently, the equilibrium price $p^*$ and profits $\pi^*$ are lower with selling competition while welfare $W^*$ is higher. Agents thus prefer an organizational structure where they are employed by several principals rather than by a monopolistic firm, employing all of them. The winners from antitrust policy, e.g. merger control, are thus not only the consumers, but also the agents employed on the market.

4 Seller Competition with an Endogenously Determined Number of Agents

Until now, the number of agents was exogenously given and assumed to be equally distributed across the two seller firms. In order to endogenously determine the number of agents in each firm, we now allow firms to hire an unequal number of agents. The first-order conditions of the agents’ maximization problem lead to the same efforts $e^*_i(s_1, s_2)$ by all $n_i$ agents in firm $i$ where

$$e^*_i(s_1, s_2) = \frac{s_i(1 + s_j)}{1 + (n_i + 1)s_i + (n_j + 1)s_j + (n_i + n_j + 1)s_is_j}, \quad i = 1, 2, \quad i \neq j,$$

depends positively on the own firm’s revenue share $s_i$, but negatively on the rival firm’s revenue share $s_j$ and also negatively on the numbers $n_i$ and $n_j$ of agents, employed either by the own or by the rival firm.

Anticipating these equilibrium effort levels, the profit functions of the two principals are

$$\pi_i(s_1, s_2) = (1 - s_i)n_i e^*_i(1 - n_i e^*_i - n_j e^*_j), \quad i = 1, 2, \quad i \neq j.$$

The principals $i = 1, 2$ simultaneously maximize their profits with respect to $s_i$ and $n_i$. Maximization with respect to $s_i$ and imposing symmetry again leads to condition (2). Differentiation with respect to $n_i$ gives

$$\frac{\partial \pi_i}{\partial n_i}\bigg|_{s_i=s^*, n_i=n^*} = (1 - s^*)e^*(1 - 2n^*e^*)^2$$

$$= \frac{s^2(1 - s^2)}{[1 + (2n^* + 1)s^*]^3} > 0, \quad i = 1, 2,$$

such that both principals want to hire as many workers as possible. As agents always have an incentive to equally distribute over firms, this result justifies the symmetry
assumption used in section 3 where the number $n$ of agents in each firm can now be interpreted as half of the whole relevant labor force.

If the number of agents in the relevant workforce goes to infinity, the number of agents employed by the firms can be endogenously determined by accounting for a binding participation constraint for the agents, given by $U \geq \bar{U} > 0$. The symmetry of the participation constraints again ensures that both firms employ the same number of agents. For example, if $\bar{U} = 0.005$, it can be seen from Table 1 that the monopolistic firm will employ only $n = 4$ agents, while it becomes obvious from Table 2 that in the duopoly case both firms will hire $n = 3$ agents each so that in total $2n = 6$ agents will be employed. This increasing number of agents has a counteracting effect on the revenue share $s^*$ and the effort level $e^*$ of agents so that the total effect is, in general, unclear even if all of our numerical solutions indicate the positive effects to dominate. Since the participation constraint is equally binding in both cases, agents are indifferent between working in a monopolistic or in a duopolistic firm. Nevertheless, due to the incentives of agents, the equilibrium market structures still consists of two symmetric rivals. As with non-binding constraints, welfare $W^*$ increases due to seller competition.

5 Conclusion

We presented a model featuring strategic interaction not only within a firm but also between firms where both, principals and agents of firms, compete with each other. In our view, intuition based on models dealing either only with intrafirm or only with interfirm competition can lead us astray.

Our analysis has shown that the usual results of (duopolistic) sales competition become questionable when not only principals but also their agents interact strategically. For a given number of agents we have shown that these prefer an industry organization where they are employed by several firms rather than by one single firm. The incentive provided by the revenue-dependent compensation schemes increases due to seller competition, leading to higher effort, larger output, and a lower market price. The winners from antitrust policy are thus not only the consumers but also the agents employed by competing firms.
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