Technology Adoption and the Selection effect of Trade*

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Abstract

Recent productivity studies suggest the reallocation of output across plants (*between* effect) and the productivity growth at individual plants (*within* effect) are both important sources of productivity growth at the industry level. Interestingly, recent evidence has shown that trade liberalization is related to both effects. While a trade model with firm heterogeneity can explain the between effect, it can not reproduce the within effect. We add to this model the option for firms to costly adopt more productive technologies and show that plant productivity actually rises in response to lower trade costs. The selection effect of trade - that some firms will be forced to exit following trade liberalization - favours the reallocation of output across incumbent firms (between effect) and contributes to raise the exporters' market share. Therefore, a greater scale of operation amplifies the firm's return from costly productivity-enhancement investments and leads a greater proportion of firms to undertake process-innovations (within effect).

Keywords: International Trade, Technology Adoption, Productivity

JEL codes: F12, F15, L11, O33, O47

1 Introduction

Longitudinal micro-data has revealed *i)* the reallocation of output across plants (*between effect*) and *ii)* productivity growth in the individual plants/firms are two relevant sources of productivity growth at the industry level (*within effect*).[1]

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[1] See Bartelsman and Doms (2000) for a recent review of the studies using the Longitudinal Research Database (LRD). See Bernard, Eaton, Jensen and Kortum (2003) for evidence on the degree of heterogeneity across firms in productivity as well as in innovation activities and export performances in nearly all industries examined. Finally the role of trade in the success and failure of firms in developing countries is reviewed by Tybout (2000).

The first effect is at the heart of the recent literature on heterogenous-firm models pioneered by Melitz (2003) and Bernard et. al. (2003). These models predict heterogenous responses to reduced trade costs across firms, including entry into exporting by some and increased failure by others. As a result, when trade costs fall, industry productivity rises both because low-productive non-exporting firms exit and because high-productive firms are able to expand through exporting. In these models, it is the reallocation of activity across firms - not intra-firms productivity growth - that boosts industry productivity.

In contrast, the aim of our paper is to stress the gains via the second microeconomic channel (*ii*), focusing on endogenous technology adoption within firms, but still building on a heterogenous firms modelling setup. We show that plant productivity actually rises in response to lower trade costs, a result beyond the existing literature and motivated by the empirical relevance that within-plant productivity improvements play in the productivity growth of an industry.

For instance, the right shift of the Canadian productivity distribution of manufacturing firms in 1996 compared to 1988 following the Canada-U.S. FTA documented in Trefler (2005) can be ascribed to both effects. Low productive firms that either exit or downsize following trade liberalization shrink the left tail of the distribution in 1996 relative to 1988, while high productive firms expanding their foreign sales through exporting contribute to the fatter right tail of the (size weighted) distribution in 1996 (*between effect*).

This does not exhaust the contribution of trade to aggregate productivity gains at the industry level. As reported in Trefler (2004), U.S. trade concessions to Canada has led to increases in productivity at surviving plants, contributing considerably to the thicker right tail of the distribution in 1996 too (*within effect*).

This effect is what Foster, Haltiwanger and Krizan (FHK, henceforth, 2001) call the "*within*" effect in their decomposition of the aggregate productivity growth and it constitutes the bulk of overall labour productivity growth in industrial economies. Likewise - as studied by Bustos (2005) - Argentinian exporters have adopted more innovative technologies after Argentina's trade liberalization of the 90s and - as reported by Bernard, Jensen and Schott (2006) - plant-productivity improvements are associated to declining industry-level trade costs in the US manufacturing industry.

Moreover, all these studies reveal that within-plant productivity growth was stronger among the group of exporters and among the most export oriented industries. This suggests that there is selection on the basis of innovation status and leads us to model firm's heterogeneity in productivity levels, so that the innovation type can be identified and her responses to trade reforms analyzed. This can not be achieved in the simpler Krugman (1980) setup, as all firms are equally productive and no firm-selection on the innovation status is possible.

We add a technology-adoption choice into the Melitz (2003)'s framework. After entry into the industry, all firms have the option to implement a more productive technology at the expense of higher "implementation" costs or adoption costs.

We think broadly of the adoption of a new technology, including the introduction of a new management, the re-organization of labour, the qualification and training of employees and leading to the reduction of the unit-cost of production. Hence, the intra-firm productivity increase is modeled as a costly investment within the firm to reduce its marginal cost of production and we shall assume there are no technological spillover across firms, as in Cohen and Klepper (1996a) and (1996b). This implies that the return of a process innovation due to a reduction of the variable costs is positively related to the number of internal applications which depends on the firm's scale.

Trade liberalization entails an improved and/or new access to product markets as well as an increased number of competitors. As a result, domestic exporters increase their combined market share, as they conquer part of the exiting firm's market as well as they gain a freer access to foreign markets. Therefore, by raising the scale of production of some exporters, trade strengthens their incentive for vertical innovation. This leads a group of exporters, who ex-ante were not productive enough to perform vertical R&D, to raise their productivity.

This is the new and main result of our model and, mostly important, not only holds true in the transition from autarky to trade (i.e. when a country first opens to trade), but it also applies when transportation costs - a proxy for trade barriers.- are reduced. Hence, it applies to incomplete steps of trade liberalization or partial tariff reforms, of which the Canada-US Free Trade Area (CUSFTA) and Argentinian trade liberalization of the 90s are two examples. This result allows to relate our model to the available evidence in Bernard (2006), Trefler (2004) and Bustos (2005).

The model is closely related to Bustos (2005), although our motivation and aim are different from hers. In common, they have the relation between the engagement of a firm in trade with the adoption of a more productive technology. Indeed, in both models firms are confronted with the option of adopting an alternative technology to the current employed, featuring a lower variable cost, but a higher fixed cost. While in Bustos (2005), the alternative technology is common to all firms, in our framework the alternative technology is firm-specific, matching the evidence on site-to-site variations in the success of implementing new technologies (e.g. Coming (2007) and Bikson et. al. (1987)).

In this respect, our model is similar to Helpam, Melitz and Yeaple (HMY henceforth, 2004) where the *proximity-concentration* trade-off determines whether a firm opts for FDI or exporting as a mode to serve a foreign market. In our framework, the trade off being between *efficiency-implementation costs* and shaping the firm's choice between two its alternative technologies (modes of production).

A second important difference with Bustos (2005) is that we present a general equilibrium set up rather than resting on a partial equilibrium approach. The new insight is that trade can both favour or deter technology-adoption as opposed to always favour it as it occurs in the partial equilibrium analysis. On one hand, trade lowers the cost to benefit ratio of implementing a more productive technology because it increases the access to foreign markets and therefore it increases the total demand for a firm's product. On the other, trade increases competition on the goods market (i.e. lower the demand for the firm's product) and it is a costly activity, putting grater pressure on the scarce input resources. This leads to a higher real wage and, overall, to a higher cost to benefit ratio of technology implementation. The latter effect - which is offsetting the former positive effect of trade - is absent in the partial equilibrium analysis.

We shall show the former can dominate the latter and therefore, when trade costs fall, productivity can increase at the plant level, in particular among the low-productive exporters.

This is one difference with Yeaple (2005) where all exporters adopt necessarily the more innovative technology and therefore, no selection on the basis of innovation status is possible. In his model, the reduction of transportation costs can only lead the domestic producers to adopt an innovative technology.

This model has some feedback for productivity studies, which are hardly related to trade. Our model suggests that a greater degree of openness in the trading relations can be partly responsible for the importance of the "*within*" component for the productivity growth in industrialized countries, as reported by Bartelsman, Haltiwanger and Scarpetta (BHS henceforth, 2004).

Finally, Baldwin and Nicoud (2005) have recently questioned that the positive effect of trade on aggregate productivity derived in a static model of trade maps into a dynamic growth effect. They highlight a static versus dynamic trade-off in terms of productivity gains: freer trade raises the aggregate productivity level through the selection effect, but at the same time it also rises the cost of creating new varieties since the expected survival probability into the industry is smaller. In turn, productivity growth slows down. Gustafsson and Segerstrom (2006) have shown that this result crucially depends on the strength of knowledge spillover assumed in the R&D technology. Our model suggests that were firms performing vertical innovation, the selection effect could generate productivity growth by forcing the least efficient firms out of the market and reallocating market shares across the most productive firms. Indeed, higher market shares incentive process-innovation leading to productivity growth.

The paper is organized as follows. Section 2 presents the model in the closed economy to be compared with the open economy in Section 3. This comparison is illustrative of the effects of trade on the aggregate productivity growth to be confronted with the available evidence. Section 4 discusses some drawbacks of the model and possible solutions to them. Finally the last section concludes.

2 The Closed Economy

In this section we extend Melitz (2003) to incorporate technology adoption.

Preference

Our economy is populated by a continuum of households of measure L, whose preferences are given by the standard C.E.S. utility function:

$$U = \left[\int_{\omega \in \Omega} [q(\omega)]^{\rho} d\omega \right]^{1/\rho}$$

where the measure of the set Ω represents the mass of available goods, $0 < \rho < 1$. Each household is endowed with one unit of labour which is inelastically supplied at the given wage w. The maximization of utility subject to the total expenditure $R = \int_{\omega \in \Omega} p(\omega) q(\omega) d\omega$ yields the demand function for every single variety ω:

$$q(\omega) = A \left[p(\omega) \right]^{-\sigma} \tag{1}$$

where A represents the demand level which is exogenous from the point of view of the individual supplier and P is the price index of the economy, given by:

$$A = \frac{R}{\int\limits_{\omega \in \Omega} [p(\omega)]^{1-\sigma} d\omega} = \frac{R}{P^{1-\sigma}}$$

$$P = \left[\int\limits_{\omega \in \Omega} [p(\omega)]^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

whereas

$$\sigma = 1/(1-\rho) > 1$$

is the elasticity of substitution across varieties.[2]

Technology

Each variety is produced by a single firm according to a technology for which the only input is labour. The total amount of labour required to produce the quantity $q(\omega)$ of the final good ω is given by

$$l(\omega) = f_D + cq(\omega) \tag{2}$$

where f_D is the fixed labour requirement and $c \in [0, \overline{c}]$ the firm-specific marginal labour requirement[3].

Entry - Exit

There is a large (unbounded) pool of prospective entrants into the industry and prior to entry, all firms are identical. To enter the industry, a firm must make an initial investment, modeled as a fixed cost of entry $f_E > 0$ measured in labour units, which is thereafter sunk. An entrant then draws a labour-per-unit-output coefficient c from a known and exogenous distribution with cdf $G(c)$ and density function $g(c)$ on the support $[0, \overline{c}]$. Upon observing this draw, a firm has three options. Like in Melitz (2003), it may decide to exit or to produce. If the firm does not exit and/or produces, it bears the fixed overhead labour costs f_D. Additionally to Melitz (2003), by investing f_I units of labour, it can opt for adopting a more productive technology and produce at a lower cost γc ($\gamma < 1$). Ultimately, it is a choice among a *well established* technology ("*baseline*") - characterized by low "implementation" costs - normalized to 0 - and variable costs of production c - and, an *innovative one* - featuring lower variable costs (γc), but higher fixed cost of adoption (f_I). The trade-off being between *efficiency-implementation costs*, much like of the *proximity-concentration* trade off for horizontal FDI in HMY (2004).

We are assuming that technological uncertainty and heterogeneity of the Melitz-type relates to what we have called a "*baseline*" technology, reflecting that firms have to learn about their market and their productivity before they can plan to improve it. Having found out about their idiosyncratic productivity, all firms face the option of adopting an alternative technology, what we have referred

[2] A is an endogenous variable to be determined in equilibrium, but it is a constant from the point of view of an individual supplier because of the monopolistic competition assumptions. Indeed, each variety supplier ignores that her behavior can affect the price or the quantity index, and therefore it takes A as given when it maximizes its profits.

[3] Clearly, this technology exhibits increasing return to scale. f_D can be thought as all those activities like marketing or setting up a sales network which are independent of the scale of production. Then, it can be seen as the fixed cost of serving the domestic market. The inverse of c is a measure of a firm's productivity in the production process.

to as the "*innovative*" one. While the extra fixed cost is the same for each firm, the reduction in variable cost is proportional to the firm's idiosyncratic "marginal cost draw" given from its own entry. Since the Melitz-type entry leads to heterogeneity in variable cost, the technological option results also differently attractive for different firms, relative to their "*baseline*" technologies. This could be rationalized as some firms being more successful than others in implementing the new technology. Indeed, technology-adoption requires an active engagement of the adopter - namely a series of investments undertaken by the adopter - beyond the selection of which technology to adopt. These investments are often label "technology implementation process" which are in the data the main source of site-to-site variations in the success (productivity) of the adopter, better implementation makes new technologies more productive.[4]

The consumers may benefit from this form of innovation in the form of a reduction of good prices. We shall refer sometimes to this reduction of costs in the production stage with an abuse of terminology as *process* or *vertical innovation* .

Finally, as in Melitz (2003) every incumbent faces a constant (across productivity levels) probability δ in every period of a bad shock that would force it to exit.

Prices and Profits

A producer of variety ω with labour-output coefficient c faces the demand function (1) and charges the profit maximizing price:

$$p(\omega) = \frac{\sigma}{\sigma - 1} wc \equiv p_D(c) \tag{3}$$

where $\frac{\sigma}{\sigma-1}$ is the constant markup factor and w is the common wage rate, hereafter taken as the numeraire ($w = 1$). The effective price (3) charged to consumers by non-innovator is higher than the price $p_I(c) = \gamma p_D(c)$ charged by an innovator. Substituting this and (3) in (1), the output of a non-innovator is:

$$q(\omega) = A \left[\frac{\sigma}{\sigma - 1} c \right]^{-\sigma} \equiv q_D(c) \tag{4}$$

and likewise, $q_I(c) = \gamma^{-\sigma} q_D(c)$ for an innovator. Therefore, the profit of firm type D (producer with a "traditional" technology) and firm type I (firm with innovative technology) are:

$$\pi_D(c) = \frac{r_D(c)}{\sigma} - f_D = Bc^{1-\sigma} - f_D \tag{5}$$

$$\pi_I(c) = \frac{r_I(c)}{\sigma} - f_D - \delta f_I = B(\gamma c)^{1-\sigma} - f_D - \delta f_I \tag{6}$$

where $r_s(c) = p_s(c) q_s(c),\ s = D, I$ is the revenue of firm type s and $B = (1/\sigma)\frac{R}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$ is taken as a constant by a single producer and it represents the level of demand in the country. The innovation cost f_I into the profit function is weighted by the exogenous probability of exiting, because the innovation decision occurs after firms learn about their productivity c and since there is no additional uncertainty or time discounting other than the exogenous probability of exiting, firms are indifferent between paying

[4] See Comin (2007) and Bikson et. al. (1987).

the one time investment cost f_I or the per-period amortized cost δf_I. We shall adopt the latter notation for analytical convenience.

For illustrative purpose, let us consider in figure 1 the profit profiles associated to the two possible technology choice. From the prospect of a single firm, (5) and (6) are linear in $c^{1-\sigma}$ which can be interpreted as a firm's productivity index: the higher it is, the greater the productivity of a firm.[5] Given the fix overhead cost of innovation and that the profit of an innovator is always steeper than a non-innovator's one, technology adoption will be profitable only for high-productivity firms. Firms with draws below $(c_o)^{1-\sigma}$make negative profit and have to exit, while firms with productivity index above $(c_o)^{1-\sigma}$ entry successfully. Only a fraction of these firms ($c^{1-\sigma} \geq (c_I)^{1-\sigma}$), perform also process-innovation.

Using (4) and (3), we have the ratio of any two firms's output and revenues only depend on the ratio of their productivity levels:

$$\frac{q(c_1)}{q(c_2)} = \left[\frac{c_1}{c_2}\right]^{-\sigma}, \quad \frac{r(c_1)}{r(c_2)} = \left[\frac{c_1}{c_2}\right]^{1-\sigma} \tag{7}$$

(7) has some interesting implications. First, dividing numerator and denominator of the quantity ratio by Q and the numerator and the denominator of the revenue ratio by R, we can conclude that relative market shares of the firms depends only on the cost ratio and are independent of aggregate variables. Second, $r_I(c)/r_D(c) > 1$, that is rent increases more than proportionally following the introduction of process innovations.[6]

Denote by M_I and M_D respectively the mass of active innovator and domestic (non-innovator) producers, where

$$M_I = \frac{G(c_I)}{G(c_o)} M \tag{8}$$

$$M_D = \frac{G(c_o) - G(c_I)}{G(c_o)} M \tag{9}$$

and M is the mass of incumbent firms in the economy. $\frac{G(c_I)}{G(c_o)}$ ($\frac{G(c_o)-G(c_I)}{G(c_I)}$) is the ex-ante (prior to entry) probability of being an innovator (non innovator). In other words, it represents the probability for a potential entrant to innovate (to entry). By the law of large numbers, it also represents the fraction of innovating (not-innovating) firms in the economy.

$M = M_I + M_D$ is also the total mass of available varieties to the consumers in this closed economy.

2.1 Equilibrium in a closed economy

We are interested in a stationary equilibrium where the aggregate variables must also remain constant over time. This requires a mass M_e of new entrants in every period, such that the mass of successful entrants, $M_e G(c_o)$, exactly replaces the mass δM of incumbents who are hit by the bad shock and exit, as in Melitz (2003).

[5] B is an endogenous variable of the model and it is a non linear function of c. However, from a single firm's prospect, B is taken as given and therefore, it can be treated as a constant. This graph can not be used for comparative statistic or to pin down equilibrium values, but it is useful to understand the behavior of a firm with a productivity draw c.

[6] Note that $r_I(c)/r_D(c) = \gamma^{1-\sigma} r_D(c)/r_D(c) = \gamma^{1-\sigma} > 1$, since $\sigma > 1$ and $\gamma < 1$.

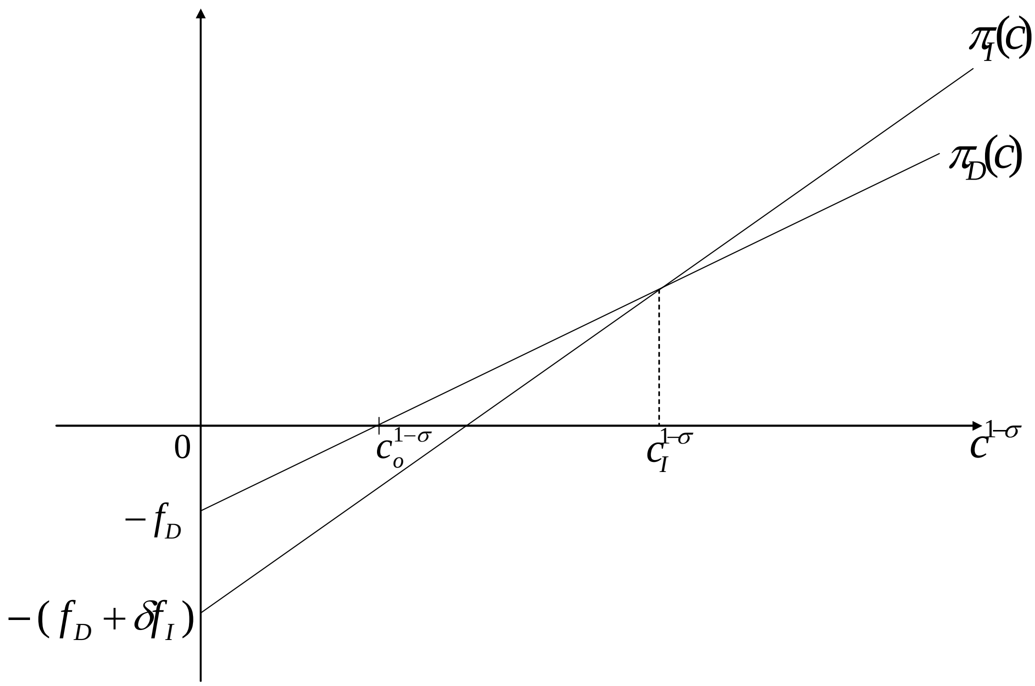

Figure 1: Profits from producing and innovating on the domestic market.

The equilibrium entry cost-cutoff c_o and innovation cost-cutoff c_I must satisfy:

$$\pi_D(c_o) = 0 \Longleftrightarrow B\,(c_o)^{1-\sigma} = f_D \tag{10}$$

$$\pi_I(c_I) = \pi_D(c_I) \Longleftrightarrow (\gamma^{1-\sigma} - 1)B\,(c_I)^{1-\sigma} = \delta f_I \tag{11}$$

Since their productivity is unrevealed upon entry, firms will compare the expected profit in the industry with the entry cost, taking into account the possibility of being hit by a bad shock. Free entry ensures the following equality :

$$\sum_{t=0}^{\infty}(1-\delta)^t\left[\int_0^{c_I}\pi_I(c)dG(c) + \int_{c_I}^{c_o}\pi_D(c)dG(c)\right] = f_E$$

or:

$$\delta f_E = \int_0^{c_I}\pi_I(c)dG(c) + \int_{c_I}^{c_o}\pi_D(c)dG(c) \tag{12}$$

which states that firms equate the per-period expected profit from entering and the equivalent amortized per-period entry cost. To develop a better intuition of (12), let us denote by $\overline{\pi}$ the average industry profit and note that $\int_0^{c_I}\pi_I(c)dG(c) + \int_{c_I}^{c_o}\pi_D(c)dG(c) = G(c_o)\overline{\pi}$ - in words, the expected average profit in the industry is the average profit in the industry ($\overline{\pi}$) times the ex-ante probability of entry ($G(c_o)$) (see (40) in the appendix), so that (12) becomes:

$$\overline{\pi} = \frac{\delta f_E}{G(c_o)} \tag{13}$$

It shows that following an increase in the per-period entry cost δf_E, firms are willing to enter if they can expect either a higher per period average profit or greater chances of entry (higher c_o).

(10) to (12) characterize the equilibrium cost-cutoffs c_o and c_I as well as B.

Combining (10) with (11) we have the relation between the innovation and the entry cutoff:

$$(c_I)^{1-\sigma} = \frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{f_D}(c_o)^{1-\sigma} = \Psi (c_o)^{1-\sigma} \tag{14}$$

where $\frac{\delta f_I}{\gamma^{1-\sigma}-1}$ is the cost to benefit ratio of innovation. The numerator is the per-period cost of innovation while the denominator represents the revenue differential of innovation per unit of revenue initially earned. It follows that a necessary and sufficient condition to have selection into the innovation status is $\Psi > 1$, which is assumed to hold throughout since the empirical evidence suggests that only a subset of more productive firms undertakes process innovations[7].

Given (14), (12) is a function of only c_o. The equilibrium is depicted in figure 2, where the flat line δf_E crosses the LHS of (12) which is monotonically increasing from 0 to infinity in c, as proved in the appendix. The graph clearly highlights that c_o has to rise when the fixed cost of entry increases, as discussed above.

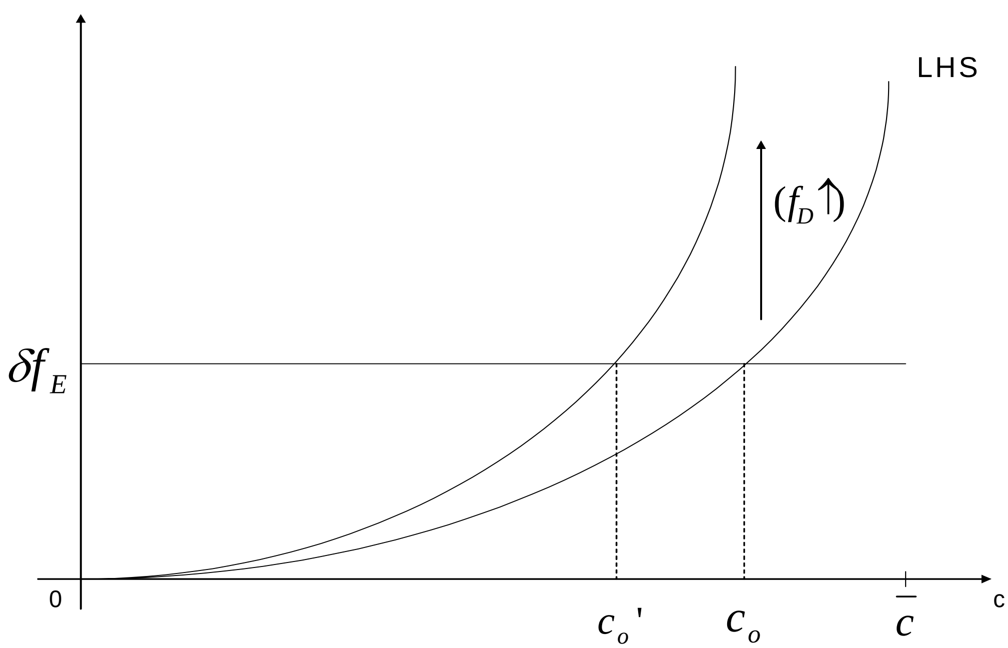

Figure 2: Determination of the equilibrium entry cost cutoff as given by the Free Entry Condition

More interesting is an increase of f_D - the degree of increasing return to scale - or, alternatively, conceivable as the cost of staying in the industry. Its increase makes survival into the industry harder for all firms and, therefore, it ultimately determines selection in our economy. This thought experiment has strong analogies to the effects of trade liberalization in our open economy analyzed next, as - in common - they both have that they make survival of domestic firms harder (selection effect). More

[7] See for instance Parisi et. al. (2005) for evidence on Italian firms and Baldwin et al. (2004) for evidence on Canada.

specifically, a greater f_D lowers Ψ, but it also shifts up the LHS curve in fig. 2, so that it reduces the entry cost-cutoff to $c_o^{'}$ (see(41) in the appendix). Overall, the effect of an increase in f_D on the innovation productivity cutoff $(c_I)^{1-\sigma}$ is ambiguous since Ψ is lower, but $(c_o)^{1-\sigma}$ is larger. This ambiguity is a specific-feature of a general equilibrium model, whereas in partial equilibrium the effect of f_D would be well determined and would affect the economy only through Ψ. The intuition comes from inspecting (5) and (6). A larger f_D reduces the profits of all firm types in the economy for any given c, forcing the least productive firms out of the market given that they are unable to recoup the increased fixed cost of operation. *Ceteris paribus* (for a given c_I), the price index P - reflecting firms interactions and competition in the monopolistic-competitive market - increases (see the appendix) because of firm exiting and reduced competition at the industry level. Incumbent firms, including innovators, benefit in terms of higher profits, as captured by a higher B. This is the pro-innovation effect through selection behind the reduction of Ψ. Selection reduces the mass of firms in the market, making possible for incumbent firms to expand their output and, consequently, increase their profits. In turn, higher profits incentive technology adoption and trigger entry by prospective firms responding to higher potential returns associated with a good productive draw. The increased labour demand by the more productive firms and new entrants puts upward pressure on the labour-factor market, leading to a higher real wage $(1/P)$. This feeds back to firms' profits, reducing them again through the B. This is the general equilibrium effect via the factor market behind the rise in $c_o^{1-\sigma}$ and it would be absent in partial equilibrium since the equilibrium wages are unchanged.

These two offsetting forces on the innovation activity will also be at play in the more complex scenario of an open economy which undergoes through trade liberalization.

f_D has ambiguous effects also on the number of varieties in the economy, whereas in Melitz (2003) increasing f_D unambiguously reduces the number of firms in the the industry. As shown in the appendix, the number of varieties is:

$$M = \frac{R}{\overline{r}} = \frac{L}{\sigma(\overline{\pi} + f_D + \frac{G(c_I)}{G(c_o)}\delta f_I)} \tag{15}$$

so that when f_D rises, a larger $\overline{\pi}$ and $G(c_o)$ contribute to reduce the number of varieties[8]. However, only when c_I rises, the total number of firms unambiguously declines.

There is an other difference between our economy and the economy in Melitz (2003), namely the entry productivity cutoff level is higher in this setting.[9] The possibility to innovate allows the most efficient firms that perform process innovation to "steal" market shares to the least efficient firms for which is harder to survive into the market. Consequently, our economy is more efficient, because some varieties are produced at a lower cost, but less varied because some varieties have disappeared. This trade-off has been well emphasized in the growth literature (see Peretto (1998) and more recently Gustafsson and Segerstrom (2006)).

[8] Recall that a larger f_D entails a lower c_o. A lower c_o translates into higher $\overline{\pi}$ - by (13) - and lower $G(c_o)$.

[9] The proof of this result has been left to the appendix.

3 The Open Economy

Let us assume that the economy under study can trade with other $n \geq 1$ symmetric countries. We will assume that trade is not free, but it involves both fixed and variable costs, since free trade could simply be analyzed by doubling L in the closed economy. One can think of the fixed cost associated to trade as the cost of customizing its own variety to the regulations and tastes of foreign countries as well as of creating sale-networks. The variable trade costs are trade barriers such as transportation costs imposed by distance. We follow a long tradition in the trade literature and model these variable costs in the iceberg formulation: $\tau > 1$ units of a good must be shipped in order for 1 unit to arrive at destination.

Finally, the symmetry of countries is required to ensure that factor price equalization holds and countries have indeed a common wage which can be still taken as the numeraire. Alternatively, a freely traded homogenous good produced under constant return to scale could be introduced to pin down its price and thus the wage to unit in all countries. The symmetry assumptions also ensures that all countries share the same aggregate variables.

Prices,Profits and Firm-Types

The variable costs of trade are naturally reflected into the price charged by the domestic exporters into foreign markets. By symmetry, the imported products are more expensive than domestically produced goods due to transportation costs. As a result, the effective consumer price for imported products from any of the n countries is:

$$p_X(c) = \tau p_D(c) \tag{16}$$

while an exporter who has opted for process innovation charges:

$$p_{XI}(c) = \gamma p_X(c) \tag{17}$$

Analogously, the profits of an exporter and an innovator-exporter in a foreign market are[10]:

$$\pi_X(c) = \tau^{1-\sigma} B c^{1-\sigma} - \delta f_X \tag{18}$$

$$\pi_{XI}(c) = (\gamma\tau)^{1-\sigma} B c^{1-\sigma} - \delta f_X \tag{19}$$

where δf_X is the amortized per-period fixed cost of the overhead fixed cost f_X that firms have to pay (in units of labour) to export to foreign markets.

The following table summarizes the profit function for all possible firm-types with productivity c.

type	Domestic Producer	Exporter
Non Innovator	$\pi_D(c)$	$\pi_D(c) + n\pi_X(c)$
Innovator	$\pi_I(c)$	$\pi_I(c) + n\pi_{XI}(c)$

[10] $r_S(c) = p_S(c)q_S(c),\ S = X\ ,XI$. Note that $r_{XI}(c) = \gamma^{1-\sigma} r_X(c) = \tau^{1-\sigma} r_I(c)$ as well as $r_X(c) = \tau^{1-\sigma} r_D(c)$. So, $\pi_X(c) = \frac{r_X(c)}{\sigma} - \delta f_X = \frac{\tau^{1-\sigma} r_D(c)}{\sigma} - \delta f_X = \tau^{1-\sigma} B c^{1-\sigma} - \delta f_X$ and $\pi_{XI}(c) = \frac{r_{XI}(c)}{\sigma} - \delta f_X = \frac{(\gamma\tau)^{1-\sigma} r_D(c)}{\sigma} - \delta f_X = (\gamma\tau)^{1-\sigma} B c^{1-\sigma} - \delta f_X$.

Note we account for the entire overhead production cost in the domestic profit (see (5) and (6)). This choice is uninfluential for the equilibrium as all firms (domestic producers and exporters) will produce also for the domestic market and incur f_D upon staying into the industry.

No firm will ever export and not also produce for its domestic market. Indeed, any firm would earn strictly higher profits by also producing for its domestic market since the associated variable profit $r_D(c)/\sigma$ is always positive and the overhead production cost f_D is already incurred. Then, all exporters' profits can be separated into the portion earned domestically ($\pi_D(c)$ or $\pi_I(c)$) and on each of the foreign market ($\pi_X(c)$ or $\pi_{XI}(c)$). Moreover, since the export cost is assumed equal across countries, a firm will either export to all n countries in every period or never export.

Finally, not all four types can coexist simultaneously in the economy, but which firm type is active will depend on the kind of selection. The empirical evidence suggests that exporting and innovation are performed by the most productive firms (lowest cost levels), while domestic producers are typically smaller, less innovative and less productive. Accordingly, we shall focus on the selections with the exporters or the innovators being the most productive types. In selection BW in figure 3, exporting is relatively cheaper than innovating and therefore only the more productive exporters can undertake vertical innovation: an innovating firm is necessarily an exporter (XI-type), but there are exporters that are not innovators (X-type).[11] Indeed, from (18) and (19) it easy to check that if the X-type is making positive profit from exporting, then also the XI-type does necessarily so. However, no innovator would produce and innovate just for the domestic market (no I-type) because given her high productivity she would give up positive profits from not meeting the foreign demand.

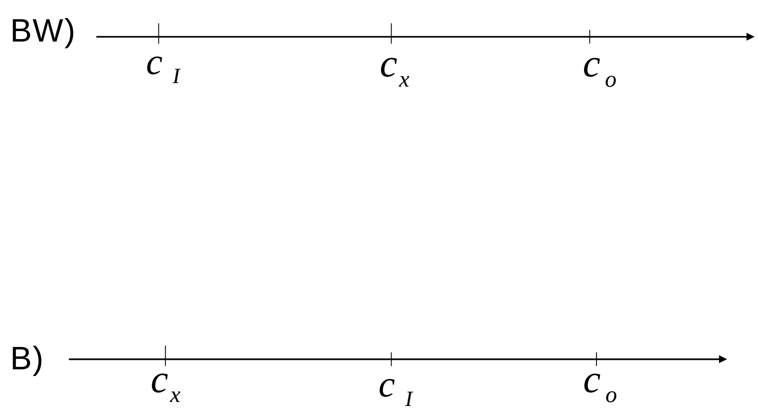

Figure 3: Plausible selections

On the contrary, in selection B only a fraction of incumbents innovate (I-type) and only a subset of innovators become exporters (XI-type). No firm will ever export without innovating (no X- type). Indeed, firms that can take advantage of profit opportunity abroad are already innovating on the domestic

[11] This is different from Yeaple (2005) where the firm type adopting the innovative technology is also necessary an exporter. In other words, the exporting firms coincides with the innovative types and therefore, no selection on the basis of innovation status is possible.

market. Therefore they will exploit their innovative technology to serve the foreign market as well.

BW is interesting because the marginal innovating firm is an exporter and trade is likely going to affect its innovation decision. B represents the other side of the same coin: the marginal innovating firm is a domestic producer and therefore, innovation is mostly determined by domestic factors and will less likely respond to trade liberalization.

Given the aim of the paper, we focus closely on selection BW where trade induces within-plant productivity changes besides allocative effects of market shares. Roughly stated, trade will have "*between*" and "*within*" effects on productivity growth (from here BW). Then, we turn to discuss briefly selection B and highlight why trade is not influential on plants' innovation activity. In this equilibrium, trade affects productivity only through allocative effects - *between* effect (form here B).

3.1 Selection BW

Let us denote by M_D the mass of active incumbent firms with a local dimension only, by M_X the mass of exporting not innovating firms and by M_{XI} the mass of exporting and innovating firms. The sum of all these firms ($M_D + M_X + M_{XI} = M$) gives the mass of incumbent firms in any country. The mass of non-innovating incumbent firms in any country is $M_{NI} = M_D + M_X$, while $M_T = M_D + n(M_X + M_{XI})$ gives the total mass of varieties available to consumers in any country. Let $p_{rD} = [G(c_o) - G(c_X)]/G(c_o)$, $p_{rX} = [G(c_X) - G(c_I)]/G(c_o)$, $p_{rXI} = [G(c_I)]/G(c_o)$ be the probability of becoming each type conditional on being an incumbent.

The equilibrium - BW

We are again interested only in a stationary equilibrium where all aggregate variables are constant over time. The stability condition imposes the entrants into the industry replaces exactly exiting firms, i.e. $\delta M = M_e G(c_o)$. Note that the equilibrium value of the aggregate variable Q, R, and therefore A and B as well as of the entry cutoff c_o is different in this equilibrium from the closed economy one. Nevertheless we stick to same notation as they are defined in the same way.

Cutoffs in **equilibrium BW** must satisfy the following conditions:

$$\pi_D(c_o) = 0 \Leftrightarrow \frac{r_D(c_o)}{\sigma} = B\,(c_o)^{1-\sigma} = f_D \tag{20}$$

$$\pi_X(c_X) = 0 \Leftrightarrow \frac{r_D(c_x)}{\sigma} = B c_X^{1-\sigma} = \frac{\delta f_X}{\tau^{1-\sigma}} \tag{21}$$

$$\pi_I(c_I) + n\pi_{XI}(c_I) = \pi_D(c_I) + n\pi_X(c_I) \Leftrightarrow \frac{r_D(c_I)}{\sigma} = B\,(c_I)^{1-\sigma} = \frac{\delta f_I}{(\gamma^{1-\sigma} - 1)(1 + n\tau^{1-\sigma})} \tag{22}$$

Thus the parameter restriction that sustains this equilibrium ($c_I \le c_X \le c_o$) where only exporters perform process innovation must satisfy:

$$\frac{\delta f_I}{(\gamma^{1-\sigma} - 1)} \frac{1}{(1 + n\tau^{1-\sigma})} \ge \delta f_X \tau^{\sigma-1} \ge f_D \tag{23}$$

This condition requires that the innovating is relatively more expensive than exporting. That is, the foreign markets should be fairly accessible, otherwise serving them would result extremely costly and it could be afforded exclusively by the most productive firms.

$\frac{\delta f_I}{(\gamma^{1-\sigma}-1)}$ is equivalent to the cutoff for innovation for the closed economy: the same assumptions that guarantees selection on the basis of innovation status in the closed economy (i.e., $\Psi \geq 1$) ensures that this term is positive and bounded away from zero in the open economy. Recall that this term represents the cost to benefit ratio of innovation. Importantly, in the open economy we have an extra term given by $\frac{1}{(1+n\tau^{1-\sigma})}$ which is unity in the closed economy (set $n = 0$ or $\tau \to \infty$). The denominator represents precisely the further revenue differential associated to innovation on each of the foreign markets that become available with trade.

We like to think of n as the number of countries into the trading network sharing a common code of rules as it could be for the WTO membership. Then, it represents a measure of the world's openness to trade, as for a low n very few countries have trading relations. $\phi = \tau^{1-\sigma} \in [0, 1]$ is commonly referred in the literature as an index of the freeness of trade with values closer to 1 indexing freer trade.

Clearly, trade liberalization that come in the form of either freer trade (greater ϕ) or greater world openness (larger n) can affect process innovation weighing upon the return of innovation.

(20) to (22) give a system of 3 equations in 4 unknowns (c_o,c_X,c_I,B). We can use the FE condition to close this system and uniquely determine the entry cutoff. The FE condition ensures the equivalence between expected entry profit and entry cost:

$$\int_{c_X}^{c_o} \pi_D(c)dG(c) + \int_{c_I}^{c_X} (\pi_D(c) + n\pi_X(c))dG(c) + \int_{0}^{c_I} (\pi_I(c) + n\pi_{XI}(c))dG(c) = \delta f_E \tag{24}$$

Combining appropriately the three conditions for the cutoff points ((20) to (22)), the relation between the cutoffs can be written explicitly as:

$$(c_I)^{1-\sigma} = \frac{\delta f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})} \frac{1}{f_D} (c_o)^{1-\sigma} = \Psi^f (c_o)^{1-\sigma} \tag{25}$$

$$(c_I)^{1-\sigma} = \frac{\delta f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})} \frac{1}{\delta f_X \tau^{\sigma-1}} c_X^{1-\sigma} = (\Psi_X^f) c_X^{1-\sigma} \tag{26}$$

$$c_X^{1-\sigma} = \frac{\delta f_X \tau^{\sigma-1}}{f_D} (c_o)^{1-\sigma} \tag{27}$$

Note that $\Psi = \Psi^f(1 + n\tau^{1-\sigma})$ and $\Psi_X^f = \Psi^f f_D/\delta f_X \tau^{\sigma-1}$. $\Psi \geq \Psi^f$ - namely, the cost to benefit ratio is smaller in the trading equilibrium than in autarky - reflects that trade and vertical innovation are related: new market opportunities abroad induce exporters to expand their scale of operation and the benefits of cost-reducing innovation are spread on a greater number of units sold, while the up-front cost of innovation is unchanged.[12] Comparing (25) with (14) shows that the distance between the entry productivity index cutoff and the innovation productivity index cutoff is always smaller in the trading equilibrium than in autarky ($\Psi^f < \Psi$), as trade reduces the relative cost of innovation. Hence, trade (for positive n and non-prohibitive transportation cost τ) reduces, *ceteris paribus,*the innovation productivity cutoff $(c_I)^{1-\sigma}$and therefore it boosts *within-plant innovation.* This is the partial equilibrium effect described also in Bustos (2005).[13]

[12] Only for prohibitive trade barriers ($\phi = 0$) or a close world ($n = 0$), $\Psi = \Psi^f$.

[13] This situation would describe an industry within the economy which is small enough to affect the equilibrium price index of the economy, and therefore, real wages and where no entry and exit takes place.

However, this is not enough for concluding the proportion of incumbents undertaking productivity innovation will be larger after trade. In general equilibrium, trade affects also the entry productivity cutoff $(c_o)^{1-\sigma}$ which results higher in the trading equilibrium than in autarky, as it is shown in the appendix.

In this equilibrium, two forces are affecting the innovation cost cutoff when the economy opens to trade:

i) the selection effect of trade reduces the incentive to perform process innovation because entry is less likely and survival more difficult in a more competitive environment - lower c_o. Trade increases competition on the domestic market and forces the least productive producers out of the market (selection effect). The most hurt are obviously the domestic firms that produce exclusively for the national market whose product demand is reduced without being compensated by the expansion of their product demand on the foreign markets.

ii) conditional on being an incumbent, the benefit of cost-reducing innovation is higher after trade because the selection effect and the scale effect together increase exporters' total market shares. Thus, some incumbent will start performing vertical innovation - $\Psi^f < \Psi$.

The overall effect of trade on innovation is ambiguous and depends on the relative strength of these pushing and deterring factors of process innovation, similarly to the effect of an increase in f_D above. Although the proportion of incumbents is reduced (lower c_o), the proportion of innovating firms among them will raise (higher c_I) if the *ii)* dominates *i)*, namely if the adjustments through the extensive margin of innovation dominate those through the extensive margin of trade.

In order to shed some light on which effect dominates, we use a specific parametrization for $G(c)$. We shall show that the net outcome of these two offsetting forces is a higher proportion of firms performing process innovation with freer trade.

Assuming that the productivity draws $(1/c)$ are distributed according to a Pareto distribution with low productivity bound $(1/\overline{c})$ and $k \geq 1$, the c.d.f of cost draws c is given by:

$$G(c) = \left(\frac{c}{\overline{c}}\right)^k, \;\; k > \sigma - 1, \;\; k > 2. \tag{28}$$

This formulation has been used widely in many extensions of Melitz (2003) because it allows to derive closed form solutions for the cutoff levels.[14] k is a shape parameter indexing the dispersion of cost draws. $k = 1$, corresponds to the uniform distribution. As k increases, the distribution is more concentrated at higher cost level and firms' heterogeneity is reduced. $k > 2$ ensures that the second moment of the distribution is well defined, while $k > \sigma - 1$ ensures the first moment of the truncated distribution ((31) and (32) in the Appendix) exists and is well defined. With this assumption, we are able to prove trade liberalization favours technology adoption by some exporters, as established in the following proposition.

[14] See for example Melitz and Ottaviano (2005).

Proposition 1 *Denote with c_I^A (c_I^f) the equilibrium innovation cost cutoff in autarky (in the open economy). If (28) and (23) hold, then the innovation cost cutoff in the open economy is larger than in autarky (i.e $c_I^A < c_I^f$)*

Proof. *See appendix.* ■

Interestingly the reallocation of output across plants induced by trade - *between effect* - is playing a key role and is related to f_X. In absence of it and with CES preferences, trade liberalization does not have an impact on c_I. Without selection to export markets ($f_x = 0$), all firms exports and perfectly compensates for the loss in the domestic market shares with gains in foreign market shares, since the increase in each firm's market size after trade is exactly offset by the rise in the number of competitors. This can be easily checked inspecting the equilibrium conditions. With $f_x = 0$, (20) becomes $(1 + n\tau^{1-\sigma})Bc_o^{1-\sigma} = f_D$ which together with (22) and (24) characterize the equilibrium and determines (c_o, c_I, B). It is easy to show that such equilibrium is equivalent to the autarky one described by (10)-(12), so that no firms using the baseline technology opts to implement the innovative technology after engaging in trade. However, when $f_X > 0$, there is selection on the export-status with only some firms engaging in exporting, while the least productive ones having only a domestic dimension. This means that, following trade liberalization, the increase in market size for the exporting firms is not longer offset by the raised number of competitors. As opposed to $f_x = 0$, some domestic exporters are enjoying a larger slice of the foreign market (and higher revenues from foreign market) as they are not facing the competition from the actual domestic producers (previously also exporting) and, at the same time, (by symmetry) are confronted with less foreign competitors on the national market. This is the basic economic intuition behind *ii)* above and it is strictly related to the existence of fixed trade costs.

Nevertheless, firms willing to engage in trade and incurring f_X, as well as firms switching to the "*innovative*" technology and incurring f_I exacerbates the competition for the scarce labour input pushing up the real wage, making survival tougher, and exporting and innovating more costly (*i*, above). This deterring effect of innovation occurs through the input-factor market and, therefore, only in a general equilibrium setting. It would be necessarily absent in a partial equilibrium approach since the equilibrium wage in the industry are unaffected, as in Bustos (2005).

Yet, trade translates into net gains for the most productive non-innovating exporting firms, inducing them to implement the innovative technology, as we can conclude from showing that *ii* is dominating *i*. As low productive domestic firms exit, their market shares are reallocated to the more productive surviving incumbents, and thus, also to some domestic exporters (*extensive margin effect* or *selection effect*). This effect adds up to the *intensive-margin effect* or *scale effect* - that following trade liberalization, some exporters have increased their market share abroad. As a result, their combined market share (the sum of the domestic and foreign market shares) enlarges. Since a larger scale of operation is associated to a greater return of the "technological option", a larger fraction of them finds profitable to implement the innovative equipment. In other words, trade affects the extensive margin of innovation inducing exporting firms that are not as productive as former innovators, to adopt more productive technologies.

We would expect that the reduction of transportation costs which lead to trade creation in this model have similar effects on innovation, consistently with the evidence in Bernard et. al. (2006). This is established in the following Lemma.

Lemma 2 *Assume (28) and (23) hold,* $dc_I/d\tau \leq 0$*.*

Proof. *See in the appendix* ■

Its relevance is that trade liberalization taking the form of partial tariff reform, as often it is in practice, induce similar positive effect on process innovation. For instance, we can evaluate the effects of Canada-US FTA (CUSFTA) on within firm performances. τ in (25) is the transportation cost faced by Canadian manufacturing firms exporting to US. The model predicts US tariff concessions granted to Canada - a reduction of τ - after the FTA would induce some Canadian exporters to innovate, as they can take advantage of a lower cost to benefit ratio. This is consistent with the evidence shown in Trefler (2004). The numbers are quite substantial: "*U.S. tariff concessions raised labor productivity by 14 percent or 1.9 percent annually in the most impacted, export-oriented group of industries*". Bustos (2005) find evidence of adoption of innovative technology by Argentinean manufacturing exporting firms following the substantial trade liberalization of the country in the 90s. Interestingly, firms adopting the innovative technology are the high productive non-innovating exporters, while the low productive exporters keep the "traditional" technology even after trade liberalization, providing empirical support for the relevance of selection BW analyzed here.[15]

Summing up, by increasing the scale of production of some of the exporters, trade increases what Cohen and Klepper (1996) call the "ex ante" output - the firm's output when it conducts process innovation. This, in turn, raises firms' incentive to innovate and triggers process-innovation, productivity increments and market share growth at firm level (see (7)). This is consistent with Baldwin and Gu (2003) and Trefler (2004) who find that within-firm productivity increments have occurred mostly among exporters. Moreover, Baldwin (2004) finds empirical support for such casual link: vertical innovation is a main determinant of productivity growth and productivity growth induces market share growth[16].

Finally, the reduction of transportation costs has contrasting effect on c_X too. A reduction of trade barriers have a direct effect and lowers the exporting productivity cutoff $c_X^{1-\sigma}$ (see (27)), but also an indirect effect through $(c_o)^{1-\sigma}$ which rises this threshold. The following lemma shows that the direct effect dominates the indirect one.

Lemma 3 *Assume (28) holds,* $dc_X/d\tau \leq 0$*.*

Proof. *See the appendix* ■

[15] Also in Yeaple (2005), lower transportation costs induce a greater adoption of the innovative technology. However, no exporters retain the old technology as found in Bustos (2005).

[16] Baldwin (2004) finds Canadian process-innovators had productivity growth that was 3.6 percentage points higher than Canadian non-process innovators (table 9). Moreover, a within-firm productivity increment of 10% relative to the industry average translate into almost 2% gain in the firm's market share (table 12).

In the context of CUSFTA, this lemma predicts that some Canadian manufacturing firms which are not as productive as established exporters, will also start to serve the US market in virtue of the American preferential tariff reform. Interestingly, Baldwin et al. (2003) find evidence of this.

3.2 Selection B

We shall just show that trade in this equilibria can not affect the extensive margin of innovation as for selection BW. The non-innovating firms are only the D-type, while the innovating firms are the I-type and the XI-type, but only the latter are present on international market. There is no X-type.

The cutoff conditions for **equilibrium B** are:

$$\pi_D(c_o) = 0 \tag{29}$$

$$\pi_I(c_I) = \pi_D(c_I) \tag{30}$$

$$\pi_{XI}(c_X) = 0 \Leftrightarrow Bc_X^{1-\sigma} = (\tau\gamma)^{\sigma-1}\delta f_X$$

which imply that the necessary and sufficient condition for $c_X \leq c_I \leq c_o$ is:

$$\delta f_X \tau^{\sigma-1} \geq \frac{\delta f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma} \geq f_D\gamma^{1-\sigma}$$

This equilibrium is characterized by a trading cost relatively higher than the innovating one. High variable and fixed cost of exporting make trading a very expensive activity performed only by the most productive firms.

Note also that (29) and (30) imply the same relation among the innovation and the entry cutoff as in the closed economy given by (14). Indeed, the marginal innovating firm is not an exporter and the transition from autarky to trade leaves the cost to benefit ratio of innovation unchanged. That is, trade liberalization can not affect and stimulate firms' innovation investments because it has no impact on Ψ. In other words, the extensive margin of innovation responds to lower trade barriers uniquely through the selection effect; consequently, a raise in $c_o^{1-\sigma}$ raises $c_I^{1-\sigma}$ as well and depresses vertical innovation.

3.3 Final Remarks

The model has implications on the aggregate productivity level. As in Melitz (2003), the industry average productivity will be rising in the long run by means of the selection effect which spells the least efficient firms out of the market - *between effect* . Moreover, in our model trade will rise the average industry productivity through a further channel, namely the *within effect* (Proposition 1 and Lemma 2). Following trade liberalization, some of the exporters opt for implementing a more efficient technology, improving their productivity level. The right shift of the Canadian productivity distribution of manufacturing firms in 1996 compared to 1988 following the Canada-U.S. FTA documented in Trefler (2005) can be interpreted as the combination of the *between effect* and of the *within effect.* Low productive firms - below the industry average - that either exit or downsize following trade liberalization - *between component* - determine a thinner left tail of the distribution in 1996 relative to 1988. Analogously, the reallocation of market shares favouring high productive firms has contributed to a fatter right tail of the distribution of 1996.

The exporters who have raised their plant productivity (within component) significantly determine the increased mass on medium and high productivity levels for the distribution in 1996 relative to the one in 1988.

Moreover, such liberalization encourages also new Canadian exporters that are less productive than old Canadian exporters to enter the US market (Lemma 3). This must reduce the industry average productivity as the expansion in the US market increases the market share of lower productivity new exporters.

Finally the model suggests that trade liberalization and the geography of a country can interact each other: the same trade liberalization may induce different innovation outcomes depending on the location of a country.

Moving from B to BW, the cost of exporting relative to the cost of innovating decreases. This means that the effect trade has on the process innovation will be differentiated according to the level of transportation cost. We shall interpret high transportation cost as a proxy for the remoteness of the Home economy from the main exporting markets or, more generally, as the level of trade barriers faced by the Home country.

If in the transition from autarky to trade, the country is fairly remote and faces selection B, then process innovation performed will be reduced, as discussed above. On the contrary, if the country is close to the exporting markets and selection BW is possible, process innovation increases.

4 Caveats and Further Research

We have modeled the process innovation very simply as a binary decision - adopt/not adopt the new productive technology. The benefit and the cost of innovation are known and exogenously given. This introduces two major limitations.

First, more innovation in this economy is measured by the changes in the proportion of firms innovating and therefore is related uniquely to the extensive margin of innovation. In other words, the intensity of innovation is out of the model as firms do not decide upon their productivity target.

Second, all innovators improve their productivity in the same proportion. This means that the mass of firm with cost levels in the range $[\gamma c_I, c_I]$ has measure zero and the ex-post innovation cost distribution of incumbent-firms has a hole.

One way around the latter problem which preserves the innovation decision as exogenous would be introducing γ as a continuous random variable. Firms would pay the cost of innovation to draw a γ.

Instead, we are currently working to make the innovation decision endogenous: firms that opt for vertical innovation, choose optimally their γ balancing the benefits with the costs of innovation. Not only this avenue would solve the problem of the hole in the distribution, but it also allows to analyze how both the intensive and the extensive margin of innovation respond to trade liberalization.

Indeed, trade would affect both who is innovating and how much each firm is innovating. In equilibrium BW, the *within* effect would not be comprised of only the new innovators, but also of the former innovators investing more intensively in productivity increments. Interestingly, in equilibrium B, trade may continue to be unrelated to the extensive margin on innovation, but it still could affect the intensive margin inducing some of the innovators to innovate more. Thus, the dichotomy *within* and *no-within* effect proper of equilibrium BW and B is a specific feature of our setup and would not survive under this modification. Trade would affect the industry productivity growth through both the *within* and the *between* effect in both equilibrium. However, the degree of importance of the within effect would be different across the two equilibrium and only in equilibrium BW trade can likely weigh upon the extensive margin of innovation.

In spite of all these limitations, this set up highlight in a simple way the trade forces related to the within-firm productivity changes. Moreover, it is able to generate some predictions that are consistent with the available empirical evidence.

5 Conclusion

The paper introduces process innovation into the Melitz (2003) framework. As in Melitz (2003), trade has a selection effect on firms forcing the least productive ones out of the market and reallocating market shares to the more productive ones. Although this contributes to the aggregate productivity growth, it is not exhaustive of the effects of trade on productivity. We showed trade can favour the adoption of an innovative technology, especially among exporters.

One could think that fiercer competition implied by trade can reduce the incentive for innovation. This is certainly true for low productive domestic firms whose survival possibilities have decreased together with their market shares. Instead, exporters compensate the loss of market shares in the domestic market with gains in market shares in foreign markets. As they expand their scale of production, their incentive for process innovation strengthens and some of them introduce a more productive technology.

In productivity studies, this is the so called *within* effect - some of the incumbent firms update their productivity - and it is a main source of labour productivity growth in industrialized countries. This is the new insight of the model: trade contributes to the industry productivity growth through the *within* effect besides through the *between* effect. More generally, a greater openness in the trading relations can justify the finding of the great importance of the within component for the industry productivity growth, as recently documented.

We have shown that this effect is the net outcome of two offsetting forces: one favouring innovation and related to one side of trade liberalization, namely the opportunity of market expansion; the other one, deterring innovation and related to the other side of the coin, namely a tougher import competition. In particular, it was highlighted that the second effect is specific to general equilibrium as it comes through the interactions with the input-factor market.

Finally, geography plays an important role. Trade liberalization can depress vertical innovation (equilibrium B) for remote countries, while it can boost process-innovation (equilibrium BW) for countries closely located to the core of the exporting markets.

6 Appendix

6.1 Appendix A - Closed Economy

We first proceed with the aggregation to define the aggregate variables of the economy that firms take as given in their decisions (see (5) and (6)). Then, we turn to the analysis of the equilibrium and determine the entry cost-cutoff and the number of varieties.

6.1.1 Aggregation

Let us denote by $\mu_D(c)$ and $\mu_I(c)$ respectively, the cost distribution of domestic producers and active innovator prior to innovation. These "ex-ante"-innovation cost distributions are truncated distribution of $g(c)$:

$$\mu_D(c) = \begin{matrix} \frac{g(c)}{G(c_o)-G(c_I)} & , & c_I < c < c_o \\ o & & otherwise \end{matrix} \tag{31}$$

$$\mu_I(c) = \begin{matrix} \frac{g(c)}{G(c_I)} & , & 0 < c < c_I \\ o & & otherwise \end{matrix} \tag{32}$$

and will be used repeatedly in the aggregation. They are not affected by the exogenous productivity δ which affects all firms equally and it is independent of the productivity level. Finally, in order to derive the aggregated variable, it is useful to introduce some synthetic measures of the productivity index as the following averages:

$$\widetilde{c_D}^{1-\sigma} = \int_{c_I}^{c_o} c\mu_D(c)dc \tag{33}$$

$$\widetilde{c_I}^{1-\sigma} = \frac{1}{G(c_I)}\int_0^{c_I} c^{1-\sigma}\mu_I(c)dc \tag{34}$$

$$\widetilde{c}^{1-\sigma} = \frac{1}{M}\left[M_I(\gamma\widetilde{c_I})^{1-\sigma} + M_D\widetilde{c_D}^{1-\sigma}\right] \tag{35}$$

(33) is the weighted average productivity index within the subgroup domestic producers, while (34) is the "ex ante"-innovating weighted average productivity index among the subgroup of inventors[17]. Given that process innovation is simply modeled as a fixed proportional reduction in the cost level, the "ex-post" innovation weighted average productivity index will also be increased proportionally and be $(\gamma\widetilde{c_I})^{1-\sigma}$. Therefore, (35) is the weighted average productivity index of the economy which is an opportunely weighted average of the averages prevailing in each subgroup. All these measures are independent of the number of firms. $\widetilde{c}$ is the most aggregated productivity-index and completely summarizes all aggregate variables of the model derived below.

Using (3) for both a domestic and an innovator producer and the definition of the price index, it is

[17] Given that domestic (non-innovators) domestic producers do not increment their productivity level, the ex-ante and ex-post innovation cost distribution of domestic producers coincide.

possible to define the price index for this economy as:

$$\begin{aligned} P^{1-\sigma} &= \int_0^{c_I} M_I [p_I(c)]^{1-\sigma} \mu_I(c) dc + \int_{c_I}^{c_o} M_D \left[p_D(c)\right]^{1-\sigma} \mu_D(c) dc \\ &= (1/\rho)^{1-\sigma} \left[M_I \gamma^{1-\sigma} \widetilde{c_I}^{1-\sigma} + M_D \widetilde{c_D}^{1-\sigma} \right] \\ &= M \left[\frac{\widetilde{c}}{\rho} \right]^{1-\sigma} = M \left[p_D(\widetilde{c})\right]^{1-\sigma} \end{aligned} \tag{36}$$

Similarly,

$$\begin{aligned} R &= \int_0^{c_I} M_I r_I(c) \mu_I(c) dc + \int_{c_I}^{c_o} M_D r_D(c) \mu_D(c) dc \\ &= M_I r_I(\widetilde{c_I}) + M_{NI} r_D(\widetilde{c_D}) \\ &= M_I \gamma^{1-\sigma} r_D(\widetilde{c_I}) + M_{NI} r(\widetilde{c_D}) \\ &= M \left[\frac{M_I}{M} \gamma^{1-\sigma} r_D(\widetilde{c_I}) + \frac{M_{NI}}{M} r(\widetilde{c_D}) \right] = M r_D(\widetilde{c}) = M \overline{r} \end{aligned} \tag{37}$$

and

$$\begin{aligned} \Pi &= \int_0^{c_I} M_I \pi_I(c) \mu_I(c) dc + \int_{c_I}^{c_o} M_D \pi_D(c) \mu_D(c) dc \\ &= \frac{1}{\sigma} \left[\int_0^{c_I} M_I r_I(c) \mu_I(c) dc + \int_{c_I}^{c_o} M_D r_D(c) \mu_D(c) dc \right] - M f_D - M_I \delta f_I \\ &= M \frac{r_D(\widetilde{c})}{\sigma} - M f_D - M_I \delta f_I = M \underbrace{\left[\frac{r_D(\widetilde{c})}{\sigma} - f_D - \frac{M_I}{M} \delta f_I \right]}_{\pi_D(\widetilde{c})} = M \underbrace{\left[\frac{\overline{r}}{\sigma} - f_D - \frac{M_I}{M} \delta f_I \right]}_{\overline{\pi}} \end{aligned} \tag{38}$$

where $\overline{r}$ and $\overline{\pi}$ are the average revenue and profit in the economy.

In a proof below, we shall use the average profit in autarky and we shall use the convention that the variable with superscript A denote the equilibrium variables in the closed economy equilibrium. For example the average profit in autarky will be:

$$\overline{\pi}^A = \frac{\overline{r}^A}{\sigma} - f_D - \frac{M_I^A}{M^A} \delta f_I \tag{39}$$

where $\overline{r}^A$ is (37) evaluated at the equilibrium cost cutoff c_o^A and c_I^A.

Note we can use the first line of (38) together with (8), (9), (31) and (32) to rewrite (12) as:

$$\int_0^{c_I^A} \pi_I(c) dG(c) + \int_{c_I^A}^{c_o^A} \pi_D(c) dG(c) = G(c_o^A) \overline{\pi}^A \tag{40}$$

which is (13).

6.1.2 Determination of the equilibrium

$\mu_I(c)$ and $\mu_D(c)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from a common distribution and δ is independent of the innovation status. These distributions depend exclusively on the cutoffs points for entry and innovation.

Using (38) together with (8) and (9), it is possible to express (12) as (13), which can be further refined and express in terms of solely c_o. Insert (10) and (11) into (12), replace $\delta f_I = \Psi(\gamma^{1-\sigma} - 1) f_D$ and rearrange terms to get:

$$\begin{aligned}
\delta f_E &= B\left[\gamma^{1-\sigma} G(c_I)\widetilde{c_I}^{1-\sigma} + (G(c_o) - G(c_I))\widetilde{c_D}^{1-\sigma}\right] - f_D G(c_o) - \delta f_I G(c_I) \\
&= G(c_I)\gamma^{1-\sigma}\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} f_D - G(c_I)\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} f_D - G(c_I)\Psi(\gamma^{1-\sigma} - 1) f_D + \\
&\quad + G(c_o)\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} f_D - G(c_o) f_D \qquad (41) \\
&= f_D\left[j_D(c_o) + \gamma^{1-\sigma} j_I(c_o)\right]
\end{aligned}$$

where

$$j_D(c_o) = G(c_o)\left[\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} - 1\right] - G(c_I)\left[\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} - \Psi\right] \qquad (42)$$

$$j_I(c_o) = G(c_I)\left[\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} - \Psi\right] \qquad (43)$$

6.1.3 Existence and Uniqueness of the equilibrium in the closed economy

Proposition 4 *Under autarky, the equilibrium exists and is unique.*

Proof. *We shall prove that the RHS of (41) is monotonically increasing in c_o on the domain $[0, \overline{c}]$, so that c_o is uniquely determined by the intersection of the latter curve with the flat line δf_e in the $[0, \overline{c}]$ space. Recall that $\widetilde{c_I}$ is a function of c_I (see (34)), which, in turn, is a function of c_o by (14). Let us define $\Lambda = \Psi^{1/1-\sigma}$. Note that (14) implies:*

$$\frac{\partial c_I}{\partial c_o} = \Psi^{1/1-\sigma} = \Lambda$$

and (34) implies:

$$\begin{aligned}
\frac{\partial}{\partial c_o}\left(\frac{\widetilde{c_I}^{1-\sigma}}{c_o^{1-\sigma}}\right) &= \frac{\frac{\Psi^{1/1-\sigma} g(c_I)\left[(c_I)^{1-\sigma} - \widetilde{c}_I^{1-\sigma}\right](c_o)^{1-\sigma}}{G(c_I)} - \widetilde{c_I}^{1-\sigma}(1-\sigma)(c_o)^{-\sigma}}{(c_o)^{2(1-\sigma)}} \\
&= \Lambda\frac{g(c_I)}{G(c_I)}\left[\left(\frac{c_I}{c_o}\right)^{1-\sigma} - \left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\right] - \frac{\widetilde{c_I}^{1-\sigma}(1-\sigma)}{(c_o)^{1-\sigma} c_o} \\
&= \Lambda\frac{g(c_I)}{G(c_I)}\left[\Psi - \left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\right] - \left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\frac{1-\sigma}{c_o}
\end{aligned}$$

It follows:

$$\begin{aligned}
\frac{\partial j_I(c_o)}{\partial c_o} &= \frac{\partial}{\partial c}\left\{G(c_I)\left[\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} - \Psi\right]\right\} \qquad (44) \\
&= g(c_I)\left(\frac{\partial c_I}{\partial c}\right)\left[\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} - \Psi\right] + G(c_I)\left[\frac{\partial}{\partial c}\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\right] \\
&= g(c_I)\Lambda\left[\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} - \Psi\right] + G(c_I)\left\{\Lambda\frac{g(c_I)}{G(c_I)}\left[\Psi - \left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\right] - \left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma}\frac{1-\sigma}{c_o}\right\} \\
&= -\frac{1-\sigma}{c_o} G(c_I)\left(\frac{\widetilde{c_I}}{c_o}\right)^{1-\sigma} \geq 0
\end{aligned}$$

Using (33) and following similar steps we get:

$$\frac{\partial}{\partial c}\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} = \frac{g(c_o)}{G(c_o)-G(c_I)}\left[1-\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma}\right] + \frac{g(c_o)\Lambda}{G(c_o)-G(c_I)}\left[\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} - \Psi\right] - \left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma}\frac{1-\sigma}{c_o}$$

$$\begin{aligned}\frac{\partial j_D(c_o)}{\partial c_o} &= \frac{\partial j_D(c_o)}{\partial c_o}\left\{G(c_o)\left[\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma}-1\right] - G(c_I)\left[\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma}-\Psi\right]\right\} \\ &= -\frac{1-\sigma}{c_o}\left[G(c_o)-G(c_I)\right]\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma} \geq 0\end{aligned} \tag{45}$$

(44) and (45) ensure that the RHS of (41) is an increasing function of c_o. Furthermore, $\lim_{c_o\to\overline{c}} j_I(c_o)=\infty$, and $\lim_{c_o\to\overline{c}} j_D(c_o) = a < \infty$, so that $\lim_{c_o\to\overline{c}}\left[j_D(c_o)+\gamma^{1-\sigma}j_I(c_o)\right]=\infty$. In order to show that the RHS of (41) goes to 0 as c_o goes to 0, I will follow Melitz (2003) and show that the elasticities of $j_I(c_o)$ and $j_D(c_o)$ are positive and $j_I(c_o)$ is always bounded away from 0.

$$\frac{\partial j_I(c_o)}{\partial c_o}\frac{c_o}{j_I(c_o)} = -(1-\sigma)\left[1+\frac{\Lambda}{j_I(c_o)}\right] \geq -(1-\sigma)$$

$$\frac{\partial j_D(c_o)}{\partial c_o}\frac{c_o}{j_D(c_o)} = -(1-\sigma)\left[\frac{\left(G(c_o)-G(c_I)\right)\left(\frac{\widetilde{c_D}}{c_o}\right)^{1-\sigma}}{j_D(c_o)}\right] \geq 0$$

Therefore the RHS of (41) is monotonically increasing in the space $(0,\overline{c})$ and it must cross the horizontal curve δf_E only once. The equilibrium c_o exists and it is unique. ■

Once the unique c_o is determined, (31) and (32) can be determined as well as (33) to (35). By (14) follows c_I, while by (13) follows $\overline{\pi}$. However, to determine the aggregate variables, we have to compute the number of varieties.

6.1.4 Determination of the number of varieties

Labour can be employed in three activities: product innovation, process innovation and production. The labour used for product innovation is the labour used by new entrants for investment purposes and amounts to L_e units. L_p is the labour devoted to produce a variety or make its productive process more efficient. By full employment $L = L_e + L_p$. The market clearing condition for product innovation is $L_e = M_e f_e$, since each of the new M_e entrants pays f_e units of labour. Domestic producers and Innovators pay their workers out of revenues. Thus, the aggregate payment to production workers must match the difference between aggregate revenue and profit:

$$wL_p = L_p = R - \Pi$$

The stability condition $M_e G(c_o) = \delta M$ together with (13) imply:

$$L_e = M_e f_e = \frac{\delta M f_e}{G(c_o)} = M\overline{\pi} = \Pi$$

Then, the labour market clearing conditions implies $L = L_e + L_p = \Pi + R - \Pi = R$, that is the aggregate revenue consists of the aggregate consumers' expenditure and it is exogenously limited by the country size. Then, from (37) follows (15), with the understanding that the superscript A denotes the autarky equilibrium value of these variables. The cutoffs and the number of varieties pin down all other aggregate variables and complete the characterization of the unique stationary equilibrium in the closed economy.

6.2 Appendix B - Comparison of our entry cutoff with Melitz's (2003) in the closed economy

Proposition 5 *Let denote c_M^* as the cutoff level of marginal cost found in Melitz (2003) for the closed economy. Then we have that:*

$c_0 < c_M^*$

Proof. *Since (10) and $R = L$ are common to both models, the ratio of the entry cost-cutoff is given by:*

$$\frac{c_M^*}{c_0} = \frac{P_M^*}{P}$$

where

$$P_M^* = \left(\int_0^{c_I} (p(c))^{1-\sigma} g(c)dc + \int_{c_I}^{c_M^*} (p(c))^{1-\sigma} g(c)dc \right)^{\frac{1}{1-\sigma}}$$

$$P = \left(\int_0^{c_I} (\gamma p(c))^{1-\sigma} g(c)dc + \int_{c_I}^{c_0} (p(c))^{1-\sigma} g(c)dc \right)^{\frac{1}{1-\sigma}}$$

Assume that:

$c_0 > c_M^*$

This implies that :

$$\left(\int_0^{c_I} (p(c))^{1-\sigma} g(c)dc + \int_{c_I}^{c_M^*} (p(c))^{1-\sigma} g(c)dc \right)^{\frac{1}{1-\sigma}} < \left(\int_0^{c_I} (\gamma p(c))^{1-\sigma} g(c)dc + \int_{c_I}^{c_0} (p(c))^{1-\sigma} g(c)dc. \right)^{\frac{1}{1-\sigma}}$$

and, rearranging terms, we have:

$(1 - \gamma^{1-\sigma}) \int_0^{c_I} (p(c))^{1-\sigma} g(c)dc > \int_{c_I}^{c_0} (p(c))^{1-\sigma} g(c)dc. - \int_{c_I}^{c_M^*} (p(c))^{1-\sigma} g(c)dc.$

which is not possible since, $\gamma < 1, \sigma > 1, c_0 > c_M^*$.

Q.E.D ■

6.3 Appendix C - Open economy - selection BW

6.3.1 Aggregation

Let $\mu_D(c) = g(c)/[G(c_o) - G(c_X)]$, $\mu_X(c) = g(c)/[G(c_X) - G(c_I)]$, $\mu_{XI}(c) = g(c)/[G(c_I)]$ denote the distribution of cost level in each subgroup prior to innovation.

Defining $\widetilde{c_D}^{1-\sigma}$ and $\widetilde{c_I}^{1-\sigma}$similarly as above and letting $\widetilde{c_X}^{1-\sigma} = \int_{c_I}^{c_X} c^{1-\sigma}\mu_X(c)dc$ and $M_D\widetilde{c_D}^{1-\sigma} + M_X\widetilde{c_X}^{1-\sigma} = M_{NI}\widetilde{c_{NI}}^{1-\sigma}$, the price index is a weighted average of within-group average productivity indexes given by:

$$P^{1-\sigma} = \frac{M_T}{\rho^{1-\sigma}} \underbrace{\left\{ \frac{1}{M_T} \left[M_{NI}\widetilde{c_{NI}}^{1-\sigma} + M_{XI}\gamma^{1-\sigma}\widetilde{c_I}^{1-\sigma} + n\tau^{1-\sigma}(M_X\widetilde{c_X}^{1-\sigma} + M_{XI}\gamma^{1-\sigma}\widetilde{c_I}^{1-\sigma}) \right] \right\}}_{\widetilde{c}^{1-\sigma}} \quad (46)$$

where $\widetilde{c}^{1-\sigma}$ is again the weighted average productivity index of the economy.

Similarly,

$$\begin{aligned} R &= M_{NI} r_D(\widetilde{c_{NI}}) + M_{XI}\gamma^{1-\sigma} r_D(\widetilde{c_I}) + n(M_X r_X(\widetilde{c_X}) + M_{XI}\gamma^{1-\sigma} r_X(\widetilde{c_I})) \\ &= M \underbrace{\left[(1-p_{rXI}) r_D(\widetilde{c_{NI}}) + p_{rXI}\gamma^{1-\sigma} r_D(\widetilde{c_I}) + n(p_{rX} r_X(\widetilde{c_X}) + p_{rXI}\gamma^{1-\sigma} r_X(\widetilde{c_I}))\right]}_{\overline{r}} \\ &= M_T r_D(\widetilde{c}) \end{aligned} \tag{47}$$

where $M_S = M p_{rS}$, $S = NI, X, XI$ was used.

Finally, the overall average - across all domestic firms - of combined profit is very similar and given by:

$$\begin{aligned} \bar{\pi} &= \Pi/M = (1-p_{rXI})\pi_D(\widetilde{c_{NI}}) + p_{rXI}\gamma^{1-\sigma}\pi_I(\widetilde{c_I}) + n(p_{rX}\pi_X(\widetilde{c_X}) + p_{rXI}\gamma^{1-\sigma}\pi_{XI}(\widetilde{c_I})) \\ &= \frac{\overline{r}}{\sigma} - f_D - \frac{G(c_I)}{G(c_o)}\delta f_I - \frac{G(c_X)}{G(c_o)} n\delta f_X \end{aligned} \tag{48}$$

where the last equality follows from substituting for the π's in the first line and using the expression for $\overline{r}$.

In a proof below, we shall use the average profit in the BW equilibrium and we shall use the convention that the variable with superscript f denote the equilibrium variables in the open economy equilibrium. For example the average profit in the trading equilibrium will be:

$$\begin{aligned} \bar{\pi}^f &= \Pi/M = (1-p_{rXI})\pi_D(\widetilde{c_{NI}}) + p_{rXI}\gamma^{1-\sigma}\pi_I(\widetilde{c_I}) + n(p_{rX}\pi_X(\widetilde{c_X}) + p_{rXI}\gamma^{1-\sigma}\pi_{XI}(\widetilde{c_I})) \\ &= \frac{\overline{r}^f}{\sigma} - f_D - \frac{G(c_I^f)}{G(c_o^f)}\delta f_I - \frac{G(c_X)}{G(c_o^f)} n\delta f_X \end{aligned} \tag{49}$$

with $\widetilde{c_D}^{1-\sigma}$, $\widetilde{c_I}^{1-\sigma}$, $\widetilde{c_X}^{1-\sigma}$, $\widetilde{c_{NI}}^{1-\sigma}$ as well p_{rD}, p_{rX}, and p_{rXI} are evaluated at the equilibrium cost cutoff c_o^f,c_I^f.

6.3.2 Existence and Uniqueness of the trading equilibrium

(20) to (22) as well as (25) and (26) allow us to rearrange the FE conveniently for the characterizing the equilibrium as a function of only c_o and c_X:

$$\begin{aligned} \frac{\delta f_E}{G(c_o)} &= \left\{(1-p_{rXI})\left[\frac{\widetilde{c_{NI}}}{c_o}\right]^{1-\sigma} + p_{rXI}\gamma^{1-\sigma}\left[\frac{\widetilde{c_I}}{c_o}\right]^{1-\sigma} - 1\right\} f_D - \delta f_I p_{rXI} + \\ &\quad + \left\{\frac{p_{rX}}{p_{rEXP}}\left[\frac{\widetilde{c_X}}{c_X}\right]^{1-\sigma} + \frac{p_{rXI}}{p_{rEXP}}\gamma^{1-\sigma}\left[\frac{\widetilde{c_I}}{c_X}\right]^{1-\sigma} - 1\right\} p_{rEXP} n\delta f_X \\ \delta f_E &= \left[l_{NI}(c_o) + \gamma^{1-\sigma} l_I(c_o)\right] f_D + \left[l_{NI}(c_X) + \gamma^{1-\sigma} l_I(c_X)\right] n\delta f_X \end{aligned} \tag{50}$$

where $p_{rEXP} = G(c_X)/G(c_o)$ and

$$l_{NI}(c_o) = G(c_o)\left[\left[\frac{\widetilde{c_{NI}}}{c_o}\right]^{1-\sigma} - 1\right] - G(c_I)\left[\left[\frac{\widetilde{c_{NI}}}{c_o}\right]^{1-\sigma} - \Psi^f\right]$$

$$l_I(c_o) = G(c_I)\left[\left[\frac{\widetilde{c_I}}{c_o}\right]^{1-\sigma} - \Psi^f\right]$$

$$l_{NI}(c_X) = G(c_X)\left[\left[\frac{\widetilde{c_X}}{c_X}\right]^{1-\sigma} - 1\right] - G(c_I)\left[\left[\frac{\widetilde{c_X}}{c_X}\right]^{1-\sigma} - \Psi_X^f\right]$$

$$l_I(c_X) = G(c_I)\left[\left[\frac{\widetilde{c_I}}{c_X}\right]^{1-\sigma} - \Psi_X^f\right]$$

Proposition 6 *Assume (23) holds. In the open economy, the equilibrium arising under selection BW exists and is unique.*

Proof. We proceed similarly as in the proof for the closed economy and we shall prove that the RHS of (50) is monotonically increasing in c_o on the interval $[0, \overline{c}]$. By (44), $l_I(c_o)$ is monotonically increasing in c_o and $l_I(c_X)$ is monotonically increasing in c_X from zero to infinity on $c \in [0, \overline{c}]$. In turn, c_X is increasing in c_o from (27). Similarly by (45), $l_{NI}(c_o)$ and $l_{NI}(c_X)$ are monotonically increasing from 0 to infinity respectively in c_o and c_X belonging to $[0, \overline{c}]$. Hence, the RHS of (50) is a monotonic increasing function from 0 to ∞ in the $[0, \overline{c}]$ space, while the LHS is a flat line. The equilibrium cost-cutoff level c_o must then be unique. ■

6.3.3 Comparison of the entry cost-cutoff in autarky and in trade

To compare the equilibrium entry cost-cutoff of autarky c_o^A with the one arising in the BW-equilibrium c_o^f, it is useful to re-arrange (50) in a more convenient way as:

$$\delta f_E = \left[j_D(c_o^f) + \gamma^{1-\sigma} j_I(c_o^f)\right] f_D + \Gamma \tag{51}$$

where

$$\Gamma = \left\{\left[\frac{\widetilde{c_X}}{c_X}\right]^{1-\sigma} - \frac{G(c_I^f)}{G(c_X)}\left[\frac{\widetilde{c_X}}{c_X}\right]^{1-\sigma} + \gamma^{1-\sigma}\frac{G(c_I^f)}{G(c_X)}\left[\frac{\widetilde{c_I}}{c_X}\right]^{1-\sigma} - 1\right\} G(c_X) n \delta f_X \geq 0$$

The first term of the RHS in (51) is exactly the same as in the closed economy. If Γ were 0, (41) and (51) would yield the same solution, i.e. $c_o^f = c_o^A$. Since Γ is positive the curve representing the RHS of (51) must lie above the curve representing the RHS of (41), implying a lower entry cost-cutoff in the trading equilibrium than in the autarky equilibrium. That is, $c_o^f \leq c_o^A$.

6.3.4 Proposition 1 - In *BW*, trade increases the proportion of firms performing process-innovation

Proposition *1. If (28) and (23) hold, then the innovation cutoff in the open economy is lower than in autarky (i.e $c_I^f < c_I^A$).*

Proof. First, use the expressions for $\mu_D, \mu_X, \mu_{XI}, p_{rX}, p_{rXI}$ to rewrite (24) as:

$$\bar{\pi}^f = \frac{\delta f_E}{G(c_o^f)} \tag{52}$$

where $\bar{\pi}^f$ is (49).

Using (13) and (52) combined with (28) it is possible to write the ratio of the average profit in the trading equilibrium to the average profit in autarky as:

$$\frac{\bar{\pi}^f}{\bar{\pi}^A} = \left(\frac{c_o^A}{c_o^f}\right)^k$$

Use (14) and (25) to get:

$$\left(\frac{c_I^A}{c_I^f}\right)^k = (1+n\tau^{1-\sigma})^{\frac{k}{1-\sigma}}\left(\frac{\bar{\pi}^f}{\bar{\pi}^A}\right)$$

Let us make some convenient transformations. Recalling that:

$$\Psi^f = \frac{\delta f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})f_D} \tag{53}$$

we define $\Lambda = \Psi^{\frac{1}{1-\sigma}}$, $\Lambda^* = \Psi^{f\,\frac{1}{1-\sigma}}$,and $\Lambda^* = \alpha\beta$ where:

$$\alpha = \left(\frac{\tau^{1-\sigma} f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})f_X}\right)^{\frac{1}{1-\sigma}}$$

$$\beta = \left(\frac{\delta f_X}{f_D}\tau^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \tag{54}$$

(50) can then be expressed as a function of the parameters of the model:

$$\begin{aligned}\bar{\pi}^f &= \left[\frac{k}{k+1-\sigma}\left[(1-\Lambda^{k+1-\sigma})+\gamma^{1-\sigma}\Lambda^{k+1-\sigma}\right]-1\right] f_D - \delta f_I \Lambda^k + \\ &\left[\frac{k}{k+1-\sigma}\left[(1-\alpha^{k+1-\sigma})+\gamma^{1-\sigma}\alpha^{k+1-\sigma}\right]-1\right]\beta^k n\delta f_X\end{aligned}$$

Using the definition of Λ and rearranging terms we get:

$$\begin{aligned}\bar{\pi}^f &= \frac{k}{k+1-\sigma}\left[(\gamma^{1-\sigma}-1)\Lambda^{k+1-\sigma}\right] f_D - \delta f_I\Lambda^k + \frac{\sigma-1}{k+1-\sigma} f_D + \\ &\frac{k}{k+1-\sigma}\left[(\gamma^{1-\sigma}-1)\Lambda^{k+1-\sigma}\right]\beta^{\sigma-1} n\delta f_X + \frac{\sigma-1}{k+1-\sigma}\beta^k n\delta f_X\end{aligned}$$

Using (54) and the fact that $\Lambda^* = (1+n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}\Lambda$:

$$\begin{aligned}\bar{\pi}^f &= \left[\frac{k}{k+1-\sigma}\left[\Lambda^{1-\sigma}(\gamma^{1-\sigma}-1)\right] f_D - \delta f_I\right]\Lambda^{*k} + \frac{\sigma-1}{k+1-\sigma} f_D \\ &+\frac{\sigma-1}{k+1-\sigma}\beta^k n\delta f_X\end{aligned} \tag{55}$$

and expanding (41):

$$\begin{aligned}\bar{\pi}^A &= \left[\frac{k}{k+1-\sigma}\left[\Lambda^{1-\sigma}(\gamma^{1-\sigma}-1)\right] f_D - \delta f_I\right]\Lambda^k + \\ &\frac{\sigma-1}{k+1-\sigma} f_D\end{aligned}$$

Then,

$$\left(\frac{c_I^A}{c_I^f}\right)^k = \frac{(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}A + B + C}{(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}A + (1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}B} \tag{56}$$

where

$$A = [\frac{k}{k+1-\sigma}\left[\Lambda^{1-\sigma}(\gamma^{1-\sigma}-1)\right] f_D - \delta f_I\Lambda^k \tag{57}$$

$$B = \frac{\sigma-1}{k+1-\sigma} f_D \tag{58}$$

$$C = \frac{\sigma-1}{k+1-\sigma}\beta^k n\delta f_X \tag{59}$$

We have to show that :

$$\frac{c_I^A}{c_I^f} < 1$$

which implies:

$$((1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}-1)B > C$$

Substituting (58),(59), the inequality becomes:

$$(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}} > 1+\left(\frac{\delta f_X}{f_D}\right)^{\frac{k+1-\sigma}{1-\sigma}}\tau^{-k}n$$

To show that this inequality holds true, note that $\beta < 1$ implies:

$$\left(\frac{\delta f_X}{f_D}\right)^{\frac{k+1-\sigma}{1-\sigma}}\tau^{-k}n < n\tau^{1-\sigma} \Rightarrow 1+\left(\frac{\delta f_X}{f_D}\right)^{\frac{k+1-\sigma}{1-\sigma}}\tau^{-k}n < 1+n\tau^{1-\sigma}$$

It follows:

$$(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}} > (1+n\tau^{1-\sigma}) > 1+\left(\frac{\delta f_X}{f_D}\right)^{\frac{k+1-\sigma}{1-\sigma}}\tau^{-k}n$$

since $k > \sigma - 1$ is assumed. ■

6.3.5 Lemma 2

Lemma *2. Assume (28) and (23) hold. Trade liberalization will have positive effects in innovation, i.e.* $dc_I/d\tau \leq 0$.

Proof. Combining (25) with (28), we get that:

$$G(c_I) = \Psi^{\frac{k}{1-\sigma}} G(c_o^f)$$

Substitute (52) and (53) into this expression to get:

$$(G(c_I))^{-1} = (1+n\tau^{1-\sigma})^{\frac{k}{1-\sigma}}\bar{\pi}^f\Theta = f$$

where Θ is a constant independent of τ, so that we shall ignore it because it does not affect the derivative. Totally differentiating both sides of this expression w.r.t. τ, we obtain the following:

$$\frac{dc_I}{d\tau} = \frac{\frac{df}{d\tau}}{\frac{d(G(c_I))^{-1}}{dc_I}}$$

Since the denominator is negative, it is enough to show $\frac{df}{d\tau} > 0$ for $\frac{dc_I}{d\tau} < 0$. Use (55), (57) to (59) and recall $\Lambda^* = (1+n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}\Lambda$ to expand f in the following way:

$$f = A + \frac{B}{(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}} + \frac{C}{(1+n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}}$$

where A, B, C are defined as in (57) to (59) and A is independent of τ. Using (54) and (59), it is convenient to express $C = \lambda\phi^{\frac{k}{\sigma-1}}$, with $\phi \equiv \tau^{1-\sigma}$, so that:

$$\frac{df}{d\phi} = \frac{\lambda\phi^{\frac{k+1-\sigma}{\sigma-1}}(1+n\phi)^{\frac{k}{\sigma-1}} - (1+n\phi)^{\frac{k+1-\sigma}{\sigma-1}}\phi^{\frac{k}{\sigma-1}}\lambda n - Bn(1+n\phi)^{\frac{k+1-\sigma}{\sigma-1}}}{(1+n\phi)^{\frac{2k}{\sigma-1}}}$$

Rearranging terms:

$$\frac{df}{d\phi} = \frac{\left[\lambda\phi^{\frac{k}{\sigma-1}}\frac{1}{\phi(1+n\phi)} - B\frac{n}{1+n\phi}\right]}{(1+n\phi)^{\frac{k}{\sigma-1}}}$$

Since we are deriving f with respect to ϕ (instead of τ) and $\sigma > 1$, the numerator is negative (i.e. $\frac{df}{d\tau} > 0$) iff:

$$\phi^{\frac{k+1-\sigma}{\sigma-1}} < \frac{B}{\lambda}n$$

and substituting for the values of B and λ, we get:

$$\phi = \frac{f_D}{\delta f_X}$$

and:

$$\tau \leq \left(\frac{\delta f_X}{f_D}\right)^{\frac{1}{1-\sigma}}$$

which satisfies our parameter restrictions (23). ■

6.3.6 Lemma 3

Lemma *3. Assume (28) holds. c_x is monotonically decreasing in τ and f_x.*

Proof. Combining (27) with (28) gives the following equality:

$$G(c_X) = \beta^k G(c_o)$$

Substitute (52) and (54) into this expression to get:

$$(G(c_X))^{-1} = \zeta \tau^k \bar{\pi}^f = g \tag{60}$$

where $\zeta = \left(\frac{\delta f_X}{f_D}\right)^{\frac{k}{\sigma-1}}$ is constant with respect to tariffs.

Proceeding similarly to the proof above, we take the total differential of both sides of (60) w.r.t. τ, so that the response of the exporting cost cutoff to changes in the transportation costs is given by:

$$\frac{dc_X}{d\tau} = \frac{\frac{dg}{d\tau}}{\frac{d(G(c_X))^{-1}}{dc_X}}$$

Since the denominator is negative, we need to prove $\frac{dg}{d\tau} > 0$ for $\frac{dc_X}{d\tau} < 0$. Substituting (55),(57),(58),(59) into (60), g is a function given by:

$$g = \zeta\tau^k(1 + n\tau^{1-\sigma})^{\frac{k}{\sigma-1}}A + \zeta\tau^k B + \zeta\tau^k C$$

or, substituting for the value of C, g can be conveniently expanded as:

$$g = \zeta(\tau^{k(\sigma-1)} + n\tau^{(k-1)(\sigma-1)})^{\frac{k}{\sigma-1}}A + \zeta\tau^k B + \Phi$$

where $\Phi = \frac{(\sigma-1)n}{k+1-\sigma}(\delta f_X)$. It follows $\frac{dg}{d\tau} > 0$.

To prove that $dc_X/df_X \leq 0$ we totally differentiate both sides of (60) w.r.t. f_X and obtain the following:

$$\frac{dc_X}{df_X} = \frac{\frac{dg}{df_X}}{\frac{d(G(c_X))^{-1}}{dc_X}}$$

Note that $\frac{dg}{df_X} > 0$ as $\frac{d\zeta}{df_X} > 0, \frac{d\omega}{df_X} > 0$ and A, B are independent of f_X. Recalling that the denominator is negative, it follows that $\frac{dc_X}{df_X} \leq 0$ - Q.E.D. ■

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