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**The Proximity-Concentration  
Trade-Off in a Dynamic Framework**

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# The Proximity-Concentration Trade-Off in a Dynamic Framework

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- Comments are welcome -

## Abstract

This paper presents a dynamic framework which implements risk as a continuous variable into the proximity-concentration trade-off concept. Additionally firms have the possibility to postpone their investment decision which gives them the possibility to collect further information about the volatile variable over time. On the basis of the real option theory (Dixit and Pindyck, 1994) an investment plan under uncertainty is derived. In contrast to static models firms postpone their investment decision although positive returns can be achieved. For specific risk values the model predicts, in the presence of a foreign direct investment choice, the export strategy can be rejected although it is dominating the FDI project and although it is worthier than its option value. The results of the model undermine empirical findings which analyze the impact of continuous variables on export and FDI patterns.

*Keywords:* Export, FDI, Uncertainty, Real Option Approach

*JEL classification:* D81, D92, F17, F21, F23, F31

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## 1. Introduction

The international economic integration of the world has been increasingly influenced by international trade and foreign direct investments in the post world war two era. According to the UNCTAD data (2006) since then domestic companies have steadily increased their exports and foreign plant shares (horizontal FDI) to access new markets for their products. Besides the persistent growth of exports and horizontal FDI, two additional striking developments can be identified in empirical data. Since the early 1980s the growth expansion of FDI inflows exceeded that of exports in every year until today (Navaretti and Venables 2004). The major share of FDI inflows originated in developed countries and were attracted by the same (Markusen, 2002). However this last development has changed its nature since 2003, as global FDI inflows have maintained their growth only because developing countries have started to attract relatively more FDI inflows whereas developed countries experienced a reduction in their inflow growth rates (UNCTAD-Statistics, 2006).

Given the increasing importance of exports and FDI, economic analyses focusing on these two elements of international economics have gained impetus. The first influential strand of explanation was the *Ownership, Location and Internalization Advantage* framework which was developed by John Dunning (1977, 1981). With the surge of FDI in the 1980s economists started to implement the OLI framework into formalized analytical models emphasizing different aspects of the three possible advantages. Among them were Horstman and Markusen (1987), Markusen and Venables (1998,2000), Brainard (1993), Helpman (1984, 1985), Ethier and Markusen (1996), Ethier (1986). These models are either static general equilibrium or static partial equilibrium models. Common to the first four mentioned models is the assumption of different cost structures between export oriented companies and multinational enterprises (MNE) which have been considered as the driving force behind FDI. Brainard (1993) e.g. considers a two country, two

sector model in which exporters are confronted with higher variable costs than foreign direct investors due to transport cost. However the domestic production expansion for exports is associated with scale economies. Whether a company should serve a foreign market as an exporter or via a FDI solution therefore depends on the trade-off between scale advantages in the domestic country and the proximity advantages in the foreign country. The author names this hypothesis the *proximity-concentration trade-off* (henceforth PCT). In a cross section analysis between the USA and 26 countries Brainard (1997) proves the empirical significance of his hypothesis and concludes:

*The proximity-concentration hypothesis predicts that firms should expand horizontally across borders whenever the advantage of access to the destination market outweighs the advantages from production scale economies.* (Brainard, 1997)

The next influential strand of analytical models which explain export and FDI behavior, appeared under the umbrella of the so called *New New Trade Theory*, referring to monopolistic competition models which include uncertainty over the productivity of firms that intend to enter new markets.<sup>1</sup> Based on the milestone work of Marc J. Melitz (2003), Helpman, Melitz and Yeaple (2004) develop a model in which firms chose between an export and FDI solution to serve a foreign market, in the presence of the proximity-concentration trade-off. However in contrast to earlier models, firms don't know their productivity performance until they execute the respective investment (domestic, export and FDI). Once the companies are involved in one of the three possible investment strategies, they finally experience their productivity. Based on the described ex ante uncertainty over productivity the model predicts that the most productive firms will become foreign direct investors, less productive one will export and the lesser productive one will stay domestic sellers. The least productive companies will disappear from the markets. The authors are analysing U.S. exports and affiliate sales data

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<sup>1</sup> A concise literature overview of the latest developments in the new new trade theory is presented by Helpman (2006).

covering 38 countries and 52 manufacturing sectors and are able to prove the significance of their model.

As risk and sunk costs are crucial elements in investment decisions of investors, their implementation into the latest models is a major step forward. However taking further empirical literature into account which deals with export and FDI decision associated with risk, it turns out, that the type of the incorporated risk is crucial for reasonable inferences. In the former model Helpman, Melitz and Yeaple consider risk as a one time shock component. Once the companies enter the markets, uncertainty disappears. In contrast neat investment models generally take risk as a time dependent variable (continuous phenomenon) into account, such as volatile prices in new markets or exchange rate volatility.

Bernard and Jensen (2004) examine export developments for U.S. companies between 1987 and 1992, a period with a high depreciation of the dollar. The variables which are taken into account are the volatility of exchange rates, foreign income growth and productivity growth. Their major finding is that primarily the change in exchange rates and foreign income growth are the dominant source for the export boom in the considered period, whereas productivity aspects play a minor role. In consideration of these findings trade models seems to overestimate the importance of productivity as the major determinant of exports and therefore additional variables should be taken into account as complementary aspects. Égert and Morales-Zumaquero (2007) analyze the impact of exchange rate volatility on export developments in less developed countries and conclude, that an increase in exchange rate volatility appears to depress exports. Similar findings are presented by Esquivel and Larrain (2002) concerning the impact of exchange rate volatility on FDI and exports. The authors examine the currency volatility of the three major economic powers (USA, Japan and Germany) and relate them to exports and FDI flows into developing countries. They are able to show a negative correlation between forex volatility and FDI. Besides the unanimous negative effect of exchange rate volatility – a continuous variable – on

exports and FDI, Bernard and Jensen as well as Égert and Morales-Zumaquero emphasize that the negative impact of forex volatility is transmitted with some delay. Bernard and Jensen relate this delay associated with increased volatility to sunk costs of entry.

As empirical research is pointing out the importance of additional continuous variables for the analysis of export and FDI patterns besides productivity, the development of an appropriate model might contribute to a better understanding of the international economic developments. Besides the implementation of continuous volatile variables and sunk costs, the adequate model should also contain the possibility of delaying export and FDI decisions, since the above mentioned empirical results provide such a pattern. McDonald and Siegel (1986) provide a financial model which combines sunk costs, volatile variables and timing to determine the optimal investment decision of an investors. Their framework became known as the *real option approach* which has been extend among others by Dixit and Pindyck (1994). Based on this dynamic framework the underlying paper develops a partial equilibrium trade model with a stochastic process and derives the proximity-concentration trade-off. In contrast to the former models investors are not only confronted with the choice between exporting and FDI but have also the possibility to postpone the investment. Based on the contingent claims approach it is possible to derive the fair value of an investment associated with the risk, equivalent to the stochastic process behind the exchange rate volatility of the real investments (export and FDI). In financial economics this fair value of a risky return is identified as the *option value* of an investment. The optimal investment at any time is derived by comparing the values of the three different investment possibilities (export, FDI, postponement) with respect to the comprised risk. Furthermore in equilibrium equal to Helpman, Melitz and Yeaple several cutoff values are derived for the state variable (exchange rate value) which describe the trigger points for the three different investment opportunities.

## 2. Theoretical Framework

There is one risk neutral investor who intends to serve a new foreign market with her output  $y$ . The foreign country can either be served by exports or by a new foreign plant (horizontal FDI). The production function for both investment choices is given by the concave Cobb-Douglas function (1) with labor  $l$  as the only input factor. There is no labor supply constraint and  $y$  provides the output for each period  $t$  with an infinite investment horizon  $T$ .

$$y(l) = l^\theta \quad \text{with } 0 < \theta < 1. \quad (1)$$

In contrast to Bernard (1993) the investor is confronted with decreasing returns to scale in both investment choices since  $0 < \theta < 1$ . Output prices  $p$  are given exogenously on the foreign market (price taker) and are certain. There is also no uncertainty about the demand on the foreign market. The optimal output in each term  $t$  can be sold completely in the foreign market. Labor costs  $w$  are assumed to be equal and constant in both investment scenarios. In the export scenario iceberg transport costs occur.<sup>2</sup> The produced output in the domestic country  $y^D$  shrinks down by the constant factor  $(1 - \tau)$  if it is transferred to the foreign market and therefore the sold amount  $y^E$  on the foreign market is given by

$$y^E = \tau y^D \quad \text{with } 0 < \tau < 1. \quad (2)$$

For the export investment the corresponding profit flows (cash flows) in each term are derived from the maximization problem

$$\pi_t(p_t, w_t, \tau_t) = \max_{l_t} p_t \tau_t l_t^\theta - w_t l_t \quad \text{s.t.} \quad y_t^E = \tau_t y_t^D \quad \text{s.t.} \quad y_t^D = l_t^\theta. \quad (3)$$

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<sup>2</sup> The transport cost technology is given by  $c(\tau) = \tau y^D$ .

As a result the labor input demand function in period  $t$  is given by

$$l_t = \left( \frac{\theta p_t \tau_t}{w_t} \right)^{\frac{1}{1-\theta}} \quad (4)$$

and the instantaneous supply function by

$$y_t^D(l_t) = \left( \frac{\theta p_t \tau}{w_t} \right)^{\frac{\theta}{1-\theta}}. \quad (5)$$

Clearly if transport costs increase,  $\tau$  decreases and as a result the optimal supply of the good decreases. Finally the perpetual cash flows in the export scenario in each period  $t$  turn out to be

$$\pi_t(p_t, w_t, \tau_t) = (1 - \theta) \left( \frac{\theta \tau_t}{w_t} \right)^{\frac{\theta}{1-\theta}} p_t^{\frac{1}{1-\theta}}. \quad (6)$$

It is possible to rewrite the cash flows in equation (6) with respect to total variable costs  $c^E$  and  $c^F$ . Since in the FDI scenario no transport cost accrue ( $\tau = 1$ ), total variable costs are equal to labor cost ( $c^F = w$ ) whereas in the export scenario total variable costs are given by  $c^E = \frac{w}{\tau}$  and equation (6) can be restated as

$$\pi_t^i(p_t, c_t^i) = (1 - \theta) \left( \frac{\theta}{c_t^i} \right)^{\frac{\theta}{1-\theta}} p_t^{\frac{1}{1-\theta}} \quad \text{with } i \in \{E, F\} \quad (7)$$

with the superscript  $F$  referring to the FDI solution and  $E$  to the export solution. Equation (6) demonstrates clearly if transport costs accrue then the cash flows in each period are declining since  $\tau$  is decreasing. As a result given the equal labor costs in both countries the cash flows from the export solution will be smaller than from the FDI solution. Equation (7) provides an alternative interpretation. As transport cost accrue total variable cost  $c^E$  increase and therefore the cash flows decrease whereas the variable cost in the FDI solution don't change. One can conclude if labor costs are equal in both countries and only in the export



scenario transport cost are accruing then

$$\pi_t^E(p_t, c_t^E) < \pi_t^F(p_t, c_t^F). \quad (8)$$

The first part of the right hand side in equation (6) consists as assumed only of constant values and therefore it can be summarized to

$$\pi(p_t) = Z_t p_t^\kappa \quad (9)$$

with

$$Z_t = (1 - \theta) \left( \frac{\theta \tau_t}{w_t} \right)^{\left( \frac{\theta}{1-\theta} \right)} \quad \text{and} \quad \kappa = \left( \frac{1}{1 - \theta} \right).$$

The cash flows in equation (9) are convex in goods prices which is a standard result if the production function has a concave curvature.<sup>3</sup> The economic intuition behind this profit structure is, to possess the ability of an instantaneous input adjustment if goods prices increase or decrease. Therefore equation (9) is also known as instantaneous profit function.

### **The Proximity-Concentration Trade-Off Under Certainty**

Although the underlying economic frame does not assume increasing returns to scale in the domestic plant opposed to the *New New Trade Theory*, a proximity-concentration trade-off can still appear if particular cost structures are prevailing. As the FDI solution is associated with a greenfield investment in the foreign country it is reasonable to assume higher fixed costs  $I^F$  for the foreign plant than fixed costs  $I^E$  for the domestic plant expansion (exports). The total cost

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<sup>3</sup> Varian (1992) provides a concise proof for this result.

structure is given by <sup>4</sup>

$$I^E < I^F \quad (10)$$

$$c^E > c^F. \quad (11)$$

Given the costs of the two investment choices and the perpetual cash flows, it is possible to calculate the value  $v(p)$  of each investment if the opportunity cost is known. In the underlying model  $\delta_c$  is assumed to be the exogenous discount rate without a deeper specification so far. Furthermore the two investments' values are expressed in domestic currencies since profits are repatriated. The exchange rate  $e$  is assumed to be fixed and as a consequence there is no uncertainty over prices, with  $p$  as the good price measured in the domestic currency and  $p^f$  as the good price in foreign currency

$$p = ep^f. \quad (12)$$

The value functions of the export and foreign direct investment choices are given by

$$v^E(p) = \frac{Z^E(p)^\kappa}{\delta_c} - I^E \quad (13)$$

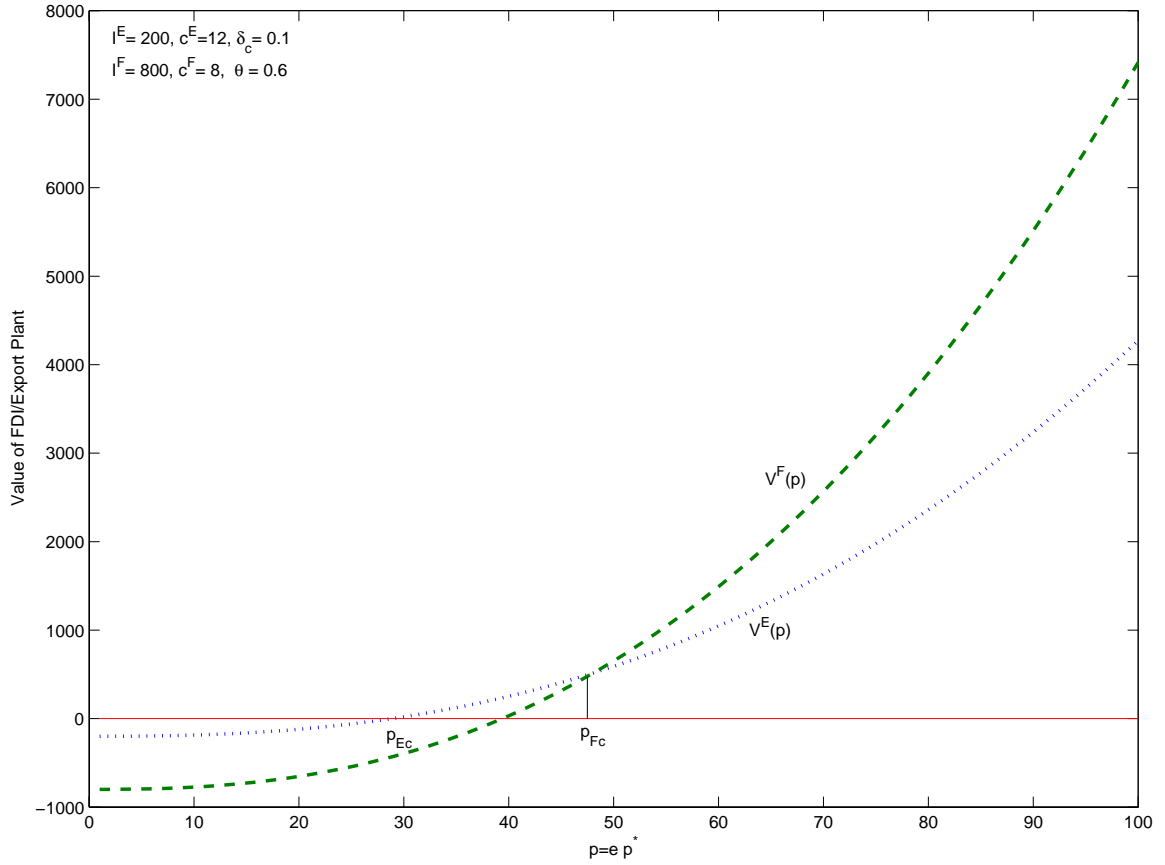
and

$$v^F(p) = \frac{Z^F(p)^\kappa}{\delta_c} - I^F. \quad (14)$$

Figure (1) depicts these two value function for specific parameter values with respect to the good price. For prices below the cutoff price  $p_{Ec}$  none of the two investment strategies is worth to be started since the cash flows are not covering the fixed cost and the project values are both negative. For prices between the two cutoff points  $p_{Ec}$  and  $p_{Fc}$  clearly the export solution is dominating the FDI solution. Due to the lower fixed costs  $I^E$  the average costs are lower than in the FDI case and therefore the investor should serve the foreign market by exports.

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<sup>4</sup> Variable costs in the export scenario are higher due to transport costs, as shown above.



**Figure 1:** Proximity-Concentration Trade-Off (Certain Case)

If the good's price exceeds  $p_{Fc}$  the FDI solution dominates the export solution since the lower variable costs show there advantage. In such a case the investor must serve the market through a foreign plant.

Given the decreasing average costs in the available investment strategies the proximity-concentration trade-off can be reformulated as:

***proposition 1:***

*Firms should expand horizontally across borders whenever the advantage of lower variable costs due to the lack of transport costs outweighs the advantage of lower fixed costs of a domestic production expansion.*

### 3. Investment Choice Under Uncertainty

So far there was no uncertainty concerning the repatriated profits of foreign sales since the exchange rate was assumed to be constant over time. Obviously the assumption of constant exchange rates is not valid for many bilateral relations and given the empirical results of Bernard and Jensen (2004) as well as those of Esquivel and Larrain (2002) a theoretical analysis of export and FDI decisions of investors under uncertain exchange rates can contribute to these findings.

The effects of volatile exchange rates on risk neutral investors can be twofold. Volatility can generate an appreciation of the exchange rate and therefore investors can expect higher foreign cash flows. Henceforth such an expectation is called the *risk driven appreciation*. On the other hand volatility can lead to a depreciation of the exchange rate which reduces foreign cash flows if they are repatriated. Henceforth such an expectation is called *risk driven depreciation*. One crucial question in the presence of these two expectation is, which effect will dominate and influence the final investment decision. Before the theoretical framework of section two is extended by risk, I first present a simpler case to establish the tools for the final analysis in the next section.

In contrast to the previous section the cash flows are assumed to depend only on  $p$  instead of  $p^k$  as in equation (13) and (14).

It is assumed that the exchange rate  $e$  follows a geometric Brownian motion

$$de = \alpha e dt + \sigma e dz \quad \text{with} \quad dz = \epsilon \sqrt{dt}. \quad (15)$$

Risk appears therefore because of the volatility of exchange rates. Since the repatriated profits are calculated on the basis of equation (12) and foreign prices are assumed to be certain, the uncertainty behind the prices measured in domestic currency will be the same as in equation (15). Therefore henceforth the analysis

will use the prices in domestic currencies  $p$  with

$$dp = \alpha p dt + \sigma p dz \quad \text{with} \quad dz = \epsilon \sqrt{dt} \quad (16)$$

as the uncertain variable in the model. In equation (16)  $\alpha$  is the expected growth rate of the price (e.g. due to macroeconomic developments) and  $\sigma$  is the variance parameter.  $dz$  represents a Wiener process and is responsible for the uncertainty in the product prices  $p$ .  $\epsilon$  is a randomly distributed variable with the mean of zero and a standard deviation of one (standard normal distribution). Therefore  $E(dz) = 0$  and  $E[(dz)^2] = dt$ .<sup>5</sup>

Given the uncertain price development in  $p$ , an investor who aims to receive profits for her project, is no longer confronted with a simple investment choice between exports and FDI, based on a traditional net present value (NPV) comparison. Additionally the investor can postpone the investment decision by a certain period to gather additional information about the behavior of the uncertain variable. Clearly gathering information by waiting is associated with return losses since the investment is not taking place. Simultaneously the waiting strategy offers the possibility to observe the behavior of the volatile variable and therefore the respective profit maximization can deliver a higher optimum. McDonald and Siegel (1986) name this additional value which can be achieved by waiting the *option value of an investment*. They derive an investment rule which includes the option value of a project and it turns out that the fair value of an investment must be not only higher than its investment cost (Marshallian rule) but much higher. One major challenge within the described frame is the determination of the option value  $F(p)$  of an investment. The objective of the remaining part is the formal derivation of the investment rule including the option value for the underlying export and FDI projects under uncertainty.

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<sup>5</sup>  $E$  refers to the expected value.

### The Fair Value Of A Risky Asset

One possibility to calculate the fair value of an investment including the option value  $F$  is offered by the *contingent claims valuation*. This approach assumes that the final good of a project is traded on capital markets and  $F$  can be replicated by using the uncertain price of that final good. Of course not every good which is sold in foreign countries is traded on capital markets and therefore the replication method would be only applicable to a restricted set of investments. However even if the final good of a real investment is not available on capital markets, the replication method can be applied to evaluate the fair value of the real project based on other assets or a portfolio of assets which comprise the same risk pattern as the real investment. Dixit and Pindyck (1994) refer to this approach as *asset spanning*. Both methods are common approaches in economics to derive the value of an option and appendix A and B present the algebraic solutions for these methods.

In the underlying problem the value of the two projects (export and FDI) are risky because their value  $v$  depends on a stochastic variable  $p$ . Therefore the diffusion process behind the value  $v$  could be derived from the volatile prices  $p$  by using the mentioned methods. As a result the option value  $F(v(p))$  of the two projects could be determined. However this nested approach turns out to deliver very complicated results. Therefore a third alternative is used here which results in the same investment rules as the replication and asset spanning method.

A riskless portfolio  $\Theta$  is constructed by

1. holding one unit of the option  $F(p)$
2. go short  $n$  units of an asset, which contains the same risk return pattern as equation (16)  $\rightarrow$  asset spanning:  $n = F'(p)$ <sup>6</sup>
3. the short position will require a payment of  $\delta F'(p)p$  for each period  $dt$ .

<sup>6</sup> Appendix B provides an analytical prove, that  $n = F'(p)$  is the optimal short position.

A crucial assumption about the asset which is used to span the risk of the real investment is, that it pays no dividend. In other words its expected return is given by  $\mu$  and results only from its capital gain.

Since this constructed portfolio  $\Theta$  is riskless, its return must be equal to a riskless return  $r[F(p) - F'(p)]dt$ , with  $r$  as the relative return of a riskless treasury bond. This can be formulated as

$$dF(p) - F'(p)dp - \delta F'(p)pdt = r [F(p) - F'(p)] dt. \quad (17)$$

$dF(p)$  can be substituted by using Ito's lemma

$$dF = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial p} dp + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} \sigma^2 p^2 dt. \quad (18)$$

The result for the option value  $F(p)$  is a second order differential equation which is linear in its dependent variable and its derivatives

$$\frac{1}{2} \sigma^2 p^2 F''(p) + (r - \delta)pF'(p) - rF(p) = 0.^7 \quad (19)$$

Therefore this homogeneous equation has a guess solution consisting of any two linearly independent solutions

$$F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2} \quad (20)$$

with

$$\beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}} > 1 \quad (21)$$

$$\beta_2 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left[ \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right]^2 + \frac{2r}{\sigma^2}} < 0. \quad (22)$$

Based on equation (20) it is possible to formulate investment rules for an investor. By taking these rules into account it is possible to determine the constants  $A_1$ ,

<sup>7</sup> The effect of  $F_t(p)$  is neglected, since in the underlying continuous case  $dt$  approaches zero.

$A_2$  and the threshold value  $p^*$ , which triggers the real investment.

The first conditions is given by

$$F(0) = 0. \quad (23)$$

It simply state that the option  $F(p)$  should be worthless if the price of the underlying asset is equal to zero. Since  $\beta_2$  is negative, condition (23) can only be true if  $A_2 = 0$ . As a result the guess solutions for equation (20) is reduced to

$$F(p) = Ap^\beta. \quad (24)$$

Two additional conditions are necessary to determine the trigger price  $p^*$  and the parameter  $A$ . These conditions are derived by considering the option value  $F(p)$  at the threshold price  $p^*$ . First in equilibrium the value of the option  $F(p^*)$  must be equal to the net value of the real investment  $v(p^*) - I$ .

$$F(p^*) = v(p^*) - I \quad (25)$$

Equation (25) is referred to as the *matching condition*. Additionally for optimality the derivative of the option value must be equal to the derivative of the real investment value

$$F_p(p^*) = v_p(p^*). \quad (26)$$

Equation (26) is referred to as the *smooth-pasting* condition or *higher-order contact*. If the two functions were not smooth at the trigger price  $p^*$  a better maximum would be available. By using these conditions it is possible to determine the cutoff price for the underlying uncertain investment at which the option value of the project is equal to the real investment. Precisely explained at  $p^*$  the investor is indifferent whether she should still postpone the investment or not. However for prices bigger than the cutoff price clearly the real investment should be initi-



ated.

Before the optimal investment rule can be derived it is necessary to have a closer look on the value of the real investment  $v(p)$  given the risk. An investor who holds the real investment associated with the risk in (16) over a period  $dt$  will expect the following returns

1. The expected appreciation of the price ( $\alpha$ )
2. Dividend ( $\delta$ )

The investor will therefore expect a total return of

$$\mu = \delta + \alpha. \quad (27)$$

Equation (27) represents an expected return rate which compensates the owner of the considered investment given its risk, described by equation (16).

Obviously risk is one major aspect within this valuation concept and should be therefore defined in a more rigorous manner. In the following risk refers always to nondiversifiable risk, because with reference to the capital asset pricing model (CAPM), diversifiable risk can be eliminated by constructing appropriate portfolios. Given a market portfolio  $M$  and a riskless bond, it is possible to determine the appropriate return for any risk rate on the considered financial market.

Once the return for the market portfolio  $M$ 's risk rate is known, it is possible to determine the risk premium for any asset on the market, based on the covariance or correlation between the market portfolio  $M$  and the respective asset (Sharpe, 1964).

$$\mu_A = r + \Lambda \sigma_A \rho_{MA} \quad (28)$$

with

$$\Lambda = \frac{(\mu_M - r)}{\sigma_M} \quad (29)$$

Equation (28) states, given the correlation coefficient  $\rho$  between the market portfolio return and the considered investment return, and given  $\Lambda$  (the market price of risk), the expected total return rate of the considered asset is a sum of the riskless rate and a respective risk premium.  $\mu$  therefore represents also the *risk-adjusted* discount rate, which will be of importance below.

Given the so far assumed simplified cash flows  $p$  of equation (16) it is possible to calculate the expected present value  $v(p)$  by using the risk adjusted discount rate  $\mu$ .<sup>8</sup> As the expected cash flows are given by  $E(p_t) = pe^{\alpha t}$  the risk adjusted value of the real investment is given by

$$v(p) = \int_0^{\infty} pe^{\alpha t} e^{-\mu t} dt = \frac{p}{\delta}. \quad (30)$$

The interpretation of equation (30) is as follows. If the option of the investment is kept alive and the project is postponed, the investor won't receive the dividend payments of the real investment. Therefore  $\delta$  appears as opportunity cost and can be used to evaluate the risk adjusted real investment value  $v(p)$ .

With the appropriate value of the real investment it is possible to formulate the final investment rule for a risky project including its option value. The functional

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<sup>8</sup> In the underlying case speculative bubbles are ruled out.

forms of the matching and smooth pasting condition deliver

$$Ap^\beta = \frac{p}{\delta} - I \quad (31)$$

$$\beta Ap^{\beta-1} = \frac{1}{\delta}. \quad (32)$$

Solving equation (31) for  $A$  results in

$$A = \frac{p^{1-\beta}}{\delta} - Ip^{-\beta}. \quad (33)$$

Substituting  $A$  into equation (32) provides the solution for the equilibrium price  $p^*$  which determines the execution of the real investment.

$$\begin{aligned} \beta p^{\beta-1} \left[ \frac{p^{1-\beta}}{\delta} - Ip^{-\beta} \right] &= \frac{1}{\delta} \\ \beta - 1 &= \frac{\beta \delta I}{p} \end{aligned}$$

The cutoff price  $p^*$  results as

$$p^* = \left[ \frac{\beta}{\beta - 1} \right] \delta I. \quad (34)$$

Calculating the value  $v(p)$  of the real investment on the basis of the cutoff price  $p^*$  and equation (30) provides the threshold value  $v^*$  which can be interpreted as the optimal investment rule under uncertainty.

$$v(p^*) = \left[ \frac{\beta}{\beta - 1} \right] I \quad (35)$$

The parameter  $A$  of the option value function is then

$$A = I^{1-\beta} \frac{(\delta\beta)^{-\beta}}{(\beta-1)^{1-\beta}} \quad (36)$$

It is easier to interpret the economic intuition behind equation (35) if a numerical example is presented. Assume that the investments cost of a project is  $I = 1$  with a volatility of the cash flows of  $\sigma = 0.2$ . The riskless interest rate is  $r = 0.05$  and  $\delta = 0.05$ . With these parameter values  $\beta = 2.16$  and the investment rule states  $v^* = 1.86 I$ . Therefore the underlying risky investment should be executed if its value is at least 1.86 times higher than the corresponding costs  $I$ . Clearly this is a huge difference to the Marshallian rule which states that an investment should be put into effect if the value of the project covers the investment costs  $I$ .

## 4. Export And FDI Choice Under Uncertainty

After presenting the general procedure of how to determine the option value of an investment with a simple risky cash flow pattern  $p$ , it is possible to assess the export and FDI decision of a risk neutral investor within the theoretical framework of section 2. A switching strategy in form of becoming first an exporter and then a foreign direct investor is excluded. The investor can choose either to serve the market as exporter or by founding a foreign plant. For the ease of reference the cash flows of the two projects are stated again.<sup>10</sup>

$$\pi^i(p, c^i) = (1 - \theta) \left( \frac{\theta}{c^i} \right)^{\frac{\theta}{1-\theta}} p^\kappa \quad \text{with } i \in \{E, F\} \quad \text{and} \quad \kappa = \frac{1}{1-\theta} \quad (37)$$

Obviously the cash flows in each term have a convex shape in the price  $p$  as the the exponent  $\kappa$  is bigger than 1 due to the concave production technology. Since the optimal investment rule is derived based on the matching and smooth

<sup>9</sup> Appendix D shows the derivation of  $A$ .

<sup>10</sup>The time index  $t$  is omitted since an infinite time horizon is considered

pasting conditions in equation (25) and (26), it is necessary to determine the risk adjusted value  $v(p)$  of the underlying investments. In the first step it is necessary to calculate the expected growth rate of  $p^\kappa$  which is named  $\alpha'$ .

$$\frac{d(p^\kappa)}{p^\kappa} = \text{relative returns} \quad (38)$$

By using Ito's lemma this can be stated as

$$\frac{d(p^\kappa)}{p^\kappa} = \frac{[\kappa p^{\kappa-1} dp + \frac{1}{2} \kappa(\kappa-1) p^{\kappa-2} \sigma^2 p^2 dt]}{p^\kappa} \quad (39)$$

where  $dp$  represents the geometric Brownian motion (16). Substituting  $dp$  by equation (16) delivers

$$\frac{d(p^\kappa)}{p^\kappa} = \frac{[\kappa p^{\kappa-1} (\alpha p dt + \sigma p dz) + \frac{1}{2} \kappa(\kappa-1) p^{\kappa-2} \sigma^2 p^2 dt]}{p^\kappa} \quad (40)$$

$$= \kappa (\alpha dt + \sigma dz) + \frac{1}{2} \kappa(\kappa-1) \sigma^2 dt \quad (41)$$

$$= \left( \alpha \kappa + \frac{1}{2} \kappa(\kappa-1) \sigma^2 \right) dt + \kappa \sigma dz. \quad (42)$$

Finally the expected growth rate  $\alpha'$  for cash flows of the shape  $p^\kappa$  with  $E(dz) = 0$  is given by

$$\alpha' = E \left( \frac{dp^\kappa}{p^\kappa} \right) = \left( \alpha \kappa + \frac{1}{2} \kappa(\kappa-1) \sigma^2 \right) dt. \quad (43)$$

Therefore the expected value  $v(p)$  of the underlying real investment with a calculated growth rate  $\alpha'$  can be determined if its risk adjusted rate of return  $\mu'$  is known.

$$v(p) = Z p^\kappa e^{(\alpha \kappa + \frac{1}{2} \kappa(\kappa-1) \sigma^2) t} e^{(-\mu') t} \quad (44)$$

Appendix F provides the proof that the risk adjusted rate of return  $\mu'$  for the cash flows  $p^\kappa$  are given by

$$\mu' = r + (\mu - r) \kappa \quad (45)$$

with  $\mu$  as the expected return of cash flows following the geometric Brownian motion in equation (16). As earlier stated the total expected returns  $\mu$  of an investment are generated by its growth rate  $\alpha$  and the dividend payments  $\delta$  which are both assumed to be constant. Substituting  $\mu = \alpha + \delta$  in equation (45) delivers

$$\mu' = r + (\alpha + \delta)\kappa - r\kappa \quad (46)$$

$$= (1 - \kappa)r + (\alpha + \delta)\kappa. \quad (47)$$

As a consequence it is possible to determine the expected present value  $v(p)$  of the real investment calculated on the basis of the risk adjusted expected return

$$v(p) = Zp^\kappa e^{\alpha't} e^{-\mu't} \quad (48)$$

$$= Zp^\kappa e^{-\delta't}. \quad (49)$$

Differently expressed the expected risk adjusted returns  $\mu'$  of an investment with  $Zp^\kappa$  as cash flows must be generated by their growth rate  $\alpha'$  and the adjusted dividends  $\delta'$ .

$$\mu' = \alpha' + \delta' \quad (50)$$

Finally the risk adjusted value of the real investment turns out to be

$$v(p) = Zp^\kappa e^{(\alpha\kappa + \frac{1}{2}\sigma^2\kappa(\kappa-1))t} e^{-(1-\kappa)r - \alpha\kappa - \delta\kappa)t} \quad (51)$$

$$= Zp^\kappa e^{(\frac{1}{2}\sigma^2\kappa(\kappa-1) - r + \kappa r - \delta\kappa)t} \quad (52)$$

$$v(p) = \frac{Zp^\kappa}{r - \kappa(r - \delta) - \frac{1}{2}\sigma^2\kappa(\kappa - 1)} \quad (53)$$

with the risk adjusted discount rate as

$$\delta' = r - \kappa(r - \delta) - \frac{1}{2}\sigma^2\kappa(\kappa - 1). \quad (54)$$

As it can be seen for a production technology with constant returns to scale ( $\kappa = 1$ ) the risk adjusted discount rate  $\delta'$  is equal to the dividend payments  $\delta$  of the investment. The fair value of the real investment with a convex cash flow structure turns out to be risk sensitive. Holding the dividend payments  $\delta$  constant, as assumed, an increase in the volatility  $\sigma$  of prices decreases the risk adjusted discount rate  $\delta'$  and therefore increases the expected value of the investment. Technically this result is driven by the convexity of the underlying function since its expected value will become higher according to Jensen's inequality. Therefore I refer to this result as *convexity-effect*. Given such a structure an investor will have a higher incentive to execute an investment the higher the price volatility is. One could interpret this result as a risk driven appreciation.

### The Optimal Investment Rule In The Underlying Framework

Once the value  $v(p)$  of the real investment with the cash flow pattern  $p^\kappa$  is known, it is possible to derive the threshold values for the export and FDI choices of an investor. For an investment which is volatile due to the price volatility in equation (16) the corresponding general value function  $F(p)$  is given by

$$F(p) = Ap^\beta. \quad (55)$$

For a detailed derivation see section 3. The corresponding optimality conditions are therefore given by

$$F(0) = 0 \quad (56)$$

$$Ap^\beta = \frac{Zp^\kappa}{\delta'} - I \quad (57)$$

$$\beta Ap^{(\beta-1)} = \frac{\kappa Zp^{\kappa-1}}{\delta'} \quad (58)$$

where equation (57) and (58) represent the matching and smooth pasting condi-

tions. Under these conditions the optimal cutoff price turns out to be

$$p^* = \sqrt[\kappa]{\frac{\beta}{\beta - \kappa} I \frac{\delta'}{Z}} \quad (59)$$

with

$$\beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \quad (60)$$

and

$$A = Z \left( \left( \frac{\beta I \delta}{(\beta - \kappa) Z} \right)^{\kappa-1} \right)^{\kappa-\beta} \delta^{-1} - I \left( \left( \frac{\beta I \delta}{(\beta - \kappa) Z} \right)^{\kappa-1} \right)^{-\beta} \quad (61)$$

Appendix E provides a concise derivation of the value  $p^*$ . The corresponding investment rule for the underlying production structure is therefore given by

$$v(p^*) = \frac{\beta}{\beta - \kappa} I. \quad (62)$$

Clearly if the volatility  $\sigma$  of the prices  $p$  increase, the parameter  $\beta$  decreases. Simultaneously  $\frac{\beta}{\beta - \kappa}$  increases and the threshold price  $p^*$  increases as well. The same effect drives up the expected investment value  $v(p^*)$ . As it can be seen the demanded real investment value is much higher than the investment costs  $I$  since the wedge  $\frac{\beta}{\beta - \kappa}$  is bigger than one. In other words, an investor who includes the option value  $F(p^*)$  in her assessment will demand higher exchange rate prices (appreciation) if their volatility increases. This can be interpreted as a risk driven depreciation of a real investment which is a countermovement to the earlier presented risk driven appreciation. Since the effect can be explained by observing  $\beta$ , I refer to this result as the  $\beta$ -effect.

Given the assumed different cost structures for the export and the FDI choice of an investor with  $I^E < I^F$  and  $c^E > c^F$ , the optimal investment strategies for each investment can be formulated separately including the option values.

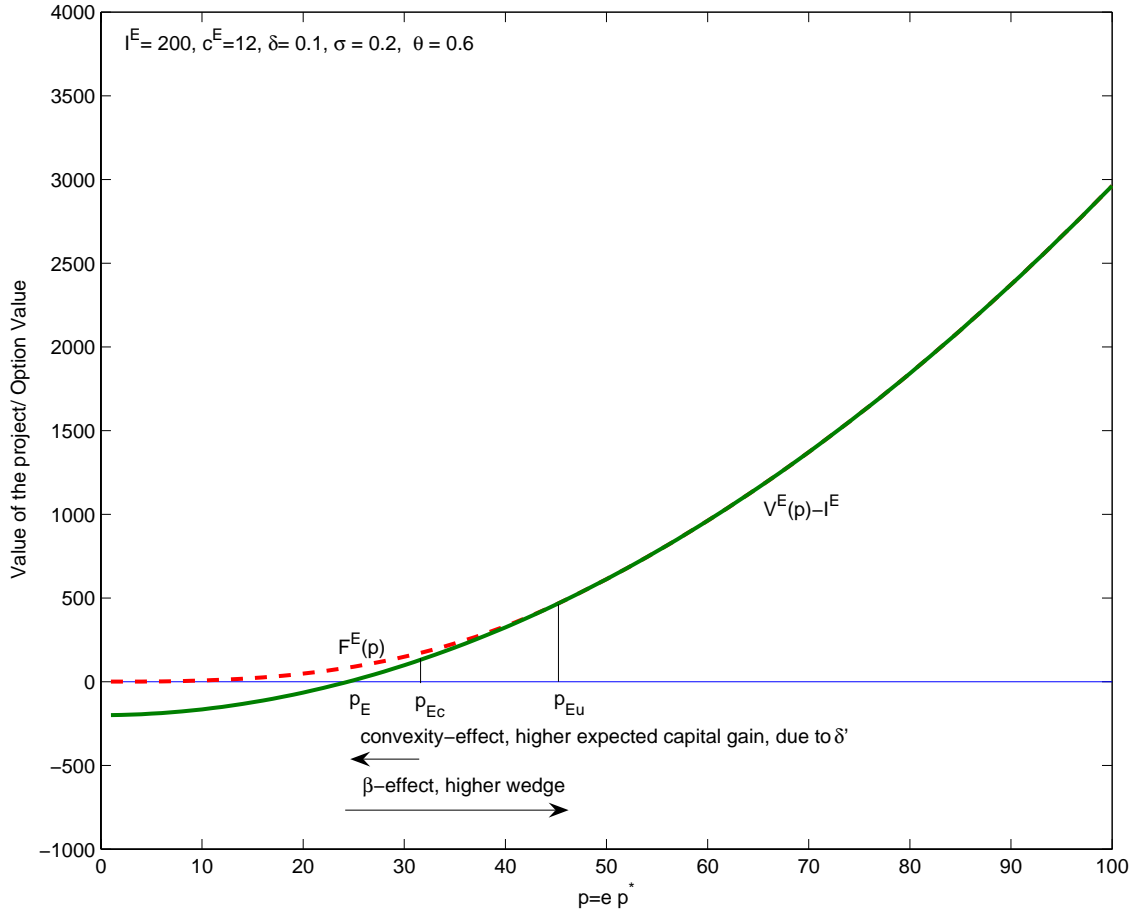


$$A(p_i)^\beta = \frac{Z^i(p_i)^\kappa}{\delta'} - I^i \quad (63)$$

and

$$\beta A(p_i)^{\beta-1} = \frac{\kappa Z^i(p_i)^{\kappa-1}}{\delta'} \quad (64)$$

with  $i \in \{E, F\}$



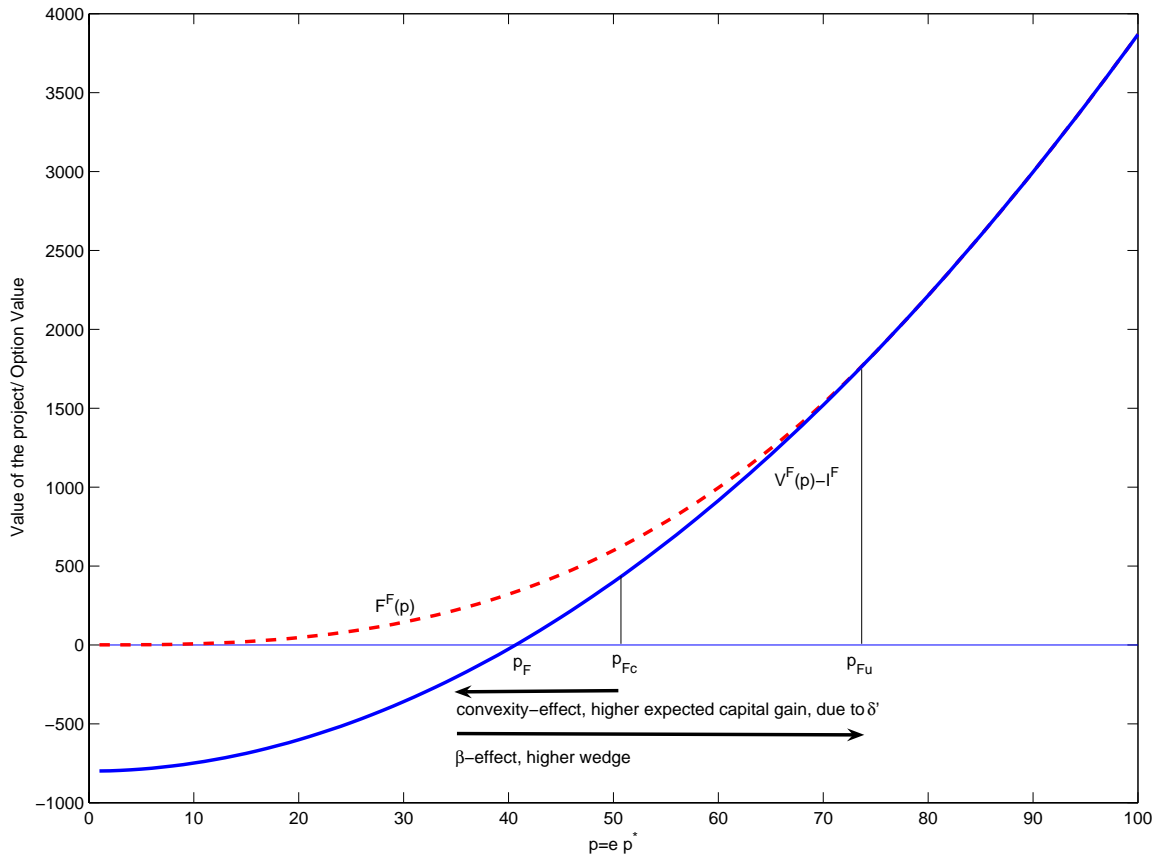
**Figure 2:** Threshold price  $p_E$  for Export under uncertainty

In figure (2) the price level  $p_{Ec}$  represents the cutoff price under certainty which was derived in figure (1). Under certainty the investor should expand her domestic output for exports if prices are higher than  $p_{Ec}$ . The introduction of uncertainty

has two effects which are influencing the cutoff price, namely the convexity and  $\beta$ -effect. In the figure (2) the continuous line represents the expected value of the export project. Due to the convexity-effect, the value function  $v(p)$  is shifted up as the price volatility increases. In a scenario where the option value  $F(p)$  of the investment is neglected, the investor would become an exporter if the prices are higher than  $p_E$ . The dashed line represents the option value of the export strategy and according to the optimality conditions an investor should execute an investment if  $F(p)$  is tangent to the expected investment value  $v(p)$ . This is the case for prices bigger than  $p_{Eu}$ . Obviously the  $\beta$ -effect increases the cutoff price of the export strategy and a risk neutral investor will postpone the investment until the price level is bigger than the new cutoff price  $p_{Eu}$ . The crucial result in figure (2) is that the  $\beta$ -effect is bigger than the convexity-effect. Therefore uncertainty leads to an investment which takes place at higher prices and implicitly later than under certainty. Figure (3) depicts the expected value function  $v^F(p)$  for the FDI strategy and the corresponding option value  $F(p)$ , based on the same parameter values as earlier. The effects within this strategy have the same pattern as in the previous export scenario. A risk neutral investor should postpone her investment until the price level  $p_{Fu}$  is reached. The new cutoff price is significantly higher than under certainty.

Finally it is possible to analyze the investment strategy of the risk neutral investor who can choose between the export strategy, FDI and the postponement of each strategy. Figure (4) depicts the value functions  $v^E(p)$  for the export strategy and  $v^F(p)$  for the FDI strategy as continuous lines. The corresponding option values are represented by the dashed lines  $F^E(p)$  and  $F^F(p)$ .<sup>11</sup> The resulting cutoff prices provide the following investment plan. If the price  $p$  measured in domestic currency is smaller than  $p_E$  the investor should wait and neither of the two investment strategies is executed, since the option values of both investments are higher than their expected values  $v^i(p)$ . For prices between  $p_E$  and  $p_{E2}$  the ex-

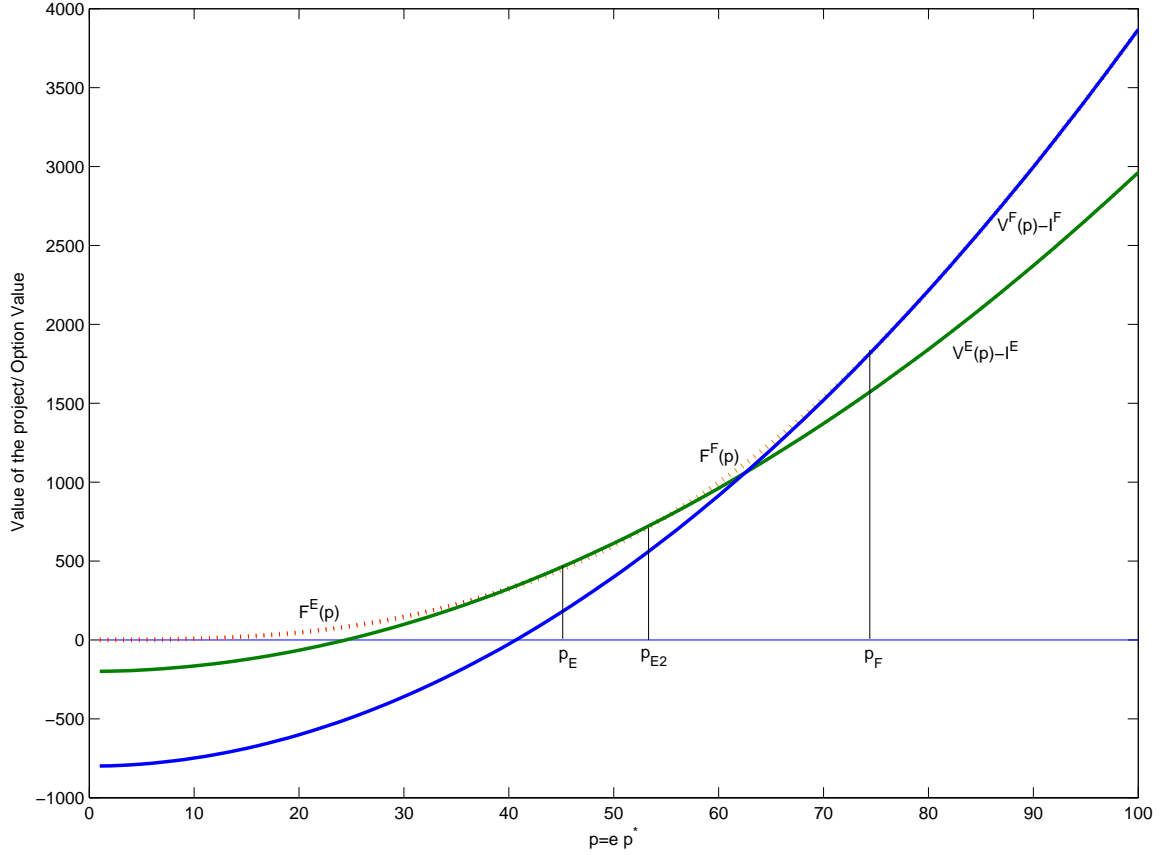
<sup>11</sup>The underlying parameter values are the same as before.



**Figure 3:** Threshold price  $p_F$  for FDI under uncertainty

port strategy turns out to be dominant because it is bigger than the option value of both strategies and also higher than the expected value of the FDI strategy. Therefore the investor should serve the foreign market through exports. However if the prices are between the cutoff levels  $p_{E2}$  and  $p_F$  then the investor should postpone both projects since the option value of the FDI strategy is the highest value function. The economic intuition behind this price range is as follows. By waiting, the investor has the chance to observe the market and gather additional information concerning the FDI strategy. Given the price volatility in  $p$  the FDI strategy offers a potential higher return than the export strategy and therefore waiting is rational.<sup>12</sup> Unfortunately it is not possible to determine the time span

<sup>12</sup>Such a strategy excludes strategic interaction between firms. It is assumed that there is no disadvantage if a firm enters a country later



**Figure 4:** Export or FDI under uncertainty

which is reasonable for the postponement of the export strategy.<sup>13</sup> In principle it is possible that  $p$  stays in this critical price range and in such a case waiting would become too expensive. However given the price volatility  $\sigma$  in equation (16) the probability of such a price behavior is low. The next sections provides an example in which exporting is postponed although it is profitable. In the long run waiting costs are easily covered by the higher FDI returns.

Finally for prices bigger than  $p_F$  the investor should serve the foreign market by a greenfield investment (FDI). By renaming the option functions of the export and FDI strategy into  $F^E = Ap^\beta$  and  $F^F = Bp^\rho$  it is possible to present a formal investment rule.

<sup>13</sup>It is important to bear in mind that the investor has not the possibility of switching from export to FDI. The investment strategy is either export or FDI. Otherwise the problem must be analyzed in a different way.

$$F_E(p) = \begin{cases} Ap^\beta & \text{if } p \in \{p, p_E\} \\ V^E(p) - I^E & \text{if } p \in \{p_E, p_{E2}\} \end{cases} \quad (65)$$

$$F^F(p) = \begin{cases} Bp^\rho & \text{if } p \in \{p, p_E\} \text{ and } \{p_E, p_{E2}\} \\ V_F(p) - I_F & \text{if } p \in \{p_F, \infty\} \end{cases} \quad (66)$$

Comparing the proximity-concentration trade-off under uncertainty with a certain case provides additional inferences. Five major results can be stated.

1. For an investment with convex profit flows uncertainty provides as such higher incentives to invest. However taking the option value of it into account decreases the incentives to invest. (the convexity-effect is smaller than the  $\beta$ -effect.)
2. Continuous risk increases the trigger price for an export strategy. The price range in which no export is done increases.
3. Risk also increases the trigger price for an FDI strategy. The price range in which foreign direct investments are dismissed increases.
4. Even if the export expansion provides positive profits, for certain price levels it is rational for an investor to postpone the export investment decision since potential higher FDI profits can be achieved.
5. Implicitly the export and the foreign direct investment will take place later than under certainty.

The first three results are well analyzed aspects and standard results in the real option theory. Additionally they provide an explanation for the empirical findings of Bernard and Jensen (2004) as well as for Égert and Morales-Zumaquero (2007). Exchange rate volatility has a prohibitive impact on export and FDI

decisions, as long as investors can postpone their decisions for a certain period. The fourth result is a new result within the proximity-concentration trade-off. Although the export expansion would provide profits for an investor, for certain prices she won't become an exporter. Instead she will observe the volatile prices and become a foreign direct investor if the upper cutoff price is reached.

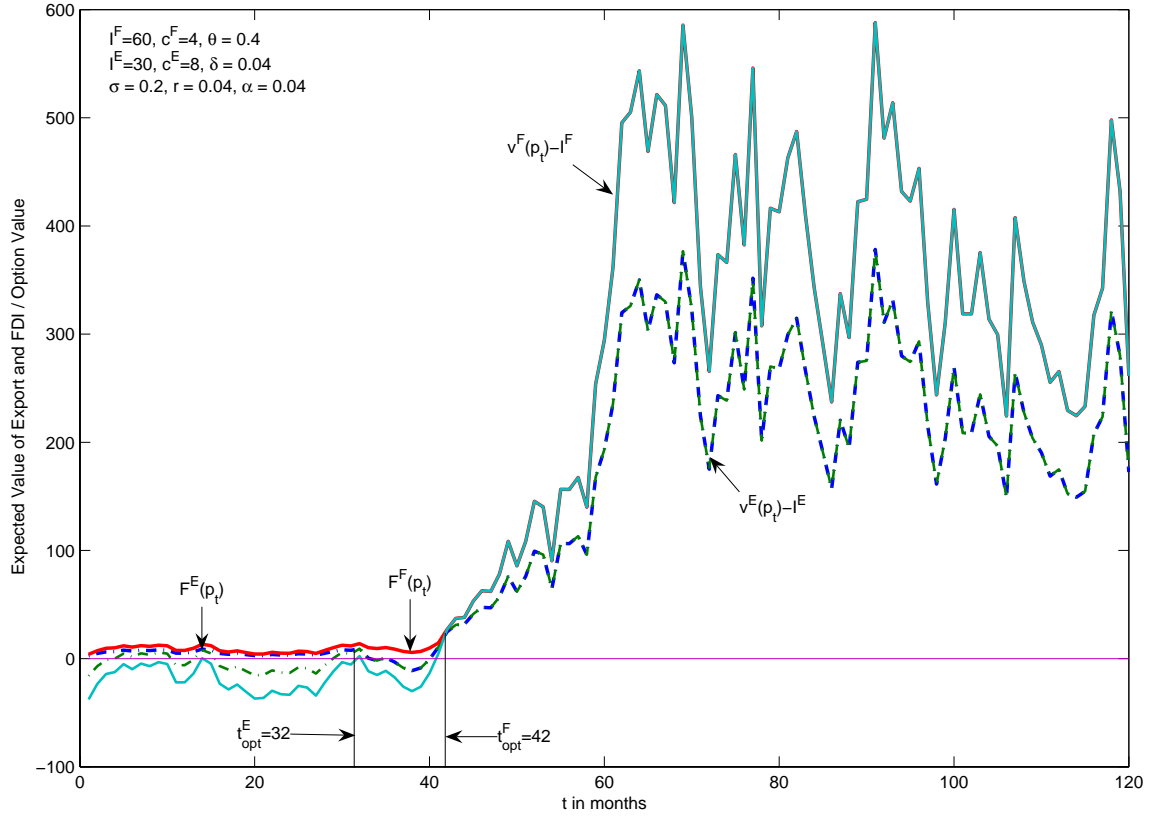
***proposition 2:***

*Firms should choose between serving a market as exporter or foreign direct investor by calculating the expected real investment values including the respective option values. Even if the export strategy dominates the expected real FDI profits the implementation of the option values might suggest a postponement of exports whenever potential higher returns can be achieved within the FDI strategy due to price volatility.*

## 5. The Timing OF Export And FDI

Within the real option framework investors are confronted with critical price values (cutoff prices) which determine the optimal investment strategy. However these cutoff prices don't provide an explicit timing suggestion since the incremental time variable disappears in the theoretical analysis. The lack of the timing component can be analyzed only in a simulation which leads to arbitrary results. Still such a simulation gives additional insights about the trade-off between waiting (information gathering) and forgone profits. Figure (5) shows a sample path for the expected investment values of the export and FDI strategies  $v^i(p) - I^i$  and the corresponding option values  $F^i(p)$ . The price changes are gauged monthly and the domestic price level  $p_t$  is given by

$$p_t = p_{t-1} \left(1 + \frac{\alpha}{12}\right) dt + p_{t-1} 0.2 \sqrt{\frac{1}{12}} \epsilon_t. \quad (67)$$



**Figure 5:** The timing of export and FDI

At each time  $t$  a random number  $\epsilon_t$  is drawn from a normal distribution with zero mean and a unit standard deviation. An initial price level  $p_0 = 1.5$  is assumed.<sup>14</sup> By using the earlier derived value functions it is possible to compare the two investment strategies over time. Under the assumed parameter values the export strategy would provide positive returns after 5 months already. However as its option value  $F^E(p_t)$  (upper dashed line) is bigger than its net investment value (lower dashed line), a rational investor postpones the investment. It turns out that the postponement is a good choice as the price development in the following months is negative and the export strategy would provide losses. After 32 months the first matching condition ( $F^E(p_t) = v^E(p_t)$ ) is fulfilled and an investor should serve the foreign market as an exporter if exporting was the only option. However

<sup>14</sup>Investors will have a price expectation on the new market. In the simplest case  $p_0$  will be equal to the domestic sales price.

once the export decision is established in the underlying model it is not possible to switch to FDI as an alternative mode. Therefore the investor has to consider the potential value of the FDI strategy. At that time clearly the option value of the FDI strategy dominates the export strategy and therefore the investor should postpone the investment decision although exporting generates profits. Finally after 42 months the second matching conditions appears and the best strategy turns out to be the foreign direct investment strategy. Furthermore if one compares the profits of the export and FDI strategies from that time on it turns out that the FDI strategy provides significantly higher returns and recovers the forgone export gains between the 32<sup>nd</sup> and 42<sup>nd</sup> month easily.

## 6. Conclusion

In the underlying model the export and FDI decisions of an investor have been analyzed on the basis of the proximity-concentration trade-off. In contrast to the New New Trade Theories which use the proximity-concentration hypothesis likewise to explain export and FDI behavior like Helpman, Melitz and Yeaple (2004), my model takes risk not as a one time shock effect into account but as a continuous phenomenon. Since foreign sales are confronted with foreign exchange rate, the model includes the exchange rate in form of a geometric Brownian motion over an infinite time. In contrast to the static models, investors have the possibility to postpone their investment decisions, since competitive interaction is assumed to be not existent. As a result the investment choice of a risk neutral investor turns out to be highly influenced by the volatility of the exchange rate. Due to the convexity of the cash flows in the underlying model, volatility turns out to increase the incentive to invest earlier in a market. Whether as an exporter or a foreign direct investor depends on the price level. However simultaneously the inclusion of the option value into the investment decision turns out to erase the positive effect of risk because waiting becomes valuable as additional information



concerning the volatile exchange rate can be collected. In the underlying model the first effect (convexity-effect) is always dominated by the incentive decreasing effect ( $\beta$ -effect). Therefore exchange rate volatility turns out to increase the postponement of export and FDI projects until the price level reaches a specific cutoff point. Since the investor in the underlying model has only the choice between either export or FDI, the implementation of the option value provides an additional result which is not existent in the prevailing trade models. Even if the export profits turn out to be higher than the FDI returns, the model predicts that the investor will still observe the market instead of becoming an exporter for certain price levels. This result is based on the value of waiting during which additional information can be collected. A trade-off between the value of waiting and the forgone export profits appears which is generated by the volatile exchange rate.

The theoretical results of the model are coinciding with the empirical results of models which analyze the export and FDI decision including the exchange rate. Bernard and Jensen (2004) conclude that an increase in the exchange rate volatility (in their case, through depreciation of the dollar) exports started to increase, however after a time lag. The incentive decreasing effect of risk has been proved as well by Égert and Morales-Zumaquero (2007). Given this congruency between the theoretical result of the model and the empirical findings the paper contributes to a better understanding of export and FDI patterns over time.

## Appendix

### A. The structure of the replicated portfolio

The option value  $F(u)$  of an investment with profit flows  $\pi(u)$  can be determined by using the tradeable good of the project which contains the same risk and return pattern as the investment.<sup>15</sup>

Assume that the project is associated with the following geometric Brownian motion

$$du = \alpha u dt + \sigma u dz. \quad (68)$$

In the first step a *riskless portfolio* is constructed by using the tradeable good and a riskless treasury bond. Specifically one Euro is invested into a treasury bond and simultaneously  $n$  units of the firms output are bought on the market. This portfolio has a value of  $(1 + nu)$  Euros. If this portfolio is held by an investor for a period of  $dt$ , the returns are:

1.  $r dt$  generated by the bond
2.  $n\delta u dt$  the dividend, generated by the traded good
3. additionally there is a capital gain of  $n du$

The total return of the portfolio for a period  $dt$  is given by

$$\delta n u dt + n \alpha u dt + n u \sigma dz + r dt. \quad (69)$$

The total relative return of the portfolio is given by

$$\frac{[r + nu + (\delta + \alpha)] dt}{1 + nu} + \underbrace{\frac{nu \sigma dz}{1 + nu}}_{\text{risky part}}. \quad (70)$$

<sup>15</sup>The underlying derivation is based on Dixt and Pindyck (1994).

where the righthand side of equation (70) describes the risky part of the total return. Now consider the hypothetical return from holding the investment in the project with a value of  $F(u)$  over the same period of  $dt$ . The payoff structure associated with the investment is containing

1. the costs of the investment  $F(u)$
2. received dividends in form of the profit flows  $\pi(u)dt$  which are certain, since they are known at the initial decision
3. capital gain  $dF(u)$ .

The capital gain  $dF(u)$  can be calculated by using Ito's lemma:

$$dF = \left[ F_t(u) + \alpha u F_u(u) + \frac{1}{2} \sigma^2 u^2 F_{uu} \right] dt + \sigma u F_u(u) dz. \quad (71)$$

In the following the effect of  $F_t(u)$  will be neglected since it is infinitesimal small. The resulting relative return of the investment is given by

$$\frac{[\pi(u) + \alpha u F_u(u) + \frac{1}{2} \sigma^2 u^2 F_{uu}] dt}{F(u)} + \underbrace{\frac{\sigma u F_u(u) dz}{F(u)}}_{\text{risky part}} \quad (72)$$

The replicated portfolio will have the same risk pattern as the investment project if the risk associated parts of the relative returns in equation (70) and (72) are equal:

$$\frac{nu \sigma dz}{1 + nu} = \frac{\sigma u F_u(u) dz}{F(u)} \quad (73)$$

$\Rightarrow$

$$\frac{nu}{1 + nu} = \frac{u F_u(u)}{F(u)} \quad (74)$$

Equation (74) represents our assumption that the Wiener processes behind the

traded goods and the project itself are identical. If the replicated portfolio contains the same risk as the project, it must deliver the same relative return.

$$\frac{[\pi(u) + \alpha u F_u(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}]}{F(u)} = \frac{r + nu(\delta + \alpha)}{1 + nu} \quad (75)$$

According to condition (74) we receive by substitution

$$\frac{\pi(u) + \alpha u F_u(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}}{F(u)} = \frac{r}{1 + nu} + \frac{u F_u(u) (\delta + \alpha)}{F(u)}. \quad (76)$$

It can be shown that

$$\begin{aligned} \frac{r}{1 + nu} &= \frac{r(1 - nu) + nu}{1 + nu} \\ &= \frac{r}{(1 + nu)} - \frac{rnu}{1 + nu} + \frac{rnu}{1 + nu} \\ &= r - \frac{rnu}{1 + nu} \end{aligned} \quad (77)$$

and according to equation (74)

$$\frac{r}{1 + nu} = r \left[ 1 - \frac{u F_u(u)}{F(u)} \right] \quad (78)$$

Combining this result with equation (76) leads to

$$\frac{\pi(u) + \alpha u F_u(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}}{F(u)} = r \left[ 1 - \frac{u F_u(u)}{F(u)} \right] + \frac{u F_u(u) (\delta + \alpha)}{F(u)}. \quad (79)$$

Simplification delivers

$$\pi(u) + \alpha u F_u(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}(u) = rF(u) - ru F_u(u) + (\alpha + \delta)u F_u(u) \quad (80)$$

$$\pi(u) + \alpha u F_u(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}(u) - \alpha u F_u(u) - \delta u F_u(u) + ru F_u(u) - rF(u) = 0 \quad (81)$$

$$\pi(u) + \frac{1}{2}\sigma^2 u^2 F_{uu}(u) - rF + (r - \delta)u F_u(u) = 0 \quad (82)$$

Finally a second order differential equation results which is linear in the dependent variable  $F(p)$  and its derivatives. Therefore the option value of an investment can be solved by any linear combination of two independent combinations.

$$F(u) = Au^\beta \quad (83)$$

with

$$\beta = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}. \quad (84)$$

## B. Valuation by asset spanning

The replication of a so called equivalent portfolio is conditioned on the fact that the output of the firm is traded on a financial market. However this must not be the case. In such a situation it is still possible to determine the appropriate value of a firm or project if one assumes, the financial markets are efficient and any risk return relationship can be acquired by an investor. Instead of replicating a riskless portfolio, the investor is supposed to *span* a portfolio which is riskless. Under such conditions a project can be valued in a similar way as in the replication method (Dixit and Pindyck 1994).

Instead of assuming that the output itself is trade on the financial market, it is assumed that there exists an asset or a portfolio of assets which contain the same risk pattern as the project which is supposed to be valued. This asset spans the risk of the considered project.  $X$  is the market price of the spanning asset and is given by:

$$dX = A(p, t)Xdt + S(p, t)Xdz. \quad (85)$$

It is assumed that the stochastic fluctuation of the spanning asset price  $X$  is perfectly correlated with the stochastic fluctuation of the output prices  $p$ . Differently expressed, the two Wiener processes behind  $dp$  and  $dX$  must be equal ( $dz_X = dz_p$ ). Furthermore the two coefficients  $A(p, t)$  and  $B(p, t)$  are functions of the output price  $p$ , which simply points out that the considered asset price contains the same information as the output price.

Furthermore the spanning asset pays a dividend rate of  $D(p, t)$  over a period  $dt$  for one invested Euro. Holding one Euro invested in the asset over a period  $dt$  delivers a total return of

$$[D(p, t) + A(p, t)] dt + B(p, t) dz. \quad (86)$$

As explained earlier according to the CAPM the appropriate expected return for an investment is given by

$$\mu_X(p, t) = r + \Lambda \rho_{p,M} B(p, t). \quad (87)$$

The coefficient  $B(p, t)$  is representing the standard deviation of the asset price and since the asset price and the output price are perfectly correlated, the correlation coefficient between the asset price and the market portfolio price is equal to the coefficient between the output price and the market portfolio price.  $\Lambda$  is representing the market price of risk whereas  $r$  is the return rate of a riskless asset. Under these assumptions the total return of the spanning asset in equilibrium is given by

$$\mu_X(p, t) = D(p, t) + A(p, t) \quad (88)$$

the sum of the dividend and the growth rate  $A(p, t)$ . Otherwise an investor would be able to generate infinite profits by arbitrage.

### The portfolio structure

An investor is supposed to hold a portfolio which consists of an investment in a project ( $F(p, t)$ ) and of  $n$  units of short positions in the asset  $X$ . The value of this portfolio corresponds to  $[F(p, t) - nX]$  Euro and it is hold over a period of  $dt$ . At the end of the period the investor has the following payoffs:

1. For the short position a dividend of  $D(p, t) p dt$  must be payed.
2. The project generates a cash flow of  $\pi(p, t)dt$ .
3. There is a capital gain of  $dF - ndX$ .

The capital gain can be expressed according to Ito's lemma as

$$dF - ndX = F_t + \left[ aF_p p + \frac{1}{2} b^2 F_{pp} p^2 - nAX \right] dt + [bF_p - nBX] dz \quad (89)$$

where  $a$  and  $b$  represent the coefficient of the stochastic process for the output price.

To be able to compare the latter portfolio with the riskless asset, it is necessary to eliminate the prevailing risk in it. This can be achieved by an appropriate choice of short positions in  $X$ . Equation (89) shows that the appropriate amount of short positions in the asset  $X$  for a riskless portfolio is given if

$$n = F_p \frac{b}{BX}. \quad (90)$$

After the elimination of the risk the expected return of the new portfolio must be equal to an equivalent riskless investment. The riskless return over a period  $dt$  is given by

$$r [F(p, t) - nX] dt. \quad (91)$$

Therefore the equilibrium condition is given by

$$dF - n dX + \pi(p, t) - nD(p, t)X dt = r [F(p, t) - nX] dt. \quad (92)$$

Substituting  $n$  by equation (90) leads to



$$\begin{aligned}
F_t + \left[ aF_p p + \frac{1}{2}b^2 F_{pp} p^2 - F_p \frac{b}{B(p,t)} A(p,t) \right] dt + \pi(p,t) - D(p,t) F_p \frac{b}{B(p,t)} dt \\
= rF dt - rF_p \frac{b}{B(p,t)} dt
\end{aligned} \tag{93}$$

and

$$F_t + \frac{1}{2}b^2 F_{pp} p^2 + aF_p p + \pi(p,t) - \frac{D(p,t) b F_p}{B(p,t)} - \frac{A(p,t) b F_p}{B(p,t)} - rF + \frac{r b F_p}{B(p,t)} = 0 \tag{94}$$

$$F_t + \frac{1}{2}b^2 F_{pp} p^2 + aF_p p + \pi(p,t) - \frac{b}{B(p,t)} F_p (r - (A(p,t) + D(p,t))) \tag{95}$$

with

$$\mu_X = A(p,t) + D(p,t). \tag{96}$$

For the simplest case where  $A = a = \alpha$  and  $B = b = \sigma$  and  $D = \delta$  and neglecting  $F_t$  since  $dt$  approaches zero in the continuous case, equation (93) becomes

$$\pi(p) + \frac{1}{2}\sigma^2 p^2 F_{pp}(p) - rF + (r - \delta)p F_p(p) = 0 \tag{97}$$

which is the same result as in the replication approach.

## C. Solution of a homogeneous differential equation

Given the second order differential equation (19)

$$\frac{1}{2}\sigma^2 p^2 F''(p) + (r - \delta)pF'(p) - rF(p) = 0$$

it is possible to state a general guess solution of the form

$$F(p) = Ap^\beta \tag{98}$$

since the differential equation is linear in the dependent variable  $F$ . Substituting the guess solution in equation (19) results in the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta - 1)Ap^\beta + (r - \delta)\beta Ap^\beta - rAp^\beta = 0 \tag{99}$$

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + (r - \delta)\beta - r = 0. \tag{100}$$

This quadratic equation is often called the fundamental quadratic equation and can be reformulated as

$$\Psi = \frac{1}{2}\beta^2 - \frac{1}{2}\beta + \frac{(r - \delta)}{\sigma^2}\beta - \frac{r}{\sigma^2} = 0. \tag{101}$$

The two solutions for equation (101) are given by

$$\beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left[\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{102}$$

$$\beta_2 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left[\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \tag{103}$$

Therefore the proper shape of the guess solution is given by

$$F(p) = A_1p^{\beta_1} + A_2p^{\beta_2}. \tag{104}$$

However, due to the first optimality condition

$$F(0) = 0 \tag{105}$$

the second solution with  $\beta < 0$  can be neglected. Otherwise the condition is not fulfilled.

The total differential of the fundamental quadratic equation  $\Psi$  delivers some important comparative results.

As the volatility  $\sigma$  increases,  $\beta_1$  will decrease. This has an important impact on the wedge in equation (62), since  $\frac{\beta}{\beta-\kappa}$  will increase and therefore the expected trigger value of the investment will increase, too.

## D. The calculation of the option parameter A

Given the homogeneous differential equation

$$\frac{1}{2}\sigma^2 p^2 F''(p) + (r - \delta)pF'(p) - rF(p) = 0 \quad (106)$$

for the Brownian motion

$$dp = \alpha p dt + \sigma p dz \quad (107)$$

the guess solution is represented by

$$F(p^*) = Ap^\beta. \quad (108)$$

The cutoff price for cash flows  $p$  has been determined as

$$p^* = \frac{\beta}{\beta - 1} \delta I. \quad (109)$$

Solving equation (31) for  $A$  and substituting  $p$  provides

$$A = \left( \frac{\beta}{\beta - 1} \delta I \right)^{1-\beta} - I \left( \frac{\beta}{\beta - 1} \delta I \right)^{-\beta} \quad (110)$$

$$= \left( \frac{\beta}{\beta - 1} \right) \left( \frac{\beta}{\beta - 1} \right)^{-\beta} \delta^{-\beta} I^{1-\beta} - I^{1-\beta} \delta^{-\beta} \left( \frac{\beta}{\beta - 1} \right)^{-\beta} \quad (111)$$

$$A = I^{1-\beta} \frac{(\delta\beta)^{-\beta}}{(\beta - 1)^{1-\beta}}. \quad (112)$$

## E. The threshold price $p^*$

Given the optimality conditions

$$F(0) = 0 \tag{113}$$

$$Ap^\beta = \frac{Zp^\kappa}{\delta'} \tag{114}$$

$$\beta Ap^{(\beta-1)} = \frac{\kappa Zp^{\kappa-1}}{\delta'} \tag{115}$$

the cutoff price  $p^*$  which determines the investment threshold, can be calculated as follows. Solving equation (114) for  $A$  provides

$$A = \frac{Zp^{\kappa-\beta}}{\delta'} - Ip^{-\beta}. \tag{116}$$

Substituting  $A$  in equation (115) provides

$$\beta p^{\beta-1} \left( \frac{Zp^{\kappa-\beta}}{\delta'} - Ip^{-\beta} \right) = \frac{\kappa Zp^{\kappa-1}}{\delta'} \tag{117}$$

$$\frac{\beta Zp^{\kappa-1}}{\delta'} - \beta Ip^{-1} = \frac{\kappa Zp^{\kappa-1}}{\delta'} \tag{118}$$

$$Zp^\kappa = \frac{\beta}{\beta - \kappa} I\delta' \tag{119}$$

$$p^* = \sqrt[\kappa]{\frac{\beta}{\beta - \kappa} \frac{I}{Z} \delta'} \tag{120}$$

## F. Risk adjusted rate of return

For the underlying geometric Brownian motion

$$dp = \alpha p dt + \sigma p dz \quad (121)$$

the total expected return is given by

$$\mu = \alpha + \delta. \quad (122)$$

However the cash flows of the considered companies with the concave production technology are given by  $Zp^\kappa$ , as a convex function. What is the risk adjusted rate of return for these type of cash flows? By using Ito's lemma

$$dF(p, t) = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial p} dp + \frac{1}{2} \frac{\partial^2 F}{\partial p^2} (dp)^2 \quad (123)$$

it is possible to reformulate the total expected returns as

$$E \left( \frac{dp^\kappa}{p^\kappa} \right) = \frac{\kappa p^{\kappa-1} dp + \frac{1}{2} \kappa (\kappa - 1) p^{\kappa-2} \sigma^2 p^2 dt}{p^\kappa}. \quad (124)$$

Substituting  $dp$  provides

$$E \left( \frac{dp^\kappa}{p^\kappa} \right) = \frac{\kappa p^{\kappa-1} (\alpha p dt + \sigma p dz) + \frac{1}{2} \kappa (\kappa - 1) p^{\kappa-2} \sigma^2 p^2 dt}{p^\kappa} \quad (125)$$

$$= \kappa (\alpha dt + \sigma dz) + \frac{1}{2} \kappa (\kappa - 1) \sigma^2 dt \quad (126)$$

$$= (\kappa \alpha + \frac{1}{2} \kappa (\kappa - 1) \sigma^2) dt + \kappa \sigma dz. \quad (127)$$

In appendix C the fundamental quadratic equation for the underlying problem was derived as

$$\Psi = \frac{1}{2} \beta^2 - \frac{1}{2} \beta + \frac{(r - \delta)}{\sigma^2} \beta - \frac{r}{\sigma^2} = 0. \quad (128)$$

In the case of convex cash flows the fundamental quadratic equation for the homogeneous part of the corresponding differential equation is given by

$$\zeta = \frac{1}{2}\kappa^2 - \frac{1}{2}\kappa + \frac{(r - \delta)}{\sigma^2}\kappa - \frac{r}{\sigma^2} = 0. \quad (129)$$

This equation can be transformed into

$$\frac{1}{2}\sigma^2\kappa(\kappa - 1) = r - (r - \delta)\kappa. \quad (130)$$

Putting relation (130) into equation (127) leads to

$$(\kappa\alpha + r - (r - \delta)\kappa) dt + \kappa\sigma dz. \quad (131)$$

From equation (122) alpha is given as

$$\alpha = \mu - \delta \quad (132)$$

which can be substituted in equation (131). As a result the expected total return of the convex cash flows is derived as

$$E\left(\frac{dp^\kappa}{p^\kappa}\right) = (\kappa(\mu - \delta) + r - (r - \delta)\kappa) dt + \kappa\sigma dz \quad (133)$$

$$= (\mu\kappa + r - r\kappa)dt + \kappa\sigma dz \quad (134)$$

$$= (r + (\mu - r)\kappa)dt + \kappa\sigma dz. \quad (135)$$

Since  $E(dz) = 0$  the expected total return of the convex cash flows is given by

$$\mu' = r + (\mu - r)\kappa. \quad (136)$$

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