The Propensity to Patent
in Oligopolistic Markets

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Abstract
We analyze the strategic protection decision of an innovator between a patent and secrecy in a setting with horizontally differentiated products. By introducing the patenting decision into the well known circular city model, the impact of the disclosure requirement linked to a patent application as well as the problem of legally inventing around a patent can be taken into account. Asymmetry in the circular market leads to a consumer migration effect. We find that secrecy may be the innovator’s profit maximizing strategy whenever the mandatory disclosure of information enhances the market entry of competitors.

Keywords: patenting decision, secrecy, disclosure requirement, patent breadth, horizontal product differentiation, circular city

JEL Classifications: L13, L24, O34

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1 Introduction

In their seminal paper Horstmann et al. (1985) were the first to question the common assumption to the literature that every innovation is patented. Opposing the formerly stylized fact that the number of innovations and patents could be seen as equivalent measures of a firm’s R&D output, Horstmann et al. (1985) find that the propensity to patent (the proportion of innovations that are actually patented) actually lies somewhere between zero and one. Empirical studies strongly support this result: Analyzing data from the 1993 European Community Innovation Survey (CIS) for up to 2849 R&D-performing firms Arundel (2001) finds that a higher percentage of firms in all size classes rates secrecy as more valuable than patents. In their empirical study on the patenting decision of U. S. manufacturing firms Cohen et al. (2000) find an increased emphasis on secrecy as a reason not to patent as compared to an earlier study by Levin et al. (1987). Cohen et al. (2000) isolate the two key reasons for firms not to patent as (i) the amount of information disclosed in a patent application and (ii) the ease of legally inventing around a patent. In this paper we treat both aspects interdependently.

To do this we perpetuate the results obtained in Horstmann et al. (1985) by introducing the possibility of patenting into an oligopolistic model of horizontally differentiated products. We assume that a drastic product innovation is released on a new market where rivals may enter with non-infringing products as patent protection is not perfect. While Horstmann et al. (1985) incorporate the disclosure requirement by assuming that the revelation of enabling information makes imitation more profitable (see Horstmann et al. (1985), p. 849), in our setting we are able to gain a further insight into the effectiveness of the disclosure requirement. Assuming that the information revealed due to the disclosure requirement reduces competitor’s market entry costs, inventing around is facilitated so that possibly more firms are able to enter the market due to a patent. Thus the positive effect of patent protection may be opposed by a negative effect of the required disclosure.

In the present paper we analyze the influence of varying intensities of the disclosure requirement’s impact on the patenting decision of the innovator. We find that the innovator will patent as this is more profitable than secrecy as long as a patent is sufficiently broad. This result holds even in both limit cases, when either the disclosure effect is absent or when the proprietary knowledge is fully disclosed. Whenever initial market entry costs are very high and thus form a natural barrier to entry, patenting becomes needless and thus the innovator chooses secrecy. Note that the parameters which positively influence the propensity to patent influence social welfare in a contrary way: On the one hand, while the protective effect increases
the advantageousness of a patent, it diminishes social welfare by mitigating competition between firms. On the other hand, the impact of the disclosure effect, which has a detrimental effect on patenting, enhances social welfare by imposing knowledge diffusion.\footnote{1} This leads us to the conclusion that policy attempts which yield at improving the patent system to enhance social welfare should by undertaken with great care, as they could possibly lead to an unintended decline of patent applications.

Since the thought-provoking impulse of Horstmann et al. (1985) many attempts have been made to analyze the patenting decision. Some of these approaches rely on the assumption that the disclosure requirement does not come to effect until a patent expires. Only then the enabling knowledge incorporated in the patent application can be used by competitors so that anyone skilled in the art is able to produce and market the formerly protected innovation.\footnote{2} Various approaches incorporate the disclosure requirement in a more adequate way. Empirically the extent of the disclosure requirement depends on factors such as policy decisions, the use of patent applications as a means to obtain technological knowledge input, and the industry specific usability of knowledge spillover.\footnote{3} Thus it is straightforward to assume that the impact of the disclosure requirement is exogenously given. Nevertheless, variations of this parameter may lead to changes in the interplay of the counter effects of patenting (protection versus disclosure) which in the end may result in an alteration of the propensity to patent. Thus implementing the possibility of a varying extent of the disclosure requirement may reveal interesting insights.\footnote{4} Introducing patent protection into a setting with horizontally differentiated products goes back to Klemperer (1990). The main focus of his paper is

\footnote{1}{These effects of patenting on social welfare are well known and straightforward. Yet an extensive welfare analysis of the innovator’s patenting behavior in this model setting is available from the author on request.}

\footnote{2}{Waterson (1990), Gallini (1992), Takalo (1998), Denicolò, Franzoni (2004a,b), and Bessen (2005) analyze the patenting decision but do not allow for an immediate impact of the disclosure requirement.}

\footnote{3}{Arundel et al. (1995) find that the importance of information disclosure as a reason not to patent varies throughout different industry sectors. This points to the fact that the impact of the disclosure requirement differs subject to the respective industry sector in which an innovative firm operates.}

\footnote{4}{In the work of Scotchmer, Green (1990), Erkal (2005) and Zaby (2009) the extent of the disclosure requirement remains fixed whereas in Harter (1994), Bhattacharya, Guriev (2006) and Aoki, Spiegel (2009) the impact of the required disclosure may vary. However, the later contributions do not explicitly focus on the consequences that a varying impact of the disclosure requirement has on the counter effects of patenting and in the end on the propensity to patent. Instead they consider the influence of alternative filing procedures on the propensity to patent (Aoki, Spiegel (2009)) or the choice of alternative licensing contracts (Bhattacharya, Guriev (2006)).}
to analyze a patent’s optimal design with regard to its length and breadth, whereas the patenting decision per se is not considered. This is accomplished by two subsequent papers: while Waterson (1990) focusses on a comparison of fencepost versus signpost patent systems with regard to social welfare, in a succeeding paper Harter (1994) examines the propensity to patent accounting for a disclosure effect. The major drawback of his modeling approach is that only one potential competitor profits from the merits of the mandatory disclosure. This fact, which largely delimitates the impact of the disclosure requirement, in the end leads Harter (1994) to conclude that there is no causal relation between the required disclosure and the propensity to patent. Economic intuition suggests the opposite: If the disclosure of information leads to decreasing market entry costs, this may enable an increasing number of firms to enter the market. A fact which the inventor will anticipate in his decision to patent. The following analysis confirms this intuition.\(^5\)

Our analysis proceeds as follows. In Section 2 we introduce the strategic protection decision between a patent and secrecy into a setting with horizontally differentiated products. The considered three stage game is solved backward, beginning with the analysis of the price competition on the last stage of the game in Section 2.1, proceeding with the market entry decisions on the second stage of the game in Section 2.2 and finally the innovator’s patenting decision on the first stage of the game in Section 2.3. Section 3 concludes. All Proofs can be found in the Appendix.

### 2 The Model

Assume that one firm has successfully accomplished a drastic product innovation and decides to release the new product immediately. As this innovative firm owns the proprietary knowledge concerning the innovation, it will be monopolist in the new market as long as no other firm successfully invents. The new product may be varied horizontally in its product characteristics which are assumed to be continuously distributed on a circle of unit-circumference. The innovator (and any other entering firm) can only offer one variant of the good. We denote the total number of firms that operate in this differentiated oligopoly as \(N = n + 1\), consisting of the innovator and \(n\) entering firms. Consumers are assumed to be uniformly distributed over the circle, with density normalized to one. The preference of a consumer is denoted by \(x \in [0, 1]\) and we assume without loss of generality that the innovator

\(^5\)Two related papers analyze the impact of licensing in a differentiated duopoly (Wang, Yang (2003), Poddar, Sinha (2004)). Neither paper considers the patenting decision per se, as both assume that the innovator has already patented his discovery.
of the new product is located at the beginning of the circle, \( x_p = 0 \). If a consumer cannot buy a good according to his preference he incurs a disutility that rises quadratically with the distance between his preferred good and the offered good. We will refer to this disutility as mismatch costs. Each consumer purchases one unit of the good as long as his net utility is weakly positive, \( U_x = v - p_z - (x - x_z)^2 \geq 0 \) where \( x_z \) represents the location of firm \( z \) on the circle. We assume throughout the paper that the reservation price \( v \) lies within the range \( 5/16 \leq v < 3/4 \) which assures that only in the case of monopoly, \( N = 1 \), some consumers prefer the outside option. For \( N > 1 \) all consumers buy one unit of the good choosing the variant which is closest to their respective preference.

The structure of the model is as follows: on the first stage of a three-stage game the innovator, already located in the new market, decides whether to patent his innovation or to keep it secret, \( \sigma^1 = \{\phi, s\} \). A patent protects a given range of product space on the unit circle against the entry of rival firms. The extent of protection is defined by the breadth of the patent, \( \beta \in [0, 1] \), which is exogenous.\(^6\) We assume that the protected product space is situated symmetrically around the location of the patentee’s product. Without loss of generality we set \( x_p = 0 \) so that this point on the circle defines the middle of the protected product space, see Figure 1. From there patent protection covers \( \beta/2 \) of the neighboring product space on either side of the innovation.

\[ \text{Figure 1: Patent breadth} \]

On the second stage potential rivals simultaneously decide whether to enter the new market, given the patenting decision of the innovator, \( \sigma^2 = \{\phi, s\} \). Patent breadth can also be interpreted as a strategic decision variable of the innovator, see Yiannaka, Fulton (2006).
Upon entry all firms face market entry costs. These can be understood as the costs necessary to achieve the capability to produce a variant of the new product. If the innovator decides to patent his discovery, according to patent law he is required to disclose sufficient information so that anyone skilled in the art is able to reproduce the patented product. Although his competitors are not allowed to copy the protected product, they have the possibility to invent around the patent as long as patent breadth does not deter entry completely, $\beta < 1$. Whenever a rival decides to enter the market despite of a patent, he profits from the disclosed information: achieving the capability to enter the new market is now easier and thus less costly. If we denote market entry costs in the case of secrecy by $f_s$, then in the case of a patent they decrease to $f_\phi$ with $f_\phi \equiv \lambda f_s$, $0 \leq \lambda \leq 1$, where $\lambda$ is a measure for the impact of the disclosure requirement which may differ subject to specific market conditions.\footnote{See Footnote 3 for the empirical motivation of this assumption.} Concerning the location of firms, we will use the well established principle of maximum differentiation meaning that firms will locate as far away from each other as possible to soften price competition.\footnote{Kats (1995) shows that this principle leads to a subgame perfect Nash equilibrium in a price then location game in a circular market.} Thus, if secrecy prevails firms will locate equidistantly on the unit circle. With a patent potential entrants cannot freely locate on the unit circle due to the range of protected product space. Still, they will try to move as close as possible to their profit maximizing, equidistant locations. Consequently, in the case of a patent, when the choice of location is restricted to the product space $1 - \beta$, the direct neighbors of the patentee will locate at the borders of the patent and all other entrants will locate equidistantly between them.\footnote{It is easy to check that with a patent even for $N^\phi = 3$ the incentive to soften price competition leads the entering firms to choose locations as far away from each other as possible so that they locate at the patent’s borders.} On the third stage all firms in the new market compete in prices, $\sigma^3_{\rho, N} = p$.

2.1 Price Competition

To find the subgame perfect Nash equilibrium, we solve the game by backward induction, setting off with the last stage. Here we have to distinguish the cases:

(i) the innovator has not patented, $\sigma^1_\rho = \{s\}$,

(ii) the innovator has patented $\sigma^1_\rho = \{\phi\}$
We will consider the cases subsequently, starting with case (i).

(i) the innovator has not patented $\sigma^1_\rho = \{s\}$

In the case that the innovator refrains from patenting and chooses secrecy to protect his innovation, our model simplifies to the well known Salop (1979) model of a circular city which we will briefly analyze in the following: All firms are symmetric so that it suffices to analyze the decision of one representative firm denoted by $k$. By assumption the outside option only plays a role in the case of monopoly, a market structure that will result if market entry costs are extremely high. We will turn to this case later. With moderate market entry costs, every consumer in the non-protected market buys one unit of the differentiated product from the firm that offers the variant which is closest to his preferences. The consumer indifferent between buying from firm $k$ or a neighboring firm, lets say $m$, thus can be found by equating the respective utilities he has by buying from either of them, $U(k) = v - p_k - (|\hat{x}^k|)^2 = v - p_m - (|1/N - \hat{x}^k|)^2 = U(m)$. Solving for $\hat{x}^k$ we get

$$\hat{x}^k = \frac{(p_m - p_k)}{2N^s} + \frac{1}{2N^s}$$ (1)

and can derive the demand for a representative firm operating in the market as $D_k = 2\hat{x}^k = (p_m - p_k)N^s + 1/N^s$. Standard computations then yield equilibrium prices,

$$p^* = 1/(N^s)^2,$$ (2)

and profits

$$\pi^*_n = 1/(N^s)^3 - f_s$$ (3)

for the $N^s$ entering firms. Note that the profit of the innovator amounts to

$$\pi^*_\rho = 1/(N^s)^3$$ (4)

as he does not face market entry costs.

In the case that only the innovator offers the innovative product due to extremely high market entry costs, we assume that consumers may not buy it if their preferences strongly differ from the characteristics of the offered product. The outside option they prefer may for example be an antecessor product of the innovation. Imagine the time immediately after the innovation has been placed in the market. Some consumers have a strong preference for it, others rather stick with less innovative products offered outside of the
market. As soon as the innovation is copied by other firms and offered in
differentiated versions, mismatch costs go down and consumers may decide
to buy the innovative product rather than an outside option. Technically
speaking, consumers will prefer to buy from the innovator in the case \( N^* = 1 \)
as long as \( v - p_\rho - (\hat{x}^{\rho, \text{out}})^2 \geq 0 \). Solving for \( \hat{x}^{\rho, \text{out}} \) we get
\[
\hat{x}^{\rho, \text{out}} \leq \sqrt{v - p_\rho},
\] (5)
where \( \hat{x}^{\rho, \text{out}} \) is the consumer indifferent between buying from the patentee or
buying the outside option.
This defines the innovator’s demand for the case \( N^* = 1 \) as \( D_\rho = 2\hat{x}^{\rho, \text{out}} \) so
that he maximizes his profits \( \pi^s_\rho = p_\rho 2\hat{x}^{\rho, \text{out}} \) by setting the price \( p_\rho = 2v/3 \).
His profits then amount to \( 10 \pi^s_\rho = 4v^3/3 \sqrt{3} \). (6)

(ii) the innovator has patented \( \sigma^1_\rho = \{ \phi \} \)

Now let us turn to case (ii) and look at the situation when the innovator
decides to protect the new product by a patent. As long as the breadth of
the patent is rather moderate, \( \beta/2 < 1/N^{\phi} \), the patent does not influence the
location of rival firms and the symmetric result derived above emerges. Note
though, that due to the assumption that the disclosure requirement lowers
market entry costs, \( f^{\phi} < f^{s} \), more firms than in the case without a patent
might enter the market. We will turn to this fact later. If the protectional
degree of the patent is high,
\[
\frac{\beta}{2} \geq \frac{1}{N^{\phi}},
\] (7)
equidistant location on the entire circumference of the circle is no longer
possible as the patent restricts the locations for entering firms to the product

\[ ^{10} \text{As the outside option should restrict the demand of the innovator to } D^s_\phi < 1 \text{ as long as } N^s = 1, \text{ the preference parameter } v \text{ has to meet the condition } 2\sqrt{v}/(3) < 1. \text{ Solving for } v \text{ we get } v < 3/4 \text{ as the lower bound of the preference parameter.}
\]
For \( N^s = 2 \) the additional firm \( i \) will locate at the opposite of the innovator at \( x_i = 1/2 \).
The indifferent consumer between \( i \) and \( \rho \) can be found by substituting \( N^s = 2 \) in equation
(1) as \( \hat{x}^{\rho, i} = 1/4 \). Prices and profits can be derived by inserting \( N^s = 2 \) in equations (2)
and (3). We get \( p_\rho = p_i = p^*_N|_{N^s=2} = 1/4 \) and \( \pi^s_\rho = 1/8, \pi^s_i = 1/8 - f_s \). As the
outside option should be of no interest for the indifferent consumer \( \hat{x}^{\rho, i} \), the condition
\( v - p^*_N|_{N^s=2} - (\hat{x}^{\rho, i})^2 \geq 0 \) has to be met. Inserting \( p^*_N|_{N^s=2} = 1/4 \) and \( \hat{x}^{\rho, i} \) as derived above,
the critical condition simplifies to \( v \geq 5/16 \) so that the domain of the preference parameter
that narrows the outside option’s relevance to the case \( N^s = 1 \) is \( 5/16 \leq v < 3/4 \).
space \(1 - \beta\). We will define patents in a setting where patent breadth, \(\beta\), fulfills condition (7) as *restrictive* patents. The following figure depicts firm’s locations with \(N^\phi = 4\) for the cases (a) that the patent is not restrictive \((\beta < 1/2)\), and (b) that the patent is restrictive \((\beta \geq 1/2)\).

![Figure 2: Firm’s locations with a patent, \(N^\phi = 4\)](image)

In the case that the innovator patents, firm’s neighborhoods are no longer uniform, but are dependent on the respective location of a firm. To distinguish firm’s locations we will refer to the left and right neighbor of the innovator as firms \(i\) and \(j\). Further we will denote the first right (left) neighbor of \(i\) (\(j\)) by \(i+1\) (\(j+1\)), the second by \(i+2\) (\(j+2\)) and so on. Consequently, with a restrictive patent an equilibrium can no longer be derived by analyzing a representative firm, as the respective neighborhood of a firm now plays a crucial role for its pricing decision. We have to distinguish three types of firms, differing by their respective neighborhood:

a) the patentee has a uniform neighborhood consisting of firms \(i\) and \(j\)

b) the „border“ firms \(i\) and \(j\) have an non-uniform neighborhood with the patentee on the one side and either each other or, if \(n > 2\), a non-patentee, non-border firm \(i + 1\) or \(j + 1\) on the other side

c) a non-patentee, non-border firm \(i + \kappa, \kappa \geq 1\) always has a non-uniform neighborhood \((i + \kappa - 1\) to the left, \(i + \kappa + 1\) to the right side) as long as it is not the firm with the greatest distance to the patentee. For this firm we need to distinguish two cases that depend on the number of non-patentee firms \(n\)
• if \( n \) is even, which we will denote by \( n^e \), then the firm furthest away from the patentee is firm \( i + (n^e/2 - 1) \) and its neighborhood is non-uniform: to the left firm \( i + (n^e/2 - 2) \), to the right firm \( j + (n^e/2 - 1) \)

• if \( n \) is uneven, \( n^u \), then the firm furthest away is firm \( i + (n^u - 1)/2 \) and its neighborhood is uniform: to the left firm \( i + (n^u - 3)/2 \), to the right firm \( j + (n^u - 3)/2 \).

As all non-patentee firms are ex-ante symmetric they will come to the same decision whenever facing the same neighborhood. Thus, if an even number of firms enters, every firm has a symmetric “partner” that faces the same neighborhood. In the following, we will refer to this as semi-circle symmetry. If an uneven number of firms enters the market then the firm located furthest away from the patentee has no symmetric “partner”, we will refer to this case as semi-circle asymmetry.

As we are analyzing the last stage of the game we take the number of firms that have entered the market as given. Due to the fact that the neighborhood of every firm is crucial for its individual demand and thus pricing decision, we will have to distinguish the indifferent consumer between every pair of firms, say \( y \) and \( z \). From the viewpoint of firm \( y \) the indifferent consumer will be denoted by \( \hat{x}_{y,z} \), from the viewpoint of its neighbor \( z \) it will be denoted by \( \hat{x}_{z,y} \). By standard computations the location of the indifferent consumer can be found by equating the respective utilities a consumer realizes by buying from either of its neighboring firms.

We will set off deriving the demand for the different types of firms, starting with the patentee. The indifferent consumer situated between the patentee, \( \rho \), and his left neighbor, \( i \), is situated at \( \hat{x}_{\rho,i} \) and can be found by equating the respective utilities the consumer realizes by buying from either of the firms

\[
 p^\phi_\rho + (\hat{x}_{\rho,i})^2 = p^\phi_i + \left( \frac{\beta}{2} - \hat{x}_{\rho,i} \right)^2
\]

\[
 \hat{x}_{\rho,i} = \frac{p^\phi_i - p^\phi_\rho}{\beta} + \frac{\beta}{4}. \tag{8}
\]

Necessarily the patentee’s left and right neighbor are semi-circle symmetric so that the indifferent consumers on both sides of the patentee are located at the same distance \( \hat{x}_{\rho,i} = \hat{x}_{\rho,j} \). Thus the patentee’s demand is given by

\[
 D^\phi_\rho = 2\hat{x}_{\rho,i}. \tag{9}
\]
If a firm has a non-uniform neighborhood the indifferent consumers on either side are not located equidistantly. This is the case for the patentee’s neighbors, $i$ and $j$. As they are semi-circle symmetric it suffices to derive the demand for one firm, say $i$.

Due to its non-uniform neighborhood firm $i$’s demand consists of two different parts: On the one hand all consumers between firm $i$ and the indifferent consumer to its left, $\hat{x}_{i,\rho}$, will buy from firm $i$. On the other hand, all consumers between firm $i$ and the indifferent consumer to its right, $\hat{x}_{i,i+1}$, will buy its product.

Thus the demand of the firm amounts to

$$D^\phi_i = \hat{x}_{i,\rho} + \hat{x}_{i,i+1} \quad \text{with} \quad \hat{x}_{i,i+1} = \frac{(p_{i+1}^\phi - p_i^\phi)(n-1)}{2(1-\beta)} + \frac{1-\beta}{2(n-1)}. \quad (10)$$

Note that whenever the difference between the firm’s prices is high, the consumer indifferent between buying from firm $i$ or firm $i+1$ is no longer situated in-between firm $i$ and firm $i+1$ but is located to the left of firm $i$, as then $\hat{x}_{i,i+1} < 0$. This at first sight surprising result is quite intuitive: due to the relatively low price firm $i+1$ offers, even consumers situated in the proximate neighborhood of firm $i$ prefer to buy the neighboring firm’s product as the higher mismatch costs they face by doing so are overcompensated by the lower price firm $i+1$ offers. We will refer to this shift in demand as consumer migration effect.\footnote{For a deeper analysis of this effect see Lemmata 1 and 2 in Section 2.2.}

Last we need to calculate the demand for the non-patentee, non-border firms, $i+\kappa$, $\kappa \geq 1$. As mentioned earlier we have to distinguish whether the number of non-patentee firms in the market is even or uneven. If it is even, $n^e$, then the neighborhood of firm $i+\kappa$, $\kappa \in [1, n^e/2-1]$ is non-uniform. The demand of firm $i+\kappa$ given $n^e$ thus amounts to

$$D(n^e)^\phi_{(i+\kappa)} = \hat{x}_{(i+\kappa),(i+\kappa)-1} + \hat{x}_{(i+\kappa),(i+\kappa)+1}. \quad (11)$$

Now let us turn to the case where the number of non-patentee firms is uneven, $n^u$. Then the range of firms $i+\kappa$ changes to $\kappa \in [1, (n^u-1)/2]$. For ease of exposition let us denote the firm with the furthest distance to the patentee by $i+k_{\text{max}}$ with $k_{\text{max}} = (n^u-1)/2$. As all firms $i+\kappa < i+k_{\text{max}}$ have non-uniform neighborhoods their demand is equal to $D(n^e)^\phi_{(i+\kappa)}$. Due to the assumption that firms locate equidistantly within the non-protected product space, the location of firm $i+k_{\text{max}}$ is exactly opposite to that of the patentee so that $x_{i+k_{\text{max}}} = 1/2$. Other than the neighboring firms, this firm faces a uniform neighborhood and thus for an uneven number of firms the demand...
for a non-patentee, non-border firm is given by

\[ D(n_u)_{i+\kappa}^\phi = \begin{cases} D(n_e)_{i+\kappa}^\phi & \forall \kappa < \frac{n-1}{2} \\ \frac{2}{\hat{x}_{i+\kappa_{\text{max}},(i+\kappa_{\text{max}})-1}} & \forall \kappa = \frac{n-1}{2} \end{cases} \]  

Having derived the respective demand functions for the different firm locations, we can now turn to the price reaction functions of the firms. Again we will look at the patentee first. His profits are \( \pi_i^\phi = p_i^\phi D_i^\phi - f_i^\phi \). Inserting the demand function from equation (9) and carrying out the optimization we get

\[ p_i^\phi(p_i) = \frac{\beta(n-1)}{2\Gamma} p_i^\phi + \frac{(1-\beta)}{\Gamma} p_i^\phi + \frac{\beta(1-\beta)}{4(n-1)} \]  

as the patentee’s price reaction function.

The semi-circle symmetric border-firms \( i \) and \( j \) face positive market entry costs so their profits amount to \( \pi_i^\phi = p_i^\phi D_i^\phi - f_i^\phi \). Their price reaction functions can be derived as

\[ p_i^\phi(p_i, p_{i+1}) = \frac{\beta(n-1)}{2\Gamma} p_i^\phi + \frac{(1-\beta)}{\Gamma} p_i^\phi + \frac{\beta(1-\beta)}{4(n-1)} \]  

with \( \Gamma \equiv 2 + \beta(n-3) \). Analogously the price reaction functions of the non-patentee, non-border firms \( i + \kappa \) with \( \kappa \geq 1 \) can be derived as

\[ p_{(i+\kappa)}^\phi(n_e) = \frac{p_{(i+\kappa)+1}^\phi + p_{(i+\kappa)-1}^\phi}{4} + \frac{1}{2} \left( \frac{1-\beta}{n-1} \right)^2 \]  

for an even number of non-patentee firms and

\[ p_{(i+\kappa)}^\phi(n_u) = \begin{cases} p_{(i+\kappa)}^\phi(n_e) & \forall \kappa < \frac{n-1}{2} \\ \frac{p_{(i+\kappa)+1}^\phi + 1}{2} \left( \frac{1-\beta}{n-1} \right)^2 & \forall \kappa = \frac{n-1}{2} \end{cases} \]  

for an uneven number of non-patentee firms.\(^{12}\) This completes the analysis of the last stage of the three stage game so that we can go one step backward and look at the simultaneous market entry decisions of the non-patentee firms.

\(^{12}\)Note that for the case that the breadth of the patent tends to zero, \( \beta \to 0 \), meaning that all firms are able to locate equidistantly, the reaction functions \( p_{(i+\kappa)}^\phi(n_u) \) and \( p_{(i+\kappa)}^\phi(n_e) \) simplify to \( p^\phi |_{\beta=0} = 1/N^2 \) which corresponds to the price choice in the case without a patent, see equation (2).
2.2 Market Entry

The analysis of the market entry decisions again needs to distinguish the cases (i) the innovator has not patented and (ii) the innovator has patented. Recall that even if the innovator patents, his competitors have the possibility to enter the market by inventing around the patent. As market entry costs are lowered due to the information disclosure patenting requires, it might be that more firms are able to enter with patent protection than with secrecy.

(i) the innovator has not patented $\sigma^1_\rho = \{s\}$

Whenever the innovator decides to keep his discovery secret the analysis of the market entry decisions of his rivals corresponds to the well known Salop result: the number of firms entering the market can be derived by solving the zero-profit condition $\pi_n^s = 0$ of a representative firm for $n$. Using (3) we get

$$(n^s)^0 = (1/f_s)^{1/3} - 1. \quad (17)$$

(ii) the innovator has patented $\sigma^1_\rho = \{\phi\}$

If we turn to case (ii) and assume that the innovator has patented his innovation on the first stage of the game, we can no longer pin down the market entry decisions in one zero-profit condition. Due to the asymmetric neighborhoods of firms the analysis of market entry becomes somewhat more complex. In the following we will derive the critical thresholds of market entry costs $f_\phi$ that yield market structures varying from $N^\phi = 1$ to $N^\phi \to \infty$. As the patentee always operates in the market himself the total number of firms consists of him and the number of entering firms. In the case that the innovator has patented we denote the entering rival firms by $n^\phi$ so that $N^\phi = n^\phi + 1$.

To ease notation we simply use the respective number of firms operating in the market as subscript, so the subscript 1 stands for the case $N^\phi = 1$ and so on.

If the patentee is the only firm in the market that offers the innovative product, $n^\phi = 0$, the patent has no protective effect. Consequently, his profits are the same as in the case of secrecy, $\pi^s_{n^1,1} = \pi^s_{n^1,1}$ see equation (6). The case $n^\phi = 0$ will occur whenever it is too costly for the patentee’s rivals to enter the market with a variant of the innovative product. Thus the innovator’s monopoly will prevail as long as market entry costs are higher than a critical threshold at which a potential entrant would realize zero profits.
Note that this condition does not sufficiently define the exact number of entering firms, as market entry costs could be low enough to allow more than one rival firm to enter the market. For a sufficient definition of the number of entering competitors a lower bound for market entry costs has to be defined, where it is just not profitable for an additional firm to enter. Necessarily the potential entrant(s) with the lowest profits is (are) decisive for the critical threshold defining the number of entering firms. Following economic intuition this must be the firm(s) located at the furthest distance to the patentee which is due to the following fact: The border firms $i$ and $j$ are able to set the highest prices of all non-patentee firms, as they face a relatively large mass of consumers situated between themselves and the patentee. This positive price effect of patent protection is passed on to every other neighbor, but it gets weaker the further away from the patentee a firm is located.

Whenever the number of entering firms, $n^\phi$, is even, all rivals have a semi-symmetric partner and thus the profits of the two firms located at the greatest distance to the patentee define the lower bound of market entry costs. Whenever the number of entering firms is uneven, the firm located furthest away from the patentee has no semi-symmetric partner and thus the lower bound of market entry costs is given by its profits. Given the lower threshold for market entry costs, the number of entering firms in general is sufficiently defined by

$$f_{\phi, N^\phi} \geq f_{\phi} > f_{\phi, N^\phi+1}.$$  

In the following we will describe in detail the derivation of the critical boundaries for $N^\phi = [2, 3, 4]$ as then the computation of all cases $N^\phi > 4$ should be obvious.

Suppose now that one additional firm, say $i$, enters the market, $n^\phi = 1$, so that $N^\phi = 2$ firms compete against each other. Recall from equation (7) that $\beta/2 \geq 1/N^\phi$ has to be fulfilled for a restrictive patent. For $N^\phi = 2$ this condition changes to $\beta \geq 1$. As we defined $\beta \in ]0, 1[$ this condition can never be fulfilled meaning that a patent is never restrictive. Thus – following the assumption of maximum differentiation – the entering firm locates at the opposite of the patentee, $x_i = 1/2$. As prices are equal in equilibrium, the consumer indifferent between buying from firm $i$ or from the patentee can be found by substituting $N^\phi = 2$ in equation (1) as $\bar{x}_{\rho,i} |_{N^\phi = 2} = 1/4$.

Prices and profits can then be derived as $p_{\rho,2} = p_{i,2} = 1/4$ and $\pi_{\rho,2}^\phi = 1/8$, $\pi_{i,2}^\phi = 1/8 - f_\phi$. The critical threshold where an entering firm realizes zero profits is thus given by $f_{\phi,2} = 1/8$ so that the necessary condition for a
market structure with $n^\phi = 1$ is
\[ f_{\phi} < f_{\phi, 2} \equiv 1/8. \] (18)

Thus the market structure with one entering rival is defined by market entry costs
\[ f_{\phi, 2} \geq f_{\phi} > f_{\phi, 3}. \]

For the case that two additional firms, $i$ and $j$, enter the market, $n^\phi = 2$, the condition for a restrictive patent changes to $\beta \geq 2/3$. If $2/3 \leq \beta < 1$, the patent restricts the product space where the two entering competitors can choose to locate to $1 - \beta$. Whenever $i$ and $j$ enter, they have a non-uniform neighborhood with the patentee to their left (right) and each other to their right (left). Thus in the price reaction function of a non-patentee firm derived in equation (14) we can set $i + 1 = j$. Due to semi-circle symmetry we know that $p^{\phi}_i = p^{\phi}_j$. Using $p^{\phi}_\rho$ from equation (13) we can derive the equilibrium prices
\[ p^{\phi}_{\rho, 3} = \frac{\beta(4 - 2\beta - \beta^2)}{8(3 - 2\beta)} \] (19)

and
\[ p^{\phi}_{i, 3} = \frac{\beta(1 - \beta)(1 - \beta/4)}{3 - 2\beta} \] (20)

so that profits amount to
\[ \pi^{\phi}_{\rho, 3} = \frac{\beta(4 - 2\beta - \beta^2)^2}{32(3 - 2\beta)^2} \] (21)
\[ \pi^{\phi}_{i, 3} = \frac{(4 - \beta)^2(2 - \beta)(1 - \beta)\beta}{32(3 - 2\beta)^2} - f_{\phi}. \] (22)

Consequently, the critical threshold for market entry costs in the case $N^\phi = 3$ is
\[ f_{\phi, 3} \equiv \frac{(4 - \beta)^2(2 - \beta)(1 - \beta)\beta}{32(3 - 2\beta)^2} \] (23)

and the case with two rivals entering the market is sufficiently defined by
\[ f_{\phi, 3} \geq f_{\phi} > f_{\phi, 4}. \]

To derive $f_{\phi, 4}$ we need to look at the case where three firms enter simultaneously, $n^\phi = 3$ and $1/2 \leq \beta < 1$. Recall that as the number of entering firms
is uneven, one firm does not have a semi-symmetric partner, for \( n^\phi = 3 \) this is firm \( i + 1 \). It’s price reaction function can be derived by inserting \( \kappa = 1 \) into equation (16). Note that the right neighbor of firm \( i + 1 \) is firm \( j \) so that we have \( p^\phi_{(i+\kappa)+1} = p^\phi_j \). The price reaction function then simplifies to

\[
p^\phi_{(i+1),4} = \frac{p^\phi_j}{2} + \frac{(1 - \beta)^2}{8}.
\]

As \( i \) and \( j \) are semi-circle symmetric, in equilibrium we must have \( p^\phi_{i,4} = p^\phi_{j,4} \). Simple computations then yield equilibrium prices and profits. Decisive for the critical threshold of market entry costs is the profit of firm \( i + 1 \) which is located at the furthest distance to the patentee. We have

\[
f^\phi_{i,4} \equiv (1 - \beta) \frac{1}{32}.
\]

In the same manner the critical thresholds for market entry costs can be derived for all market structures \( N^\phi \geq 4 \).\(^{13}\)

Last let us turn to the limiting case \( f_\phi \to 0 \), meaning that we have free entry, \( n^\phi \to \infty \). The price reaction function of the patentee will not change as it is independent of \( n^\phi \), see equation (13). The case is different for the non-patentee firms: in the limit case the border firm’s price reaction function as derived in equation (14) degenerates (using De L’Hôpital) to \( p^\phi_i \big|_{n^\phi \to \infty} = p^\phi_{i+1}/2 \). In the limit, price competition between firms will become so tough that they end up setting a price according to their cost, in our case \( p^\phi_{i+\kappa} = 0 \). This means that all non-patentee firms will set the same price and have zero-profits.\(^{14}\) We can derive the patentee’s optimal price choice in the limiting case by inserting \( p^\phi_i = 0 \) in equation (13). This yields the price \( p^\rho_i \big|_{n^\rho \to \infty} = \beta^3/8 \) with the corresponding profits

\[
\pi^\phi_i \big|_{n^\phi \to \infty} = \frac{1}{32} \beta^3.
\]

We will turn to the question whether a patent is profitable with extremely low market entry costs in the next section.

Equipped with these results we are now able to take a closer look at the consumer migration effect mentioned earlier.\(^{15}\) Due to the asymmetric equilibrium prices demand may shift from a border firm, say \( i \), to its neighbor

\(^{13}\)The respective outcomes for the cases \( N^\phi \in [1, 6] \) are summarized in Table 1 in the Appendix.

\(^{14}\)Klemperer (1990) comes to the same conclusion.

\(^{15}\)Naturally the argumentation concerning the consumer migration effect holds for both border firms, \( i \) and \( j \) and their respective neighbors \( i + 1 \) and \( j + 1 \). For the ease of exposition we refer to firm \( i \) in the following.
$i + 1$ as the consumer indifferent between buying from either firm is no longer located in-between the firms, but beyond the location of firm $i$, as depicted in the following figure.\textsuperscript{16}

We find that the factors leading to consumer migration correspond to the factors which strengthen price competition between the non-patentee firms. As more firms enter in the non-protected product space the distance in-between firms decreases and price competition becomes fiercer so that lower prices result. Increasing breadth of a patent affects prices in the same way: as the non-protected product space becomes narrower, firms move closer together and again the intensified price competition leads to decreasing prices. The following Lemma summarizes these results.

**Lemma 1** Consumer migration takes place whenever price asymmetry is sufficiently high. The effect is higher the more firms are operating in the market and the broader a patent is.

The consequence of the consumer migration effect is that even consumers situated in the proximate neighborhood of a border firm prefer to buy the neighboring non-border firm’s product so that the border firm’s demand necessarily decreases. As the following Lemma states, consumer migration will never reduce the border firm’s demand to zero.

**Lemma 2** A border firm’s demand is positive for every restrictive patent.

\textsuperscript{16}In the case $N = 5$ patent breadth needs to exceed $\beta_5^{\text{me}} = \frac{1}{9}(-5 + \sqrt{115})$ for consumer migration to occur. See the Proof of Lemma 1.
From the above Lemma we can deduce that consumer migration only influences the innovator’s patenting decision indirectly by driving the border firm’s pricing decisions. Since their demand will always be positive, consumers located in the proximate neighborhood of the innovator will never have the incentive to migrate to firm \( i + 1 \). Technically speaking we have \( \hat{x}_{p,i} < \beta/2 - |\hat{x}_{i,i+1}| \) so that consumers migrate only from the border firms to their non-patentee neighbors.

### 2.3 The Patenting Decision

On the first stage of the three-stage game the innovator decides whether to patent his innovation or to keep it secret, \( \sigma^1_\rho = \{\phi, s\} \). His patenting decision is driven by two opposing effects. On the one hand a patent protects part of the market, \( \beta \), from the entrance of rival firms (protective effect), on the other hand the disclosure requirement linked to a patent may lead to decreasing market entry costs for potential rivals, possibly making market entry profitable for a larger number of firms than with secrecy (disclosure effect). Recall from above that we define the reduction of market entry costs as \( f_\phi \equiv \lambda f_s \). In the following we distinguish two cases: either the disclosure requirement has an impact, \( 1 \geq \lambda > \lambda^N \), or it has no impact, \( \lambda^N \geq \lambda \geq 0 \).\(^{17}\)

Whenever the disclosure requirement has no impact the reduction of market entry costs is too small to change the number of entering firms so that patenting will either lead to \( N^\phi = N^s \), or will even reduce the number of firms in the market, \( N^\phi < N^s \). If the disclosure requirement has an impact it leads to a sufficient decrease of market entry costs to make market entry profitable for a larger number of rival firms, \( N^\phi > N^s \). Intuitively it should be that whenever patent protection is intense (\( \beta \) high), the protective effect dominates the disclosure effect and the innovator will patent. If patent breadth is rather low, the negative disclosure effect should dominate the protective effect so the innovator will refrain from patenting.

To analyze the patenting decision of the innovator it is thus crucial to know how many firms would enter the market with secrecy and distinguish how many firms would possibly additionally enter with a patent. Recall from above that the number of firms entering the market is sufficiently defined by market entry costs with \( f_N \geq f > f_{N+1} \).

The following figure illustrates the critical thresholds of market entry costs derived in Section 2.2 for alternative levels of patent breadth, \( \beta \), where the

\(^{17}\)The critical threshold \( \lambda^N \) is subject to the particular patent breadth \( \beta \) and the initial market entry costs \( f_N \) and can be derived as \( \lambda^N \equiv f_{N-1}/f_N \) where \( f_{N-1} \) is the next lower critical threshold of market entry costs.
solid lines depict the critical thresholds for the case that the innovator chooses secrecy and the dashed lines depict the critical thresholds for the case that the innovator patents.\footnote{Note that to maintain clarity we omitted $f_{s,N^*}$ for $N^* < 3$ and $N^* > 6$. The former would be located above $f_{s,3}$ and all later below $f_{s,6}$.}

Obviously $f_{\phi,N^\phi}$ and $f_{s,N^s}$ are equal up to the point where patent protection becomes restrictive, $\beta \geq 2/N^s$. All combinations of $f$ and $\beta$ that lie between two curves $f_N$ and $f_{N+1}$ lead to a situation where $N$ firms enter the market. Thus in the upper shaded area $N^\phi = 3$ firms would enter the market with a patent while with secrecy any number $N^s \geq 3$ could enter in this area. In the lower shaded area $N^\phi = 5$ firms would enter with a patent while $N^s \geq 5$ could enter with secrecy. Figure 4 shows that given market entry costs and patent breadth, a patent may lead to three different cases:

(a) due to a dominant protective effect less firms enter with a patent

(b) due to a dominant protective effect the number of firms stays unchanged,

(c) due to a dominant disclosure effect more firms enter with a patent.

Take for example the combination $\bar{f}$, $\bar{\beta}$ which leads to point $A$. With secrecy, market entry costs $\bar{f}$ allow $N^s = 4$ firms to enter the market, with a patent

$\begin{array}{c}
\text{Figure 4: Critical thresholds of market entry costs} \\
\end{array}$
only $N^\phi = 3$ firms could enter due to a strong protective effect (case (a)). If patent breadth is rather low, $\bar{\beta}$, the protective effect will only be moderate: Given the same height of market entry costs, $\bar{f}$, we are at point $B$ ($B'$) where still $N^s = 4$ but now $N^\phi = 4$ firms would enter if the innovator patented (case (b)). Now suppose that the disclosure requirement has an impact and leads to a sufficient reduction of market entry costs to change the number of entering firms. To differentiate between a high and a low impact of the disclosure requirement we assume that for our example value $\bar{\beta}$ the reduction of market entry costs with a patent is rather moderate so we come to point $C$, for the example value $\bar{\beta}$ we assume a high impact of the disclosure requirement, so that the reduction of market entry costs leads to point $D$. As $f_s = \bar{f}$ stays unchanged, with secrecy $N^s = 4$ firms would enter, but with a patent $N^\phi = 5$ firms would be able to locate in the market for both values $\bar{\beta}$ and $\bar{\beta}$ (case (c)).

To find out whether the innovator will choose to patent or to keep his innovation secret in the cases considered above, we need to compare the respective profits he can realize given the alternative combinations of market entry costs and patent breadth. In the following figure the profits of the innovator subject to $f$ and $\beta$ (see table 1) are plotted for the cases that he chooses a patent (dashed lines) or secrecy (solid lines).

Figure 5: Alternative profits of the innovator with a patent/secrecy
Let us start with the analysis of case (a) where \( N^s > N^\phi \). For our example combination \( \bar{f}, \bar{\beta} \) we need to compare the profits at points \( A^\phi \) and \( A^s \). Obviously the innovator is better off with a patent in this case, as then he realizes higher profits, \( \pi^\phi(\bar{\beta}) > \pi^s(\bar{\beta}) \). Things change in case (b) where \( N^s = N^\phi = 4 \). In the above figure we can see that the respective profits with a patent and secrecy, marked by the point \( B^s, B^\phi \) are equal as the patent is not restrictive, \( \bar{\beta} < 1/2 \). By assumption the innovator then prefers secrecy.\(^{19}\) If patent breadth increases to \( \beta' \) the patent becomes restrictive since \( \beta' > 1/2 \) and the innovator will choose to patent, see points \( B'^s \) and \( B'^\phi \). At last we turn to case (c) where the disclosure requirement has an impact so that, speaking in terms of our example, patenting leads us to the points \( BC \) or \( AD \), respectively. For the relatively low value of patent breadth, \( \bar{\beta} \), the innovator compares the profits marked by the points \( B^s \) and \( BC \) and will apparently choose secrecy as \( \pi^s(\bar{\beta}) \). Again, as patent breadth increases, patenting may become the more attractive strategy: with our example value \( \bar{\beta} \) the innovator faces \( A^s \) or \( AD \), clearly preferring to patent since \( \pi^\phi(\bar{\beta}) > \pi^s(\bar{\beta}) \).

In Figure 5 the profit function the innovator would realize in the case that three rival firms entered with a patent shows some exceptional characteristics. Compared to the profit functions for \( N^\phi > 3 \) it is the only curve that has an inner optimum for patent breadth so that for all \( \beta > \beta_\text{max} \) the patentee’s profits are downward sloping. For very high values of \( \beta \) secrecy even becomes the more attractive strategy.

This puzzling result contradicts economic intuition, as one would naturally assume that a patent is the better for its holder, the broader it’s protective level is. To discover the driving forces behind the patentee’s seemingly uncommon strategy choice in this case, let us take a closer look on how patent breadth influences his profits if \( N^\phi = 3 \). A change of \( \beta \) influences the patentee’s profits in two ways: his demand as well as his optimal price choice are altered. The following Lemma states in which way.

**Lemma 3** For \( N^\phi = 3 \) the patentee’s demand decreases as patent breadth rises, while his price rises as long as \( \beta \) does not exceed a critical threshold \( \beta_0 \).

The intuition behind the above Lemma is the following: As patent breadth increases, the border firms \( i \) and \( j \) are forced to move closer together. This intensifies price competition between them, resulting in lower prices since

\(^{19}\)If we would introduce patent costs into our model, the innovator would clearly refrain from patenting in the case that it lead to the same profits as with secrecy.
\[ \partial p^\phi_{i,3}/\partial \beta < 0. \] This in turn increases the demand of the border firms while lowering that of the patentee. Nevertheless the patentee is initially able to increase his price as the effect of the extending protected product space exceeds the negative effect of decreasing prices. Only for very high values of \( \beta \) the patentee has to match his rivals in reducing prices as else he would lose too many consumers. From this point on a further rise of patent breadth leads to decreasing profits, eventually turning secrecy into the more attractive strategy.

The following Proposition summarizes our results so far.

**Proposition 1** Whenever the disclosure requirement has no impact, \( \lambda \leq \lambda^N \), so that \( N^* \geq N^\phi \), the innovator’s protection decision depends solely on the protective effect of a patent. If

(i) \( \beta \leq 2/N^* \) the protective effect is low and the innovator always prefers secrecy

(ii) \( 2/N^* < \beta < f_{\phi,N^\phi} \) the protective effect is moderate and the innovator always prefers to patent for \( N^\phi > 3 \). For \( N^\phi = 3 \) the innovator will only patent if \( \beta < 0.915 \)

(iii) \( \beta > f_{\phi,N^\phi} \) the protective effect is high and the innovator always prefers to patent.

The above Proposition covers the situation where the disclosure requirement has no impact which leaves us to analyze the case where due to the required disclosure of the innovation more firms are able to enter the market with a patent, \( N^\phi > N^* \) (case (c)). From our example values \( \tilde{\beta} \) and \( \hat{\beta} \) we know that the impact of the disclosure requirement may lead to secrecy as well as a patent, depending on the extent of patent breadth. In Figure 5 we can see that the patent profit functions \( \pi^\phi_{\rho,N^\phi} \) for \( N^\phi > 4 \) cross at least one secrecy profit function \( \pi^\phi_{\rho,N^*} \) with \( N^\phi > N^* \). Let us call the intersection point \( \hat{\beta}_{N^*,N^\phi} \). As the patent profit functions are increasing in patent breadth, the innovator will prefer secrecy for relatively low values of patent breadth, \( \beta \leq \hat{\beta}_{N^*,N^\phi} \), and he will prefer to patent for relatively high values of patent breadth, \( \beta > \hat{\beta}_{N^*,N^\phi} \). Take for example the situation where with secrecy four firms would enter the market and with a patent six firms could enter due to the market entry costs reduction of the disclosure requirement. The relevant intersection point in this case is \( \hat{\beta}_{4,6} \). Whenever patent breadth is lower than \( \hat{\beta}_{4,6} \) the protective effect of the patent is too weak to outreach the negative
effect of the disclosure requirement and the innovator will prefer secrecy as this yields higher profits. If patent breadth exceeds the critical threshold, the protective effect overcompensates the disclosure effect and the innovator is better off with a patent. Generalizing these results we come to our next Proposition.

**Proposition 2** Whenever the disclosure requirement has an impact, \( \lambda > \lambda^N \), so that \( N^\phi > N^s \), the innovator will

(i) prefer secrecy for all \( N^s \leq 3 \)

(ii) prefer to patent for all \( N^s > 3 \) if and only if patent breadth exceeds a critical threshold \( \beta > \hat{\beta}_{N^s, N^\phi} \). Else the innovator will prefer secrecy.

Note that – keeping the number of firms entering with secrecy, \( N^s \), fixed – the critical threshold, \( \hat{\beta}_{N^s, N^\phi} \), increases as the impact of the disclosure requirement increases, i.e. more firms are able to enter with a patent. Thus we come to the following corollary.

**Corollary 1** Whenever the disclosure requirement has an impact, the propensity to patent decreases with the strength of the disclosure effect.

Last let us turn to the extreme case where the disclosure requirement has a very high impact, \( \lambda \to 1 \), so that market entry costs tend to zero and an infinite number of firms enters. From equation (26) we know that in the limit case for \( n^\phi \to \infty \) the profit of the patentee amounts to \( \pi^\phi \big|_{n^\phi \to \infty} = \beta^3/32 \). A patent will be profitable for the innovator whenever it yields higher profits than secrecy. The following Proposition states the result of this comparison.

**Proposition 3** Whenever a patent requires complete disclosure, \( \lambda \to 1 \), so that \( n^\phi \to \infty \), a patent is profitable for the innovator whenever patent breadth exceeds a critical threshold, \( \beta > 2\sqrt{4/N^s} \). For \( N^s \leq 3 \) the innovator always prefers secrecy.

Notably, even if the market will become extremely crowded with a patent the innovator will nonetheless patent whenever patent breadth is sufficiently high. Due to the strong protective effect with a high \( \beta \) the entering firms have to locate in a rather narrow area of non-protected product space which drives their prices and profits to zero. The distance \( \beta/2 \) between the patentee and each of his neighbors then allows him to set a higher price which – in the case that \( \beta \) is high enough – leads to higher profits than with secrecy where the distance to a neighbor is only given by \( 1/N^s \).
Summarizing we find that the innovator’s decision between a patent and secrecy in a differentiated oligopoly is mainly influenced by two factors: the potentially substantial change in the number of entering firms due to the disclosure requirement, and the breadth of the patent which is the determining factor for the strength of the protective effect. Whenever the disclosure requirement has a high impact, meaning that more firms will enter due to decreasing market entry costs, the innovator may prefer secrecy. If, however, the disclosure requirement plays a minor role, a patent may be profit enhancing for the innovator, as it forces his rivals to locate their products further away from him.

The following table summarizes our results for $N^s, N^s \in [1, 6]$ and $n^o \to \infty$.

<table>
<thead>
<tr>
<th>$N^s$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>$n^o \to \infty$</th>
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<tr>
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<td>$\beta \leq 0.33$</td>
<td>$\beta \leq 0.53$</td>
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</table>

Table 1: Summary of results

In the above table we find two regions where the protection decision is independent of patent breadth: in the lower left area where $N^s < N^o$ the

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\[\text{For } N^s = 2 \text{ we must have } f_{s,2} \geq f > f_{s,3} \text{ whereas for } N^o = 1 \text{ market entry costs have to fulfill } f_{o,1} \geq f > f_{o,2}. \text{ We know that } f_{o,1} = f_{s,1} \text{ and } f_{o,2} = f_{s,2} \text{ so the later condition changes to } f_{s,1} \geq f > f_{s,2} \text{ which cannot be fulfilled for any situation with } N^s > 1.\]
innovator chooses to patent and in the upper right area where $N_s \leq 3$ and $N_s < N_\phi$ the innovator chooses secrecy. In the area in-between, patent breadth is the decisive factor for the dominant effect. If $\beta$ is rather low, the protective effect is weak so that the negative effect of the required disclosure leads to secrecy. If patent breadth is rather high, the protective effect is strong enough to overcompensate the disclosure effect so that the innovator chooses to patent.

### 3 Concluding Remarks

Although there is strong empirical evidence that the disclosure requirement is a major reason for firms to refrain from patenting, the influence of a varying impact of the disclosure requirement on the propensity to patent has drawn sparse attention in the theoretical literature so far.

Our aim was to provide a framework in which the decision of an innovator between a patent and secrecy could be analyzed taking into account the possibility of inventing around by competitors as well as a varying impact of the disclosure requirement. To capture these effects we introduced the strategic protection decision of an innovator in a model of horizontally differentiated products. As here market entry costs are decisive for the number of firms which are able to enter, the disclosure requirement’s impact could be substantiated as a decrease of the initial market entry costs. Whenever the innovator patents, asymmetry is introduced in the circular market, so that it becomes crucial in which neighborhood a firm is situated. Due to the resulting asymmetric equilibrium prices we found that consumer’s may choose to buy from a non-neighboring firm whenever its price is sufficiently lower than the prices offered in the direct neighborhood of the consumer. This consumer migration effect is stronger, the more intense price competition between the non-patentee firms is. The patenting decision of the innovator is indirectly influenced by this effect as he anticipates the pricing decisions of his neighbor’s, the border firms, in setting his own price.

Our main results differ subject to the impact of the disclosure requirement: Either the influence of the disclosure requirement is such that the number of firms able to enter the market is left unchanged or it is such that the number of firms increases. Whenever the disclosure requirement has no impact, the patenting decision is solely driven by the protective effect – the broader a patent is, the higher is the innovator’s propensity to patent. Other than this, whenever the disclosure requirement has an impact, we find that the propensity to patent decreases with the strength of the disclosure effect.

A vast number of theoretical approaches concerning patents is dedicated to
the optimal design of the different dimensions constituting a patent, namely the interplay of *patent scope*, *patent length* and the *inventive step*.\textsuperscript{21} Naturally most of this literature assumes that a patent already exists and does not question an innovator’s decision on the method of appropriating his returns on research investments. The focus of the present analysis was to challenge the assumption that every innovation is patented by analyzing an innovator’s decision to patent. Our finding that the propensity to patent actually lies below unity asks for a more comprehensive approach to optimal patent design where the patenting behavior of a successful inventor is properly taken into account. Else, as the driving forces behind the propensity to patent influence social welfare contrarily, policy attempts could have the unintentional outcome of decreasing the propensity to patent.

\textsuperscript{21}An excellent survey is provided by Encaoua et al. (2006).
Appendix

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\pi_{\rho,N}^\phi$</th>
<th>$f_{s,N}^\phi$</th>
<th>$f_{\phi,N}^\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{4\sqrt{3}}{3}$</td>
<td>$f_s &gt; \frac{1}{8}$</td>
<td>$f_{\phi} &gt; \frac{1}{8}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\beta(4-2\beta-\beta^2)^2}{32(3-2\beta)^2}$</td>
<td>$\frac{1}{27}$</td>
<td>$\frac{\beta(4-\beta)^2(2-\beta)(1-\beta)}{32(3-2\beta)^2}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{n}{32}$</td>
<td>$\frac{1}{64}$</td>
<td>$\frac{(1-\beta)}{32}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{\beta(20+14\beta+11\beta^2)^2}{2\cdot6^{16}(3+2\beta)^2}$</td>
<td>$\frac{1}{125}$</td>
<td>$\frac{(1-\beta)(8+4\beta-3\beta^2)^2}{2\cdot6^{16}(3+2\beta)^2}$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{\beta(2+3\beta+3\beta^2)^2}{8(1+9\beta)^2}$</td>
<td>$\frac{1}{216}$</td>
<td>$\frac{(1-\beta)(3+2\beta-3\beta^2)^2}{16(1+9\beta)^2}$</td>
</tr>
</tbody>
</table>

Table 2: Critical thresholds of market entry costs for $N \in [1, 6]$

Proof of Lemma 1:

Consumer migration takes place whenever $\hat{x}_{i,i+1} < 0$. Solving $\hat{x}_{i,i+1} < 0$ for $\Delta p^\phi \equiv p_i^\phi - p_{i+1}^\phi$ we get $\Delta p^\phi > \left(\frac{1-\beta}{n-1}\right)^2$ as critical condition for consumer migration. Obviously the critical threshold on the right hand side decreases with the number of firms entering, $n$, and with the breadth of the patent, $\beta$. Inserting the equilibrium prices for any number of firms $N > 3$ and solving for $\beta$, the critical condition for asymmetric prices translates into a critical threshold of patent breadth. We find that consumers migrate whenever $\beta > \beta_{N}^{\text{me}}$.

Proof of Lemma 2:

First let us show that firm $i$’s demand is smallest for high $p_i^\phi$ and low $p_{i+1}^\phi$. We have $D_i^\phi = \hat{x}_{i,\rho} + \hat{x}_{i,i+1}$. From equations (8) and (10) we get $\hat{x}_{i,\rho}$ and $\hat{x}_{i,i+1}$. Now inserting the price reaction functions $p_i^\rho$ from equation (13) and $p_{i+1}^\phi$ from equation (15), assuming that due to its negligible impact on $i$’s pricing decision $p_{i+2}^\phi$ can be treated as a constant, we have $\partial D_i^\phi / \partial p_i^\phi < 0$. To analyze the influence of firm $i + 1$’s pricing decision we substitute $p_{i+1}^\rho$ and $p_{i+1}^\phi$ (see equation (14)) in $D_i^\phi$ and get $\partial D_i^\phi / \partial p_{i+1}^\phi > 0$. Thus we have that $D_i^\phi$ reaches its lowest values for high $p_i^\phi$ and low $p_{i+1}^\phi$. Next we need to show that in the case that $p_{i+1}^\phi$ reaches its minimum and $p_i^\phi$ reaches its
maximum firm $i$’s demand is still positive. This corresponds to the cases where $p_{i+1}^{\phi} = 0$ and $p_i^{\phi} = p_{i,4}^{\phi}$ as $p_{i,N}^{\phi}/\partial N < 0$. Inserting $n = 3$, $p_{i+1}^{\phi} = 0$, $p_{i,4}^{\phi} = (1-\beta)\beta/4$ and $p_{i,4}^{\phi} = \beta/8$ and solving $D_i^{\phi} > 0$ for $\beta$ we get the critical condition $\beta > 1/(3 + \sqrt{17}) \approx 0.27$. Since the patent needs to be restrictive to have an impact, for $n = 3$ it must be that $\beta > 1/2$ so that the critical condition for $\beta$ is always fulfilled. □

**Proof of Lemma 3:**

Differentiating the patentee’s profit function $\pi_{\rho,3}^{\phi} = p_{\rho,3}^{\phi} D_{\rho,3}^{\phi}$ we have

\[
\frac{\partial \pi_{\rho,3}^{\phi}}{\partial \beta} = p_{\rho,3}^{\phi} \frac{\partial D_{\rho,3}^{\phi}}{\partial \beta} + p_{\rho,3}^{\phi} D_{\rho,3}^{\phi}. \tag{27}
\]

Differentiating the patentee’s demand, $D_{\rho,3}^{\phi}$, see equation (9) with respect to $\beta$ using

\[
\frac{\partial p_{\rho,3}^{\phi}}{\partial \beta} = \frac{1}{2} \frac{\partial p_{i,3}^{\phi}}{\partial \beta} + \frac{\beta}{4} \tag{28}
\]

and inserting $p_{i,3}^{\phi}$ from equation (20), simplifying yields

\[
\frac{\partial D_{\rho}^{\phi}}{\partial \beta} = \frac{1 - 3\beta + \beta^2}{2(3 - 2\beta)}
\]

which is negative for any restrictive patent, as then $2/3 < \beta < 1$ holds. Thus the patentee’s demand decreases with patent breadth.

Now let us turn to the patentee’s price choice. Obviously the derivative of his optimal price, see equation (28), with respect to $\beta$ is positive whenever

\[
\frac{\partial p_{i,3}^{\phi}}{\partial \beta} > -\frac{\beta}{2}.
\]

It is easy to show that for $2/3 < \beta < 1$ the derivative $\partial p_{i,3}^{\phi}/\partial \beta$ (from equation (20)) fulfills this condition whenever $\beta < \beta^o$ with $\beta^o \equiv (-3 + \sqrt{105})/8$. Thus, as patent breadth rises the patentee will increase his price until the critical threshold $\beta^o$ is reached. After this point a further increase will lead to a price reduction as the negative influence of the decreasing prices of the
border firms, $\partial p_{i,3}^\phi / \partial \beta$, becomes dominant.\footnote{Note a dominant price effect only occurs in the case $N^\phi = 3$ as then prices under a restrictive patent are higher than in all cases $N^\phi > 3$. Consequently, as in the limit for $\beta \to 1$ all prices tend to zero, the downward slope of the price function $\partial p_{i, N^\phi}^\phi / \partial \beta$ is highest for $N^\phi = 3$ so that the patentee’s neighbors are affected relatively stronger than in the cases $N^\phi > 3$.}

**Proof of Proposition 1:**

(i) Due to model assumptions we have that for $\beta \leq 2/\bar{N}$ the profit functions in the respective cases patent or secrecy coincide, $\pi^\phi_{\rho, N} = \pi^s_{\rho, \bar{N}}$, in which case the innovator prefers secrecy.

(ii) Further it can be shown for all $\pi^\phi_{\rho, N^\phi}$ with $N^\phi > 3$ that $\partial \pi^\phi_{\rho, N^\phi} / \partial \beta > 0$. Since $\pi^s_{\rho, N^\phi}$ is independent of $\beta$, it must then be that $\pi^\phi_{\rho, N} > \pi^s_{\rho, \bar{N}} \forall \beta > 2/\bar{N}$. For $N^\phi = 3$ solving $\pi^\phi_{\rho, 3} - \pi^s_{\rho, 3} > 0$ for $\beta$ yields $\beta > 0.915$.

(iii) As obviously $\pi^s_{\rho, \bar{N}} > \pi^s_{\rho, N^\phi + 1}$, then $\pi^\phi_{\rho, N} > \pi^s_{\rho, N^\phi + 1}$ is always fulfilled and the innovator prefers to patent.

**Proof of Proposition 2:**

(i) In the cases $N^s < 3$ the profits of the innovator are the same with secrecy and with a patent, see equations (4) and (6). It is easy to show that $\pi^\phi_{\rho, 1} > \pi^\phi_{\rho, 2}$ holds within the domain of $v$ (see Footnote 10). Thus if $\pi^\phi_{\rho, 2} > \pi^\phi_{\rho, N^\phi}$ for $N^\phi > 2$ holds, it is never profitable to patent for $N^s < 3$. Obviously $\pi^\phi_{\rho, 2} > \pi^\phi_{\rho, 3}$ for all $\beta$. This leaves us to show that $\pi^\phi_{\rho, 3} > \pi^\phi_{\rho, N^\phi}$ with $N^\phi > 3$. Using $p^\phi$ and $D^\phi$ from equations (6) and (9) it is easy to show that $\partial \pi^\phi_{\rho, N^\phi} / \partial p^\phi_{i} < 0$ generally holds. Knowing that $\partial p^\phi_{i} / \partial N^\phi < 0$ we can conclude that $\partial \pi^\phi_{\rho} / \partial N^\phi < 0$. Consequently $\pi^\phi_{\rho, 3} > \pi^\phi_{\rho, N^\phi}$ must hold so that a patent is never profitable for $N^s < 3$.

For $N^s = 3$ we need to show that $\pi^s_{\rho, 3} > \pi^\phi_{\rho, N^\phi}$ for $N^\phi > 3$. In the limit $\beta \rightarrow \infty$ all patent profits tend to 1/32. We know that $\partial \pi^\phi_{\rho, N^\phi} / \partial \beta > 0$ with $N^\phi > 3$ so that 1/32 is the maximum value patent profits can reach. Since $\pi^s_{\rho, 3} > 1/32$ the innovator will always prefer secrecy.

(ii) In the limit $\beta \rightarrow 0$ all patent profits tend to zero and for $\beta \rightarrow \infty$ all patent profits tend to 1/32. As $\partial \pi^\phi_{\rho, N^\phi} / \partial \beta > 0 \forall N^\phi > 3$ and
\[ \pi_{\rho, N^*} < 1/32 \] for \( N^* > 4 \) all patent and secrecy profit functions must have exactly one intersection point whenever \( N^\phi < N^* \) and \( N^\phi, s > 3 \).

We get \( \hat{\beta}_{N^*, N^s} \) by solving \( \pi^*_s = \pi^*_p \) for \( \beta \).

**Proof of Proposition 3:**

Solving \( \pi^\phi_{\rho, n^\phi} \mid_{n^\phi \to \infty} > \pi^*_s \) with \( \pi^*_s = 1/(N^*)^3 \), see equation (4) for \( \beta \) yields the critical threshold \( \beta > 2^{3/4}/N^* \). Note that the right hand side is greater than unity whenever \( N^* \leq 3 \). Since \( \beta \in ]0, 1[ \), the inequality can never be fulfilled for \( N^* \leq 3 \) and thus the innovator chooses secrecy. \( \square \)
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