On the Economic Architecture of the Workplace: Repercussions of Social Comparisons amongst Heterogeneous Workers

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Abstract

We analyze the impact on a firm’s profits and optimal wage rates, and on the distribution of workers’ earnings, when workers compare their earnings with those of co-workers. We consider a low-productivity worker who receives lower wage earnings than a high-productivity worker. When the low-productivity worker derives (dis)utility not only from his own effort but also from comparing his earnings with those of the high-productivity worker, his response to the sensing of relative deprivation is to increase the optimal level of effort. Consequently, the firm’s profits are higher, its wage rates remain unchanged, and the distribution of earnings is compressed.

Keywords: Social comparisons; Heterogeneous workforce; Relative deprivation; Effort exertion; Earnings gap; Earnings compression

JEL Classification: D01; D21; J22; J24; J31; M54
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“Relative success, tested by an invidious pecuniary comparison with other men, becomes the conventional end of action. The currently accepted legitimate end of effort becomes the achievement of a [less un]favourable comparison with other men [. . .].” Thorstein Veblen (1899).

I. Introduction

There is strong and growing interest in the link between individual behavior and social comparisons.\(^1\) The recent and evolving research on happiness is a case in point.\(^2\) In this paper we link social comparisons with incentive structures within firms. To model incentive structures properly, it is necessary to understand how workers behave in the presence of externalities arising from comparisons with co-workers. Akerlof (1982) explains why firms pay wages above the market-clearing level. When workers develop a feeling for their firm, an exchange of “gifts” can occur: workers exert more effort than is required, and the firm pays wages that are higher than the workers could receive from alternative employers. Furthermore, if workers have a fellow feeling towards co-workers, the firm has to treat its workers as a group. Fellow feeling towards co-workers yields increased utility when the firm “relaxes pressure on the workers who are hard pressed; in return for reducing such pressure, better workers are often willing to work harder” (Akerlof 1982, 550). Unlike Akerlof (1982), and unlike Levine (1991), we argue that increasing the “pressure” on the less able (less

\(^1\) As early as half a century ago, Festinger (1954) pointed out that humans routinely compare their abilities with those of others. More than a century ago, Veblen (1899), as quoted above, intimated that people compare their fortunes with those of others and seek to narrow the gap by working harder.

\(^2\) Clark, Frijters, and Shields (2008) review the importance of relative considerations and explain why increasing real incomes in developed countries are not perceived to increase average happiness. Knight, Song, and Gunatilaka (2007) even find that relative income is more important for individual happiness than actual income.
productive) workers will induce them to work harder (exert more effort). Our general idea is that discontent can arise not only from low earnings but also from earnings lower than those of co-workers, and that unfavorable comparisons can induce workers to work harder.\(^3\) We thus model workers’ behavior in the context of social interactions. We study the links between workers’ behavior, the firm’s response, and the workers’ reaction to that response, and we trace out the repercussions for profits, earnings compression, and the design of the workplace.\(^4\)

We assume that workers dislike earning less than fellow workers. We do not consider the case in which low-productivity workers dislike high-productivity workers as such (Becker 1957; Schelling 1971). As in Cabrales, Calvó-Armengol, and Pavoni (2008) we assume that discontent arises only from market outcomes. Heterogeneity in levels of productivity gives rise to variation in earnings. We show that for reasonable preferences, the sense of relative deprivation which arises from a comparison of a low-productivity worker’s earnings with those of a high-productivity worker induces more effort: relative deprivation motivates the low-productivity worker to close the earnings gap with the high-productivity worker, and this is achieved by exerting more effort.\(^5,6\)

In principle, there are (at least) four basic responses to the sensing of discontent or dismay from having a wage that is lower than that of others with whom comparisons are made: exert more effort, exit (migrate), acquire better skills (enhance productivity), and sabotage the performance of others.\(^7\) The type of response that is likely to be observed

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\(^3\) Frank (1985, 30) intimates that “the evidence we have seen suggests that people’s relative economic standing strongly influences the way they feel and act.”

\(^4\) Whereas social comparisons are at the heart of our model, they are not critical to the rank-order tournament environment modeled by Lazear and Rosen (1981) and Rosen (1986). In their environment, the engine that drives effort is prizes. Workers’ expected income consists of a prize to a winner and a prize to a loser. Workers seek to maximise their expected income net of the cost of exerting the effort that yields that income. In our model, the engine that drives effort is the higher income of co-workers: workers seek to maximize their expected utility net of the cost of exerting effort, where utility is derived from a rise in their own income and a fall in the income of the co-workers whose earnings are higher than theirs. While the rank tournament environment is applicable to workers comparing themselves to an exogenously set “fixed standard” (Lazear and Rosen 1981, 848), our model builds on the idea that the effort eliciting comparison is endogenous. It follows that if social comparisons were to be incorporated in rank-order tournament models, the workers who sense “relative deprivation” (defined below) will be predicted to exert more effort than the workers who do not sense “relative deprivation.” It is possible that this omission accounts for the experimental finding of Bull, Schotter, and Weigelt (1987) that the rank-order tournament theory under-predicts the effort supplied by the high cost or the disadvantaged agents.

\(^5\) Frank (1985, 19) notes that “people want to be like those they consider above them, they show little evidence of wanting to be like those they regard as substantially inferior.”

\(^6\) We neglect the possibility that high income earners are affected by social comparisons from below. However, if this effect is present, such that workers do not like to earn more than their co-workers, then a counteracting effect might prevail as high achievers reduce their effort. As long as the inequality from above impacts more strongly than the inequality from below, our main insights will still hold qualitatively.

\(^7\) In our own work (several references are provided in footnote 14 below), we have studied the first three responses.
depends on individual perceptions and capabilities, the nature of the economic and social environment, the set of opportunities, the time frame, and the social and cultural norms. It is beyond the scope of the current paper to seek to model an overarching theory of the determination of which type of response will be adopted. In particular, in this paper we do not cover responses such as skill formation and exit (migration). The former response may not be feasible if skills are exogenous (or are very costly to acquire, or can be acquired only in the long run). The second response requires an elaboration of alternative environments, the modeling of belief structures regarding the behavior of others, and the modeling of costs. We do seek, however, to examine closely one type of response - exertion of effort - and link this type with the economic design of the firm. Perhaps out of the plethora of responses, a modification of effort is the one that a firm can engineer most easily and most swiftly.

We show that compared to a baseline model (where no relative deprivation is felt), firms can increase their profits by “engineering” a sense of relative deprivation. Moreover, the more relative deprivation plays a role in shaping a worker’s utility, the more earnings are compressed. These results differ from those obtained by others. For example, rather than analyzing social interactions between co-workers, Dur and Glazer (2008) study optimal contracts when a worker envies his employer, bearing in mind that his effort contributes to the employer’s wealth. Dur and Glazer (2008) conclude that in order to elicit a given level of effort from a more envious worker, either the employer has to increase the worker’s base salary or bonus salary, or the employer has to relax the effort requirement. In a way, envy is an emotion evoked by relative deprivation. In our model, however, when the worker senses relative deprivation, the firm could increase his relative deprivation by reducing his wage rate while leaving the effort exerted by him unchanged. In addition, Dur and Glazer (2008) find that the employer’s profits decline when a worker’s envy increases. We provide conditions under which the opposite holds.

Utilizing a principal-agent model, Fang and Moscarini (2005) consider the effect of wage differentiation on the morale of workers, which in turn affects the workers’ incentive to exert effort. Morale is affected if workers misjudge their own ability when ability is revealed through differentiated wage contracts. Unlike in our model, in the model of Fang and Moscarini (2005, 751), workers compare relative earnings “on the perception that they have about their own skills.” That is, if the firm’s belief regarding workers’ ability coincides with the workers’ perception of their own ability, a relative comparison of earnings will have no effect on morale. Fang and Moscarini (2005, 751) distinguish between two possible cases: “If effort and ability are complements in production, then workers who suffer a loss in morale are
discouraged from exerting effort. …The other workers are further encouraged and work harder. …If instead effort and ability are substitutes, negative and positive morale effects switch places: a loss in morale induces the affected workers to try even harder, to compensate the lack of ability; while those who gain morale now believe their natural talent to be sufficient for a good performance at lower effort….” The effect of a loss in morale on effort exerted within “the substitution case” in the model of Fang and Moscarini (2005) is akin to the effect of the sensing of relative deprivation on the effort extended by a low-productivity worker in our model, although we do not consider an effect on the effort supplied by the high-productivity worker who is not relatively deprived. While in the setting of Fang and Moscarini (2005), non-differentiation in wages is applied in order to retain morale, in our setting employers should pay wage rates aligned to workers’ levels of productivity and reap the ensuing profit returns.

Earnings compression is often linked to teamwork and is rationalized by its effect of reducing non-cooperative behavior (as in tournament models). The earnings compression result identified in our model suggests that the feeling of relative deprivation is another reason for pay compression.

Frank (1984a, 1984b, 1985) studies wage determination when status considerations matter. Workers who value status highly could gain more status by working for a firm that pays a lower wage rate. And those who value status little will accept low-ranking positions if paid more than the value of what they produce. “Status payments” lead to earnings compression. Nickerson and Zenger (2008) argue that compressed wages can result from the actions of management. In a similar vein, Akerlof and Yellen (1990) assume that workers have a sense of fairness and reduce their effort if the actual wage falls short of the fair wage. Wage compression occurs when one group receives high payments, and the payments of the other groups are adjusted so that the wages of the latter groups are not perceived as unfair. In our model, however, wage compression arises when the workers themselves adjust their level of effort.

Our model implies that the workplace layout or configuration is an important arena for setting the firms’ profits. When a low-productivity worker senses relative deprivation and seeks to quell that feeling by applying more effort at no cost to the firm, the firm may “engineer” such a response by positioning the low-productivity worker next to the high-productivity worker.

As already noted, workers can take a variety of actions to mitigate an unfavorable income gap. Nickerson and Zenger (2008) consider a menu of such actions. When perceived
inequity arises, the resulting feeling - which Nickerson and Zenger (2008) call “envious emotions” - drives workers to reduce effort, depart, engage in sabotage, or withdraw cooperation. These actions impose costs on the firm - the “social comparison costs.” In response, the management can compress rewards, revise production technologies, or shift boundary conditions (for example, by outsourcing tasks). Nickerson and Zenger (2008) provide several interesting examples. However, all their illustrations refer to “unfair payments:” either one team gets paid more than other teams in the same plant, or there are different compensation systems within the same company, or unequal payments for equal tasks, or different payments for equal positions. It is hard to deny that in the settings illustrated by Nickerson and Zenger (2008), social comparison costs might arise. It seems, however, that these settings all refer to inequity. Conceivably, a worker will feel that equity holds sway whenever his ratio of inputs (time, effort) to outcomes (wages) is equal to that which he perceives pertains to others (Adams 1963). What happens, however, when workers compare their rewards with those of co-workers who earn more, even when inequity does not pervade the workplace? As we note below (footnote 11), inequity does not pervade our setting. And we employ the concept of relative deprivation which, while encompassing social comparisons, is shown to lead to different outcomes from those feared by Nickerson and Zenger (2008). In sum, whereas Nickerson and Zenger (2008) emphasize the adverse repercussions for the firm of workers’ “engagement” in social comparisons, we identify a possible benefit.

II. Analysis

Consider a firm with two workers: a high-productivity worker, \( s \), and a low-productivity worker, \( u \). The output of a worker, \( q_i \), \( i = s,u \), depends on \( e_i \), the level of effort exerted by the worker, and on a quality parameter, \( \vartheta_i \), that converts effort into output. Thus, the adjusted-for-quality output of worker \( i \) is given by

\[
q_i = \vartheta_i e_i, \ i = s,u.
\]

To allow for heterogeneity in workers’ productivity, we naturally assume, and without loss of generality, that \( \vartheta_s < \vartheta_u = 1 \): the quality of the output of the high-productivity worker \( s \) is superior to the quality of the output of the low-productivity worker \( u \). For example, a low-
productivity mushroom harvester collects mushrooms contaminated with soil, while a high-productivity harvester collects perfectly clean mushrooms. Even if both workers exert the same level of effort, say each toils to pick one kilogram of mushrooms, their output adjusted for quality differs because “soiled” mushrooms sell for a lower price than clean mushrooms. As yet another example, consider salesmen: in a given time span and when negotiating with the same number of customers, a low-productivity salesman is likely to conclude fewer sales than a high-productivity salesman.

We study environments in which workers are in close proximity to each other, perform comparable tasks, and are aware of the earnings of fellow workers. Output is measurable, and a worker’s effort directly affects output.

A. A Baseline Model

To model a worker’s behavior, we construct a “reaction function” that determines the optimal level of effort exerted by a utility maximizing worker for a given wage rate. Consider a worker $i$ who decides how much effort, $e_i$, to exert for a given wage rate, $w_i$. The worker derives utility from earnings (income), $w_i e_i$, and disutility from exerting effort. Let $c > 0$ be a parameter that converts effort into utility; it measures the intensity of the “pain” or displeasure from exerting effort, and is assumed to be the same for both workers. The worker’s utility function, $\nu$, is given by

$$\nu_i(e_i) = w_i e_i - c e_i^2.$$  

We assume that exerting an additional unit of effort when the level of effort is high is worse than exerting an additional unit of effort when the level of effort is low. Hence the term $e_i^2$. Maximization of $\nu_i(e_i)$ yields an optimal level of effort

$$e_i(w_i) = \frac{w_i}{2c}.$$  

Given the reaction function (3), and normalizing the market price of a unit of output as one, the profits, $\pi$, of the firm that employs the two workers are given by
\[ \pi = e_s(w_s) + \partial^2 e_u(w_u) - (e_s(w_s) - (e_u(w_u)) w_s + e_u(w_u) w_u) \]

\[ = \frac{w_s}{2c} + \frac{\partial^2 e_s(w_s)}{2c} - \frac{w_s}{2c} w_i - \frac{w_u}{2c} w_u. \] (4)

The first order conditions for optimal profit are

\[ \frac{\partial \pi}{\partial w_s} = \frac{1}{2c} w_s - \frac{w_s}{c} = 0, \] (5)

and

\[ \frac{\partial \pi}{\partial w_u} = \frac{\partial^2 e_u}{2c} - \frac{w_u}{c} = 0. \] (6)

The Hessian matrix of \( \pi \), given by

\[ H(\pi) = \begin{bmatrix} \frac{\partial^2 \pi}{\partial w_s^2} & \frac{\partial^2 \pi}{\partial w_s \partial w_u} \\ \frac{\partial^2 \pi}{\partial w_u \partial w_s} & \frac{\partial^2 \pi}{\partial w_u^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{c} & 0 \\ 0 & -\frac{1}{c} \end{bmatrix}, \] (7)

is negative-definite (minors are \( M_1 = -\frac{1}{c} < 0 \), and \( M_2 = |H| = \frac{1}{c^2} = 0 > 0 \)). Therefore, \( \pi \) is jointly strictly concave with respect to both wage rates. Consequently, the profit maximizing wage rate for the high-productivity worker is uniquely determined as the solution to (5), that is,

\[ w_s^* = \frac{1}{2}, \] (8)

and the profit maximizing wage rate for the low-productivity worker is uniquely determined as the solution to (6), that is,

\[ w_u^* = \frac{\partial^2 e_u}{2}. \] (9)

The optimal levels of effort exerted by the high-productivity worker and by the low-productivity worker thus follow from (8) and (9), and are given, respectively, by

\[ e_s^* = e_s(w_s^*) = \frac{1}{4c}, \] (10)

and

\[ e_u^* = e_u(w_u^*) = \frac{\partial^2 e_u}{4c}. \] (11)
Therefore, the firm’s maximal profits are
\[
\pi^* = \frac{1}{4c} + \frac{\vartheta_u^2}{4c} - \frac{1}{8c} - \frac{\vartheta_u^2}{8c} = \frac{1 + \vartheta_u^2}{8c}.
\] (12)

**CLAIM 1.** The firm’s maximal profits are inversely related to the intensity of the workers’ displeasure from exerting effort.

**Proof.** \[\frac{\partial \pi^*}{\partial c} = -\frac{1 + \vartheta_u^2}{8c^2} < 0.\]

Claim 1 implies that firms that provide pleasing working conditions and thereby reduce workers’ disutility from exerting effort will reap higher profits compared to firms that provide awkward or uninviting working conditions.

We next compare the earnings of the high-productivity worker with the earnings of the low-productivity worker. The earnings gap between the workers, \(g\), is given by
\[
g^* = w^*e^* - w^*_ue^* = \frac{1}{8c} - \frac{\vartheta_u^2}{8c} = \frac{1 - \vartheta_u^2}{8c}.
\] (13)

**CLAIM 2.** When the intensity of the displeasure from exerting effort increases, the earnings gap shrinks.

**Proof.** \[\frac{\partial g^*}{\partial c} = -\frac{1 - \vartheta_u^2}{8c^2} < 0, \text{ because } \vartheta_u^2 < 1.\]

An increase in \(c\) reduces the effort level exerted by the high-productivity worker relatively more, and because the high-productivity worker’s earnings are higher, the absolute decrease in his earnings is bigger than the absolute decrease in the earnings of the low-productivity worker. This results in a reduced earnings gap.\(^{10}\) This outcome is based on the premise that workers are motivated solely by their (material) self interest. We next consider the possibility that workers’ behavior is shaped also by interpersonal comparisons.

\(^{10}\) Claim 2 relies on our assumption (made prior to displaying equation (2)) that the displeasure from exerting effort of the low-productivity worker is equal to the displeasure from exerting effort of the high-productivity worker. Even if this assumption were to be relaxed, that is, even if the displeasure arising from exerting effort of the high-productivity worker would be higher than the displeasure arising from exerting effort of the low-productivity worker, Claim 2 will go through, provided that \(\vartheta_u\) is sufficiently small.
B. The Baseline Model Augmented by Relative Deprivation (RD)

The earnings of the low-productivity worker are lower than the earnings of the high-productivity worker. Because of this earnings gap (and because of the presumed corresponding gap in consumption possibilities), the low-productivity worker may experience “relative deprivation,” in which case, in addition to experiencing disutility from the exertion of effort, the low-productivity worker derives disutility from the unfavorable comparison of his earnings with the earnings of the high-productivity worker.\(^{11}\) Thus, let the utility function of the low-productivity worker be

\[
\tilde{v}_u = w_u \tilde{e}_u - \alpha RD - c \tilde{e}_u^2
\]

where RD denotes the relative deprivation sensed by a low-productivity worker from comparing his earnings with the earnings of the high-productivity worker, and where a tilde denotes variables in the RD setting. The coefficient \(\alpha > 0\) determines the weight accorded to RD in the utility function of the low-productivity worker.\(^{12,13}\)

It is instructive to briefly explain what the term relative deprivation stands for. A helpful way of accomplishing this is to refer to Runciman (1966, 10): “We can roughly say that [a person] is relatively deprived of X when (i) he does not have X, (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X (whether or not this is or will be in fact the case), (iii) he wants X, and (iv) he sees it as feasible that he should have X.” This succinct exposition of the relative deprivation concept also helps define the confines of our analysis: we are interested in studying effort responses, and, as already noted, assume away other possible responses such as sabotaging the performance of others or investing in skill acquisition.

Let RD be measured as the proportion of those who earn more (in our case, this proportion is \(\frac{1}{2}\) times their mean excess income (in our case, this excess income is the earnings gap between the workers).\(^{14}\) This formula is derived from a simple definition of the concept of relative deprivation.

\(^{11}\) When equity is defined by the ratio of a worker’s input to outcome being the same as that of co-workers, inequity does not pervade because from equations (8) through (11) it follows that \(\frac{e_x}{w_x} = \frac{e_y}{w_y} = \frac{1}{2c}\).

\(^{12}\) The baseline model can be viewed as a special case of the extended model when \(\alpha = 0\).

\(^{13}\) How much RD matters depends on a variety of considerations. Frank (1985) underscores the intensity of interaction, and our treatment environment measures up to Frank’s (1985, 57) depiction: “in firms in which co-workers perform their tasks largely independently of one another, one’s rank among one’s co-workers should matter less than it does in a firm in which interactions among co-workers are more extensive.”

\(^{14}\) This measure of relative deprivation was proposed by Yitzhaki (1979) and axiomatized by Bossert and D’Ambrosio (2006) and Ebert and Moyes (2000) who in turn followed the seminal work of Runciman (1966). Since the 1960s, a considerable body of research evolved, demonstrating empirically that interpersonal
Consider a population $N$ of $n$ individuals whose incomes are $x_1 \leq x_2 \leq ... \leq x_n$, where $n \geq 2$. The relative deprivation, $RD$, of an individual whose income is $x_i$, $i=1,...,n-1$, is defined as

$$RD_N(x_i) \equiv \frac{1}{n} \sum_{k=i+1}^{n} (x_k - x_i),$$

and it is understood that $RD_N(x_n) = 0$. Let $F(x_i)$ be the fraction of those in the population whose incomes are smaller than or equal to $x_i$. Let us denote by $k_i$ the smallest $k \in [i+1,n]$ for which $x_k > x_i$. That is, $k_i$ is the index of the first individual to the right of $x_i$ in the ordered distribution whose income is strictly higher than $x_i$. Since for different $i$'s there can be different corresponding $k$'s, we use the term $k_i$. Then, we have that

$$RD_N(x_i) \equiv \frac{1}{n} \sum_{k=i+1}^{n} (x_k - x_i) = \frac{1}{n} \sum_{k=k_i}^{n} (x_k - x_i)$$

$$= \left[1 - F(x_i)\right] \sum_{k=k_i}^{n} \left(\frac{1}{n}\right)(x_k - x_i)$$

$$= \left[1 - F(x_i)\right]E(x - x_i|x > x_i).$$

That is, the relative deprivation of an individual whose income is $x_i$ is the fraction of those in the population whose incomes are higher than $x_i$, times their mean excess income.

This RD measure is more concrete than the general concepts of envy and inequity, and is more refined than the ordinal measure rank: in an ordered income distribution, for example, the RD measure says that given one’s rank, the rank is more unfavorable if those holding higher ranks are farther apart, and it records a higher value if the proportion of those to one’s right rises, even when their incomes and number do not. (In Appendix C we show that the key effort exertion result of our model holds also when a more general measure of RD is employed.) Then, assuming that each worker observes fully and accurately the earnings of the other worker, the low-productivity worker’s RD is

$$RD = \frac{1}{2}(w_i \bar{z}_i - w_o \bar{z}_o). \quad (15)$$

Comparisons of income (that is, comparisons of the income of an individual with the incomes of higher income members of his reference group) bear significantly on the perception of well-being, and on behavior. (For a recent review see Clark et al. 2008). One branch of this body of research has dealt with migration. Several studies have shown empirically that a concern for relative deprivation impacts significantly on migration outcomes (Stark and Taylor 1989; Stark and Taylor 1991; Quinn 2006; Stark, Micevska, and Mycielski 2009). Theoretical expositions have shown how the very decision to resort to migration and the choice of migration destination (Stark 1984; Stark and Yitzhaki 1988; Stark and Wang 2007), as well as the assimilation behavior of migrants (Fan and Stark 2007), are modified by a distaste for relative deprivation.
Inserting (15) into (14) yields
\[ \tilde{v}_u = w_u \tilde{e}_u - \frac{\alpha}{2} (w_s \tilde{e}_u - w_u \tilde{e}_u) - c \tilde{e}_u^2. \] (16)

Maximization of (16) with respect to \( \tilde{e}_u \) yields the optimal level of effort exerted by the low-productivity worker for a given wage rate:
\[ \tilde{e}_u(w_u) = \frac{w_u(2 + \alpha)}{4c}. \] (17)

Because the high-productivity worker senses no relative deprivation, his utility function remains as in (2), and his optimal effort level is as in (3), that is,
\[ \tilde{e}_s(w_s) = \frac{w_s}{2c}. \] (18)

CLAIM 3. The stronger the concern of the low-productivity worker for relative deprivation, the higher the increase in the level of effort exerted by the workforce.

Proof. \[ \frac{\partial \tilde{e}_u(w_u)}{\partial \alpha} = 0, \quad \frac{\partial \tilde{e}_u(w_u)}{\partial \alpha} = \frac{w_u}{4c} > 0. \Box \]

Holding constant the wage rate of the low-productivity worker, when this worker senses relative deprivation, his optimal level of effort increases. This effort response can be seen nicely upon decomposing the worker’s effort into two components. From (17), the worker’s effort is the effort level as in (3), plus \( \frac{w_u \alpha}{4c} \). This second component represents the contribution of relative deprivation to the worker’s effort. What drives the higher effort is an aspiration to narrow the earnings gap and thereby to increase utility. Absent the sensing of relative deprivation, the firm would need to increase the wage rate in order to elicit more effort.

Because \( \tilde{e}_u(w_u) > e_s(w_u) \), we can already expect that there will be a corresponding increase in profits. For instance, the firm could reduce \( w_u \) such that the output of the low-productivity worker would not change, and thereby also reduce the wage bill for the low-productivity worker. What wage rates will the firm choose then to maximize its profits? Will the earnings gap increase, decrease, or not change at all? To this end, we further investigate the firm’s behavior.

The profit function is
\[ \tilde{\pi} = \tilde{\pi}(w_s) + \theta \tilde{\pi}(w_u) - (\tilde{\pi}(w_s)w_s + \tilde{\pi}(w_u)w_u) \]
\[ = \frac{w_s}{2c} + \theta \frac{w_u(2+\alpha)}{4c} - \frac{w_s}{2c}w_s - \frac{w_u(2+\alpha)}{4c}w_u. \]  

(19)

The optimal wage rates paid to a high-productivity worker, \(w_s^{**}\), and to a low-productivity worker, \(w_u^{**}\), are determined, respectively, by

\[ \frac{\partial \tilde{\pi}}{\partial w_s} = \frac{1}{2c} - \frac{w_s}{c}, \]  

and by

\[ \frac{\partial \tilde{\pi}}{\partial w_u} = \frac{\theta(2+\alpha)}{4c} - \frac{w_u(2+\alpha)}{2c}. \]  

(20)  

(21)

The Hessian matrix of \(\tilde{\pi}\), given by

\[ H(\pi) = \begin{bmatrix}
\frac{\partial^2 \pi}{\partial w_s^2} & \frac{\partial^2 \pi}{\partial w_s \partial w_u} \\
\frac{\partial^2 \pi}{\partial w_u \partial w_s} & \frac{\partial^2 \pi}{\partial w_u^2}
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{c} & 0 \\
0 & -\frac{2+\alpha}{2c}
\end{bmatrix}, \]

(22)

is negative-definite (minors are \(M_1 = -\frac{1}{c} < 0\), and \(M_2 = |H| = \frac{2+\alpha}{2c^2} > 0\)). Therefore, \(\tilde{\pi}\) is jointly strictly concave with respect to both wage rates. Solving for \(w_s\) and \(w_u\), we obtain, respectively,

\[ w_s^{**} = \frac{1}{2}, \]  

(23)

and

\[ w_u^{**} = \frac{\theta}{2}. \]  

(24)

and that the corresponding levels of effort exerted are

\[ e_s^{**} = \tilde{e}_s(w_s^{**}) = \frac{1}{4c}, \]

(25)

and

\[ e_u^{**} = \tilde{e}_u(w_u^{**}) = \frac{\theta(2+\alpha)}{8c}. \]  

(26)
By comparing (24) with (9), we see that when the low-productivity worker senses and responds to relative deprivation, the firm will elect \textit{not} to reduce the wage rate of the low-productivity worker (nor, for that matter, to increase the wage rate of the low-productivity worker).

The profits of the profit-maximizing firm when the low-productivity worker senses relative deprivation, $\pi_{RD}^{**}$, are given by

$$\pi_{RD}^{**} = \frac{1}{4c} + \frac{d_u^2(2+\alpha)}{8c} - \frac{1}{16c} \cdot \frac{d_u^2(2+\alpha)}{16c} = \frac{2}{16c} + \frac{d_u^2}{16c}.$$  \hfill (27)

**CLAIM 4.** The firm’s optimal profits are \textit{higher} the stronger the concern of the low-productivity worker for relative deprivation.

\textit{Proof.} \quad \frac{\partial \pi_{RD}^{**}}{\partial \alpha} = \frac{d_u^2}{16c} > 0. \quad \Box

Because $\alpha > 0$ then, from comparing (17) and (3) while bearing in mind from (24) and (9) that the low-productivity worker’s wage rate does not change, an increase in profits is apparent; when low-productivity workers become more “productive” for a given wage rate, the firm benefits. Comparing (27) and (12) reveals that when the low-productivity worker senses relative deprivation, the firm’s profits increase by

$$\pi_{RD}^{**} - \pi^* = \frac{2}{16c} + \frac{d_u^2(2+\alpha)}{16c} - \frac{1}{8c} \cdot \frac{d_u^2(2+\alpha)}{16c} = \frac{d_u^2\alpha}{16c}.$$  \hfill (28)

From (28) we can infer that holding the wage rates intact, a gain in profits could come about from a lower $c$, or from (the presence of, and thereby) a higher $\alpha$. This result is informative when tinkering with $c$ can be done only to a little extent or is quite costly, whereas affecting relative deprivation by a redesign of the workplace aimed at intensifying cross-worker comparisons could be relatively easy. When the technology of production allows teamwork, our model suggests employing mixed-productivity teams. In a different setting, the firm could place a low-productivity worker and a high-productivity worker in the same office. Furthermore, wages of (co-)workers can be made public.

We next consider $g_{RD}$, the earnings gap within the framework of the augmented model. We get that when RD matters,

$$g_{RD}^{**} = e_{s}^{**}w_{s}^{**} - e_{u}^{**}w_{u}^{**} = \frac{1}{8c} - \frac{d_u^2(2+\alpha)}{16c} = \frac{2 - d_u^2(2+\alpha)}{16c}. \hfill (29)$$
To ensure that we will never end up with a negative gap between the earnings of the high-productivity worker and the earnings of the low-productivity worker (thereby rendering the high-productivity worker relatively deprived) or, for that matter, with a negative gap between the output of the high-productivity worker and the output of the low-productivity worker, we assume that \( \alpha < \frac{2}{\vartheta^2} - 2 \).

CLAIM 5. The earnings gap is lower the more relative deprivation “counts.”

Proof. \( \frac{\partial g_{ro}^{**}}{\partial \alpha} = -\frac{\vartheta^2}{16c} < 0. \) \( \square \)

Claim 5 shows that the sensing of relative deprivation gives rise to compressed earnings: the effect of the joint behavior of the “dismayed” low-productivity worker and the firm on the inequality in the distribution of earnings within the firm is to reduce the inequality. Because low-productivity workers might sense relative deprivation in diverse intensities, earnings are more compressed the more relative deprivation plays a role in a worker’s utility. This can also be inferred by looking at the effect of \( \alpha \) on the change in the earnings gap. This change is given by

\[
\begin{align*}
g_{ro}^{**} - g^* &= \left( \frac{2-\vartheta^2(2+\alpha)}{16c} \right) - \left( \frac{2-2\vartheta^2}{16c} \right) = -\frac{\vartheta^2 \alpha}{16c} < 0, \\
\end{align*}
\]

which tells us that a higher \( \alpha \) gives rise to a larger decrease in the earnings gap.

In summary, our model suggests that in the presence of a concern for relative deprivation, the inequality in the intra-firm distribution of earnings is lower, and the firm’s profits are higher.

III. Discussion: the Paper that is Closest to Ours

Cabrales et al. (2008) study the earnings structure in a model in which workers have social preferences. They define the individual’s reference group as consisting of people who work in the same firm and who have had similar career histories within the firm. Cabrales et al. (2008, 67) maintain that “in the absence of frictions and with social preferences, …, firms hire only from one skill pool.” When considering social preferences and social friction, the firm can either increase wages to compensate for unskilled workers’ disutility, or segregate skills. Like Cabrales et al. (2008), we assume that the individual’s reference group consists of people in the immediate vicinity - in our case, co-workers. Neither we nor Cabrales et al.
consider “status:” having a higher income than others does not confer utility. Taking into account social preferences, individuals derive disutility if they earn less than other individuals within their reference group.

There is no reason for productivity segregation in our model; on the contrary, firms have an incentive to match high-productivity and low-productivity workers, because low-productivity workers apply more effort than when employed by themselves. Firms reap gains they do not have to pay for through higher wage rates, so firms benefit from employing a mixed pool of workers. We show that the incorporation of social preferences and the associated behavioral response leads to increases in earnings for some workers. Cabrales et al. (2008, 73) do not take into account the effect of social preferences on the effort exerted by the “lower types,” and they explicitly leave out the choice of the level of effort, noting that this is “something that is beyond the scope of this paper.” Thus, our analysis complements the analysis of Cabrales et al. (2008) because in our model, workers explicitly choose the level of effort (that is, the choice of the level of effort is endogenous). Thus, what Cabrales et al. (2008) term a wage increase unrelated to productivity may simply reflect the higher level of effort exerted by the low-productivity workers, as suggested by our analysis.

Like Cabrales et al. (2008), we find that social concerns bring about a compression of earnings. However, the underlying mechanism differs, because in our case an increase in earnings induced by relative deprivation is the crucial factor. Cabrales et al. (2008) also find that wages are downwardly flexible. In their model, workers’ earnings decrease if firms pay less. In our model, workers’ earnings will decrease endogenously, for example, when the high-productivity worker leaves the firm.

IV. Implications, Limitations, and Supplementary Evidence

A. Implications

Taking a cue from the literature on mechanism design, we ask: if the firm’s desired outcome is to maximize profit, what mechanism could the firm design (which architecture of the workplace could the firm select) to attain that outcome? The workplace layout or configuration is an important arena for shaping the firm’s profit. Consider a duty-free store layout (or, for that matter, the layout of any retail store) with two entrances, a cashier at each entrance, and a pay structure based on the value of the sales generated by the cashier. Our model predicts that the positioning of the two cashiers will impact on the level of effort that they will exert and thereby, and in a well-predicted way, on the store’s profits. If one entrance
is sealed off and the two cashiers are assigned to work next to each other, the less able cashier will feel relative deprivation and will therefore increase his or her effort. If both are equally able, pairing them will affect neither effort nor profit.

Our model has an interesting implication for the productivity profile of a firm’s new hires. Suppose that the firm seeks to expand its production, and to this end considers hiring additional workers. And suppose that the firm is pleased to learn what we have pointed out in Section II of the paper. When RD matters (and is defined as in Section II. B.), the hiring of an additional low-productivity worker (or even very low-productivity worker) will lower the optimal level of effort supplied by the incumbent low-productivity worker because the proportion of high-productivity worker(s) in the firm’s workforce will decline from one half to one third. The converse holds if the firm hires a high-productivity worker: the optimal effort supplied by the low-productivity worker will rise. If the firm seeks to expand its production while retaining the prevailing incentives intact, and if that expansion requires input of low-productivity workers, the firm may just as well outsource. It follows then that by retaining an environment which encourages the desired level of effort, outsourcing is more likely when the additional operations are low productivity intensive than when they are high productivity intensive.

Assume that there are two similar firms with equal distributions of workers’ productivities, except that one firm, the N firm, employs only natives whereas the other firm, the NM firm, employs natives as well as migrants. Suppose that natives compare themselves with each other, and that migrants compare themselves with other migrants, but not with natives. Because low-productivity natives compare themselves with high-productivity natives, our model predicts that earnings will be compressed in firm N. If, overall, migrants are less productive than natives, that is, if migrants are the low-productivity workers in firm NM, and if migrants compare themselves only with other migrants (and, therefore, do not sense as much relative deprivation as the low-productivity natives in firm N), our model predicts that there will be less earnings compression (or more earnings stretching) in the NM firm (cf. Claim 5). That the earnings of migrants within firm NM are lower than the earnings of comparable (in terms of levels of productivity) native workers in firm N, is not then the outcome of anti-migrant discrimination. As a corollary, and as implied by Claim 4, if at no extra cost a firm can obtain a higher profit by eliciting more effort from low-productivity native workers than from low-productivity migrants, the firm will prefer to hire the former. This preference arises, though, from cool economic considerations, not from diehard
discrimination. A similar consideration could apply to US settings in which the N firm employs only whites, whereas the NM firm employs whites as well as blacks.

B. Limitations

The technology of production impacts on the extent to which output is team-produced, on the spatial proximity of workers, and on the intensity with which workers interact (Nickerson and Zenger 2008). The configuration of the firm as dictated by the production technology shapes the social comparisons space. The social architecture, in turn, affects the cost of accessing information about peers’ performance and their productivity. For instance, workers who are in close proximity and perform similar tasks are likely to identify each other as members of their reference group and compare their earnings with each other (Nickerson and Zenger 2008). Merely by reducing the physical distance between workers, management can increase interaction and information sharing. Choosing a production technology that departs from the most efficient production technology in order to increase relative deprivation can impose its own cost, however. If this effect is operative, it diminishes the increase in profits predicted by our model. Furthermore, the precision with which individuals can measure their own performance as well as the performance of others will affect our outcome. Again, from the perspective of management, difficulties in measuring individual performance, for example in team production, reduce the effectiveness of strategies to increase effort, when (due to these same difficulties) such an increase in effort is not rewarded by an increase in earnings.

Our model is more likely to hold when recognition, evaluation, and remuneration of performance are fairly easy. In variance with our model, the more wages are based on performance standards determined by the peer group, the stronger the incentive to undermine co-workers, as is also the case when rewards are based on relative performance. Under the assumptions of our model, a tournament structure may not lead to an increase in effort when the probability of promotion of high-productivity workers is higher than the probability of promotion of low-productivity workers. And when output strongly depends on luck and the receipt of promotion is a pure game of chance, workers of both types might withdraw effort.

In the environment modeled by us, there is tension between the preferences of low-productivity workers, and the preferences of firms: low-productivity workers will not like to be teamed up with high-productivity workers, because such a matching will inflict relative deprivation; firms will want to pool together low-productivity workers with high-productivity workers precisely for this very same reason, because the response to the ensuing relative
deprivation will increase profits. In our analysis, we have disregarded the worker’s outside option, or exit. If low-productivity workers can exercise free choice, they will seek to avoid working together with high-productivity workers: absent frictions, low-productivity workers will prefer complete segregation. In a competitive labor market, all firms will prefer to employ only high-productivity workers. Since the workforce is heterogeneous, some firms will end up employing (only) low-productivity workers, as long as profits are yielded. According to our model, by employing a mixed pool of workers, firms stand to reap profits that they do not have to pay for. Firms employing low-productivity workers have an incentive to employ high-productivity workers. These firms will be willing to pay wages above the market-clearing level to attract high-productivity workers. Therefore, if we allow both competition and some friction, our basic argument can still hold.

C. Supplementary Evidence

Although we have not conducted a controlled experiment to measure the signs and elasticities of the effects highlighted by our model, we can nonetheless enlist evidence garnered from the empirical analyses of others that appears to align with the predictions of our model.

Hamilton, Nickerson, and Owan (2003) found that the adoption of teams in a garment factory increased productivity, on average, by 14 percent, and that more heterogeneous teams were more productive than teams of the same average ability. Hamilton et al. (2003) also report that when the team structure was first introduced, the teams attracted the relatively high-productivity workers. To the extent that the insights obtained in our analysis of a two-worker firm could be applied to multiple-worker firms, we would have predicted that behavior: because a low-productivity worker would sense relative deprivation on joining a team of high-productivity workers, he would not be keen to join the team if he did not have to. Hamilton et al. (2003, 493) report one more result: “high-ability workers appear to improve team productivity more than low-ability workers do.” If the most able team member is replaced by a still higher-ability individual, all other members of the team would sense heightened relative deprivation and increase their effort. If the least able team member is replaced by a more able individual who still remains the least able team member, no other team member will be exposed, as a consequence, to more relative deprivation and, as long as the wage rate paid to the least productive worker remains unchanged, the supply of effort will not change either. If, following the replacement of the least able team member by a slightly more able worker, the wage rate of the marginally more productive, new least able worker is
increased in recognition of this worker’s higher productivity, the supply of effort of the two-
worker team will increase. To see this, we revisit equation (26). Because

\[ e_u^{**} = \frac{\vartheta u (2 + \alpha)}{8c}, \]  

(31)

and because

\[ \frac{\partial e_u^{**}}{\partial \vartheta_l} = \frac{2 + \alpha}{8c} > 0, \]  

(32)

it follows that the team’s supply of effort will increase.

Similar support for the predictions of our model can be found in Falk and Ichino (2006). In a controlled field experiment they obtain evidence that peer effects raise overall average productivity, and that low-productivity workers are more influenced by peers than high-
productivity workers. This is in line with our model assumption that the low-productivity worker but not the high-productivity worker is influenced by the consideration of relative deprivation. Falk and Ichino (2006) conclude that output maximizing firms should mix low and high-productivity workers rather than group together workers of similar levels of productivity.

Bandiera, Barankay, and Rasul (2010) sought to unravel how workers’ social ties in the workplace affect performance. Drawing on data from a farm that employs fruit harvesters who are paid a piece rate per kilogram of picked fruit, Bandiera et al. (2010) find that the productivity of less able workers increases significantly when working alongside more productive co-workers with whom the less able workers interact socially, as compared to working alongside co-workers who are not part of the less able workers’ social network. Using our terminology, the former co-workers constitute the reference group of the less able workers, whereas the latter do not. The firm is reported to benefit from the prevalence of socially-based incentives.

Mas and Moretti (2009) study peer effects in the workplace. They investigate how the productivity of a worker is affected by the productivity of co-workers. Using scanner data from a large supermarket chain, they find strong evidence of productivity spillovers associated with the introduction of highly productive workers into a shift. The productivity of the high-productivity workers is not affected by the presence of low-productivity workers, however. This aligns with our basic idea that incentives arise from looking up (to the right), not down (to the left). Furthermore, productivity increases when the highly productive workers are those with whom the workers interact frequently. This too is in line with our basic
concept of a reference (or comparison) group. It is hard, however, to tell what exactly runs through workers’ minds when they tune up their effort - whether the motive is as postulated by us (a desire to curtail relative deprivation) or an aspiration not to sense shame from trailing behind others. The findings of Mas and Moretti (2009) suggest that an optimal organization of the workplace ought to recognize the benefits from mixing different levels of productivity (“engineering” a variance in productivities).

Osterman (2006) finds that High Performance Work Organization (innovative work systems such as teams, quality programs, and job rotation) has a positive impact on wages for core blue-collar manufacturing employees, even after controlling for education. Our model suggests a mechanism that explains why this happens.

V. Conclusion

How and when can a reconfiguration of the workplace increase a firm’s profits without the firm changing its wage rates? We developed the following line of reasoning. Taking into account social preferences, we postulated that workers derive disutility when they earn less than fellow workers. Sensing relative deprivation affects the optimal level of effort chosen by a worker. Relative deprivation motivates a low-productivity worker to close the earnings gap with a high-productivity worker, and this is achieved by exerting a higher level of effort. The increase in the level of effort is not the result of an increase in the wage rate for the low-productivity worker, as would have been implied by the baseline model. We point to a novel way of inducing workers who are relatively less productive to exert more effort. We also show that the more relative deprivation plays a role in shaping a low-productivity worker’s utility, the more earnings are compressed. Our model predicts that the worker’s sense of relative deprivation and consequent response will have a positive effect on the firm’s profits.

Appendix A
Modifying the Utility Function

Our results hold when in the utility function displayed in equation (2) we replace the coefficient 2 with any number $n > 1$. Namely, if instead of

$$v_i(e_i) = w_i e_i - ce_i^2$$

we were to use the utility function

$$v_i(e_i) = w_i e_i - ce_i^n, \quad n > 1,$$
then we would derive qualitatively exactly the same results as when using (2).

Appendix B

A Piece Rate Compensation Scheme

It can be shown that we will derive the very same results as those of our baseline model and of our augmented model when instead of an hourly wage scheme, a piece rate compensation scheme is in place.

Consider once again a firm with two workers: a high-productivity worker, $s$, and a low-productivity worker, $u$. The output of a worker, $q_i$, $i = s,u$, depends on $e_i$, the level of effort exerted by the worker, and on a quality parameter, $\vartheta_i$, that converts effort into output. Thus, the adjusted-for-quality output (the produced goods, or the sales) of worker $i$ is given by

$$q_i = \vartheta_i e_i, \quad i = s,u. \quad (A2)$$

To allow for heterogeneity in workers’ productivities, we assume, that $\vartheta_s < \vartheta_u = 1$.

I. A Baseline Model

Consider a worker $i$ who decides how much effort, $e_i$, to exert for a given piece rate, $w$. The worker derives utility from earnings (income), $w \vartheta_i e_i$, and disutility from exerting effort. Let $c > 0$, a parameter that converts effort into (dis)utility, be the same for both workers. The worker’s utility function, $v_i$, is given by

$$v_i(e) = w \vartheta_i e_i - c e_i^2. \quad (A3)$$

Maximization of $v_i(e)$ yields an optimal level of effort

$$e_i(w) = \frac{w \vartheta_i}{2c}. \quad (A4)$$

Given the reaction function (A4), and normalizing the market price of a unit of output as one, the profits, $\pi$, of the firm that employs the two workers are given by

$$\pi = e_s(w) + \vartheta_s e_s(w) - e_u(w) w + \vartheta_u e_u(w) w$$

$$= \frac{w}{2c} + \vartheta_s \frac{w \vartheta_u}{2c} - \frac{w}{2c} - \vartheta_u \frac{w \vartheta_u}{2c} w. \quad (A5)$$

The first order condition for optimal profit is
\[
\frac{1}{2c} + \frac{\varphi^2_w}{2c} - \frac{w}{c} - \frac{w \varphi^2_u}{c} = 0. \tag{A6}
\]

Because \( \frac{\partial^2 \pi}{\partial w \partial w} = -\frac{1 + \varphi^2_u}{c} < 0 \), the profit maximizing wage rate for both workers is uniquely determined as the solution to (A6), that is,\( w^* = \frac{1}{2} \). \tag{A7}

The optimal levels of effort exerted by the high-productivity worker and by the low-productivity worker thus follow from (A7), and are given, respectively, by\[
e^*_s = e_s(w^*) = \frac{1}{4c}, \tag{A8}
\]
and \[
e^*_u = e_u(w^*) = \frac{\varphi^2_u}{4c}. \tag{A9}
\]

Therefore, the firm’s maximal profits are\[
\pi^* = \frac{1}{4c} + \frac{\varphi^2_u}{4c} - \frac{1}{8c} - \frac{\varphi^2_u}{8c} = \frac{1 + \varphi^2_u}{8c}, \tag{A10}
\]
just the same as in (12).

Correspondingly, the earnings gap between the workers, \( g \), is given by\[
g^* = w^* e^*_s - w^* e^*_u \varphi^2_u = \frac{1}{8c} - \frac{\varphi^2_u}{8c} = \frac{1 - \varphi^2_u}{8c}, \tag{A11}
\]
which is the same as (13).

II. The Piece Rate Compensation Scheme Baseline Model Augmented by Relative Deprivation (RD)

As before, let RD be measured as the proportion of those who earn more, times their mean excess income. Then, \[
RD = \frac{1}{2} (w \tilde{e}_s - w \varphi^2_u \tilde{e}_u). \tag{A12}
\]
Expanding (A3) to incorporate (A12) yields
\[ \tilde{v}_u = w \varphi \tilde{u}_u - \frac{\alpha}{2} (w \tilde{e}_u - w \varphi \tilde{u}_u) - c \tilde{e}_u^2. \] (A13)

Maximization of (A13) with respect to \( \tilde{e}_u \) yields the optimal level of effort exerted by the low-productivity worker for a given piece rate:

\[ \tilde{e}_u(w) = \frac{w \varphi (2 + \alpha)}{4c}. \] (A14)

Because the high-productivity worker senses no relative deprivation, his utility function is as in (A3), and his optimal effort level is as in (A4), namely

\[ \tilde{e}_s(w) = \frac{w}{2c}. \] (A15)

The profit function is

\[ \tilde{\pi} = \tilde{e}_s(w) + \varphi \tilde{e}_u(w) - (\tilde{e}_s(w) w + \varphi \tilde{e}_u(w) w) \]

\[ = \frac{w}{2c} + \frac{\varphi^2 w (2 + \alpha)}{4c} - \frac{w}{2c} - \frac{\varphi^2 w (2 + \alpha)}{4c} w. \] (A16)

The optimal wage rate paid to a high-productivity worker and to a low-productivity worker is determined by

\[ \frac{\partial \tilde{\pi}}{\partial w} = \frac{1}{2c} + \frac{\varphi^2 (2 + \alpha)}{4c} - \frac{w}{c} - \frac{\varphi^2 w (2 + \alpha)}{2c}. \] (A17)

Solving for \( w \), we obtain

\[ w^{**} = \frac{1}{2}, \] (A18)

and the corresponding levels of effort exerted are

\[ e_s^{**} = \tilde{e}_s(w^{**}) = \frac{1}{4c}, \] (A19)

and

\[ e_u^{**} = \tilde{e}_u(w^{**}) = \frac{\varphi (2 + \alpha)}{8c}. \] (A20)

The profits of the profit-maximizing firm when the low-productivity worker senses relative deprivation, \( \tilde{\pi}^{**} \), are given by
\[
\pi^{**}_{RD} = \frac{1}{4c} + \frac{\varphi^2(2+\alpha)}{8c} - \frac{1}{8c} - \frac{\varphi^2(2+\alpha)}{16c} = \frac{2 + \varphi^2(2+\alpha)}{16c}.
\] (A21)

We next consider \( g^{**}_{RD} \), the earnings gap within the framework of the model augmented by RD. We get

\[
g^{**}_{RD} = \varphi^{**} w^* - \varphi^{**} w^* = \frac{1}{8c} - \frac{\varphi^2(2+\alpha)}{16c} = \frac{2 - \varphi^2(2+\alpha)}{16c}.
\] (A22)

Because we derive the same solutions for \( e^*_s, e^*_u, \pi^*, g^*_s, e^*_s, e^*_u, \pi^{**}_{RD} \), and \( g^{**}_{RD} \) as in Section II, Claims 1-5 do not change.

**Appendix C**

**A General Form of Relative Deprivation (RD)**

That the sensing of RD will increase the level of effort exerted by a low-productivity worker can be shown to hold even in a fairly general model. We are able to draw several conclusions (for example, that profits will increase), in line with the conclusions drawn in the model presented in the body of the paper even when RD is measured heuristically. However, with the “general form” we cannot construct a reaction function, so we do not know how exactly the firm will react.

**I. A Basic General Model**

Consider the same setting as before: let there be two workers, a high-productivity worker, \( s \), and a low-productivity worker, \( u \). Let each worker’s earnings, \( E_i(e_i) \), \( i = s, u \), be a function of the worker’s effort, \( e_i \), where \( E'_i(e_i) > 0 \) and \( E''_i(e_i) \leq 0 \). Let the cost of extending this effort be \( C_i(e_i) \), where \( C'_i(e_i) > 0 \) and \( C''_i(e_i) > 0 \). The worker’s utility function, \( v_i \), is given by

\[
v_i(e_i) = E_i(e_i) - C_i(e_i), \quad i = s, u.
\] (A23)

The decision problem of the worker is how much effort to exert. The first order condition for a maximum of (A23) is

\[
E'_i(e_i) - C'_i(e_i) = 0.
\] (A24)

We denote the solutions to (A24) by \( e^*_i \) and \( e^*_u \).

We assume that the high-productivity worker is more productive and therefore receives higher earnings no matter how much effort the low-productivity worker exerts:
\( E_s(e) > E_u(e) \quad \forall e \quad \text{and} \quad E_s(e_u^*) > E_u(e_u^*). \) We denote the high-productivity worker’s earnings by \( \bar{E}_s \equiv E_s(e_u^*). \)

**II. Augmenting the Basic General Model by Relative Deprivation**

Because the low-productivity worker receives lower earnings than the high-productivity worker, he derives (dis)utility also from an unfavourable comparison of his earnings with the earnings of the high-productivity worker. RD denotes the relative deprivation arising from earnings comparisons, and is defined as follows

\[
RD \equiv RD(E_u(e_u), \bar{E}_s),
\]

such that \( \frac{\partial RD(E_u(e_u), \bar{E}_s)}{\partial E_u} < 0 \), and \( \frac{\partial RD(E_u(e_u), \bar{E}_s)}{\partial \bar{E}_s} > 0 \).

Combining (A23) and (A25) we have that

\[
\frac{\partial RD(E_u(e_u), \bar{E}_s)}{\partial e_u} = \frac{\partial RD(E_u(e_u), \bar{E}_s)}{\partial E_u} E_u'(e_u) < 0.
\]

We can rewrite the utility function of a low-productivity worker to reflect his concern for RD:

\[
v_u(e_u) = E_u(e_u) - \alpha RD(E_u(e_u), \bar{E}_s) - C_u(e_u),
\]

where \( \alpha \) is the weight that this worker attaches to RD.

The optimally chosen level of effort, \( e_u^{**} \), is given by

\[
E_u'(e_u) - \alpha \frac{\partial RD(E_u(e_u), \bar{E}_s)}{\partial e_u} - C_u'(e_u) = 0.
\]

We assume that the second order condition for a maximum, \( v'_u(e_u^{**}) < 0 \), holds. This requires that \( E_u^{**}(e_u) - C_u^{**}(e_u) < \alpha \frac{\partial^2 RD(E_u(e_u), \bar{E}_s)}{\partial e_u \partial e_u}. \)

CLAIM A1. The sensing of relative deprivation increases the level of effort exerted: \( e_u^{**} > e_u^* \).

**Proof.** We prove this claim by contradiction. Suppose otherwise, that is, suppose that \( e_u^{**} \leq e_u^* \).

Because \( C_u'(e_u) > 0 \), \( C_u^{**}(e_u) > 0 \), \( E_u'(e_u) > 0 \), and \( E_u^{**}(e_u) \leq 0 \), we have that

\[
E_u'(e_u^{**}) - C_u'(e_u^{**}) \geq E_u'(e_u^*) - C_u'(e_u^*) = 0,
\]

(A29)
and because \( \frac{\partial RD(E_u(e_u^*), \bar{E}_s)}{\partial e_u} < 0 \) we have that

\[
E'_u(e_u^{**}) - \alpha \frac{\partial RD(E_u(e_u^{**}), \bar{E}_s)}{\partial e_u} - C'_u(e_u^{**}) > 0, 
\]

(A30)

which contradicts (A28). □

CLAIM A2. A higher coefficient \( \alpha \) - an increase in the weight that the low-productivity worker attaches to RD in his utility function - elicits a higher level of effort by the low-productivity worker.

**Proof.** Because from (A28),

\[
\Omega \, de_u^{**} = \frac{\partial RD(E_u(e_u^{**}), \bar{E}_s)}{\partial e_u} \, d\alpha = 0, \tag{A31}
\]

we have that

\[
\frac{de_u^{**}}{d\alpha} = \frac{1}{\Omega} \frac{\partial RD(E_u(e_u^{**}), \bar{E}_s)}{\partial e_u} > 0, \tag{A32}
\]

where \( \Omega \), the derivative of the left hand side of (A28) with respect to \( e \) is simply the second order condition, and hence it must be negative at the optimum. □

Because \( e_u^{**} > e_u^* \), we can already expect that there will be a corresponding increase in profits. The incentive to exert effort yielded by a concern for relative deprivation could substitute for the wage as a tool aimed at eliciting effort. Consequently, a firm with an RD-sensitive worker could cut wages without hurting profits.
References


