Strategic Misrepresentation of Information:
Implications of Biased and Diverse Information in Capital Markets

Inaugural Thesis

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\( \tilde{v} \) .................................................. Terminal Firm Value (per Share)
\( \tau_v \) .................................................. Precision of Firm Value
\( \tilde{u} \) .................................................. Manager’s Interest in Share Price
\( \tau_u \) .................................................. Precision of Interest in Share Price
\( \mu_u \) .................................................. Expectation of Interest in Share Price
\( c \) .................................................. Marginal Costs of Biasing
\( r \) .................................................. Reported Terminal Firm Value (per Share)
\( \tau_r \) .................................................. Precision of Report
\( b \) .................................................. Bias in Report
\( P \) .................................................. Price (per Share)
\( Q \) .................................................. Aggregate Net Order Flow
\( V \) .................................................. Trading Volume
\( N \) .................................................. Number of Informed Traders
\( x_i \) ........................................... Demand of Informed Trader \( i \in \{1, \ldots, N\} \)
\( \tilde{e}_i \) ................................ Noise in Information of Informed Trader \( i \in \{1, \ldots, N\} \)
\( \tau_e \) .................................................. Precision of Noise
\( M \) .................................................. Number of Liquidity Traders
\( \tilde{y}_j \) ........................................... Demand of Liquidity Trader \( j \in \{1, \ldots, M\} \)
\( \sigma_y^2 \) .................................................. Variance of Liquidity Trading
\( \rho \) .................................................. Risk Aversion of Uninformed Traders
\( \tilde{w}_j \) ................................ Premium for Immediate Execution of Uninformed Trader
\( j \in \{1, \ldots, M\} \)
\( \sigma_w^2 \) \ldots Variance of Immediate Execution Premia
\( S_j \) \ldots Surplus of Liquidity Trader \( j \in \{1, \ldots, M\} \)
\( \alpha \) \ldots Manager's Equilibrium Strategy Coefficient
\( \beta_0, \beta_r, \beta_Q \) \ldots Informed Traders' Strategy Coefficients
\( \delta_0, \delta_r, \delta_Q \) \ldots Market Maker's Strategy Coefficients
\( \gamma \) \ldots Uninformed Traders' Strategy Coefficient
\( \Psi \) \ldots Price Efficiency
\( \Theta \) \ldots Expected Profit of one Informed Trader (not \( i \))
\( \Theta_L \) \ldots Expected Profit of one Uninformed Trader (not \( n \))
\( U_a, U_p \) \ldots Ex-Ante and Ex-Post Utility of the Manager
Preface

Today’s predominant disclosure philosophy calls for the full disclosure of all decision relevant information by companies.\footnote{The concept of ‘full and fair disclosure’, calling for a disclosure of all material facts relating to securities publicly offered and sold, can be traced back to the U.S. Securities Act of 1933. The basic philosophy is still prevailing at least in U.S. standard setting, see Levitt (1998b).} More publicly available information, the argument goes, limits the adverse selection problem by reducing information asymmetry among heterogeneous investors. Thus, the liquidity in capital markets will be increased and the cost of capital decreased in consequence, to the benefit of all.

But is more disclosure really always preferable? Two arguments can be brought forward which shed a different light on the above line of argument. Firstly, decision relevant information necessarily involves future-oriented, manipulable information. Secondly, supposing that information processing skills differ among investors, a disclosure may inherently create information asymmetries instead of relieving them.

In such a scenario, it becomes important to know who generates information (and with which incentives), how that information is disseminated in capital markets and incorporated into market prices and finally, whether the overall effect of such releases of information is to level the informational playing field or vice versa. If a disclosure does not make the playing field more leveled, it also becomes unclear what the economic purpose of disclosure really is.

The following formal analysis investigates four alternative market settings which simultaneously include a manager’s incentives regarding the share price of his firm on the one hand and a group of heterogeneously informed common-stock investors on the other hand. This allows us to calculate a number of market characteristics and to identify scenarios under which more disclosure is preferable and others under which it is not.
Chapter 1

Introduction

"Well, today, I’d like to talk to you about another widespread, but too little-challenged custom: earnings management. This process has evolved over the years into what can best be characterized as a game among market participants. A game that, if not addressed soon, will have adverse consequences for America’s financial reporting system.”

1.1 Financial Reporting

The burst of the stock market bubble of the 1990s led to a number of spectacular bankruptcies throughout the world. At the same time, accounting scandals of previously unheard-of dimensions reached the public, forcing many companies to restate their earnings while diminishing investors’ trust in the measures of corporate performance provided by the accounting system. Managers stated aggressive or even fraudulent earnings to hide operative problems and to fulfill the expectations of analysts. Arthur Levitt, at that time president of the Securities and Exchange Commission (SEC), addressed these mechanisms which he called ”a game among market

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1See Levitt (1998).

2The SEC is the U.S. federal agency whose primary responsibility lies in administering the federal securities laws and the establishment of disclosure requirements for public companies.
participants” in an influential speech at the New York University.

Financial accounting is the process whereby the economic activities of an organization are measured, summarized and communicated to entities outside the organization. One purpose of financial reports is to provide information that is useful for making economic decisions, such as buying or selling firm shares. It aims at helping investors to identify relatively efficient and inefficient users of resources, and it aids in assessing the relative returns and risks of investment opportunities. The stock prices that result from common-stock investors’ buy and sell decisions affect management’s perceived cost of capital, since prices on secondary markets influence the terms at which companies are able to raise capital in primary markets. Thus, at an economy-wide level, the financial reporting system facilitates an efficient resource allocation and helps to create a favorable environment for capital formation by ensuring investor confidence. This line of argumentation highlights the importance of the financial reporting system for economic development.

Due to the international harmonization of accounting standards, the regulation of accounting in Germany is currently undergoing dramatic changes. The traditional set of German accounting principles is replaced by International Financial Reporting Standards (IFRS) for consolidated statements of listed companies. New institutions, such as the private standardization organization called ‘German Accounting Standards Committee’ (GASC) and its subsidiary organizations, have been founded to

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3See the preface of IASB (2003). According to Neus (2005, p. 388), the role of financial reports prepared under the German HGB may lie primarily in the assessment of tax payments and in the restriction of dividend distribution.

4On dates when a firm sells new securities, firms must release information to obviate against adverse selection: since the firm is better informed than investors, it must convince potential investors that it is not selling shares out of an awareness that the price is too high; see Leland and Pyle (1977) and Myers and Majluf (1984).

5Bhattacharya, Daouk and Welker (2003) find that an increase in earnings opacity in a country is linked to an economically significant increase in the cost of equity, and a decrease in the stock market trading of that country. See also Beck, Levine and Loayza (2000) and Levine (1997).

6The term “accounting regulation” refers to the existing laws, rules, and generally accepted accounting principles concerning the timing, content, and form of corporate financial reports as well as the regulations regarding the verification of disclosed information, e.g. auditing.

ensure a rapid adjustment to the new requirements.\textsuperscript{8} In the meantime, Germany has laid the foundation for a two-stage enforcement process consisting of the private Financial Reporting Enforcement Panel and the Federal Financial Supervisory Authority, which came into effect on July 1st, 2005.\textsuperscript{9} The IFRS themselves are currently being reformed.\textsuperscript{10}

The increasing extent of regulation by public agencies serves to ensure that companies meet certain minimum levels of financial disclosure (mandatory disclosure).\textsuperscript{11} However, an undisputed reasoning in favor of accounting regulation is currently unknown.\textsuperscript{12} Some argue that such regulation is redundant since managers have sufficient incentives to disclose financial information voluntarily, or that the existing regulation is ineffective in achieving socially-desired goals.\textsuperscript{13} Due to the complexity of the factors influencing the reporting decision (see figure 1.1 for an overview), a resolution of such disputes may well be unachievable. If the financial reporting environment were unregulated, disclosure would occur voluntarily as long as the incremental benefits to the company and its management from supplying financial information exceeded the incremental costs of providing that information.

External users of financial reporting information include (current and potential) shareholders, banks and other lenders, suppliers and customers of a firm’s products, employees, governments and their agencies (e.g. the tax ministry), and the public in general. Competitors (active or potential) are unwanted, yet not to be disregarded as

\textsuperscript{8}The legal basis for the definition of the responsibilities of a private standardization committee can be found in § 342 Handelsgesetzbuch (HGB). The German Federal Ministry of Justice recognized the GASC in the sense of § 342 on September 3rd, 1998.

\textsuperscript{9}The corresponding law, the 'Bilanzkontrollgesetz' (BilKoG), passed on October 29th, 2004, see Bundesgesetzblatt 2004 I (No. 69), page 3408.

\textsuperscript{10}See Wagenhofer (2003, pp. 108-111) for a description of current reforms.

\textsuperscript{11}The term 'disclosure' has two significations in the literature. Firstly, it can refer to any information item provided by a firm or its managers (e.g. an earnings forecast in a press release). Secondly, it may signify information released in the notes to financial statements. In the following 'disclosure' shall refer to the former significiation.

\textsuperscript{12}See Healy and Palepu (2001).

\textsuperscript{13}See Easterbrook and Fischel (1991), Watts and Zimmerman (1986), Christensen and Denski (2003, chapter 13) for reviews of such arguments.
users. Naturally, all users have different or even contradictory needs of information.\textsuperscript{14}

Wagenhofer and Ewert (2003) broadly classify the objectives of financial reports into two groups: firstly, they provide decision useful information that helps to decide, for example, about the purchase or sale of company shares, the award and terms of credits, the establishment of trading relations with, or the decision to accept a position in the company. Secondly, financial reports have a stewardship function, i.e. they provide numbers on which contractual claims can be based, such as dividend and tax payments, variable management compensation, or credit triggers in the case of an exceeding of certain balance sheet relations.

\textsuperscript{14}See paragraph 9 of the Framework of IASB (2003).
Financial reports also serve the shareholders to monitor managers’ actions, enabling them to interfere in the case of insufficient effort or expropriation. A way to confront this agency conflict that arises from the separation of ownership and control, is to grant the manager an incentive contract, for example on the basis of accounting numbers, to align his interests with those of the investors. Financial reports thus play an important role in corporate governance, which "deals with the ways in which suppliers of finance to corporations assure themselves of getting a return on their investment." The rights of investors are protected to that extent to which managers cannot obfuscate real firm performance by "managing earnings" and compliance with the accounting standards is enforced by the legal system.

The different functions of accounting information involve demands for different and possibly conflicting kinds of information. Although there are more competing characteristics, the problem can be exemplified by the trade-off between reliable and relevant information. Reliability means that the information is "free from material error and bias and can be depended upon by users to represent faithfully that which it either purports to represent or could reasonably be expected to represent." Information is relevant "when it influences the economic decisions of users by helping them [to] evaluate past, present or future events or confirming, or correcting, their past evaluations." Thus, the stewardship function of financial reports tends to call for reliable, historical information, while common-stock investors are primarily interested in relevant, future-oriented information in order to evaluate a firm’s expected future performance. Now, the more relevant the information is, the lower in general its reliability. In fact, there exists no optimal trade-off between such demands, and not even the IFRS provide clear guidance as to how to weigh them up against each

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15 Models of principal-agent relationships as in Holmstrøm (1979) generally assume the pre-existence of a court system which can freely enforce contracts written on verifiable information variables.


19 Future-oriented information must to some degree be reliable as well, since otherwise investors would disregard it in order not to be misled.
other. The usefulness of a particular accounting method necessarily depends on the users’ decision problem and their individual preferences.\textsuperscript{20}

The adoption of the IFRS in Germany (for consolidated statements) reflects a major shift towards the informational function. This implies that the core attribute of accounting numbers is seen in their "information content" for capital market investors.\textsuperscript{21} The IFRS are clear in their focus on investors: paragraph 10 of the Framework of IASB (2003) states that "as investors are providers of risk capital to the enterprise, the provision of financial statements that meet their needs will also meet most of the needs of other users that financial statements can satisfy." In what follows, this informational view of financial reports will be adopted.\textsuperscript{22}

1.2 Earnings Management

Any set of accounting principles allows the preparer of financial statements to exercise discretion.\textsuperscript{23} This involves the choice of accounting policies, such as a particular method of amortization, or the inclusion of discretionary accruals, such as provisions for credit losses or warranty costs. The usage of fair values, i.e. the valuation of an asset on the basis of its market value or the discounted present value of its future receipts, necessarily involves subjective judgements and estimates on the side of the preparer.\textsuperscript{24}

\textsuperscript{20}Demski (1973) shows the general impossibility of optimal accounting standards in a formal model. Earlier literature tried to derive these from sets of predetermined assumptions regarding the objectives of accounting in a deductive way. Watts and Zimmerman (1986, p. 7) put their disapproval of this approach in simple terms by stating that "the decision on the objective is subjective." Wagenhofer (2003, p. 4) argues that "each system has its own advantages and disadvantages, depending on its purpose, the institutional embedding, and the decision context of its users."

\textsuperscript{21}Beaver (1998) called this shift which occurred in the USA in the 30s, an "accounting revolution."

\textsuperscript{22}In the following, the term 'investor' refers to common-stock shareholders.

\textsuperscript{23}There are many textbooks on accounting discretion and earnings management based on different accounting principles: see, for example, Veit (2002) for the German HGB principles, Wagenhofer (2003, pp. 572-582) for IFRS, and Davidson, Stickney and Weil (1987) for U.S. GAAP.

\textsuperscript{24}An obvious problem with a rigid accounting system (with no room for judgment) is providing rules for all facts and circumstances. For example, such a system could specify that the allowance for uncollectibles is always 10% of receivables, that equipment is depreciated straight line over 5 years, and that all marketable securities are to be treated as if they were available for sale. In addition, new
Such discretion allows for a more specific and informative conveyance of information in financial statements. Managers can communicate their insider information to investors. However, it can naturally be expected that managers will choose accounting policies which maximize their own utility. In fact, managers can also use their discretion in an opportunistic fashion and influence the earnings figure in their favor. Thus, the reliability of the statements is reduced.

If the financial reporting process is viewed more broadly as including voluntary, non-audited disclosures, other possibilities to paint a better image of the firm’s performance arise. Less formal communication channels, such as press-releases and direct contact with analysts, allow for even more discretion. An earnings forecast per definition contains uncertain information that needs to be estimated. In press-releases, managers can choose to include information or not, or to give special emphasis to a particular piece of information (e.g. via repeating or locating it purposefully). Managers may also emphasize earnings constructs other than net income, such as “pro forma” earnings that include or exclude a variety of items from the official earnings number. Managers frequently argue that the official earnings number was influenced by extraordinary factors and therefore cannot be used to forecast. However, especially if pro-forma earnings are published in advance to the official statement release, it is difficult to see through to the ”adequate” numbers.

According to Healy and Wahlen (1999, p. 368), ”earnings management occurs when managers use judgment in financial reporting and in structuring transactions to alter financial reports to either mislead some stakeholders about the underlying performance of the company or to influence contractual outcomes that depend on situations arise regularly (debt-equity-hybrids, securitizations) requiring that new accounting rules be devised. In other words, accounting choice likely exists because it is impossible, or infeasible, to eliminate it;” see Fields, Lys and Vincent (2001).

26Excluded items can be any combination of depreciation, amortization, goodwill, or taxes, among others. See Wagenhofer (2003, p. 596).
27In March 2003, the SEC passed a resolution (Regulation G) which imposes on listed corporations a transition to the most similar GAAP-measure and an explanation by the management why it believes that the chosen non-GAAP measure represents the situation better.
reported accounting numbers.” Schipper (1989, p. 92) emphasizes in her definition that earnings management can occur in any part of the external disclosure process by defining earnings management as the “purposeful intervention in the external reporting process, with the intent of obtaining some private gain.”

Illustration 1.2 distinguishes between accounting and real earnings management, and between the legal use of accounting discretion and fraud. Accounting earnings management includes the way accounting standards are applied to the recording of given transactions and events, whereas real earnings management changes the timing or structuring of real transactions.

![Figure 1.2: A classification of earnings management (Source: Dechow and Skinner (2000)).](image)

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28The National Association of Certified Fraud Examiners (1993, p. 12) defines financial fraud as "the intentional, deliberate, misstatement or omission of material facts, or accounting data, which is misleading and, when considered with all the information made available, would cause the reader to change or alter his or her judgment or decision.” See the Treadway Commission (1987) for a thorough analysis of firms committing fraud and their governance environment.

Incentives for earnings management are ample. In general, each particular user group of a financial report can (and does) serve as a motivation for the manager to misrepresent the released information. Many of these incentives and the accompanying earnings management have been shown to exist empirically. The review articles of Healy and Wahlen (1999) as well as Schipper (1989) list a large number of such studies.

Managers may be inclined to manage earnings due to the firm’s relations with capital markets, the existence of explicit and implicit contracts, or other specific circumstances like the political and regulatory environment. Due to the focus of this thesis on common-stock investors, the following list of motivations concentrates on the firms’ and managers’ relations with capital markets (the cited papers are all empirical studies if not defined otherwise).

- In periods surrounding capital market transactions like equity offers, initial public offerings or prior to stock-financed acquisitions, managers have been shown to increase earnings. See Teoh, Welch and Wong (1998), Teoh and Welch (1998), and Erickson and Wang (1999).

- Perry and Williams (1994) find that managers decrease income prior to a management buyout. Good information may also be withheld by a seller in the course of an acquisition if the buyer’s intention with respect to the execution of the deal is uncertain, as Wagenhofer (2000) shows in a game-theoretic model.

- Burgstahler and Eames (2003) find that firms manage earnings in order to meet

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30See section 1.1.
31Contracting motivations refer to the intent of avoiding the violation of a restrictive debt covenant, of increasing earnings-based bonus awards, or to decrease tax payments.
32Managers may try to reduce political sensitivity when government intervention or union negotiations impede; they may understate the true potential when the entrance of a competitor is possible; a manager may want to make his image look better when he has a short expected tenure with the firm.
33Dechow and Skinner (2000, p. 236) argue that “academics’ research efforts should focus more on capital market incentives for earnings management.”
analysts’ forecasts. In particular, they find that managers take actions to manage earnings upward to avoid reporting earnings below analysts’ expectations. If their stock received a sell recommendation by analysts, they engage more frequently in extreme, income-decreasing earnings management. See also Abarbanell and Lehavy (2003a and b) and Kasznik (1999).

- Managers may try to lower the share price prior to stock-option awards in order to lower the exercise price for options granted. See Aboody and Kasznik (2000).

- Burgstahler and Dichev (1997) find that managers try to meet simple earnings benchmarks, such as avoiding small losses and earnings declines. See also Degeorge, Patel and Zeckhauser (1999).

The different incentives may result in a variety of earnings management patterns. These include income minimization (“taking a bath”), income maximization, income smoothing, or goal reaching. The pattern chosen by a firm may vary over time due to changes in contracts, in levels of profitability, in the CEO, capital needs, and changes in political visibility. Even at a given point in time, the firm may face conflicting needs, for example, to reduce reported income for political reasons, but smooth it for borrowing purposes. The resulting motivation is difficult to figure out exactly. Beaver (2002, p. 466) states that “these motives can operate in either opposing or reinforcing ways, often making it difficult to isolate the primary motive.”

The definition of Healy and Wahlen (1999) emphasizes the importance of the intention to deceive users of financial reports. However, the managerial intent, and managers’ accounting choices in the absence of economic incentives, are not observable. Dechow and Skinner (2000) note that “in many cases it seems difficult to distinguish earnings management from the legitimate exercise of accounting discretion.”

Under their definition, earnings management involves “to alter financial reports.”

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34 Trueman and Titman (1988) formally argue that managers have incentives to smooth earnings to reduce the variability of reported earnings and thus investors’ perceived risk.

This assumes that there is a neutral report from which the actual report deviates, but such a neutral income is not observable. One acceptable candidate for neutral income from a theoretical point of view would be economic income. But, as Beaver and Demski (1979) show, if markets are not complete and perfect, the very notion of economic income is not meaningfully defined. It follows that the magnitude of earnings management must be unknown.

Earnings management cannot indisputably be judged as good or bad. One positive aspect of earnings management is that it can be a vehicle for the communication of management’s insider information to investors, thus enabling the share price to better reflect the firm’s future prospects. Earnings smoothing, for example, can convey insider information to the market by enabling the firm to communicate its expected persistent earning power. From a contracting perspective, earnings management could be used as a low-cost way of protecting the firm from the consequences of unforeseen state realizations in the presence of rigid and incomplete contracts. Both of these perspectives lead to the interesting, and perhaps surprising, conclusion that earnings management can be "good". This is especially the case when the manager is disinterested and objective.

In practice, it is difficult to distinguish between the conveyance of objective and self-serving information. Fields, Lys and Vincent (2001) note that an accounting

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36 In complete and perfect capital markets, financial reports would not be needed anyway. See Watts and Zimmerman (1986).

37 Later chapters assume that the "true value" can be observed in a financial report when the firm reports truthfully and implements an accepted set of accounting policies and procedures to communicate the outcome of events affecting the firm. Fischer and Stocken (2004, p. 844, footnote 2) think of this accepted set of accounting policies in mathematical terms as "a previously established mapping from the space of economic events to the reporting space."

38 Barth and Elliott (1999) suggest that increasing earnings patterns reveal insider information about growth opportunities. Sankar and Subramanyam (2001) demonstrate that granting managers limited discretion to manipulate earnings allows them to communicate value-relevant information not reflected in fundamental earnings, and consequently always enhances price efficiency. In an agency setting, Liang (2004) finds that some discretion for earnings management can improve risk allocation.

method may be chosen in a self-interested attempt to increase the stock price; alternatively, the same accounting choice may be motivated by managers’ objective assessment that the current stock price is undervalued (relative to their private information). In the following, however, the focus lies on the assumption that some informed investors understand the implications of earnings management more than other, less sophisticated investors. This raises the question of how efficiently capital markets as a whole reflect the information contained in a biased financial report.

1.3 Market Efficiency with Respect to Financial Reports

Jensen (1978) defines a market to be efficient with respect to a specified information set, if it is on average not possible to earn abnormal economic profits\(^40\) by trading on the basis of that information set. Thereby, economic profits also contain costs of information acquisition and transaction costs. By considering different sets of information, different grades of efficiency with respect to these sets can be distinguished. In the following, the information set shall include all publicly available information at a given point in time (including information from financial reports).\(^41\) Beaver (1981) emphasizes the heterogeneity of information among investors by defining a market to be efficient with respect to a set of information if security prices “act as if this set were freely available to all investors.” A few informed investors may suffice to incorporate a certain set of information into the price.

Yet when exactly is information publicly available? It is evident that asymmetry and diversity of information among traders are important elements of financial markets. Traders expend resources to acquire information, just as there are active markets

\(^{40}\)Abnormal returns must be defined relative to some benchmark of normal expected return, as is, for example, provided by the capital asset pricing model. This implies that tests of market efficiency are always joint tests of market efficiency and a market model.

\(^{41}\)Fama (1970) called this the semi-strong form of efficiency.
in advisory services and other forms of information (e.g. investment newsletters). Information intermediaries, such as financial analysts and rating agencies, engage in private information production to uncover managers’ superior information. They are often able to gather, process, and evaluate financial information more economically and accurately than individual investors can, because they possess specialized skills (e.g. more sophisticated techniques of analysis) or knowledge (e.g. industry expertise) or because they have access to specialized resources provided by their organizations. Intermediaries may sell information to individual investors; the lower the price, then, the more investors will be informed. In effect, the exact definition of which information is publicly available becomes arbitrary.\textsuperscript{42}

The activities of financial analysts may provide complementary or substitute information to financial reports; they provide complementary information if they make complex information, such as pension footnote information, publicly available. To the extent that intermediaries provide alternative sources of information to investors, they are in competition with publicly available financial information.\textsuperscript{43} More comprehensive and more timely sources of information could reduce the informational content of financial reports to a point where they merely reflect factors already incorporated in stock prices. This implies that the capital market as a whole is unlikely to be misled by earnings management. If it is supposed that the share price reacts in an unbiased fashion to new information,\textsuperscript{44} a number of implications can be deduced:\textsuperscript{45}

- A switch from one accounting method to another without a direct cash flow effect, signaling effect, or consequences on incentives, would not affect security prices as long as sufficient information is given, so that the reader can convert across different policies. That is, the market would see through to the ultimate

\textsuperscript{42}See Latham (1986).
\textsuperscript{43}Complementary sources of information include macroeconomic data as published, for example, by governmental agencies (e.g. on the inflation rate, exchange rates, or the oil price), weekly production data on the industry or sales of competitors, interviews with management or competitors about sales numbers or with creditors about a firm’s credit standing, or reports in business journals on events related to the firm or the development of the industry sector.
\textsuperscript{44}Watts and Zimmerman (1986) call this the "no-effects hypothesis."
cash flow and dividend implications of accounting changes.

- It should not make any difference whether a piece of information is entered into the balance sheet, or mentioned in a footnote or in the appendix. Hence, the substance (i.e. bringing an item into the public domain) rather than the form of disclosure counts for investors in determining security prices. This argument assumes that the choice of the location has no influence on contracts, and that the format does not convey signals regarding the management’s expectations.

- Regulators should not be overly concerned about naive investors. If enough investors understand the disclosed information, this is sufficient to ensure that the market price of a firm’s shares is the same as it would be if all investors understood it. As a result, any information advantage is quickly dissipated and investors are price-protected by the efficient market.

It also follows that, if prices were not efficient with respect to publicly available information, the three implications above would not hold. Earnings management would influence market prices, so that regulators should be concerned about the effects of such biased prices on ”naive investors.”

How efficient capital markets actually are is an empirical question. Evidence shows that capital markets are highly sophisticated in interpreting new information and incorporating it in prices. However, within the last years, the belief that ”price convergence to value is a much slower process than prior evidence suggests” has acquired currency among leading academics.\footnote{See Frankel and Lee (1998, p. 315).} The post-announcement drift, to give an example for an anomaly, denotes the predictability of abnormal returns following an earnings announcement. Since the drift is of the same sign as the earnings change, this suggests that the market underreacts to information in earnings announcements. The extent of capital market efficiency remains a highly controversial issue.\footnote{See Fama (1991), Brown (1994), Kothari (2001) for review articles on this ongoing debate.} Anyway, even if prices efficiently reflect all publicly available information, managers may
possess insider information about the operation of their firms which is not available to any investor. Thus, earnings management can still be effective.

Although empirical studies show the existence of earnings management accompanying capital market transactions,\textsuperscript{48} a test setup to show that earnings management has an effect on the share price is particularly difficult. For example, the effect of a change in the estimate of service lives of depreciable assets is hard to estimate for the years after the initial change. The correct price reaction to accounting changes which would occur in an efficient market cannot be easily hypothesized if indirect cash flow effects (for example via deferred tax payments), signaling effects, or incentive consequences are present. These alternative explanations make it difficult to reject a maintained hypothesis of market efficiency. Therefore, studies often proceed in an indirect manner:

- Aboody (1996) investigates whether investors value the pricing consequences of recognized information (i.e. included in earnings) differently from disclosed information (i.e. mentioned in footnotes) in the oil and gas industry in the United States, and shows that the form of disclosure indeed plays a role.

- The market seems not to respond fully to certain balance sheet information. Rather, it may wait until the balance sheet information shows up in earnings or cash flows before reacting. Ou and Penman (1989) derive a list of financial ratios and form a predictor for the following year’s earnings changes, and then applied the latter to devise an investment strategy for their sample firms. They find that their strategy earns a significant, abnormal return for the years 1965 to 1972.

- Sloan (1996) separates reported net income into operating cash flows and accrual components. He argues that an efficient market should react more strongly to a dollar of good news in net income if that dollar originates from operating cash flows, because accruals reverse in future periods. However, the market

\textsuperscript{48}See section 1.2.
did not respond more strongly to good news in earnings the greater the cash flow component relative to the accrual component. See also Xie (2001) for an overvaluation of discretionary accruals.

- Dechow, Sloan and Sweeney (1996) find that there are significant adverse capital market reactions to SEC enforcement actions. When SEC enforcement actions are published, share prices fall on average by 9%, indicating that the capital market is not able to identify earnings management by itself.

- Wyatt (1983) reports that management behaves as if it does not believe the efficient market hypothesis. Wyatt (1983) documents several transactions in which management appears to care about the accounting method and is willing to incur costs in order to structure a transaction so as to have a desired effect on the reported results.\textsuperscript{49}

Another reason why earnings management is problematic, even if prices were efficient, is the heterogeneity of investors. Suppose that there are some market participants who observe that earnings management occurs and make corresponding adjustments to arrive at what they see as the appropriate earnings numbers, because they have low cost access to the requisite information and are reasonably sophisticated in their information processing abilities. Because of the objective to provide a "level playing field" for all investors, regulators cannot ignore the possibility that certain investors rely completely on earnings numbers reported on the face of the income statement because their ability to process more sophisticated (i.e. footnote) information is limited.\textsuperscript{50} In fact, the regulation of insider trading, public disclosure of information, and the organization of financial markets are undertaken in response to the welfare implications of the allocation of public and private information.\textsuperscript{51}

\textsuperscript{49}This could also be explained by irrational behavior on the managers' side.
\textsuperscript{50}Lev (1989) states that earnings are widely believed to be the premier information item in financial statements. This may be due to an irrational earnings orientation).
\textsuperscript{51}See Admati (1989).
1.4 The Social Value of Information in Models of Competitive Trading

This section narrowly surveys rational expectations models of trading in competitive markets analyzing the value of private and public information for investors. These models do not directly consider earnings management, but the "quality" of a piece of information is indicated by the precision of the corresponding random variable. This thesis also focuses on the value of information for investors but in a non-competitive setting. The competitive models are covered here in order to show the limitations of this approach and to motivate the choice of non-competitive models in the remainder.

Suppose there are a number $N > 1$ of investors, who trade once in a single asset (one-shot trading). Additional information has a social value unconditionally, if it makes some investors better off but no investor in the market worse off in terms of expected utility before the realization of the signal (Pareto improvement). The ex-ante criterion is expected to yield more desirable outcomes on average, but does not necessarily generate more desirable outcomes for each investor.

To formally capture the interaction between information and the welfare of investors, the Walrasian equilibrium concept can be used. In a one-period setting, it considers a round of trading in both a risky and a risk-less asset at the beginning, with an uncertain payoff for the risky and a certain payoff for the risk-less asset at the end of the period. Investors are endowed with assets and may differ in their risk aversion. A competitive equilibrium consists of demands for each investor and a

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52 In a single-person setting, the Blackwell theorem states that the individual investor would always prefer more detailed information to less detailed one. This result is intuitive insofar as more information could simply be disregarded by the investor and thus never leads to a lower expected utility. In fact, he achieves the highest utility with a perfect information system that reveals the exact state. A formal description can be found in Christensen and Feltham (2003, pp. 93-100).

53 Another method of defining social value is to compare the utility among investors of the allocation after the reception of the signal, i.e. on the basis of the ex-post distribution. There are other criteria: for example, the aggregate costs and benefits criterion considers the difference between the sum of the incremental costs borne by all individuals and the sum of the incremental benefits across all individuals. See Verrecchia (1982b, p. 8).

54 See Mas-Colell, Whinston and Green (1995), Part IV, for an introduction.

55 In competitive models, all traders take prices as given.
market clearing price, at which the sum of demands is equal to the sum of endowments. Under the given assumptions, in equilibrium, each investor holds a fraction of the total supply which exactly equals his risk tolerance relative to the aggregate risk tolerances of all investors in the market. Thus, due to risk aversion, trading occurs in order to optimally share risks despite homogeneous beliefs (if the endowments were not already optimal from the beginning).

Hirshleifer (1971) and Marshall (1974) pioneered the idea that market participants may be collectively worse off in case of an anticipated disclosure, because it distorts optimal risk sharing (adverse risk-sharing effect). "As among a group of traders who would otherwise have mutually insured against fire, a conclusive message (as to whose houses would actually burn down) would negate the possibility of mutually advantageous risk-sharing through insurance." Wagenhofer and Ewert (2003, pp. 80-88) present the extreme scenario where each investor is strictly worse off even with a complete information system which reveals the terminal payoff.

Following models discussed extensively when the adverse risk-sharing effect allows for a Pareto-improvement or not in a variety of settings within the competitive framework. For example, if there had been a prior round of trade, so that investors would already have shared their risks optimally, an additional disclosure would not affect investors negatively. Verrecchia (2004) notes that financial disclosures of firm performance like annual reports or quarterly results are "routine events that are widely anticipated." Hence, investors are able to trade in advance of a disclosure. The seminal paper of Diamond (1985) derives a positive value of public information despite adverse risk sharing. He uses the rational expectations approach and includes the aggregation of private information in prices, in addition to the public signal.

The rational expectations approach proposes equilibrium prices and demands, in which investors learn about the private information of others by observing the equilibrium price. The term "rational expectations equilibrium" as a reference to the Walrasian structure is misleading to the extent that expectations are also rational

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56 See Hirshleifer and Riley (1979, p. 1401).
in the original sense of Muth (1961) in Kyle’s model, which will be used in later chapters.\textsuperscript{58} Muth (1961) originally called expectations ”rational”, when the economy generally does not waste information, i.e. when expectations depend specifically on the structure of the entire system. However, in the literature, rational expectations models of trade are established to denote those equilibra in which investors condition their expectations on the price at which markets are cleared.\textsuperscript{59} The rational expectation-based equilibrium concept is not uncontroversial. Dye (2001, p. 201) notes that ”traders are assumed to simultaneously observe the asset’s price, extract some of the other traders’ information about the asset from that price, and then trade on this price, all at the same time.”

In early rational expectations models, as for example in Grossman (1976), the private information is redundant given the price (information paradox).\textsuperscript{60} These difficulties are resolved by introducing ”noise” into prices, i.e., an exogenous, unobservable source of uncertainty that causes prices to vary in addition to the variations induced by private information. The primary approach for accomplishing this has been to assume that the aggregate supply of the risky asset is exogenously random.\textsuperscript{61} This noise, then, makes prices only partially revealing because traders cannot disentangle the price change due to the noise component from that change which is due to the positive private information of other investors. The diversity of private information combined with the noise included the price imply that every investor uses both private information and the equilibrium price to create expectations.

As Hellwig (1980) points out, each trader’s private signal realization affects the

\textsuperscript{58}Brown and Zhan (1997) propose the name ’limit-order markets’ for these equilibria, since the demands are conditioned on the price.


\textsuperscript{60}Formally, the price becomes a sufficient statistic for each investor’s private information. See Christensen and Feltham (2003, pp. 78-81).

\textsuperscript{61}However, the number of outstanding shares of a stock and the number of potential traders in a securities market is fixed. Dye (2001) notes that an economically plausible interpretation of a random per capita endowment is an ”average aggregate unknown volume of trade requested by traders whose demand for the security under study is infinitely inelastic with respect to changes in the price of the security.”
price even though investors are assumed to act as price-takers. He therefore considers the limit of Grossman’s (1976) equilibrium as the number of agents grows to infinity. Then the effect of the error terms in individual signals on the equilibrium price becomes negligible. The price reflects only what is common to many signals, and the competitive price-taking assumption makes more sense than in the finite-agent economy.\textsuperscript{62}

In Grossman and Stiglitz’s (1980) influential model, each agent decides whether to acquire, at a given cost, a particular piece of information. As more traders are informed, the (incremental) value of information declines, since the price reveals more of it to the uninformed. In equilibrium, the proportion of the traders that are informed is such that the value of information exactly equals its cost.\textsuperscript{63} One conclusion is that an overall equilibrium with costly, endogenous information acquisition does not exist if markets are informationally efficient (Grossman-Stiglitz paradox).\textsuperscript{64} In Grossman and Stiglitz (1980), all investors are made worse off if they have the opportunity to acquire information. In fact, the utilities of all investors would be maximized, if they colluded and made binding commitments not to acquire any private information.\textsuperscript{65}

The suggestion that there may exist too much incentive to acquire private information raises the question of how to prevent this. Diamond’s (1985) idea is to

\textsuperscript{62}Admati (1985) extends Hellwig’s (1980) model to many risky assets. In consequence, some of the intuitive results are no longer valid. For example, an asset’s equilibrium price might be decreasing in its own payoff or increasing in its own supply. A higher price might be bad news for the asset’s payoff and so on. This is the case because a price change in one asset can provide information about other risky assets.

\textsuperscript{63}The equilibrium informed fraction is such that all investors are indifferent between being informed or uninformed. Therefore, the model does not allow for a comparison of the welfare effects of information among investors.

\textsuperscript{64}Hellwig (1982b) qualifies this result. In a multi-period setting, where uninformed investors only learn from past prices, he shows that prices may be “nearly informationally efficient” although there is a positive cost of information acquisition.

\textsuperscript{65}Such a proscription of private information acquisition would be difficult to enforce and there would be huge incentives to cheat for an investor if he considered himself the only informed trader. According to Verrecchia (2004, p. 151) the proscription “seems a difficult policy to reconcile with unregulated markets and the free flow of goods, services, and information commonly touted in free markets and democratic societies.”
substitute public disclosure for private information-gathering. Information acquisition decisions generally depend on the amount of public information available. Firms can reduce incentives to acquire private information by releasing public information. Diamond (1985) characterizes the public information-release policy that maximizes the ex-ante expected utility of speculators in a model based on Hellwig (1980) as well as Grossman and Stiglitz (1980). Because the firm can generally produce and release the information more cheaply than can the aggregate of all traders, and since (in this model) ex-ante expected utility is decreasing with the precision of public information if no private information is collected (adverse risk-sharing effect), the optimal amount of public information exactly eliminates the incentives to acquire private information.

Diamond’s (1985) result is not uncontroversial:

- Public disclosure would not have the salutary effect postulated if there were an additional round of trade in the time-interval between the release of the public information and the acquisition of the private information. The question then arises whether the results are robust or “merely a consequence of an arbitrary assumption about the timing of the operations of markets.”

- A critical question is if more disclosure really leads to fewer resources being invested in private information-gathering. It may also be the case that public disclosure and pervasive investment in private information-gathering are complements, not substitutes. When public disclosure of firm performance is combined with private information about firm potential, investors who have knowledge of the latter may be able to realize excess profits.


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68 Kim and Verrecchia (1994) and Bushman et al. (1997) use a complementary relation in theoretical models.
In Indjejikian (1991), the manager of a firm acts on behalf of the investors and selects the precision of the public report so as to maximize the investors’ ex-ante expected utility. Christensen and Feltham (2003, p. 387) note that, due to the adverse risk-sharing effect, the optimal precision of the public report is zero, making it entirely uninformative so that investors can learn nothing by acquiring private information about the public report. Bushman (1991) considers a similar setting, where investors ‘shoot themselves in the foot’ by expending funds on the acquisition of private information in order to gain advantages at the cost of the other investors. He assumes that private information is supplied by a monopolist who sets the price of his investor-specific signals so as to maximize his profits. Hence, the optimal level of public signal precision is context specific. Lundholm (1991) and Alles and Lundholm (1993) concentrate on the correlation among the private signals of investors.

To summarize, the literature on the welfare effects of public information in competitive models of trading deals primarily with the puzzling insight that public information can make investors worse off due to the adverse risk-sharing effect. Due to the limited progress of this literature, the remainder of this thesis models the disclosure of information in non-competitive settings.

1.5 Outlook

"A mathematical model is little more than a device to make a good economic argument better through the imposition of rigor." \[69\]

The remainder of this thesis examines the environment sketched in this first chapter in formal models. The analysis includes a manager who discloses a financial report which contains information about the terminal value of the firm. He has the discretion of adding a bias to the report at a cost increasing with the size of the bias. Thereby, he follows his incentives to influence the share price of his firm.

The biased report is disclosed to the capital market. Here, trading takes place among informed and uninformed investors and a market maker. These market participants learn about the parts of the report to a varying extent in the different chapters. In contrast to the models reviewed in section 1.4, the traders act strategically, i.e. they take into account the effect of their own action on the market price. Finally, the terminal value of the firm is paid out as a dividend to the shareholders.

The equilibrium concept employed is the strategic Bayesian Nash Equilibrium (BNE) concept.\textsuperscript{70} The following models might be called "rational behavioral models,"\textsuperscript{71} because they feature liquidity or "noise" traders. Black (1986, p. 531) notes that "people who trade on noise are willing to trade even though from an objective point of view they would be better off not trading." Some irrationality is necessary, because there is typically no room for earnings management in models where all participants act completely rational. Other models describe earnings management patterns and corresponding market reactions with more explicit irrational behavior. Investors’ irrational behavior may be related to their inattentiveness or their naivety with respect to financial reports. In Hirshleifer and Teoh (2003), for example, investors exhibit limited attention and processing power, i.e. they take the reported earnings at face value (without considering the possibility that they might be manipulated). However, irrationality opens up the door for criticism: Lambert (2003, p. 399), for instance, argues in the corresponding discussion of Hirshleifer and Teoh (2003) that "here, you can tell any story you want about inattentive investors’ beliefs and behavior, plug it into the model, and that behavior pops out as part of an 'equilibrium'." Therefore, the following refrains from further irrational behavior.

Chapter 2 discards the manager. It reviews a model in which an exogenous financial report is given, traders act strategically, and there is heterogeneity among investors. Furthermore, the chapter introduces a benchmark model which is an extension to the seminal Kyle (1985) model. Later results will be compared with this

\textsuperscript{70}See Fudenberg and Tirole (1996) or Gibbons (1992) for introductions to game theory. If players of a game face uncertainty and hold asymmetric information, the Nash equilibrium concept generalizes to the Bayesian Nash equilibrium, provided that agents update their prior beliefs using Bayes’ rule.

\textsuperscript{71}Lee (2001) also uses this denotation.
benchmark model and its properties. Finally, the role of liquidity traders in this class of models is discussed. Their trading profits are an interesting aspect of the setting, since they are disadvantaged by the release of a public report which they do not use. An asymmetric distribution of information does not only persist between investors on secondary markets, but also between the management of a company and its investors. Thus, an analysis which does not consider the incentives of the information discloser does not look at the whole picture.

Chapter 3 introduces a manager as a player who chooses the bias of the report so as to maximize his own utility. Similar to Fischer and Verrecchia (2000), the investors are uncertain about the manager’s interest in the share price. This makes it impossible for market participants to perfectly undo the bias. In equilibrium, the manager biases the report and the market price is not identical to the true firm value. One important equilibrium mechanism is that a lower expected bias implies a higher value-relevance of the report; this higher value-relevance in turn increases the benefits from biasing for the manager. The bias also influences the trading profits of informed and uninformed investors, while the former benefit from their informational advantage vis-à-vis the market maker. Standard setters might be especially interested in the utility of the latter, due to their mandate to provide a ”level playing field” among investors. Particularly, ”financial statements should meet the needs of those with the least ability to obtain information.” However, the exogeneity of the trading motivations of the uninformed investors makes an analysis of their trading profits questionable.

Chapter 4 makes the uninformed liquidity traders endogenous. They add an idiosyncratic value to a share that reflects their willingness for immediate execution of trade. Although the basic structure of the equilibrium remains, their strategy now influences the demand of the informed investors, trading volume, and trading profits. They still lose on average by trading with the informed investors, but their utility

\footnote{A level playing field is an environment, in which all companies in a given market must follow the same rules and are given an equal ability to compete. See Lev (1988).}

\footnote{See the American Institute of Certified Public Accountants’ Study Group on the Objectives of Financial Statements (1973, p. 17).}
is positive and increases with the informativeness of the report.

Chapter 5 presents a variant in which only the informed investors learn about the report. This is the case for selective disclosures, for example, in analyst conferences. Net demand is the only source of information for the market maker turning around some of the results obtained up to that point. The liquidity now decreases in the precision of the manager’s report, because the market maker uses net demand more for valuation. This, in effect, harms the uninformed investors.

The model in chapter 6 omits the market maker. Whether he observed the report or not plays a crucial role in the other chapters. The price is now determined by balancing supply and demand. The rational uninformed investors observe the report and trade to hedge an endowment shock. An important innovation is that now the price efficiency is influenced by the activities of the uninformed investors.

In chapter 7, a paper by Fischer and Stocken (2004) is discussed. Although they also combine the models of Kyle (1985) and Fischer and Verrecchia (2000), the differences in the setup make it difficult to compare the results.

Finally, chapter 8 concludes the work. It returns to the questions posed in the preface about the economic purpose of disclosures. The chapter starts with a general discussion of arguments in favor of and against disclosure regulations. Next, possible significations of the parameters and testable hypotheses derived from the equilibria are proposed. This includes a discussion of the implications of heterogeneity among investors, and of the information asymmetry component of the cost of capital. The final section confronts the reasoning for disclosure regulations with the results of the formal models in chapters 3 to 6.
Chapter 2

The Kyle-Setting with Exogenously Given Information

Section 1.4 reviewed a literature which discusses the informational role of prices under the assumptions of perfect competition, one-shot trading, and a Walrasian auction mechanism. However, these assumptions do not describe precisely how stock markets work. This chapter starts out in section 2.1 by comparing the aforementioned price formation process with a specialist system in which large investors have the ability to influence the price.

Section 2.2 elaborates on a specific example based on Kyle (1985), which will serve as a benchmark for comparisons with the results of later chapters. Subsequently, section 2.3 presents models with similar frameworks for studying the role of information.

In the benchmark model of section 2.2, rational investors buy (or sell) the shares supplied (or acquired) by liquidity traders who trade for reasons which are independent of the price and the information acquired by the rational investors. Section 2.4 discusses the liquidity traders’ role in the price formation process and shows how they influence the price efficiency and trading profits of the informed investors.

Finally, section 2.4 lists some features of the multivariate normal distribution that is useful for later derivations.


\section*{2.1 Price Formation}

Prices on capital markets simultaneously aggregate the information contained in investors’ demands, and determine the latter’s trading profits. The literature on market microstructure formally analyzes how specific trading mechanisms affect this price formation process. Such models are used to understand the effects of different exchange structures on such variables as market depth, price volatility, informativeness of prices, and the ability of informed traders to exploit their private information.\footnote{See O’Hara (1995, p. 1).}

Real trading mechanisms for equity shares are structured in a huge variety of ways. They differ in the type of orders available, the sequence of moves, the price setting rule, and in whether their structure is competitive or strategic.\footnote{See Brunnermeier (2001, p. 60) for this classification.}

The literature covered in section 1.4 on the social value of information as presented in the accounting literature use the Walrasian equilibrium concept. Alternatively, the price formation can be described using the framework in Kyle (1985). The following section compares the features of both approaches.

\section*{The Walrasian Equilibrium concept}

The basic one-period Walrasian setting considers a round of trading in both a risky and a risk-less asset at the beginning, and with an uncertain payoff for the risky and a certain payoff for the risk-less asset at the end of the period. Traders may differ in their endowment with the assets and their risk aversion. This setting has been extended to include public and private information procession, whereby traders may differ in the precision of their private information or in their costs of acquisition of the private information.\footnote{See section 1.4.}

Equilibrium prices are determined by the intersection of demand schedules. A demand schedule specifies the number of stocks that a trader wants to buy or sell for each possible equilibrium price. In effect, investors trade conditionally on the equilibrium price; the equilibrium price can therefore be thought

\begin{itemize}
    \item \footnote{See O’Hara (1995, p. 1).}
    \item \footnote{See Brunnermeier (2001, p. 60) for this classification.}
    \item \footnote{See section 1.4.}
\end{itemize}
of as being part of the traders’ information sets.

However, the Walrasian equilibrium concept leaves unanswered, how the price formation process works. The price formation of the Walrasian equilibrium could be implemented with a fictitious Walrasian auctioneer, who aggregates traders’ demands and supplies in order to find the market-clearing price. First, the auctioneer announces a potential trading price at random, then traders determine their optimal demands at that price. No actual trading occurs until each trader has had a chance to revise his order. A new potential price is suggested, traders again revise any orders, and the process continues until no further revision takes place. Equilibrium prevails when each trader submits his optimal order at the equilibrium price, and at that price the quantity supplied equals the quantity demanded.

The Walrasian equilibrium is a competitive equilibrium, i.e. all traders act as price takers. They think that their actions have no impact on the decisions of others. This is a reasonable assumption under the condition that there is a large number of traders or if the markets are deep, i.e. liquidity is such that individual demands do not move prices quickly. In the Walrasian equilibrium, prices only reflect information which is common to many traders. If only one, or only a small group of traders has superior information, these traders could make huge profits by trading on their information, as Hirshleifer (1971) notes.

According to Kyle (1989b), in actual financial markets, the largest players probably do have the ability to influence the price (if only temporarily), because no market is infinitely liquid or infinitely deep. Large traders, such as dealers, mutual funds, and pension funds, play an increasingly important role in financial markets. Their trades

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4See O’Hara (1995, p. 4). A similar sequential auction process is used by the London gold fixing, as noted by O’Hara (1996, p. 7).
6There are two ways to formalize “a large number of traders.” Hellwig (1980), for example, presupposes a countably infinite number of traders, while Kim and Verrecchia (1991a), for example, suppose a continuum of traders.
7Hirshleifer (1971) shows this formally in a competitive, but not Walrasian model. The Walrasian setup does not allow for individual asymmetric information, since this would contradict the price taking assumption. See Hellwig (1980).
have a significant price impact.\footnote{See the discussion in the introduction of Vayanos (1999).} Traders take into account the effect they have on the market price, i.e. they act strategically. Trade which is geared at profiting from information in turn tends to eliminate the opportunity for making such profit.

\section*{Strategic Trading}

Many models analyzing the effects of the market microstructure focus on the behavior of the New York Stock Exchange (NYSE). Here, equity trading is centered around the stock specialist (or market maker), who is assigned particular stocks in which to make a market. He serves as intermediary who quotes a bid and an asked price to buy and sell the stock up to some particular trading size. In addition to all market orders (orders for immediate execution), the specialist also receives all public limit orders (orders that are contingent on price, time, etc.), all of which are kept in the specialist’s book. On the NYSE, the book is not common knowledge, although it is often available to traders to be viewed at the discretion of the specialist. All trading at the exchange goes through the specialist, although the specialist need not be a participant in every trade. A continuous auction is used throughout the trading day, in which trades occur individually.

The bid-ask-spread, i.e. the difference between ask and bid prices, has been explained with the help of the market maker’s inventory costs\footnote{See Ho and Stoll (1983) or Huang and Stoll (1997).} or purely by information asymmetry\footnote{See Glosten and Milgrom (1985), Copeland and Galai (1983), or Kyle (1985). The idea already appears informally in Bagehot (1971).} regarding the asset’s true value. The core idea of the (for the following relevant) informational explanation is that the market maker faces an "adverse selection problem", because there are informed and noise traders. Informed and uninformed or noise\footnote{Noise traders can be motivated by liquidity or life-cycle needs, or by risk aversion coupled with endowment shocks.} traders are indistinguishable for the market maker. If the market maker quotes a single price at which he is willing to both buy and sell the asset, he breaks even on average with noise traders (because their trading is random), but
loses money systematically on transactions with informed traders, because informed traders on average only buy when the price is about to rise and on average only sell when the price is about to fall. On average, therefore, the market maker loses money when he is willing to buy and sell at the same price, losses which can be traced back to adverse selection from informed traders. Even with the bid-ask-spread, the market maker always loses to informed investors despite a spread between bid and ask prices. In effect, the market maker must recoup the losses suffered in trades with the well informed by gains in trades with noise traders. These gains are achieved by setting the spread.

One such informational approach that will be used throughout this thesis is the static version of the insider trading model in Kyle (1985). The static version consists of a one-shot trading in an asset whose end-of-period payoff is normally distributed. There are three groups of traders: a single informed investor (insider), many liquidity (noise) traders, and a market maker. The uninformed trade for liquidity reasons which are exogenous to the model, while the insider tries to exploit his given private knowledge of the terminal value. The informed insider and the liquidity traders submit their orders simultaneously. The market maker only observes the net order flow, i.e. the sum of the demands of the informed and liquidity traders. He does not observe each individual order, and, thus, he does not know the informed trader’s order size. Kyle (1985) assumes that the market maker sets the execution price equal to his best estimate of the terminal value, i.e. at the expected asset value given the order flow.

The market maker breaks even on average. He loses money to the insider but makes the same amount of money from the noise traders. Consequently, the insider’s expected profit equals the noise traders’ expected trading costs. In equilibrium, half of the insider’s information is revealed after one trading round, that is, the new variance of the true value of the stock conditional on the order flow is only half of the original

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12Kyle (1984) proposes a model of a futures market for agricultural commodities. In contrast to Kyle (1985), there are \( M \) market makers, \( N \) informed traders, and liquidity traders. He then considers the limit case with infinitely many market makers.
unconditional variance in firm value.

The insider trades strategically; he finds it optimal to withhold some of his private information from prices. In equilibrium, prices do not reveal all of the private information and the insider earns a positive profit on the basis of his private information. The variance in the uncertain terminal value measures the informational advantage of the insider; his profits increase if the terminal value is ex-ante more uncertain for the market maker. A higher variance in liquidity trading (which can be interpreted as more liquidity traders participating) provides more opportunity for the insider to disguise his information based trading and thus increases his profits.

There are a number of divergences in Kyle (1985) from the mechanisms at the New York Stock Exchange:

- There is no bid-ask-spread in Kyle (1985). However, one aspect of liquidity, namely market depth, can be measured conveniently by the coefficient of net demand in the equilibrium price. Its inverse is the demand which is necessary to change prices by one unit. Due to the analogy of the bid-ask-spread with liquidity, Kyle (1984, p. 58) states that "it is not too misleading to refer to $\lambda$ [the coefficient of net demand in his notation] as the equilibrium bid-asked spread."

- Another difference is the sequence of moves. At the NYSE, the market maker is obliged to quote a price at which he takes over the order in advance of the corresponding request (quote-driven market). The market maker offers bid and ask prices to screen out different informed traders. In Kyle (1985), the market maker first observes the aggregate net demand and then sets the price at which all demands are transacted (order-driven market). The informed trader, who submits his order first, reveals some of his private information. Thus, Kyle’s (1985) setting is closer to signaling models. However, the Kyle (1985) setup is robust to a change in the order of moves.\(^\text{13}\)

\(^{13}\text{Vives (1995b) shows that a market clearing process, whereby a competitive risk neutral market making sector submits demand schedules based on public information while the informed trader}
Finally, the NYSE and Kyle (1985) differ in the price-setting rule. At the NYSE, the market maker picks off the different limit orders at different limit prices from his order book. As he "walks along the book", his execution price for the additional units becomes worse and worse (discriminatory pricing). In Kyle (1985), the price of every unit is the same (uniform pricing).

The primary difference between the rational expectations equilibrium and Kyle’s (1985) setting is that the former emphasizes prices as signals operating through a Walrasian market-clearing mechanism, whereas the market-maker approach emphasizes quantities as signals processed by the market maker. Both approaches, however, capture the basic idea that the buying and selling of traders on the marketplace leads to prices that contain information about the value of the traded asset.

2.2 A Benchmark Economic Setting

This section provides a model on the basis of Kyle (1985) similar to Kim and Verrecchia (1994). It serves as a benchmark to compare the models in chapters 3 to 6 with. Further intuition for the equilibrium can be found in these later chapters.

The model considers the trading in one risky asset. There are two dates (one period): at $t = 0$ (presence), the asset is traded at a price $P$ and, at $t = 1$ (future), the asset is liquidated with a liquidation value $v$, which is uncertain at $t = 0$. A priori, the market participants homogeneously believe the final dividend $\tilde{v}$ to be normally distributed with a mean of zero and precision $\tau_v$. The mean of zero is the consensus value of the stock based on all public information available before the beginning of

\footnotesize

14 The differences between the presented setting and that of Kim and Verrecchia (1994) are outlined in section 2.3.

15 It is assumed, that there are no transaction costs or taxes, securities are perfectly divisible, and unlimited short selling is allowed.

16 An additional risk-less asset could be eliminated by stating the return of the risky asset in terms of units of the risk-less asset.

17 A tilde over a symbol denotes a random variable, while realizations of that random variable are represented by the same symbol without a tilde.

18 The precision of a random variable is the inverse of its variance.
the game. All participants share the same initial probability distribution, and it is common knowledge that this is so. There are three types of market participants: a market maker, \( N \) profit-maximizing informed traders, and a group of liquidity traders. The informed investors can be thought of as market experts who follow the development of a firm closely, such as large shareholders, institutional investors like insurance companies, pension funds or banks (and their analysts etc.), or the managers of competing firms.

At \( t = 0 \), the informed traders and the market maker learn a noisy public signal \( r \) about the liquidation value \( v \) given by

\[
r = v + u. \tag{2.1}
\]

The noise \( u \) is the realization of a normal random variable \( \tilde{u} \) with mean zero and precision \( \tau_u \). The random variables \( \tilde{u} \) and \( \tilde{v} \) are independently distributed.

Each informed investor \( i \) receives additional information \( o_i \) about the noise \( u \) in the public report given by

\[
o_i = u + e_i. \tag{2.2}
\]

Each idiosyncratic error term \( e_i \) is the realization of a normal random variable \( \tilde{e}_i \) with mean zero and equal precision \( \tau_e \).\(^{19}\) All random variables are independent.

Each informed investor calculates his demand \( x_i \) by maximizing his expected trading profit

\[
\Theta_i = E[(\tilde{v} - \tilde{P}(x_i))x_i | r, o_i] \tag{2.3}
\]

given his information with respect to \( x_i \), and submits it to the market maker. All orders are market orders. Liquidity traders demand the exogenously given amount of \( y \) shares, whereby \( y \) is the realization of a normal random variable with mean zero and variance \( \sigma_y^2 \).

\(^{19}\)Kyle (1985) assumes that the informed trader observes the liquidating value of the firm without error.
The market maker observes the net demand\textsuperscript{20}

\[ Q = \sum_{i} x_i + y \quad (2.4) \]

and sells those shares at a price \( P \) so as to break even in expectation,\textsuperscript{21} i.e.

\[ P = E[\hat{v}|r, Q]. \quad (2.5) \]

At \( t = 1 \), the liquidating dividend \( v \) of the asset is paid out to the market participants.

The parameters \( \tau_v, \tau_u, \tau_e, \sigma^2_y \), and \( N \) are common knowledge. The mean values of \( \hat{v}, \hat{u}, \hat{e}_i \) for \( i = 1, \ldots, N \), and \( \hat{y} \) are zero. Non-zero means would simply be carried through all equations without changing the results significantly.

**The Equilibrium**

Kyle (1985) focuses on the Bayesian Nash equilibrium. To determine his respective equilibrium strategy, each participant forms a conjecture about the behavior of the other players. The analysis is restricted to linear equilibria,\textsuperscript{22} i.e. the players’ decision variables are linear in their respectively given information. The market maker conjectures that the informed traders will demand\textsuperscript{23}

\[ \hat{x}_i = \beta_r r + \beta_e \rho_i \quad \text{for} \quad i = 1, \ldots, N. \quad (2.6) \]

\textsuperscript{20}This is the aggregate net order flow. First, informed and liquidity traders trade among themselves. The market maker then only serves the order imbalance.

\textsuperscript{21}The market maker thus acts as a perfect competitor, which can be ensured by two market makers who act under Bertrand competition. He is assumed to have an unlimited inventory of cash and securities, and to face no transaction costs (including his time, inventory costs, etc.).

\textsuperscript{22}The linearity does not preclude the existence of other, nonlinear equilibria. Nonlinear equilibria are usually not studied in the literature, probably because nonlinearity makes the derivations intractable. The equilibrium derived below is unique for the class of all prices, having the conjectured linear form. Bagnoli, Viswanathan and Holden (2001) provide a comprehensive overview of the existence of linear equilibria in static Kyle (1985)-type models (and sequential trade models à la Glosten and Milgrom (1985)) for various distributions of the asset value and the liquidity trading. They show that, for a fixed number \( N \) of symmetrically informed investors, the linear equilibrium is the unique equilibrium even for a class of so-called "stable" distributions (including the normal distribution), and that the results from the standard Kyle version are due to its focus on linear equilibria, not to the assumption of normality.

\textsuperscript{23}A hat over a variable denotes a conjecture about its equilibrium form.
The informed traders conjecture that the market maker will set the price according to
\[ \hat{P} = \delta_r r + \delta_Q Q \] (2.7)
with the other \( N-1 \) informed traders also trading according to (2.6). In equilibrium, the strategies are self-fulfilling. That is, the demand \( \hat{x}_i \) maximizes the investors’ trading profit (2.3) for all investors \( i \) given the conjectured price (2.7), and the market maker breaks even according to (2.5) given the conjectured demands (2.6) of the informed traders.\(^{24}\) The equilibrium is symmetric because all informed investors play the same strategy.

**Proposition 2.2.1.** A unique linear equilibrium exists for any combination of the parameters \( \tau_v, \tau_u, \tau_e, \sigma_y^2, \) and \( N \). The equilibrium demand \( x_i \) of the informed traders and the equilibrium price \( P \) are given by
\[
x_i = \beta \left( \frac{\tau_v}{\tau_v + \tau_u} r - \alpha_i \right) \quad \text{for } i = 1, \ldots, N \quad \text{and} \quad (2.8)
\]
\[
P = \frac{\tau_u}{\tau_v + \tau_u} r + \frac{\beta^{-1}}{N + 1} \frac{1}{1 + \frac{2}{N+1} \frac{\tau_v + \tau_u}{\tau_e}} Q, \quad (2.9)
\]
whereby
\[
\beta = \sqrt{\frac{1}{N \sigma_y^2} \frac{\tau_v (\tau_v + \tau_u)}{\tau_e + \tau_v + \tau_u}}. \quad (2.10)
\]

**Proof.** The proof resembles Kim and Verrecchia (1994) who analyze a multi-period model, yet with only one period, in which a release of public information occurs. In addition, Kim and Verrecchia (1994) assume a correlation between the error terms \( \text{Corr}(\tilde{e}_i, \tilde{e}_j) = \rho \frac{1}{\tau_e} \) with \( 0 \leq \rho \leq 1 \) of the private signals and fixed costs of information acquisition \( C \). The stated results follow for \( \rho = 0 \) and \( C = 0 \). □

\(^{24}\)It is assumed that they make that conjecture because it turns out to be the correct one (i.e., it is self-fulfilling). Although traders may not be born with this intuition, they are presumed to develop it over time. In the context of a one-shot model of trading, however, this argument may sound a bit vague.
particular objective.” Since the price is set competitively, the informed investor can act as if he had monopoly power over the price. He demands such a number of shares that the expected terminal value $v$ (corresponding to the constant marginal revenues) equals the marginal costs of an extra share (see figure 2.1). In the benchmark model, the interaction between liquidity and informed demands is similar, because the $N$ informed investors act identically.

![Figure 2.1](image)

Figure 2.1: The informed investor behaves like a monopsonist. Suppose he knows the terminal value is $v$ and the price is $P(x) = a + bx$. Then the total costs are $ax + bx^2$ and the marginal costs are $a + 2bx$. The optimal demand $x^*$ is realized for $x = \frac{v-a}{2b}$. The optimal price $P^*$ is independent of $b$. (Confer figure 26.2 in Varian (2003, p. 461)).

It follows that the price is independent of the variance $\sigma^2_y$ in liquidity trading: If the liquidity trading is more variant, the price becomes less sensitive to a change in the demand, i.e. $\delta_Q$ decreases ($b$ decreases in figure 2.1). The informed investor then

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26 As Kyle (1985) notes, the situation is similar to a monopsonist who dominates the factor market and sells his products in a competitive market. See Varian (1999, p. 459-461).
increases his equilibrium demand accordingly (the intersection between \( v \) and \( a + 2bx \) moves to the right in figure 2.1), and extracts more trading profits. Consequently, the price is independent of the variance \( \sigma_y^2 \) and the liquidity traders do not influence the price efficiency.

The analogy between Kyle (1985) and the current model does not hold for the influence of the precisions of the random variables on the equilibrium, because the informational distribution is different. In Kyle (1985), only the informed investor learns the terminal value \( v \) and its variance is a measure of his informational advantage. Here, the noisy signal \( r \) on the firm value is a public information, i.e. it is also observed by the market maker. The informed investors additionally observe a noisy information \( o_i \) about the noise \( u \) in the public report. The equilibrium price increases for a higher report \( r \) and a higher net demand \( Q \) because the net demand contains additional information on the firm value via the informed trading. The informed investors trade on a difference between the expected terminal value given the report

\[
E[\tilde{v}|r] = \frac{\tau_v}{\tau_v + \tau_u} r
\]

and their private information \( o_i \). The trading aggressiveness corresponds to \( \beta \); it increases if there are less informed traders \( N \) and if the variance of noise trading \( \sigma_y^2 \) is higher. The influence of the precisions \( \tau_v \) and \( \tau_u \) is indeterminate.

The remainder of this section states and describes the influence of the parameters \( \tau_v, \tau_u, \tau_e, \sigma_y^2, \) and \( N \) on

- price informativeness,
- trading volume,
- liquidity, and
- the trading profits of informed and liquidity traders.

This section presents only a benchmark model; more detailed intuition for the marginal influences is offered in later chapters.
Price Informativeness

The price at which shares are finally traded can be obtained by inserting definitions (2.1), (2.2), (2.4), and the equilibrium demand (2.8) into the equilibrium price (2.9). The ex-post price then is

\[ P = \frac{\tau_u + \frac{N}{N + 1} \tau_v}{\tau_u + \tau_v} v + \frac{1}{N + 1} \frac{\tau_u}{\tau_u + \tau_v} u - \frac{1}{N + 1} \sum_{i=1}^{N} (u + e_i) + \frac{1}{N + 1} \frac{1}{\beta} y. \]  

(2.12)

The price approximates the terminal value \( v \) but is biased by the influence of the noise in the public information, in the private signals, and by the demand of the noise traders. It is noteworthy that, as the number of informed traders \( N \) grows large, the equilibrium price converges to the asset value \( v \). The price averages out the idiosyncratic error terms \( e_i \) of the informed traders.

Prices in theoretical models with rational, utility maximizing agents are, of course, highly efficient. The market maker assures that the public report \( r \) is correctly and fully reflected in the price. Private information is not expected to be reflected completely in the price, since the informed traders act strategically. Here, the informativeness of the price, which is measured by the reduction of uncertainty due to the observation of the price, is

\[ Var(\tilde{v}|r) - Var(\tilde{v}|P, r) = \frac{1}{\tau_u + \tau_v} \frac{N \tau_e}{(N + 1) \tau_e + 2(\tau_u + \tau_v)}. \]  

(2.13)

This incremental informativeness of the price decreases with the precisions \( \tau_v \) and \( \tau_u \) of the public report and increases with the precision \( \tau_e \) in the idiosyncratic error terms and the number \( N \) of informed traders. The informativeness is independent of the noise \( \sigma_y^2 \) created by the liquidity traders.

Because of the sequence of moves, there is uncertainty for the investors about the price at which trades are executed. The price risk, i.e. the a-priori variance in the price regarding (2.12) as a random variable, is

\[ Var[\tilde{P}] = \frac{\tau_u}{\tau_v + \tau_u} \frac{1}{\tau_v} + \frac{1}{\tau_u + \tau_v} \frac{N \tau_e}{(N + 1) \tau_e + 2(\tau_u + \tau_v)}. \]

\[ ^{27} \text{Later chapters measure the price informativeness as the remaining variability in the distribution of the asset value given the observation of the price } Var[\tilde{v}|P, r]. \text{ Due to the presence of idiosyncratic error terms, this would be intractable here.} \]
Trading Volume

The trading volume is defined as the number of shares which changes hands. Shares are traded both between investors (informed and liquidity traders) and between investors and the market maker. The first amount is half of the sum of the absolute demands \( \sum_{i=1}^{N} |x_i| + |y| \) (to avoid double counting) minus the net demand \(|Q|\). The latter amount, which is traded with the market maker, is the absolute value of the net demand \(|Q|\). The trading volume is the sum of both and equals

\[
\hat{V} = \frac{1}{2} \left( \sum_{i} |\tilde{x}_i| + |\tilde{y}| - \left| \sum_{i} \tilde{x}_i + \tilde{y} \right| + \left| \sum_{i} \tilde{x}_i + \tilde{y} \right| \right). \tag{2.14}
\]

The expected trading volume is therefore\(^{28}\)

\[
E[\hat{V}] = \frac{1}{2} \left( \sum_{i} E|\tilde{x}_i| + E|\tilde{y}| + E|\sum_{i} \tilde{x}_i + \tilde{y}| \right) \tag{2.15}
\]

\[
= \frac{1}{\sqrt{2\pi}} \left( \sum_{i} \sqrt{Var(x_i)} + \sigma_y + \sqrt{Var(\sum_{i} \tilde{x}_i + \tilde{y})} \right)
\]

\[
= \frac{\sigma_y}{\sqrt{2\pi}} \left( \sqrt{N} + 1 + \sqrt{1 + \frac{N\tau_c + \tau_u + \tau_v}{\tau_c + \tau_u + \tau_v}} \right). \tag{2.16}
\]

Higher precisions \(\tau_u\) and \(\tau_v\) increase the expected trading volume and a higher \(\tau_e\) decreases it.\(^{29}\) The expected trading volume also increases with the number \(N\) of informed traders, and the variability \(\sigma_y^2\) of liquidity trading.

\(^{28}\)The expected value of the absolute of a normal random variable equals its standard deviation times \(\sqrt{\frac{2}{\pi}}\).

\(^{29}\)Kim and Verrecchia (1994) exclude the factor \(Q = \sum_{i} \tilde{x}_i + \tilde{y}\) from the trading volume, i.e. they exclude trading with the market maker. In consequence, the formula no longer depends on the precisions \(\tau_e, \tau_u, \) and \(\tau_v\). Although it may be reasonable to exclude shares traded with the market maker, it does not fit to the definition of the volume as the number of shares which changes hands. Kim and Verrecchia (1994) go on to analyze the influence of costs of information acquisition on the expected trading volume, an analysis that would be intractable if the volume included additionally the precisions.
Since, as Christensen and Feltham (2003) argue, an explanation of the relationship between price changes and trading volume caused by information is one of the main contributions of the rational linear expectations literature, the following presents the main results of Holthausen and Verrecchia (1990).\footnote{See also Karpoff (1987) for a review of theoretical and empirical work which explores the relation between volume and price changes.}

Holthausen and Verrecchia (1990) provide intuitive insights in the expected trading volume. The setting is that of a partially revealing rational expectations model of competitive trading, in which many agents exchange a single risky asset over one period, as reviewed in section 1.4. At the beginning of the period, agents have homogeneous expectations with respect to the value of the risky asset. Exchange is motivated by agents receiving information about the liquidating value of the risky asset. Investors’ interpretation of that information contains a noise term which is common across all agents, and an agent-specific idiosyncratic noise term of the form $r_i = v + u + e_i$.\footnote{Note that in the competitive models, agents also learn something about other investors’ beliefs through the price aggregation process, yet they do not learn everything. Hence, investors have heterogeneous expectations after the information is released.}

Holthausen and Verrecchia (1990) identify two information effects: an informedness effect and a consensus effect. The informedness effect measures the degree to which agents become more knowledgeable (i.e. the precision of investors’ beliefs conditional on observing information and price), while the consensus effect measures the extent of agreement among agents at the time of an information release (i.e. the covariance between agent’s idiosyncratic error terms). It is demonstrated that price changes and trading volume are both influenced by informedness and consensus alike. Holding the other effect constant, a higher consensus among agents decreases the expected trading volume, and a higher informedness increases the expected trading volume. The specification above has no correlation among the idiosyncratic noise terms; therefore, the consensus effect cannot be observed.
Verrecchia (1993, p. 871) explains the intuition underlying price and volume reactions as follows: "Price is a type of aggregator of information that averages the opinions and information of investors in the marketplace, giving weight to each opinion on the basis of the confidence expressed by that investor through the aggressiveness of his or her demand for the asset. Through this averaging process, price yields a type of 'consensus judgment’ that ameliorates the problem created by the idiosyncratic error in each individual investor’s opinion. Volume, on the other hand, arises from differences of opinion. While volume is driven in part by the same shift in opinion captured by a change in price, volume aggregates the idiosyncratic error in individual investors’ opinions (as opposed to averaging it out, as in the case of price). Idiosyncratic errors exacerbate the volatility of volume more so than price."

**Liquidity**

Kyle (1989b) distinguishes between three aspects of market liquidity. These include the "tightness" of the market (that is, the cost of turning over a position in a very short period of time) and the "resiliency" of the market (that is, the speed at which prices tend to converge towards the underlying value of the commodity when perturbed by noise trading). However, these aspects cannot be calculated in an atemporal context with one-shot trading.

However, the "market depth", which measures the demand necessary to increase the price by one unit, can be calculated conveniently here. It is given by the inverse of the demand coefficient $\delta_Q$ in (2.7) as

$$
\frac{1}{\delta_Q} = \beta \left( (N + 1) + 2 \frac{\tau_v + \tau_u}{\tau_e} \right) = \sqrt{\frac{\sigma_y^2}{N \tau_e \tau_v + \tau_v + \tau_u}} \left( (N + 1) \tau_e + 2(\tau_v + \tau_u) \right).
$$

(2.17)

The liquidity increases with the variance in liquidity trading $\sigma_y^2$ and the precisions $\tau_v$ and $\tau_u$ of the public information. The influence of the precision $\tau_e$ in the error terms in the private signal and the number $N$ of informed investors are indeterminate.
Trading Profits

The trading in the asset is a zero sum game, i.e. the realized profits of the market maker, the informed investors and the liquidity traders sum up to zero. Since the market makers’ expected profit is zero, the expected trading profits of the sum of informed investors equals the trading losses of the uninformed liquidity traders. The expected profit of the informed traders in (2.3) is

$$\Theta = \frac{1}{N} \sigma_y^2 \delta Q \quad \text{for} \quad i = 1, \ldots, N.$$  \hspace{1cm} (2.18)

The expected profit increases with the variance $\sigma_y^2$ of liquidity trading, and decreases with the number $N$ of informed traders as well as with the market depth $\frac{1}{\delta Q}$. Hence, the expected profit of the informed traders decreases with the precisions $\tau_v$ and $\tau_u$. The influence of $\tau_e$ is indeterminate.

The expected trading loss of the liquidity traders, in turn, increases with $\sigma_y^2$, and decreases with $N$ and the market depth $\frac{1}{\delta Q}$.

As already noted by Kyle (1984), an increase in liquidity trading tends to increase the profits of the informed investors and thus induces the entry of new informed traders. From equation (2.13), it can be seen that this increase in $N$ leads to an increase in the informativeness of prices. Thus, in this framework, liquidity trading tends to stabilize prices.\textsuperscript{32}

2.3 Extensions of Kyle (1985)

The basic Kyle (1985) setting has been extended\textsuperscript{33} in a number of directions for studying market microstructure phenomena.\textsuperscript{34} In this section, the focus lies on the

\textsuperscript{32}This finding is in contrast to the competitive models of trading reviewed in section 1.4, where noise trading made the price less informative.

\textsuperscript{33}Kyle (1985) already contains an extension of the static model to a series of discrete call markets in a sequential auction setting. In this dynamic setting, the insider faces the following trade-off: if he takes on a larger position in the early periods, his early profits increase but prices in the later trading rounds worsen.

\textsuperscript{34}For a broader coverage of the market microstructure literature, see O’Hara (1995) or the survey of Madhavan (2000).
incorporation of private and public information in the price. Public information may invoke the acquisition of private information before or after its disclosure, and there is interaction between both. The Kyle (1985) setting has also been used to study other accounting-related questions.

**Private Information after Public Disclosure**

In addition to the setting considered in section 2.2, in Kim and Verrecchia (1994), investors can acquire additional information about the error in a public disclosure. There is a correlation $\text{Corr}(\tilde{e}_i, \tilde{e}_j) = \rho \frac{1}{\tau_e}$ with $0 \leq \rho \leq 1$ between the error terms of the private signals, and fixed costs $C$ of information acquisition arise. This allows making the number of informed investors endogenous. Since the expected trading profit decreases with the number $N$ of informed traders, under free entry their number will be precisely so as to turn the profits per trader to zero (disregarding integer problems):

$$\Theta(N) - C = 0. \quad (2.19)$$

It follows, then, that the number of information processors $N$ decreases with the prior precisions $\tau_v$ and $\tau_u$ of the public report. Their number increases with the variance of liquidity trading $\sigma_y^2$ and decreases with the processing costs $C$. The sign of the precision in the error term is ambiguous.

Kim and Verrecchia (1994) continue by looking at the effect of the mentioned parameters on market liquidity, volume, and price informativeness when the number of informed investors is endogenous. The market is more liquid for a more precise public report and more liquidity trading. The influence of the costs of information acquisition on the liquidity is indeterminate. The trading volume is lower for a more precise report, but increases with liquidity trading and the information acquisition costs. The incremental informativeness of the price (as defined in section 2.2 above) now, and in contrast to the benchmark model, decreases with the precision of the public report, increases with liquidity trading (which had no influence without costs of information acquisition), and decreases with the information processing costs.
Kim and Verrecchia (1994) use a multi-period extension of Kyle (1985), but with only one earnings announcement in which the mechanisms described above can be observed. There are, in addition to the market maker and the informed traders, non-discretionary and discretionary liquidity traders. The former trade a random amount on each trading date, while the latter can choose to trade at one date a given random amount. By allowing the noise traders to switch into whichever time period, Kim and Verrecchia (1994) attempt to make the trading behavior of the noise traders strategic while constraining them to the trading of a specific amount of the shares. At the time of an earnings announcement, there is more information asymmetry and, consequently, less liquidity in the market. Also, the trading volume may increase due to more informed trading, despite the decreasing liquidity.

The model of Kim and Verrecchia (1994) fits to a situation of simultaneously available private and public information. If one intends to study a posterior information acquisition, "there is no reason not to suppose that a (new) securities market opens in the time interval between the acquisition of the private information and the release of the public information." This situation has been modeled for competitive markets in Feltham and Wu (2000). They extend Grossman and Stiglitz (1980) to a situation where all rational investors first receive a public signal and where, in the second period, informed investors receive a common private signal at a fixed cost. If the informativeness of the public report is increased while reducing the incremental uncertainty resolved by the private signal, it follows that the fraction of investors who become informed is reduced. Yet what happens to the precision of the uninformed investors’ posterior belief regarding the dividend? The direct effect is that a more precise public signal reduces uncertainty about the dividend. However, as an indirect effect, less about the terminal value can be inferred from the price since less private information is reflected in it. Feltham and Wu (2000) show that in their model, the two effects exactly offset each other.\footnote{See chapter 4 for different approaches to endogenize the liquidity trading.}

\footnote{See Dye (2001, p. 192).}

\footnote{In addition, Feltham and Wu (2000) include an agency setting.}
Private Information before Public Disclosure

McNichols and Trueman (1994) study the previous acquisition of information in a model based on Kyle (1985). They examine how public disclosure affects previous private information acquisition activity. In their setting, traders with short-term investment horizons are allowed to trade on their private information prior to a public disclosure. A preference for short-term profits may be due to either capital or labor market imperfections. For instance, if portfolio managers’ performance is evaluated over short intervals, that makes long-term positions more costly to hold than those of a shorter term.

There are four relevant dates: at date 0, one informed trader acquires a noisy private information \( \theta = v + \epsilon_1 \) about the liquidating value \( v \) of the firm. Subsequently, at date 1, trading in the two assets takes place. The informed trader’s optimization problem is \( \max_x E[(P_2 - P_1)x|\theta] \). The price at date 1 is \( P_1 = E[v|x+u] \). At date 2, the firm releases some information \( z = v + \epsilon_2 \) about its liquidating value \( v \) with a known probability. The covariance between the two error terms \( \epsilon_1 \) and \( \epsilon_2 \) is \( \sigma_{12} \). Also at date 2, both the informed trader and the liquidity traders close out their positions in the firm. The price which the market maker sets at date 2 is \( P_2 = E[v|x+u, z] \). Finally, at date 3, the firm is liquidated.

McNichols and Trueman (1994) demonstrate that in this setting, public disclosure stimulates investment in prior private information acquisition. More specifically, it is demonstrated that an increase in either the probability or the precision of a public disclosure also increases the expected trading profits of the informed trader thus giving him an incentive to increase the precision of his private information. This result

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38 For another model with prior private information, see the presentation of Fischer and Stocken’s (2004) model in section 6.3.

39 Due to a correlation in error terms, a situation may exist where the informed trader takes a long (short) position in the firm’s shares when he receives unfavorable (favorable) private information. McNichols and Trueman (1994) exclude this nonintuitive possibility from their analysis.

40 This implies that the market maker cannot make additional inferences about the liquidating value of the firm from the date 2 order flow.

41 This stands in contrast to Diamond’s (1985) conclusion that the introduction of a public disclosure reduces the incentive to collect private information.
is shown to have implications for the magnitude of the pre-announcement and announcement price reactions to the disclosure. Allowing the informed trader to decide upon the precision of his private information, McNichols and Trueman (1994) show that the absolute pre-announcement price change increases both with the probability and the precision of the public disclosure. As a consequence, the magnitude of the announcement-date price change decreases with the probability of disclosure, and in some cases also decreases with its precision. This last result implies that it is not always possible to use the magnitude of the announcement-date price reaction for assessing the informativeness of a public disclosure.

The setting of private information acquisition prior to a public report has also been examined in competitive markets by Demski and Feltham (1994), and Kim and Verrecchia (1991a and 1991b). Demski and Feltham (1994) extend the Grossman and Stiglitz (1980) model. At $t = 1$, some investors acquire private information at a fixed cost, and trading with liquidity traders takes place. At $t = 2$, the public report is released and again, liquidity trading takes place. At $t = 3$, the firm pays a terminal dividend. The public report is assumed to be a sufficient statistic for the private signal. The equilibrium fraction of informed traders is similar to Grossman and Stiglitz (1980) except that the variance term for the private information is more complex, reflecting the additional riskiness of the return due to the second round of trade. If the informativeness of the public report is increased while the informativeness of the private signal remains constant, the price-informativeness at $t = 1$ decreases while at $t = 2$, trading volume decreases and price variability increases.

All of these two-period models of trading must reduce the complexity of the analysis for reasons of tractability: McNichols and Trueman (1994) assume that the informed investor closes out his position in the second trading round (from which follows that he solves only one optimization problem). Demski and Feltham (1994) assume that the public disclosure is a sufficient statistic for the private signal (from which follows that the private information need not be considered as inducing optimal trading in the second trading round).
Other Accounting-Related Papers Following Kyle (1985)

The remainder of this section addresses four further models which are based on Kyle (1985). They exemplify the variety of problems that can be addressed relating to Kyle’s (1985) basic model.

Diamond and Verrecchia (1991) make the number of market makers endogenous in the one-periodic Kyle (1985) setting. They show how limited risk-bearing capacity on the side of the risk-adverse market makers interacts with the effects of private information in the determination of security prices. Two large institutional traders are subject to future liquidity shocks that may force them to trade a particular random amount in the future. Alternatively, they may receive private information in the future. Given a fixed number of market makers, large traders would be better off with reduced information asymmetry, facing a more liquid market as a result. Yet in Diamond and Verrecchia’s (1991) setting, a countervailing effect occurs in that the reduced information asymmetry reduces the volatility of order imbalances, causing some market makers to exit. Therefore, the maximum current price is accompanied by some asymmetry of information.

Baiman and Verrecchia (1996) examine the relation between the level of financial disclosure, the firm’s cost of capital, production efficiency, and the extent of insider trading in a Kyle (1985) type of market. The level of disclosure is determined by trading off a production-efficiency effect and a compensation-subsidy effect, both of which are weakened with increased disclosure, against a market-illiquidity effect and its effect on the cost of capital, both of which are also weakened with increased disclosure. Production efficiency drops because more disclosure means that less information about the manager’s action is impounded in price, so that price-based performance measures become less efficient, agency problems increase, and output falls. The compensation-subsidy drops because more disclosure reduces the manager’s insider trading profits, thereby reducing the market-subsidy associated with hiring and paying the manager. Market illiquidity and the cost of capital drop because more

42See also Baiman and Verrecchia (1995), Stocken and Verrecchia (1999), and Neus (1999).
disclosure encourages investment by individuals who may have future liquidity needs.

Kwon (2001) extends the Feltham-Ohlson residual income valuation model\textsuperscript{43} to a situation with asymmetrically informed traders as in Kyle (1985). He thus applies the Kyle (1985) model to an infinite horizon. Since traders are asymmetrically informed in the Kyle (1985) setting, the firm value is no longer equivalent to the present value of the firm’s expected dividends. In his model, the informed investor observes a signal for the firm’s profitability which the market maker is unable to observe. The analysis identifies the equilibrium firm-value as a linear function of current book value, current residual income, and the aggregate order flow.

Zhang (2001) considers two rounds of trading, first in the primary and then, along the lines of Kyle (1985), in the secondary market. He assumes that the owner of the firm sells his shares in an initial public offering at $t = 0$, and that he makes at that time a commitment with respect to the informativeness of the public report, which his firm will release at $t = 1$. Furthermore, there are no informed investors acquiring the shares at $t = 0$, and the price is set in such a manner that an investor who believes he may be a liquidity trader at $t = 1$ expects to break even. The single informed investor chooses the informativeness of his private signal endogenously so as to maximize his utility. The marginal cost of the incremental informativeness of the private signal is thereby assumed to be increasing with the informativeness of the public report. That is, the more informative the public report, the more costly is it for the informed investor to obtain additional information. Zhang (2001) shows, for example, that the owner’s choice of public report informativeness decreases with the cost for the private signal, and increases with the noise due to liquidity trading.

\textsuperscript{43}See Ohlson (1995) and Feltham and Ohlson (1995). The model indicates that, given a dividend valuation model and clean surplus accounting, the stock price can be written as a linear function of earnings and the book value of equity.
2.4 The Role of Liquidity Trading

The Kyle (1985) setting distinguishes between two types of investors and a market maker. While the informed investors trade to maximize their profits on the basis of private information, the demand of the uninformed investors is given exogenously as the realization of a random variable. The presence of noise which the uninformed investors add to the net demand is necessary in the Kyle (1985) setting, because the equilibrium is based on the signal extraction problem of the market maker. If there were no noise in net demand, the information asymmetry between the market maker and the speculator would break down and the informed investor would have no incentive for trading.

The demand of uninformed investors may result from a number of factors: it might be liquidity motivated trading due to an idiosyncratic wealth shock, i.e. the investor might need to invest excess capital, or he might need a certain amount of money for a specific purpose, e.g. for immediate consumption or because he has a private investment opportunity. Other motivations include portfolio rebalancing, risk-exposure adjustment after preference shocks like changes in risk aversion, or tax planning. Additionally, irrational trading can occur; “People sometimes trade on noise as if it were information.” This implies that investors may act on the belief that they possess value relevant knowledge, although their information is in fact already incorporated into the price. They may have less ability to interpret financial information or may simply not follow firms closely enough so as to be able to understand the implications of information.

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44 However, in section 2.2, the realized liquidity demand forces the uninformed investor to sell or buy a fixed number of shares of the asset, independently of the resulting market price. Another drawback is the absence of other ways of satisfying the liquidity shock. Dye (2001, p. 208) enumerates that the liquidity trader cannot “go to a bank to get a loan, use his credit cards, or sell some other asset. If the liquidity trader demands shares, he cannot consider any other use of his money than to buy this firm’s shares.”

45 See Black (1986, p. 529). He describes the essential role which noise plays for equilibria in the fields of finance, econometrics, and macroeconomics.

46 “As a consequence many detailed information [in financial statements] are simply ignored and only evaluated by a few specialists...” See Wagenhofer (2003, p. 571).
Brunnermeier (2001, p. 27) distinguishes between trade on information of common and of private interest. Information of common interest, for example is information about the liquidation value of the traded asset. Information of private interest leads to noise trading with such diverse motivations as described above. A feature of private interests is that the trader’s behavior does not “influence the aggregates”, i.e., given their trading pattern, the dynamics of price formation do not depend on their exact economic motive. If motivations were systematic among many investors, it would become information of common interest.

**Is Noise Trading Good or Bad?**

According to Kyle (1989b), financial markets serve two key roles:

1. They provide liquidity to investors who want to exchange financial assets for cash or vice versa without necessarily possessing superior information.47

2. They provide informationally efficient prices. This is a prerequisite for the allocation role of prices. The terms on which companies are able to raise and invest capital depend on the share prices. Via the level of capital costs, the information procession influences the investment and production decisions of companies.48

In the benchmark setting of section 2.2, the relationship between these two roles can be analyzed. Suppose the variance $\sigma_y^2$ of noise trading is monotonically increasing with the number of noise traders.49 Using the results of section 2.2, more noise traders then increase the liquidity (see (2.17)) and the profit of the informed traders (see (2.18)), but have no influence on price efficiency (see (2.13)). However, if the number $N$ of informed investors is given endogenously (see (2.19)), the price efficiency also increases with the number of noise traders. Kyle (1989b) argues that noise traders, who use the marketplace for exchanging assets, pay for publicly revealed information in the form

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49 See chapter 3 for a formal justification.
of transaction costs or expected trading losses, which are necessary because informed traders make profits from trading in the market. Because noise trading increases price efficiency in Kyle (1989b), he argues that a regulator should attempt to increase the liquidity of the market by attracting more noise traders.\footnote{This result does not hold in the competitive rational expectations models as reviewed in section 1.4. There, more liquidity trading reduces price efficiency and destabilizes the market. Another difference is that, in competitive models, the equilibrium collapses with risk neutrality since a risk-neutral price taker demands infinitely many stocks if his estimate of the stock value exceeds the competitive price. In strategic models, the trader also takes into account the fact that the price moves against him if he demands a large quantity.}

On the other hand, the better a market processes information and the higher its informational efficiency is, the lower is the danger of the exploitation of uninformed investors. If the less informed investors realize that they are exploited systematically, this can negatively influence their willingness to buy assets on capital markets. Should they thus withdraw from capital markets due to a lack of confidence, less capital is disposable for the raising of funds.\footnote{See Franke and Hax (2004, p. 406).} However, the benchmark model does not address this effect formally.

The trading models presented in this chapter focus on the market participants’ information gathering and trading activities. The public disclosure is taken as given exogenously. However, the way in which traders act on publicly disclosed information and the manager’s incentives to disclose are clearly interdependent. A more realistic model would combine both parts in the analysis. This view is shared by Verrecchia (2001, p. 173) who notes in a review article that ”research that examines incentives to disclose in markets comprised of a single, representative trader as no more or less ‘comprehensive’ than those that endogenize the market and treat disclosure as exogenous. Both approaches only look at one piece of the overall disclosure puzzle.” The next chapter explicitly includes incentives of the firm’s manager, who selects the properties of the public information so as to maximize his own utility.
The Multivariate Normal Distribution

Some properties of the multivariate normal distribution will be used throughout this thesis.  

For \( n \geq 2 \) let \( x = (x_1, \ldots, x_n)^T \) be an \( n \)-dimensional normal random variable. Let \( \mu_i \) and \( \sigma_i^2 \) denote the mean and variance of \( x_i \) for \( i = 1, \ldots, n \) and \( \sigma_{ij} \) the covariance between two random variables \( i \) and \( j \). Then

\[
\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{pmatrix}
\]

are, respectively, the mean vector and the covariance matrix of \( X \).

For fixed \( k < n \), consider the partitions of \( X, \mu, \) and \( \Sigma \) given below:

\[
X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \quad \mu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}
\]

where

\[
X_1 = (x_1, \ldots, x_k)^T, \quad X_2 = (x_{k+1}, \ldots, x_n)^T, \quad \nu_1 = (\mu_1, \ldots, \mu_k)^T, \quad \nu_2 = (\mu_{k+1}, \ldots, \mu_n)^T.
\]

Three basic properties are:

1. The marginal distributions of \( X_1 \) and \( X_2 \) are normally distributed with mean \( \nu_1 \) and \( \nu_2 \) and covariance matrix \( \Sigma_{11} \) and \( \Sigma_{22} \), respectively.

2. A linear combination \( Y = CX + b \), where \( C \) is any given \( m \times n \) matrix and \( b \) is any \( m \times 1 \) vector, is normally distributed with mean \( C\mu + b \) and covariance matrix \( C\Sigma C^T \).

3. The conditional distribution of \( X_1 \) given \( X_2 = y_2 \) is normally distributed with mean

\[
\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (y_2 - \mu_2)
\]

and covariance matrix

\[
\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}.
\]

Tow propositions which follow from these basis properties are

**Lemma 2.4.1.** Suppose \( k = 1 \). For an invertible \((n-k) \times (n-k)\) matrix \( C \), the conditional distributions of \( X_1 \) given \( X_2 = y_2 \) and \( CX_2 = Cy_2 \) are identical, which implies for example that

\[
E[X_1 | X_2 = y_2] = E[X_1 | CX_2 = Cy_2].
\]

In this case, the two systems of equations \( X_2 = y_2 \) and \( CX_2 = Cy_2 \) are called informationally equivalent, because the solution spaces of \( X_2 = y_2 \) and of \( CX_2 = Cy_2 \) are identical.

**Lemma 2.4.2.** Let \( x_1, x_2, \ldots, x_n \) be normally and independently distributed random variables with zero means and precisions \( \tau_1, \tau_2, \ldots, \tau_n \). Defining \( \tau^* \) by

\[
\tau^* = Var^{-1}[x_1 | x_1 + x_2, x_1 + x_3, \ldots, x_1 + x_n],
\]

it is

\[
\tau^* = \sum_{i=1}^n \tau_i,
\]

\[
E[x_1 | x_1 + x_2, x_1 + x_3, \ldots, x_1 + x_n] = \sum_{i=2}^n \frac{\tau_i}{\tau^*}(x_1 + x_i).
\]
Chapter 3

Reporting Bias in the Presence of Private Information

In the benchmark setting of section 2.2, a noisy information about the asset value was available to the informed investors. Now, the information represents a financial report about a firm, and the firm’s manager determines the bias in the report according to his interest in the share price. Section 3.1 discusses the used representation of financial reports.

Section 3.2 extends the benchmark setting by making the decisions of the manager regarding the bias in the report endogenous on the basis of Fischer and Verrecchia (2000). The equilibrium and its properties, such as the bias and the precision of the report, the price efficiency or the welfare of the investors are calculated in section 3.3. Finally, section 3.4 compares the results with both the benchmark model and Fischer and Verrecchia (2000), and discusses possible extensions of the model setting.

It will turn out that the model properties remain largely similar to both the benchmark setting and Fischer and Verrecchia (2000). However, the introduction of the manager as a player introduces new and sometimes countervailing effects regarding the marginal influence of the parameters on the equilibrium. In some cases, there are new dependencies which exist neither in the benchmark model nor in Fischer and Verrecchia (2000).
3.1 Representing a Financial Report

The signal $r = v + u$ in definition (2.1) in chapter 2 can refer to a disclosure by a firm or to a signal from any other exogenous source as long as it contains some noisy information about an asset’s value. If the signal reflects a financial report on the per share value disclosed by a firm, it suggests itself to include the utility of the discloser in the analysis. Dye (2001, p. 186) records that "exogenous disclosures, by definition, are not designed to maximize anything, and in particular, are not designed to maximize the benefits of the entity making the disclosures.” However, he argues that a central premise of a theory of disclosures should be that an entity will only disclose information which is favorable to the entity.\textsuperscript{1} As Dye (2001) furthermore emphasizes, this may also result in price-decreasing disclosures.\textsuperscript{2}

From now on, $r = v + b$ shall denote a financial report on the per share value of a firm which is disclosed by the manager of the same firm. Thereby, the manager chooses a bias $b$, i.e. the difference between the actual, reported per share value $r$ and the realized, true per share value $v$. This representation assumes the manager to observe the true firm value $v$; however, it may be more realistic to let him measure the firm value with error.\textsuperscript{3} If $b$ is positive, the manager inflates the report; in the case that $b$ is negative, the manager deflates the report. This definition is broad enough to include

- annual or quarterly financial statements,
- earnings announcements or forecasts,

\textsuperscript{1}Dye relates this argument only to voluntary disclosures. But it can be reasonably extended to any disclosure whereby the entity has some discretion.

\textsuperscript{2}As examples of when managers may try to decrease the share price, Dye (2001) cites disclosers prior to option awards, prior to a management buyout, prior to union negotiations, or when managers do "not know how investors will react to their disclosures (and hence will not know what constitutes a value-increasing disclosure) or that, in some instances, firm ‘value’ may not be well defined”. See Dye (2001, p. 187). See also the examples in section 1.2.

\textsuperscript{3}Section 5.3 contains a discussion of the consequences of a measurement error $n$, i.e. the manager only learns $v + n$ instead of $v$, whereby $n$ is the realization of a normal random variable. The following refrains from the introduction of such a measurement error to not further complicate the analysis.
• presentations on analyst conferences, or

• any other disclosure, as in interviews or via the internet.

Schipper’s (1989) definition of earnings management in section 1.2 is equally wide in scope and includes the entire external reporting process. For specific examples of earnings management in financial statements, as well as in earnings announcements and earnings forecasts, confer section 1.2.

The generality of the representation has another distinctive advantage. Skinner (1994) demonstrates that the informational content of financial statements may be low for investors because other, more timely sources anticipate the information of annual statements. These more timely sources of information specifically include, for example, ad-hoc reports and earnings forecasts by the firm, whose disclosure coincides with more pronounced price changes. It is evident that managers with incentives to influence the share price will do so especially in the case of disclosures with a subsequent strong share price reaction. All of these channels of disclosures are captured by the broad definition of a financial report.

In financial reports, including financial statements, managers do not disclose an estimate of the firm value; however, the information they disclose serves as an input to the valuation. In effect, by assuming a report \( r = v + b \) about the per share value, the process of valuation is reduced to a purely statistical problem consisting of the application of Bayes rule to the disclosed number as \( E[\tilde{v}|r] \). In reality, financial reports are not given as the realization of a univariate random variable, but consist of a

---

4Lang and Lundholm (1993) and Holland (1998) find a high and positive correlation between annual report disclosures and other forms of disclosure.

5Kothari (2001, p. 151) notes that research on management forecasts collectively shows that management forecasts have informational content. Through the forecasts, management aligns investors’ expectations with superior information which the management possesses.

6Christensen and Demski (2003, p. 4) note that "if valuation is the purpose, accounting is an abject failure on a worldwide basis. Economic and accounting values, where we have data, are virtually never well aligned."
plethora of quantitative and qualitative pieces of information. Due to the complexity of the economic world, there are no strict or precise rules of how this information changes expectations about future payments. The process of valuation using data from financial reports therefore "appears to be complex, elusive, subjective and dependent on experiences." Perhaps due to these complications, the representation of the report \( r = v + u \) as a noisy signal about the firm value "is used almost exclusively in the literature", as Dye (2001) notes.

A primary motivation for managers to manipulate earnings is to influence investors' beliefs. Several models study earnings management with the above representation, whereby investors do not observe the manager’s earnings manipulation and it is costly for the manager to manipulate earnings. This literature initially assumed that the manager’s reporting incentives were common knowledge. In this case, the manager manipulates the firm’s earnings and investors rationally anticipate this manipulation which they can then filter out perfectly from the earnings report. Therefore, the manager’s manipulation of the earnings report does not affect the informativeness of the report or the efficiency of prices. Fischer and Verrecchia (2000) weaken the assumption that manager’s reporting incentives are common knowledge, and instead suppose that the manager privately pursues his or her reporting incentives. Investors are thus incapable of perfectly filtering the manager’s reporting bias and therefore earnings manipulation reduces the informativeness of the report as well.

---

7Fundamental analysis tries to identify mispriced securities. It uses financial statement information, along with industry and macroeconomic data, to forecast future stock price movements. Investors who use this approach consider past sales, earnings, cash flow, product acceptance and management performance as predicting future trends in these financial driving forces of a company’s success or failure. They then assess whether a particular stock or group of stocks is undervalued or overvalued at the current market price. For further information on statement analysis and security valuation, see Leffson (1984) or Penman (2001).

8See Schildbach (1986, p. 8).

9See Dechow, Sloan and Sweeney (1996) and the examples of section 1.2.


11Lang and Lundholm’s (2001) study provides an example thereof: because shareholders do not know which firms intend to make stock offerings and because not all firms making offerings bias their disclosure, some firms can temporarily increase their stock prices through increased disclosure. However, once the offering is announced, the stock price falls dramatically for the firm with an increased disclosure, yet very little for firms without unusual disclosure increases.
as the efficiency of stock prices.

The notion that a manager’s reporting objective is uncertain seems reasonable since, in real markets, a manager’s reporting objective at any point in time is not known entirely. Fischer and Verrecchia (2000, p. 233) state that, “at any point in time, the market does not know: the precise nature of the manager’s compensation; the manager’s time horizon; the manager’s rate of time preference and degree of risk aversion; the manager’s psychic costs associated with bias; or the level of effort or resources the manager must expend to achieve a workable bias scheme.” In their model, this uncertainty is formalized by introducing the random variable $\tilde{u}$ into the manager’s objective function.

In Fischer and Verrecchia’s (2000) model, the manager’s utility is given by

$$\tilde{u}P(b) - 0.5cb^2,$$

(3.1)

whereby $u$ is the interest of the manager in the share price $P$, and $c$ is the marginal cost of a bias $b$. The utility weighs the costs of biasing against the benefits in the form of a desired share price reaction ($P = P(b)$). The interest of the manager in the share price $u$, whose realization is not observable for investors, is the realization of a normal random variable with mean zero$^{12}$ and precision $\tau_u$. Since the main objective is to show how the manager’s interest in the stock price causes the manager to manipulate the report, it is natural to assume that his incentives regarding the price enter into his utility, without being overly specific about their concrete origin. After the disclosure of the report $r = v + b$, the capital market values the firm at its expected value given the released report

$$P = E[\tilde{v}|r].$$

The marginal costs $c$ can generally be interpreted as the searching costs, i.e. the time and effort needed to find and execute the appropriate actions, or psychic costs of biasing. Furthermore, they may stand for a number of factors depending on the

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$^{12}$In Fischer and Verrecchia (2000), $\mu_u \neq 0$ is possible. See chapter 6 for a model with $\mu_u \neq 0$. 
interpretation of the report \( r \). If the report \( r \) stands for an annual statement, the marginal costs \( c \) can measure the tightness of accounting standards or the expected punishment in the case of a detection of illegal earnings management (fraud). Yet also for misrepresentations in voluntary forecasts, a threat of litigation is present and managers may incur reputation losses. Quite naturally the characteristics of annual statements audited\(^{13}\) later will in general have a disciplinary influence on other, softer disclosures like non-audited voluntary disclosures. The disciplinary role of audited statements is analyzed in an agency setting in Arya et al. (2004). Thus, the costs \( c \) for the biasing of annual statements are also relevant for other forms of disclosures.

There are, of course, a variety of other approaches regarding the modeling of earnings management in a capital market setting. Among others, these include signaling\(^{14}\) and agency\(^{15}\) models.

The model of Fischer and Verrecchia (2000) considers only a single representative investor. In the remainder of this thesis, a price formation process along the lines of Kyle (1985) will be added to this earnings management model. The information flow is depicted in figure 3.1.

### 3.2 Introducing a Manager as Player

Consider a one-period game with a manager, a market maker, a number \( N \) of informed investors as rational players, and \( M \) liquidity traders whose behavior is given

\(^{13}\)Auditors provide investors with independent assurance that the firm’s financial statements conform to GAAP. However, they may accept biased reports, because they earn from consulting businesses with the firm. Auditors may act in the interest of the managers that hire them, rather than in the interest of the firms’ investors.


\(^{15}\)Lambert (1984) uses agency theory to show that the optimal compensation plan offered by the principal causes the manager to smooth the firm’s income. Because an explicit contractual relationship often does not exist between managers and investors in competitive equity market settings, agency studies of earnings management largely ignore the effect of the price-formation process on earnings management. In addition, in the Kyle (1985) setting, players are risk-neutral while risk-aversion is an important ingredient of agency models.
learn report and additionally the bias

Manager \rightarrow reports \rightarrow Market Maker

Value \rightarrow estimates \rightarrow demand

Informed Investors

Liquidity Traders

Figure 3.1: Information Flow in Chapter 3 (Source: Own Presentation).

exogenously.\textsuperscript{16} At the beginning, all participants homogeneously believe the uncertain firm value to be distributed as a normal random variable $\tilde{v}$ with mean zero and precision $\tau_v$.\textsuperscript{17} A higher precision $\tau_v$ means that market participants possess more precise initial knowledge about the value of the firm. All basic production decisions are taken efficiently by the manager and are implicit in $\tilde{v}$.

There are two dates, $t = 0$ and $t = 1$. The sequence of events is depicted in time line 3.2.

At $t = 0$, the manager learns the realized firm value $v$ and discloses a public report

$$r = v + b,$$

consisting of the firm value $v$ and a bias $b$. The informed investors and the market maker observe the report $r$. Only the informed investors are capable of processing the report and in addition learn the bias $b$. The market maker does not observe

\textsuperscript{16}Alternatively, the number $M$ of uninformed investors could be avoided by interpreting a higher variance in the whole group’s demand as more uninformed investors. Since the parameter $M$ can thus be normalized to 1 without changing the results its presence may seem artificial in this chapter. However, in chapter 4, the number $M$ of uninformed investors has a distinctive influence on the equilibrium and the results for $M$ will be compared with the current situation.

\textsuperscript{17}The precision of a random variable is defined as the inverse of its variance.
the bias $b$.\footnote{Kim and Verrecchia (1994, p. 44) note that ”market makers are not thought to do any fundamental analysis, such as analyzing in great detail the annual reports of the companies whose shares they trade.” Mayer (1988, p. 211) says that ”some of them do not read annual reports.”} This corresponds to his objective of making a market in shares, i.e. to follow the goals of price stability and continuity while not maximizing his own profits. Kyle (1989b, p. 155) notes that the market makers’ “profits come from the bid-ask-spread, not from speculation; they make no money by knowing where other traders would like to buy and sell, not by acquiring unique fundamental information of their own. Their information comes from observing the ‘order flow’ (i.e., by observing the trading activities of other traders in the marketplace).” There is no information with any relevance for the assessment of the value $v$ other than the information which arises directly from the disclosure. This absence of other relevant information will be the case in a short time window around the disclosure. If the prior information on $v$ in the market is already very precise (high $\tau_v$), the report is merely an echo of already publicly known information, and hence of limited use.

Next, each informed trader submits his demand $x_i$, for $i = 1, \ldots, N$, and the $M$ liquidity traders submit their demand $My$ to the market maker. The market maker, who cannot distinguish between informed and uninformed demands, determines the
price \( P \) for the shares on the basis of the public report \( r \) and the net demand

\[
Q = \sum_i x_i + My. \tag{3.3}
\]

Due to informed trading, the net demand contains additional information about the bias \( b \) in the report \( r \). The price is set efficiently as the expected firm value given the net demand \( Q \) and the public report \( r \), i.e. as

\[
P = E[\hat{v}|Q, r]. \tag{3.4}
\]

The market maker sells \( Q \) shares at a per share price \( P \) so as to break even in expectation.\(^{19}\) At \( t = 1 \), the value \( v \) of the firm is paid out to the market participants as a dividend. This assumption can be applied to enduring firms if it is supposed that, at some point in the future, the price will have converged to the fundamental value, for example due to the arrival of additional information from complementary sources.

The risk-neutral manager chooses the bias \( b \) so as to maximize his utility

\[
uP(b) - \frac{1}{2}cb^2. \tag{3.5}
\]

His utility grows with the share price \( P \) for \( u > 0 \). The coefficient \( u \) will be called the manager’s interest in the share price. It is the realization of a normal random variable \( \hat{u} \) with mean zero and precision \( \tau_u \). Consequently, it may also be negative, implying that the manager’s utility increases if the share price is lower.\(^{20}\) The market maker and the \( N \) informed investors know about the structure of the manager’s utility function, but not about the realized interest \( u \). More precise knowledge among market participants about the incentives of the manager, i.e. a higher precision \( \tau_u \), can result, for example, from meetings with managers, the gathering of information about his compensation schemes from financial statements, or assessing the likelihood that the

\(^{19}\) The zero profit assumption can be justified by assuming competition with other market makers; in fact, two competitors who act under Bertrand competition suffice.

\(^{20}\) For a rationalization of this situation, see section 1.2. See also the corresponding discussion in section 3.4.
firm will raise capital and hence manipulate earnings for reducing its cost of capital. For the manager arises a disutility of biasing which grows with the square of the bias $b$ multiplied by a constant cost parameter $c$, and which is positive by definition. Possible significations of the marginal costs $c$ are described on page 60.

The informed investors submit a market order to the market maker. Each trader $i$ chooses his demand $x_i$ so as to maximize his expected profits given the public report $r$ and the bias $b$. The expected trading profit of each investor $i$ is

$$
\Theta_i = E[(\tilde{v} - \tilde{P}(x_i))x_i | r, b] \quad \text{for } i = 1, \ldots, N.
$$

(3.6)

Investors face uncertainty both from terminal value and the price. In addition, they take into account the effect of their own demand on the price. The $M$ liquidity traders demand an exogenously given number $My$ of shares, whereby $y$ is the realization of a normally distributed random variable $\tilde{y}$ with mean zero and variance $\sigma_y^2$.

To summarize, the manager chooses the bias $b$ in the report given the firm value $v$ and his interest $u$ in the share price while the informed traders choose their demand $x_i$ for $i = 1, \ldots, N$ given the report $r$ and the bias $b$, and the market maker chooses the market price $P$ given the report $r$ and net demand $Q$. The random variables $\tilde{v}$, $\tilde{u}$, and $\tilde{y}$ are mutually independent. The structure of the game as given in (3.2) to (3.6) and the parameters $\tau_u$, $\tau_v$, $\sigma_y^2$, $c$, $M$, and $N$ are common knowledge.

The given set of strategies constitutes a Nash equilibrium, provided that each player’s strategy optimizes the player’s objective given the equilibrium strategies of the other players. The equilibrium thus consists of

- a bias $b$ which maximizes the manager’s utility, i.e.
  $$uP(b) - \frac{cb^2}{2} \geq uP(\bar{b}) - \frac{c\bar{b}^2}{2} \quad \text{for all } \bar{b},
  $$

(3.7)

- a demand $x_i$ for each informed trader $i = 1, \ldots, N$ which maximizes his expected

---

profits, i.e.

\[ E[\Theta_i(x_i)] \geq E[\Theta_i(\bar{x}_i)] \text{ for all } \bar{x}_i \text{ and all } i = 1, \ldots, N \tag{3.8} \]

and

- a market pricing function \( P(Q, r) \) which satisfies

\[ P(Q, r) = E[\tilde{v}|Q, r]. \tag{3.9} \]

In a rational expectations equilibrium, each player conjectures equilibrium strategies for the other players. Players then choose their own strategy given these conjectures. In equilibrium, the conjectures are self-fulfilling, i.e. they indeed turn out to be the optimal strategies for the other players.

The following analysis is limited to linear conjectures\(^{22}\) of the form

\[
\begin{align*}
    b &= \alpha u \tag{3.10.1} \\
    x_i &= \beta_r r + \beta_b b \text{ for } i = 1, \ldots, N \tag{3.10.2} \\
    P &= \delta_r r + \delta_Q Q. \tag{3.10.3}
\end{align*}
\]

An equilibrium thus follows from the assignment of \( \alpha, \beta_r, \beta_b, \delta_r, \text{ and } \delta_Q \) to the model parameters. The equilibrium is symmetric in the sense that all informed investors play the same strategy. No constant terms are necessary in the specification, because all expected values are zero. It also comes without a loss of generality that the bias is independent of the terminal value \( v \). These results follow immediately from the derivation of the equilibrium in the appendix to this chapter.

\(^{22}\)See the remarks in footnote 22 of section 2.2. Due to the quadratic form of the trading profits in (3.6) as implied by the linear pricing rule (3.10.3), a linear order of the informed traders is optimal even when nonlinear strategies are allowed for.
3.3 The Equilibrium

Proposition 3.3.1. For all combinations of $c > 0$, $\tau_u$, $\tau_v$, $M$, $N$, and $\sigma_y^2$, only a single linear equilibrium exists in the setting of section 3.2. The equilibrium strategies are given by

\[
b = \alpha u \quad (3.11.1)\]
\[
x_i = \beta \left( \left( 1 - \frac{\tau_r}{\tau_v} \right) r - b \right) \quad \text{for} \quad i = 1, \ldots, N \quad (3.11.2)\]
\[
P = \frac{\tau_r}{\tau_v} r + \frac{1}{N+1} \frac{1}{\beta} Q, \quad (3.11.3)\]

whereby the trading aggressiveness is

\[
\beta = \sqrt{\frac{M^2 \sigma_y^2}{N} (\tau_v + \alpha^2 \tau_u)} \quad (3.12)\]

and the precision of the public report is

\[
\tau_r = \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} \right)^{-1}. \quad (3.13)\]

The coefficient $\alpha$ is the unique root of the third-order polynomial

\[
c\alpha = \frac{1}{N+1} \frac{\tau_r}{\tau_v}. \quad (3.14)\]

Proof. See the appendix.

The equilibrium strategies display the following basic properties: since $\alpha > 0$, the amount to which the manager inflates the report correlates positively with his interest $u$ in the share price; the realized firm value $v$ has no influence on the bias in this setting. The informed investors’ demand increases with the report $r$, but is corrected by the bias $b$.\(^\text{23}\) The equilibrium price increases with the report $r$ and the net demand $Q$, i.e. a higher demand intuitively signifies good news about the fundamental firm value.

\(^{23}\)It follows from the definition of $\tau_r$ in 3.13, that $\frac{\tau_r}{\tau_v} \leq 1$ such that the coefficient of $r$ in 3.11.2 is positive.
An intuitive scenario for the equilibrium can be sketched here, before being discussed in more detail below, as arising from the interplay between the three strategic players. The manager chooses the amount of report bias in such a way that his marginal costs equal his marginal benefits. The marginal costs are given exogenously by \( c \). The marginal benefits of biasing correspond with the coefficient of \( b \) in the equilibrium price (see (3.21)). The informed investors trade proportionally to the difference between the fundamental value and the price that would result if the market maker only observed the public report, but not the order flow (see (3.17)). Thereby, the investors try to disguise their information from the market maker. Hence, they are able to trade more aggressively on this difference, provided that the precision of the terminal value given the report is higher and the market maker considers the report relatively more seriously in comparison with the net demand. If the informativeness of the report increases (excluding an effect of the number of informed traders), the informativeness of the order flow remains identical to the informativeness of the report in equilibrium. The market maker determines the price \( P \) by using direct information from the report \( r \) and from the net demand \( Q \). He uses these sources of information to the extent that their informativeness (precisions) exceeds the information previously available on \( v \).

The equilibrium is a prisoner’s dilemma type of situation. Suppose that the manager would announce to avoid earnings management, and the capital market had trust in that. Under such conditions, the introduction of a bias would enter the price immediately. Thus, since the manager cannot make a credible commitment not to manage earnings, the market rationally expects a bias and the manager has no better response other than to bias his report until the expectations about the bias are fulfilled on average. In fact, there is no equilibrium with any differentiable and non-constant price function \( P(b) \) in which the manager never manages earnings.\(^{24}\) If the price \( P(b) \) were constant, i.e. independent of the bias \( b \), the manager would not

\(^{24}\)The proof goes as follows: consider the first-order condition for a maximum of the manager’s utility (3.5) with respect to \( b \): \( u \frac{\partial P}{\partial b} - cb = 0 \). An equilibrium with \( b = 0 \) would require \( \frac{\partial P}{\partial b} = 0 \), which contradicts the presupposition that the price function is non-constant.
manage earnings because a bias would create costs for him without any benefit since the price is not affected by earnings management.

The remainder of this section analyzes

- the bias and precision of the report,
- the demand and trading volume,
- the informativeness of the price,
- the welfare of informed and liquidity traders,
- the value of the option to bias for the manager, and
- the effect of the informed traders.

The marginal influence of the parameters on these equilibrium characteristics is the first derivative of the characteristic under consideration with respect to the considered parameter. These marginal influences are stated in the following tables. If in a table the marginal influence of the costs $c$ on the precision $\tau_r$ is positive (denoted by a '+'), then $\frac{\partial \tau_r}{\partial c} > 0$ and vice versa for a '-'. The deviations are available from the author at request.

**Bias and Precision of the Report**

The bias $b$ in the public report $r$ is given by a coefficient $\alpha$ times the interest $u$ of the manager in the market price (see (3.11.1)). The coefficient $\alpha$ of the interest $u$ is given endogenously by the unique solution to polynomial 3.14: \[25 c\alpha = \frac{1}{N+1} \tau_r \tau_v. \] (3.15)

The left side of the equation corresponds to the manager’s marginal costs $cb = c\alpha u$ in (3.5) divided by $u$, while the right side equals his marginal benefit, as it is the coefficient of $b$ in the ex-post price (3.21) below. The effect of the bias on the price

\[\text{Note that this is a polynomial of third order, because } \tau_v \text{ depends on } \alpha^2.\]
decreases with the number $N$ of informed traders, and when the public report is more informative (more precise) relative to the prior information on $v$.

It follows from page 90 in the appendix that, for a positive realization of $\bar{u}$, the bias is bounded by

$$0 < b < \frac{1}{N + 1} \frac{1}{c} u. \quad (3.16)$$

If either the marginal costs $c$ of biasing or the number $N$ of informed investors grow to infinity, the bias converges to zero, in accordance with intuition.

A measure of the quality of financial reports is their precision. The choice of the bias $b = \alpha u$ indirectly influences the precision $\tau_r$ of the public report $r$ in (3.13).\(^{26}\)

The effect of a change in the parameters on the bias $b$ and precision $\tau_r$ of the report $r$ is stated in

**Corollary 3.3.2.** The bias $b = \alpha u$ increases with the interest $u$ of the manager in the firm value. The realized interest $u$ has no influence on the precision $\tau_r$ of the report. The marginal influence of the parameters on the coefficient $\alpha$ and the precision $\tau_r$ of the public report is shown in table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M^2\sigma^2_y$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$\tau_u$</td>
<td>$M^2\sigma^2_y$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>$M^2\sigma^2_y$</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

*Table 3.1: The Marginal Effect of the Parameters on the Bias $\alpha$ and Precision $\tau_r$.*

The explanation for the effects rests on a trade-off between the direct and indirect effects on the marginal costs and benefits of biasing: increasing the marginal costs $c$ of biasing directly reduces $\alpha$, whereas it increases the precision of the report $\tau_r$ and thus the benefit of biasing. However, the direct effect prevails. More informed traders $N$ lead to a less biased price (lower benefit of biasing); this effect dominates the indirect effect of a higher precision $\tau_r$, which results from less variation in the bias. If the

\(^{26}\)Formula (3.13) for $\tau_r$ follows immediately from $r = v + \alpha u$. 
prior precision $\tau_v$ of the firm value is higher, the market maker relies more on his prior information when updating his belief. This reduces the benefit of biasing and lowers $\alpha$ directly. The corresponding increase in the informativeness of $r$ (higher $\tau_r$) is subordinated. An increase in the precision $\tau_u$ of the manager’s interest in the share price directly increases the precision $\tau_r$ of the report (higher marginal benefits); this dominates the effect of a higher variance in the report due to the increased bias. The variance $M^2\sigma_y^2$ of liquidity trading has no influence for risk-neutral informed traders (for an explanation, see the paragraph on price efficiency below).

**Demand and Trading Volume**

Because the informed traders observe the bias $b$ in the report $r$, they know that the true firm value must be $v = r - b$. Nevertheless, they trade on both $r$ and $b$ in different magnitudes, since in maximizing their expected trading profits, they take into account the actions and knowledge of the market maker who only observes $r$ and $Q$, but not $b$.\(^{27}\) Although informed investors are risk-neutral and possess superior information, they strategically limit their trades.\(^{28}\) Their demand reveals information to the market maker and, in effect, reduces their trading profit which stems from the difference between the price and the terminal value.

The equilibrium demands $x_i$, for $i = 1, \ldots, N$, in (3.11.2) are

$$x_i = \beta \left( \left( 1 - \frac{\tau_r}{\tau_v} \right) r - b \right)$$

(3.2)

$$= \beta \left( v - \frac{\tau_r}{\tau_v} r \right)$$

$$= \beta(v - E[\tilde{v}|r]).$$

Thus, the informed investors trade exactly proportionally to the difference between

\(^{27}\) The information pair $(r, b)$ is thus superior to a knowledge of only $v$. This also follows from the equilibrium demand: if $v$ were sufficient, the absolute value of the coefficients of $r$ and $b$ would be identical.

\(^{28}\) In competitive models, these conditions would lead to an infinite demand.
the terminal value and the market price if the market maker observes only the report and not the net demand. If the shares are undervalued given the information contained in the report only, the informed investors buy the stock and they sell the stock in the case of an overvaluation.

**Corollary 3.3.3.** The marginal influence of the parameters on the trading aggressiveness

\[
\beta = \sqrt{\frac{M^2\sigma^2_v}{N}(\tau_v + \alpha^{-2}\tau_u)}
\]

in (3.12) is shown in table 3.2.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( c )</th>
<th>( N )</th>
<th>( \tau_v )</th>
<th>( \tau_u )</th>
<th>( M^2\sigma^2_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>?</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

**Table 3.2: The Marginal Effect of the Parameters on the Trading Aggressiveness \( \beta \).**

The informed investors try to hide their trading from the market maker. Their trading aggressiveness depends on the degree to which the market maker uses the information from net demand relative to the report. If the report \( r \) is more informative for the market maker, the informed investors can trade more aggressively. However, the overall informativeness of the price remains exactly constant, as will be shown below in (3.22). Here, the informativeness of the report is measured by the precision of the terminal value remaining after having observed the report:

\[
V ar^{-1}[\tilde{v}|\tilde{v} + \alpha\tilde{u} = r] = \tau_v + \alpha^{-2}\tau_u.
\]

**Lemma (2.4.2)**

The cost \( c \) of biasing and the precisions \( \tau_v \) and \( \tau_u \) increase the informativeness of the report and therefore allow the informed investors to trade more aggressively. If there are more informed investors \( N \), the competition between them lowers individual profits and thus their individual trading aggressiveness. However, more informed investors also increase the informativeness of the report via the less pronounced bias
and in this indirect way lets the informed investors trade more aggressively. The overall effect of \( N \) is indeterminate. If more liquidity traders are involved, the net demand is less informative for the market maker. The informed investors exploit this beneficial effect by proportionally increasing their amount of trade. In effect, the variance of the sum of the informed traders’ demands, i.e.

\[
Var \left( \sum_i \tilde{x}_i \right) \quad (3.11.2)
\]

\[
= Var \left( N \beta \left( 1 - \frac{\tau_r}{\tau_v} \right) r - b \right)
\]

\[
\quad = NM^2 \sigma_y^2 (\tau_v + \alpha^{-2} \tau_u) \left( 1 - \frac{\tau_r}{\tau_v} \right) \left( r - b \right)
\]

\[
\quad = NM^2 \sigma_y^2 (\tau_v + \alpha^{-2} \tau_u) \left( 1 - \frac{\tau_r}{\tau_v} \right) v - \frac{\tau_r b}{\tau_v}
\]

\[
\quad = NM^2 \sigma_y^2 (\tau_v + \alpha^{-2} \tau_u) \left( 1 - \frac{\tau_r}{\tau_v} \right)^2 \frac{\tau_r^2}{\tau_v} \frac{\alpha^2}{\tau_u} \left( \frac{\alpha^2}{\tau_u} + \frac{1}{\tau_v} \right)
\]

\[
\quad = NM^2 \sigma_y^2 (\tau_v + \alpha^{-2} \tau_u) \frac{\tau_r^2}{\tau_v} \frac{\alpha^2}{\tau_u} \left( \frac{\alpha^2}{\tau_u} + \frac{1}{\tau_v} \right)
\]

\[
\quad = NM^2 \sigma_y^2 \quad (3.20)
\]

increases with \( M^2 \sigma_y^2 \), so that the market maker can precisely balance out any influence of the liquidity traders on the net demand. It is thus a characteristic of Kyle (1985)-type models with risk-neutrality and without information acquisition costs, that the informed investors exactly offset any effect of the liquidity trading on the price.\(^ {29} \)

The expected trading volume \( V \), as defined in section 2.2, with the help of (3.20) immediately turns out to be \( E[\tilde{V}] = \frac{M \sigma_y}{\sqrt{2\pi}} (\sqrt{N} + 1 + \sqrt{N + 1}) \). It increases with \( N \) and \( M \sigma_y \).

\(^{29}\)The independence of the variance of liquidity trading does not hold if there are information acquisition costs for the informed traders or if risk-aversion is introduced. In Kyle (1989), a trading model without a market maker, a higher variance of liquidity trading decreases the price efficiency for risk-averse informed traders. However, with endogenous acquisition of information, \( \sigma_y^2 \) induces more informed traders to enter, so that the price informativeness increases.
Price Informativeness

The market maker observes the public report \( r \) and the net demand \( Q \). Both contain noisy information about the terminal value \( v \). The realized equilibrium price can be expressed as a linear combination of the terminal value \( v \), the bias \( b \), and liquidity trading \( y \) as

\[
P = \frac{\tau_r r}{\tau_v} + \frac{1}{N + 1} \beta Q
\]

\[
= \frac{N + \tau_r}{N + 1} v + \frac{1}{N + 1} \frac{\tau_r}{\tau_v} b + \frac{M}{N + 1} \beta y.
\]

A deviation from the realized terminal value \( v \) stems from the influence of the bias \( b \) and the liquidity trading \( y \). Nevertheless, if the cost of biasing \( c \) or the number \( N \) of informed traders grow large, the price approaches the realized firm value \( v \).\(^{30}\)

The price clearly reflects all information contained in the public report \( r \), because knowing the report, the market maker sets the price efficiently. The private information of the uninformed investors is only partially incorporated in the price, due to their strategic trading. The degree to which the price also reflects the informed traders’ private information can be measured by the uncertainty about the terminal firm value which remains after the price has been observed. Ex ante, i.e. before trading takes place, the price \( P \) in (3.21) can be seen as a normally distributed random variable\(^{31}\) which depends on the uncertain value \( \tilde{v} \), the uncertain bias \( \tilde{b} = \alpha \tilde{u} \), and liquidity trading \( M \tilde{y} \). The conditional variance of the terminal value given the price

\(^{30}\)Note that in these cases, \( \frac{\tau_r}{\tau_v} \to 1, \frac{1}{N + 1} \beta \to 0. \)

\(^{31}\)Because the price is a linear combination of normal random variables, it is itself also normally distributed. In a one period model, this implies that prices can be negative with positive probability, and hence, is problematic as a representation of equity prices for a limited liability corporation; it implies the unlimited liability of the investors. Fischer and Verrecchia (1997) show in a limited liability scenario that firm value is strictly convex in the disclosed variable.
\begin{align*}
\text{Var}[\hat{v} | \hat{P}] &= \text{Var}[\hat{v}] \frac{N + \tau_v}{N + 1} \hat{v} + \frac{1}{N + 1} \tau_v \hat{b} + \frac{1}{N + 1} \beta \hat{y} = P \\
&= \text{Var}[\hat{v}] \hat{v} + \frac{1}{N + \tau_v} \tau_v \hat{b} + \frac{1}{N + \tau_v} \beta \hat{y} = \frac{N + 1}{N + \tau_v} P \\
\text{Lemma 2.4.2} &= \frac{1}{N + 1} \tau_v + \alpha^{-2} \tau_u. \quad (3.22)
\end{align*}

The remaining variance \( \text{Var}[\hat{v} | P] \) lies between 0 and \( \frac{1}{\tau_v} \), denoting the cases of certainty about \( v \), and no additional information vis-à-vis prior knowledge on \( v \), respectively. Defining the conditional variance \( \text{Var}[\hat{v} | P] \) of the terminal value given the price as the price efficiency \( \Psi \), the influence of the parameters is as stated in

**Corollary 3.3.4.** The marginal influence of the parameters on the informational efficiency is shown in table 3.3.

<table>
<thead>
<tr>
<th>( \Psi )</th>
<th>( c )</th>
<th>( N )</th>
<th>( \tau_v )</th>
<th>( \tau_u )</th>
<th>( M^2 \sigma_{\hat{v}}^2 )</th>
<th>( 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( + )</td>
<td>( + )</td>
<td>( + )</td>
<td>( + )</td>
<td>( + )</td>
<td>( + )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

*Table 3.3: The Marginal Effect of the Parameters on the Price Efficiency \( \Psi \).*

Equation (3.19) stated the precision of the terminal value given the public report. The remaining precision determines as well the price efficiency in (3.22). The informational efficiency increases with all parameters, as does the precision given only the public report. The incremental information content of the net demand in addition to the public report can be measured by the relation between the precisions of \( \hat{v} \) given the report, and the net demand relative only to the report as

\[
\frac{\text{Var}[\hat{v} | r]}{\text{Var}[\hat{v} | Q, r]} = N + 1. \quad (3.23)
\]
Hence, informed trading increases the informational efficiency by the factor $N + 1$.\textsuperscript{32} As the discussion of the net demand revealed above, the variance of liquidity trading does not influence the price efficiency because the variance in the informed demand is proportional to the variance of liquidity trading, allowing the market maker to perfectly balance out its influence (see (3.20)).

\section*{Welfare}

The expected trading profit of the market maker is by definition zero, since he sets the price so as to break even in expectation. The informed investors only trade if their profit is expected to be positive. Since trading takes place in a pure exchange setting, the profit of the informed investors must come at the expense of liquidity traders. In fact, the expected profits sum up to zero:

$$\sum_i E[\tilde{x}_i(\tilde{v} - \tilde{P})] + ME[\tilde{y}(\tilde{v} - \tilde{P})] = 0. \quad (3.24)$$

The focus lies on the a-priori expected profit, i.e. preceding the reception of the report $r$ and the observation of the bias $b$.\textsuperscript{33} For each informed investor, this expected profit

\textsuperscript{32}This is similar to the result of the basic Kyle (1985) setting, where exactly half of the variance is eliminated by informed trading (for $N = 1$).

\textsuperscript{33}Alternatively, the amount which the informed investor would be willing to pay for obtaining his private information $b$ could be determined by comparing $E[\tilde{x}_i(\tilde{v} - \tilde{P})|r]$ with $E[\tilde{x}_i(\tilde{v} - \tilde{P})|r, b]$. However, the determination of the expected profits is intractable. It involves, for example, the calculation of $E[\tilde{u}\tilde{v}|\tilde{v} + \alpha\tilde{u} = r]$. 
\[ \Theta_i = \Theta = E[(\tilde{v} - \tilde{P})\tilde{z}_i] \]

\[ E \left[ \left( \tilde{v} - \frac{N + \tau_v}{N + 1} \tilde{v} + \frac{1}{N + 1} \tau_v \tilde{b} \right) \beta \left( \left( 1 - \frac{\tau_r}{\tau_v} \right) \tilde{v} - \frac{\tau_r}{\tau_v} \tilde{b} \right) \right] \]

\[ \beta E \left[ \left( 1 - \frac{N + \tau_v}{N + 1} \right) \left( 1 - \frac{\tau_r}{\tau_v} \right) \tilde{v}^2 - \frac{1}{N + 1} \frac{\tau_r^2}{\tau_v^2} \sigma_u^2 \right] \]

\[ \frac{1}{N + 1} \sqrt{\frac{M}{N}} \sqrt{\text{Var}[\tilde{v}|r]} \quad \text{for } i = 1, \ldots, N. \]

The remaining variance of \( v \) given the report \( r \) is stated in (3.19). The influence of the parameters on this expected profit is stated in Corollary 3.3.5.

**Corollary 3.3.5.** The marginal influence of the parameters on the expected trading profit is shown in Table 3.4.

<table>
<thead>
<tr>
<th>( \Theta )</th>
<th>( c )</th>
<th>( N )</th>
<th>( \tau_v )</th>
<th>( \tau_u )</th>
<th>( M^2 \sigma_y^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.4: The Marginal Effect of the Parameters on the Expected Informed Profit \( \Theta \).**

The expected profit of an informed investor intuitively increases with the number of liquidity traders, and decreases with the number of informed investors. In addition, it decreases when the remaining precision of the firm value \( v \) given the report \( r \) is higher, which implies a more informed market maker. This also explains the effects of \( c, \tau_v, \) and \( \tau_u \) on \( \Theta \). Although the informed investors trade more aggressively (higher \( \beta \)) if this remaining variance \( \text{Var}[\tilde{v}|r] \) increases, the profit-diminishing effect of the increased informativeness of the price dominates.

\[ ^{34}\text{Note that } E[\tilde{v} \tilde{y}] = 0 \text{ for independent random variables, and } E[\tilde{u}^2] = \frac{1}{\tau_u} \text{ because the mean of } \tilde{u} \text{ is zero.} \]
The liquidity traders are interested in a liquid market. The market depth, as defined in section 2.2, is given by the inverse of $\delta_Q$ in (3.11.3) as

$$\frac{1}{\delta_Q} = \frac{N + 1}{\sqrt{N}} M \sigma_y \sqrt{Var[\tilde{v}|r]}.$$ (3.26)

The expected trading profit $\Theta_L$ of the liquidity traders is

$$\Theta_L = E[(\tilde{v} - \tilde{P})\tilde{y}] = -\frac{M}{N + 1} \frac{1}{\beta} E[\tilde{y}^2] = -\frac{\sqrt{N}}{N + 1} \sigma_y \sqrt{Var[\tilde{v}|r]}.$$ (3.27)

The same result follows by using (3.24) in combination with (3.25). The sign of the effects of a marginal change in the parameters is opposite to table 3.4 for all parameters except the liquidity traders’ own influence via the parameter $M^2 \sigma_y^2$.

**The Value of the Option to Bias**

If the manager could not bias the public report, he would not incur the costs $\frac{1}{2} cb^2$ of biasing, yet his expected benefit would also be zero (since $E[\tilde{P}] = 0$ before learning $v$). Thus, the manager’s expected benefit represents the change from a situation with the opportunity to bias to a situation without biasing (where the utility is zero). Fischer and Verrecchia (2000) note that the expected utility in this sense represents the value of the option to bias.

When the manager determines the bias $b$, he already observed his interest $u$ in the share price. At this point in time, his expected utility in (3.5) turns out to be

$$U_p = E[\tilde{u}\tilde{P} - \frac{1}{2} c \alpha^2 \tilde{u}^2 | u] = \frac{1}{2c} \left( \frac{1}{N + 1} \tau_r \right)^2 u^2.$$ (3.28)

The manager’s utility is positive.$^{35}$ The influence of the parameters on this expected utility is stated in
Corollary 3.3.6. The marginal influence of the parameters on the expected utility $U_p$ of the manager is shown in table 3.5.

The influence of the parameters on the value of the option to bias can be explained by the relation between the marginal benefits and costs of biasing. The marginal costs $c$ of biasing have a direct negative effect on the manager’s utility, and an indirect positive effect due to an increased effectiveness of the biasing (due to a higher $\tau_r$). The overall effect is ambiguous (i.e., the marginal influence depends on the constellation of the other parameters). A higher number $N$ of informed traders decreases the extent to which the bias is reflected in the price, thus decreasing the marginal benefits. A higher prior precision $\tau_v$ of the assumed firm value makes the market maker rely more on his initial information. Interesting to note is the influence of the precision $\tau_u$. More knowledge about the manager’s interest in the price (higher precision $\tau_u$) increases the informativeness of the report, thus increasing the marginal benefits of biasing. It follows that the manager has an interest in more disclosures about, for example, his payment contracts. Liquidity trading has no influence on the price and consequently neither on the effect of the bias.

The Effect of Informed Trading

In the absence of informed trading, i.e. for $N = 0$, the net demand would not contain any information on terminal value. In effect, the model would turn out to be identical to that of Fischer and Verrecchia (2000).

The equilibrium price would be $P = E[\tilde{v}|r] = \frac{\tau_r}{\tau_v}r$ and the expected profit of uninformed investors $E[(\tilde{v} - \tilde{P})\tilde{y}] = 0$. The bias $b = \alpha u$ in (3.14) is given more simply

\[
\begin{array}{cccccc}
U_p & c & N & \tau_v & \tau_u & M^2\sigma_y^2 \\
 & ? & - & - & + & 0
\end{array}
\]

Table 3.5: The Marginal Effect of the Parameters on the Manager’s Utility $U_p$.

\footnotesize{The utility can be negative when the mean of $\bar{u}$ is non-zero. See chapter 5 for such a case.}
by \( \alpha = \frac{\tau_v}{\tau_r} \) with \( \tau_r \) as in (3.13). The price efficiency is 
\[
Var^{-1}[\hat{v}|P] = \tau_v + \alpha^{-2}\tau_u.
\]

Consequently, the bias always decreases with informed trading, just as the price efficiency is higher.\(^{36}\) However, these benefits come at the loss of the liquidity traders.

### 3.4 Discussion

This chapter has presented a reporting game with a manager, a market maker, and \( N \) informed investors as strategic players. Section 3.3 derived linear strategies for an optimal bias, informed demand, and price. These strategies depend on the realization of three random variables, namely the firm value \( \hat{v} \), the interest of the manager in the share price \( \hat{u} \), and the demand of liquidity traders \( \hat{y} \). The actions of the players intuitively influence these realizations. If the manager has an interest in a higher share price, he always adds a positive bias on the firm value in the report, and vice versa. The price reflects the good news of a higher reported value, and increases if demand for the shares rises. The informed investors trade proportionally to the expected deviation of the price from the true value in the absence of their trading. However, that the bias is independent of the realized firm value \( v \) is a consequence of the model’s assumptions, especially normality and risk-neutrality.

The setting combines two components: as in Fischer and Verrecchia (2000), a manager discloses a biased report which the capital market is not able to trace back because there is uncertainty about the manager’s interest in the share price. The capital market reflects, according to the benchmark model of section 2.2, the actions of heterogeneously informed participants. The following first compares the results of the model with its two components then discusses the model’s assumptions and some extensions.

\(^{36}\) Confer chapter 7 for further results.
Comparison of the Results

Section 3.3 calculated a number of market characteristics as well as the marginal influence of the model parameters on these characteristics. The following compares the arising marginal influences between the current setting, the model of Fischer and Verrecchia (2000) and the benchmark model of section 2.2. Table 3.6 summarizes the findings.\textsuperscript{37}

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>N</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M^2\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-..</td>
<td>-..</td>
<td>-..</td>
<td>++.</td>
<td>0..</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>++.</td>
<td>+0</td>
<td>+++</td>
<td>+++</td>
<td>0.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>..</td>
<td>.-</td>
<td>++</td>
<td>+.</td>
<td>+.</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>++</td>
<td>++</td>
<td>+.</td>
<td>++</td>
<td>0.0</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>-..</td>
<td>-..</td>
<td>-..</td>
<td>-..</td>
<td>+.</td>
</tr>
<tr>
<td>$\Theta_L$</td>
<td>+..</td>
<td>+.</td>
<td>+.</td>
<td>+.</td>
<td>-..</td>
</tr>
<tr>
<td>$U_p$</td>
<td>??</td>
<td>..</td>
<td>++</td>
<td>++</td>
<td>0..</td>
</tr>
</tbody>
</table>

Table 3.6: The marginal effect of the parameters on the equilibrium outcome in the present model, in Fischer and Verrecchia (2000), and in the benchmark model, respectively. (The symbols denote: "++" positive influence, "--" negative influence, "0" no influence, "." influence cannot be calculated because parameter or characteristic is not included in the model.)

It is noticeable that most of the results of the component models hold equally in the extended setting. The results of Fischer and Verrecchia (2000) are robust to the introduction of a market microstructure. In the same way, the introduction of a manager who chooses the bias in the report does not change the results of the benchmark model (the only exception is the influence of the number $N$ of informed investors). However, firstly, the explanation and underlying mechanism have become

\textsuperscript{37}Note that Fischer and Verrecchia (2000) do not calculate $\tau_r$; they define $\Psi$ in relative terms (implying a different influence of $\tau_v$ than stated in the table) and do not state the marginal influence of the parameters on $U_p$. For the benchmark model, note that the marginal influence of the parameters is stated for the case of no additional idiosyncratic noise, i.e. for $\tau_e \to \infty$. Remember also that the informativeness was defined incrementally over the report; this explains the differences in the influence of $\tau_u$ and $\tau_v$. 
more complex and secondly, there are entirely new dependencies at work in the combined model.

Concerning the first point, consider the influence of the number \( N \) of informed investors on the price efficiency \( \Psi \). In a direct manner, the presence of more informed investors increases the informativeness of the net demand as in the benchmark model (but not in Fischer and Verrecchia (2000)).\(^{38}\) Indirectly (and not covered in the benchmark model), the informed investors lower the marginal benefits of biasing for the manager, which lowers the bias and thereby also increases the price informativeness as in Fischer and Verrecchia (2000).\(^{39}\)

As a second example of how the complexity of the model’s underlying mechanisms and explanations has increased, consider the influence of the number \( N \) of informed investors on the precision \( \tau_r \) of the report. In the benchmark model, the report is given exogenously and is not influenced by the number \( N \) of informed investors. In the combined model, however, informed investors lower the extent of the biasing, which increases the precision of the report.

Concerning the second point on new influencing factors, all cells in table 3.6 with two points ("."') for the model component are of interest.

- Firstly, a new effect is that the marginal costs \( c \) of biasing for the manager increase the trading aggressiveness \( \beta \) of the informed investors. Higher marginal costs make the market maker consider the report relatively more seriously, rendering the market deeper. This fact is exploited by the informed investors.

- Secondly, the number \( N \) of informed investors decreases the amount of bias \( b = \alpha u \) in the report. This occurs because, by trading on their private information about the bias, the informed investors influence the marginal benefits of biasing for the manager in a negative way.

\(^{38}\)Note that, in the benchmark model, the informativeness was defined incrementally and the sign is therefore not directly comparable with the current definition of \( \Psi \).

\(^{39}\)The number \( N \) is not covered in Fischer and Verrecchia (2000). However, the effect stems from the Fischer and Verrecchia (2000) component; it is present and similar to the effect of \( c \) on \( \Psi \) in Fischer and Verrecchia (2000).
The most interesting new results, however, concern the influence of the costs \( c \) and the number \( N \) on the profits \( \Theta \) and \( \Theta_L \) of the informed and liquidity traders and the utility \( U_p \) of the manager.

- Higher marginal costs \( c \) of biasing increase the profits \( \Theta_L \) of the liquidity traders (or rather: decrease their losses), although they do not take account of the report, whose precision increases with \( c \). This effect occurs on the grounds that the market becomes more liquid, since the market maker directly considers the report relatively more in comparison with the net demand.

- Higher marginal costs \( c \) decrease the informational advantage of the informed investors in comparison with the market maker. Thus, their expected profit \( \Theta \) decreases although they trade more aggressively.

- The utility \( U_p \) of the manager decreases with the number \( N \) of informed investors, as the latter decrease his marginal benefits of biasing.

Thus, although the liquidity traders are not capable of processing the report \( r \) or simply unwilling to do so because of a liquidity need, they would still benefit from a higher cost of biasing and a more precise public report in the model via the price formation process. However, the exogeneity of their trading motives makes an interpretation of their welfare questionable. Christensen and Feltham (2003, p. 409) write that ”unfortunately, while the models [...] can be used to describe the impact of changes in the informativeness of public reports, they cannot be used to make social welfare statements in this type of analysis because the preferences of the liquidity traders are not explicitly modeled.” Similarly, Admati (1989, p. 144) states that ”since the preferences of 'noise traders' are unspecified, this interpretation is unsatisfactory for certain purposes, especially welfare analysis.” Chapter 4 addresses this issue.
The Model Setting and Extensions

The setting of section 3.2 contains a number of restrictive assumptions which make the derivation of an equilibrium tractable. The remainder of this section reviews some of these and speculates about likely changes concerning the equilibrium outcome if the assumptions are relaxed accordingly.

The introduction of nonzero mean values would not lead to additional insights due to normality and linearity. The means would be carried through all equations. A mean value of \( \mu_v \neq 0 \) would be extracted by the market maker. Nonzero expected liquidity trading \( \mu_y \neq 0 \) would be absorbed by the informed investors. For a nonzero expected interest \( \mu_u \neq 0 \) of the manager in the share price, confer chapter 5.

The risk-neutrality of the informed investors and the market maker allows for a focus on informational effects, and to abstract from the risk-sharing motivation for trading and collecting private information (see section 1.4). However, it is likewise responsible for the "artificial" result that the informed investors perfectly balance out the influence of the liquidity traders. If the informed investors were risk-averse, they would not trade as aggressively; in consequence, this could lead to a more biased report and a less informative price. However, the effect on the market depth and the profit of the liquidity traders is not evident.

It would be more realistic if the random variables were restricted to be positive values. This is the case for the firm value \( \tilde{v} \) and the share price \( \tilde{P} \) as already noted in footnote 31 to this chapter. It also holds for the interest \( u \) in the share price. It is conventional wisdom that managers prefer a higher price (e.g., in the case of stock-based compensation). The model is consistent with this objective as long as \( u \) is positive. Note, however, that \( u \) can also assume negative values because \( \hat{u} \) is

---

40"Tractability" in the current environment means (a) that in order to apply Bayes' rule, no inversion of three-dimensional matrices is necessary, especially for the calculation of price and informed demand, and (b) that the equilibrium is given by the roots of a polynomial of an order not higher than three. Otherwise, a derivation of marginal influences would turn out to be tedious.


42The normal distribution implies that the realization can be arbitrarily negative.
normally distributed.\textsuperscript{43}

Section 1.2 mentions a number of realistic scenarios in which the manager intends to deflate the report. However, the application to the current setting requires that the incentive stems from an interest in the share price (an incentive which does not relate to the share price might yield to a bias in the report and the ultimate price as well; however, the utility of the manager is directly related to the share price in the model) and that it is unrelated with the fundamental firm value.\textsuperscript{44} Fischer and Verrecchia (2000) give the following examples: (1) A manager who is a large blockholder might deflate the price if he intends to hold his shares over a long term, and, if his firm must repurchase shares in the near term to cover employee stock options. (2) Managers can benefit by driving down the price if they intend to engage in a management buyout. (3) A manager who is about to receive a new option grant may attempt to drive down the price in order to lower the strike price for the options granted.

The setting does not allow the manager to trade in the market, which can be rationalized by assuming the presence of an insider trading ban. The manager, in fact, has no informational advantage over the informed investors in this chapter. However, if he were allowed to trade, his orders needed to be considered simultaneously in his utility function, together with his biasing decision. Thus, his trading would be interdependent with his biasing decision.

However, the close similarity of the questions considered here with the regulation of insider trading is obvious if an insider is defined as a person that can better estimate the value of an asset than other persons.\textsuperscript{45} Many investors do not make use of supplied information because of a lack of expert knowledge, time or interest.\textsuperscript{46} Hirth and Neus (2001, p. 102) argue that the prohibition of insider trading would mean

\textsuperscript{43}A positive realization of $\tilde{u}$ with an arbitrarily large probability can be reached if $\tilde{u}$ had a large positive mean combined with a high precision $\tau_u$. As in chapter 5, a nonzero mean $\mu_u$ could be introduced without changing the results substantially.

\textsuperscript{44}Fischer and Verrecchia (2000) note in footnote 5 that a correlation between $\tilde{v}$ and $\tilde{u}$ would yield a situation with three equilibria in their model.

\textsuperscript{45}See Manne (1966), Fishman and Hagerty (1992), and Leland (1992) for seminal articles in the insider trading literature.

\textsuperscript{46}See Schneider (1993, p. 1430).
that information analysis and implementation is conducted by market insiders or professionals.\textsuperscript{47} “To disqualify managers from trading is to pass the informational advantage to brokers, investment banks, and others on the Street.”\textsuperscript{48} The uninformed investors would not be exploited by market insiders only if prices corrected to the efficient level without delay after the appearance of new information.

The model presumes that there are only two types of investors. In reality, investors are more heterogeneous and complex interactions take place among them. Uninformed investors can transfer a part of the investment process to intermediaries. News services and analysts sell their insights to investors.\textsuperscript{49} Other conflicts of interest arise, for example, when a bank analyst does not forward bad news about a company to investors, because the firm is among his employer’s customers as well.

The number $N$ of informed investors can be made endogenous. If their number is fixed, they are able to realize a positive expected profit which decreases with $N$. Thus, if there were fixed costs of information acquisition, the number $N$ of informed investors would be determined endogenously under conditions of free entry. Uninformed investors would acquire the private signal up to the point, where the expected trading profit turns negative, as discussed by Kim and Verrecchia (1994) in section 2.3. Introducing such costs into the current model would yield a polynomial equation for $N$ as in Kim and Verrecchia (1994), which cannot be solved explicitly, in addition to the polynomial for $\alpha$. Thus, an analysis of the marginal influences of the parameters would become intractable.

Another immediate extension would be to let the informed investors learn only a noisy piece of information $o_i = b + e_i$ about the bias, as in the benchmark model. However, this noisy version would lead to a fifth-order polynomial for $\alpha$. See chapter 5 for the introduction of an additional idiosyncratic noise term.

\textsuperscript{47}Market professionals include arbitrageurs, researchers, brokers and portfolio managers, who devote their careers to acquiring information and honing evaluate skills.
\textsuperscript{48}See Easterbrook and Fischel (1991, p. 262).
\textsuperscript{49}In Admati and Pfleiderer (1988a), an information owner can sell the information, whereas the alternative is to trade strategically on the basis of the information.
All production decisions are assumed to be taken efficiently, and the biasing decision has no effect on the realized firm value \(v\) (pure exchange). In reality, of course, there are "feedback and feedforward effects between a firm’s production activities and its stock price, and these interactions may be altered by its disclosure practices."\(^{50}\) Financial reports affect real decisions made by managers and others, rather than simply reflecting the results of these decisions. To illustrate such effects, consider the interaction between production activities and stock prices: stock prices determine both the firm’s cost of equity capital and its capacity to borrow; they hence decide whether a company will be taken over by others or can acquire others. That the interactions between production activities and stock prices can be influenced by a firm’s disclosure practices is intuitive.\(^{51}\)

Proof of Proposition 3.3.1

The derivation of an equilibrium proceeds in three steps. Firstly, the utility of the manager in (3.5) and the expected trading profits of the informed traders in (3.6) are maximized, and the efficient price condition in (3.4) is evaluated. Secondly, a comparison of the coefficients of these results with the conjectured strategies in (3.10) yields a system of equations. Thirdly, this system is solved for the equilibrium coefficients and it is shown that there always exists a unique solution.

**Step 1.** The manager maximizes his utility function (3.5) with respect to the bias \(b\). It is

\[
up - \frac{1}{2} cb^2 = u(\delta_r r + \delta_Q Q) - \frac{1}{2} cb^2 \quad (3.10.3)
\]

\[
= u(\delta_r (v + b) + \delta_Q (N\beta_r(v + b) + N\beta b + My)) - \frac{1}{2} cb^2
\]

\(^{50}\)See Dye (2001, p. 195).

\(^{51}\)If one allows for production and exchange, Kunkel (1982) shows that conditions exist under which disclosure would be preferred (the adverse risk-sharing effect otherwise prohibits a Pareto improvement) because altered production plans lead to a more efficient allocation of resources across time and firms. See Christensen and Feltham (2003), chapter 8, for a literature review of trading models with production.
First and second order conditions for a maximum with respect to $b$ are

$$u(\delta_r + N\delta Q(\beta_r + \beta_b)) - cb = 0$$

and $c > 0$.

The first order condition yields

$$b = \frac{u}{c}(\delta_r + N\delta Q(\beta_r + \beta_b)). \quad (3.29)$$

Each informed investor $i$, for $i = 1, \ldots, N$, conjectures the other $N - 1$ informed investors, the market maker, and the manager to play according to the system of conjectures in (3.10). The expected profit in (3.6) is

$$\pi_i = E[(\tilde{v} - P(x_i))x_i | \tilde{v} + \alpha\tilde{u} = r, \alpha\tilde{u} = b]$$

(3.10.3), (3.3)

$$= \left(E[\tilde{v}|r,b] - \delta_r r - \delta_Q \left(\sum_{j \neq i} E[\tilde{x}_j|r,b] + x_i + E[\tilde{y}|r,b]\right)\right) x_i$$

$$= \left(r - b - \delta_r r - \delta_Q \left(\sum_{j \neq i} E[\tilde{x}_j|r,b] + x_i\right)\right) x_i$$

By observing the report $r = v + b$ and the bias $b$, the informed investor effectively also knows the terminal value $v = r - b$. Deriving the profit with respect to $x_i$ is using the product rule

$$\frac{\partial \pi_i}{\partial x_i} = \left(r - b - \delta_r r - \delta_Q \left(\sum_{j \neq i} E[\tilde{x}_j|r,b] + x_i\right)\right) x_i + x_i \left(-\delta_Q \sum_{j \neq i} \frac{\partial E[\tilde{x}_j|r,b]}{\partial x_i} - \delta_Q\right).$$

In a Nash equilibrium, a change in an informed investor’s demand does not influence the others’ demands,\(^{52}\) i.e.

$$\frac{\partial E[\tilde{x}_j|r,b]}{\partial x_i} = 0 \quad \text{for all} \quad j \neq i \quad (3.30)$$

\(^{52}\) Hirth and Neus (2001) also use this argument in a similar setting.
The first order condition for a maximum therefore is

\[ \frac{\partial \pi_i}{\partial x_i} = r - b - \delta_r - \delta Q \left( \sum_{j \neq i} E[\tilde{x}_j | r, b] + x_i \right) - \delta Q x_i \]

(3.11.2)

\[ = r - b - \delta_r - \delta Q \left( \sum_{j \neq i} (\beta_r r + \beta_b b) + x_i \right) - \delta Q x_i \]

\[ = r - b - \delta_r x_i (N - 1)(\beta_r r + \beta_b b) - 2\delta Q x_i = 0. \]

This yields

\[ x_i = \frac{1}{2\delta Q} (r - b - \delta_r r - (N - 1)\delta Q(\beta_r r + \beta_b b)). \]

(3.31)

The market maker sets the price according to (3.4). The observation of

\[ (\tilde{v} + \alpha \tilde{u} = r, \sum_i \tilde{x}_i + M \tilde{y} = Q) \]

is informationally equivalent (using (3.10.2)) to an observation of

\[ (\tilde{v} + \alpha \tilde{u} = r, \tilde{v} - \frac{1}{N\beta_b} M \tilde{y} = -\frac{1}{N\beta_b} Q + \frac{\beta_r + \beta_b}{\beta_b} r). \]

Thus, the price (3.4) is

\[ P = \frac{\alpha^{-2} r \tau_v + N^2 \beta_b^2}{M^2 \sigma_y^2} \left( -\frac{1}{N\beta_b} \frac{\beta_r + \beta_b}{\beta_b} r \right) \]

(3.32)
Step 2. Comparing the coefficients of (3.29), (3.31), and (3.32) with the conjectured system (3.10) yields the system of equations

\[ c\alpha = \delta_r + N\delta_Q(\beta_r + \beta_b) \]  
(3.33.1)

\[ \beta_r = \frac{1}{2\delta_Q} (1 - \delta_r - \delta_Q(N - 1)\beta_r) \]  
(3.33.2)

\[ \beta_b = \frac{1}{2\delta_Q} (-1 - \delta_Q(N - 1)\beta_b) \]  
(3.33.3)

\[ \delta_r = \frac{\alpha^{-2}\tau_u + \frac{N^2\beta_b}{M^2\sigma_y^2}(\beta_r + \beta_b)}{\tau_v + \alpha^{-2}\tau_u + \frac{N^2\beta_b^2}{M^2\sigma_y^2}} \]  
(3.33.4)

\[ \delta_Q = \frac{-\frac{N\beta_b}{M^2\sigma_y^2}}{\tau_v + \alpha^{-2}\tau_u + \frac{N^2\beta_b^2}{M^2\sigma_y^2}} \]  
(3.33.5)

under the second order condition

\[ \delta_Q > 0. \]  
(3.34)

Step 3. To solve (3.33), consider \( \beta_b \) in (3.33.3)

\[ 2\beta_b\delta_Q = -1 - (N - 1)\beta_b\delta_Q \]
\[
\Leftrightarrow \quad (N + 1)\beta_b\delta_Q = -1
\]
(3.33.5)
\[
\Rightarrow \quad (N + 1)\beta_b \frac{-\frac{N\beta_b}{M^2\sigma_y^2}}{\tau_v + \alpha^{-2}\tau_u + \frac{N^2\beta_b^2}{M^2\sigma_y^2}} = -1
\]
\[
\Leftrightarrow \quad \frac{N\beta_b^2}{M^2\sigma_y^2}(-(N + 1) + N) = -(\tau_v + \alpha^{-2}\tau_u)
\]
\[
\Leftrightarrow \quad \beta_b^2 = \frac{1}{N} M^2\sigma_y^2(\tau_v + \alpha^{-2}\tau_u). \]  
(3.35)

Due to \( \delta_Q > 0 \), equation (3.33.5) shows that \( \beta_b \) must be the negative solution. Inserting \( \beta_b \) into (3.33.5) yields

\[ \delta_Q = -\frac{1}{N + 1} \frac{1}{\beta_b} \]  
(3.36)

Using these results for \( \beta_b \) and \( \delta_Q \), equation (3.33.2) is

\[ \beta_r\delta_Q(N + 1) = 1 - \delta_r \]
\[
\Leftrightarrow \quad \frac{\beta_r}{\beta_b} = \delta_r - 1 \]  
(3.37)

and equation (3.33.4) is

\[ (N + 1)\delta_r(\tau_v + \alpha^{-2}\tau_u) = \alpha^{-2}\tau_u + N(\tau_v + \alpha^{-2}\tau_u) \left( \frac{\beta_r}{\beta_b} + 1 \right) \]  
(3.38)
Inserting (3.37) into (3.38) yields

\[(N + 1)\delta_r = \frac{\alpha^{-2}\tau_u}{\tau_v + \alpha^{-2}\tau_u} + N(\delta_r - 1 + 1)\]

\[\iff \delta_r = \frac{\alpha^{-2}\tau_u}{\tau_v + \alpha^{-2}\tau_u} \] (3.39)

and inversely

\[\beta_r = \beta_b - \frac{-\tau_u}{\tau_v + \alpha^{-2}\tau_u}. \] (3.40)

Equation (3.33.1) is

\[c\alpha = \delta_r + N\delta_Q(\beta_r + \beta_b)\]

(3.36), (3.39)

\[= \frac{\alpha^{-2}\tau_u}{\tau_v + \alpha^{-2}\tau_u} - \frac{N + 1}{N + 1} \beta_b (\beta_r + \beta_b)\]

(3.35), (3.40)

\[= \frac{1}{N + 1} \frac{\alpha^{-2}\tau_u}{\tau_v + \alpha^{-2}\tau_u}\]

Hence, the solutions to \(\alpha\) are given by the roots of the third order polynomial

\[(N + 1)\tau_v c\alpha^3 + (N + 1)\tau_u c\alpha - \tau_u = 0. \] (3.41)

Its first derivative \(3(N + 1)\tau_v c\alpha^2 + (N + 1)\tau_u c\) is always positive.

Thus, the polynomial has one and only one root in \(\alpha\), since the left hand side is monotonically increasing in \(\alpha\), is negative for \(\alpha = 0\), and positive for \(\alpha = \frac{1}{N + 1} \frac{1}{c}\). Existence follows from the mean value theorem since polynomials are continuous functions.

The other coefficients \(\beta_r, \beta_b, \delta_r,\) and \(\delta_Q\) depend uniquely on \(\alpha\) so that there always exists a unique solution to the system (3.33), whereby \(\alpha\) lies in the interval

\[\left(0, \frac{1}{N + 1} \frac{1}{c}\right). \] (3.42)
Chapter 4

An Extension to Endogenous Liquidity Trading

Up to this point, liquidity trading was given exogenously as the realization of a normal random variable. Such an approach makes the analysis of their utility questionable, because the model excludes the motivations for liquidity trading. Since liquidity traders always lose money to informed traders, why do they not withdraw from the market and satisfy their liquidity need in another way?

The chapter differentiates between two different motivations for liquidity trading. Section 4.1 introduces into the setting of chapter 3 a number of rational uninformed investors who either maximize idiosyncratic liquidity preferences (section 4.2) or who hedge their endowment in the asset (section 4.3). It turns out that the basic equilibrium structure remains similar; since the informed investor perfectly balances out any influence of liquidity traders on the equilibrium price, the liquidity demand has still no effect on price informativeness and the utility of the manager.

However, the liquidity traders’ strategy and their utility critically depend on their specific motivation to trade. The discussion in section 4.4 compares the results with the scenario presented in chapter 3 and the differing results in the two scenarios of this chapter.
4.1 Rational Uninformed Investors

The economic setting remains similar to that of section 3.2. The sequence of events (see figure 4.1) changes only insofar as now, both informed and uninformed demands are rationally determined, a behavior which is anticipated by the other players in equilibrium.

![Figure 4.1: Sequence of Events in Chapter 4.](image)

Again, at $t = 0$, the manager discloses a biased report

$$r = v + b$$

(4.1)
on firm value $v$. He chooses the bias $b$ so as to maximize his utility

$$uP(b) - \frac{1}{2} cb^2.$$  

(4.2)

Competition among the informed investors was studied in chapter 3; hence, the following assumes only one informed investor ($N = 1$). This serves to reduce the notational burden. His demand maximizes the trading profit

$$\Theta = E[(\tilde{v} - \tilde{P}(x))x|r, b].$$  

(4.3)

In contrast to chapter 3, there are now $M$ rational, risk-averse liquidity traders. They have a negative exponential utility with the risk-aversion coefficient $\rho$.\footnote{See Christensen and Feltham (2003, chapter 2) on the properties of the negative exponential utility function.} If $y_j$ denotes
uninformed investor $j$’s demand for shares, then his utility $U_j$ is given by

$$U_j(\tilde{S}_j(y_j)) = -e^{-\rho \tilde{S}_j(y_j)},$$

(4.4)

whereby $S_j(y_j)$ differs in the two trading motivations:

1. The liquidity traders may possess idiosyncratic, i.e. disparate, liquidity preferences for a share: if the terminal value of the firm is $v$, then liquidity trader $j$’s private valuation of the asset is

$$v + w_j \text{ for } j = 1, \ldots, M,$$

(4.5)

whereby $w_j$ is the realization of a normal random variable $\tilde{w}_j$ with mean zero and variance $\sigma_w^2$. In (4.4), it is then

$$\tilde{S}_j(y_j) = (\tilde{v} + \tilde{w}_j - \tilde{P}(y_j))y_j.$$  

(4.6)

The idiosyncratic preferences of the liquidity traders can be motivated by disparate valuations for a share due to a heterogeneous willingness to pay for the immediate execution of the trade.\(^2\) Thus, although all liquidity traders have the same expectations regarding the firm’s terminal value, each has an idiosyncratic opportunity cost of failing to trade at a given point in time, which reflects the liquidity trader’s particular circumstances as opposed to information about the future value of the firm. Examples for private liquidity needs include cash needs in response to an emergency or an unexpected opportunity, the desire to balance a suboptimal portfolio, the desire to minimize the tracking error of an investment which was designed to meet a specific target, or the need to reduce a short position.\(^3\)

---

\(^2\)This type of preferences has been used in market microstructure models, e.g. in Garman (1976), Cohen et al. (1978, 1981), Ho and Stoll (1981), Glosten and Milgrom (1985), and in Mendelson and Tunca (2004).

\(^3\)See section 2.4 for these and other motivations of liquidity and noise trading. Wang (1994) explicitly models such a liquidity shock: in his model, uninformed investors have a high liquidity demand when they face highly profitable private investment opportunities.
2. Liquidity trading may also occur due to the wish to hedge a random endowment shock $w_j$. Thereby, $w_j$ is again the realization of a normal random variable $\tilde{w}_j$ with mean zero and variance $\sigma^2_{w}$.

In this case, the utility in (4.4) is given with

$$\tilde{S}_j(y_j) = \tilde{v}(y_j + \tilde{w}_j) - \tilde{P}(y_j)y_j.$$  \hfill (4.7)

This kind of motivation for liquidity trading has been used, for example, in Grossman and Miller (1988) and Spiegel and Subrahmanyam (1992).\(^4\)

The symbol $S_j$ will take the signification of (4.6) in section 4.2, where the equilibrium with idiosyncratic liquidity preferences is analyzed, and that of (4.7) in section 4.3, where the equilibrium with hedging motivations is analyzed. The random variables $\tilde{w}_j, j = 1, \ldots, M$ are, in both versions, mutually independent and also independent from $\tilde{v}$ and $\tilde{u}$. The variance $\sigma^2_{w}$, the mean of zero, and the number $M$ of uninformed traders are common knowledge.

To quantify the optimal demand, the uninformed investors maximize their utility (4.4) with respect to $y_j$. In doing so, they do not observe the public report $r$;\(^5\) however, they take into account the effect of their own trading on the price.

The market maker now observes the altered net demand

$$Q = x + \sum_{j=1}^{M} y_j$$  \hfill (4.8)

and the public report $r$. He sets the price efficiently at

$$P = E[\tilde{v}|Q, r].$$  \hfill (4.9)

The equilibrium strategies are now conjectured to be

$$b = \alpha u$$  \hfill (4.10.1)

$$x = \beta_r r + \beta_b b$$  \hfill (4.10.2)

$$P = \delta_r r + \delta_Q Q.$$  \hfill (4.10.3)

\(^4\)Other studies with hedging-motivated trading include Glosten (1989), Spiegel and Subrahmanyam (1995), Vayanos (1999), and Bernhardt and Massoud (1999).

\(^5\)It would be of no advantage for the uninformed investors to observe the report $r$, because due to the market maker, its informational content is fully reflected in the price.
In section 4.2, the demand of the uninformed investors is conjectured to be

$$y_j = \gamma w_j \quad \text{for} \quad j = 1, \ldots, M$$ \hspace{1cm} (4.11)

i.e. linear in the realized idiosyncratic additional utility $w_j$ for a share. In section 4.3, it is conjectured to be

$$y_j = -\gamma w_j \quad \text{for} \quad j = 1, \ldots, M$$ \hspace{1cm} (4.12)

i.e. linear in the endowment shock $w_j$ but with the opposite sign.

In equilibrium, a strategy of this form indeed turns out to maximize their utility (due to exponential utilities and the assumption of normality). The manager, the market maker, and the informed investor now take into account the strategic behavior of the $M$ liquidity traders.

### 4.2 The Equilibrium with Idiosyncratic Liquidity Preferences

Before stating the equilibrium, two helpful immediate relations are introduced. The precision $\tau_r$ of the report $r = v + \alpha u$ is again

$$\tau_r = \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} \right)^{-1}.$$ \hspace{1cm} (4.13)

The conditional variance of firm value given the report $r$ is as in chapter 3

$$\text{Var}[\tilde{v}|r] = \frac{1}{\tau_v} \left( 1 - \frac{\tau_r}{\tau_v} \right)$$ \hspace{1cm} (4.14)

$$= \frac{1}{\tau_v + \alpha^2 \tau_u}.$$ \hspace{1cm} (4.15)

The inverse $\text{Var}^{-1}[\tilde{v}|r]$ will be referred to in the following as the informativeness of the report.

**Proposition 4.2.1.** In scenario 1 of section 4.1 (idiosyncratic liquidity preferences), there is one unique equilibrium if and only if the condition

$$\text{Var}[\tilde{v}|r] < M\sigma_w^2$$ \hspace{1cm} (4.16)
holds for the parameters $\tau_v, \tau_u, M, c, \text{ and } \sigma_w^2$; thereby, $\alpha$ is the unique root of the polynomial

$$c\alpha = \frac{1}{2}\frac{\tau_r}{\tau_v}. \quad (4.17)$$

The equilibrium bias $b$, the demands $x$ and $y_j$, for $j = 1, \ldots, M$, and the price $P$ depend uniquely on $\alpha$ and are given by

$$b = \alpha u \quad (4.18.1)$$
$$x = \beta \gamma \left( 1 - \frac{\tau_r}{\tau_v} \right) r - b \quad (4.18.2)$$
$$y_j = \gamma w_j \quad \text{for} \quad j = 1, \ldots, M \quad (4.18.3)$$
$$P = \frac{\tau_r r}{\tau_v} + \frac{1}{2} \frac{1}{\beta \gamma} Q \quad (4.18.4)$$

whereby

$$\beta = \sqrt{\frac{M \sigma_w^2}{\text{Var}[\tilde{v} | r]}} \quad \text{and} \quad (4.19)$$
$$\gamma = \frac{4}{\rho} \frac{M}{2M - 1} \left( \frac{1}{\text{Var}[\tilde{v} | r]} - \sqrt{\frac{1}{M \sigma_w^2 \text{Var}[\tilde{v} | r]}} \right). \quad (4.20)$$

Proof. See the appendix.

The strategies of the manager, the market maker, and the informed investor remain similar to chapter 3. The additional factor $\gamma$ in the informed investor’s demand in (4.18.2) reflects his objective to disguise his trading by scaling up his demand in accordance with the liquidity demand. Therefore, uninformed trading does not influence the biasing decision.

The strategy of the uninformed investors is given by $\gamma w_j$. Equation (4.51.4) in the appendix shows that the coefficient $\gamma$ can also be written as

$$\gamma = \frac{1}{2 \delta Q + \rho \text{Var}[\tilde{v} - P]}. \quad (4.21)$$

The demand hence increases with the market depth $\frac{1}{\delta Q}$, decreases with the risk-aversion $\rho$ and the variance of the deviation of the equilibrium price from the terminal value.
In contrast to chapter 3, there is now not necessarily an equilibrium. If condition (4.16) does not hold, the equilibrium breaks down. The equilibrium set denotes the combinations of parameters for which condition (4.16) holds. A convenient interpretation for this circumstance is that the demand of the uninformed investors grows with $M\sigma_w^2$ and, as in chapter 3, the demand of the informed investor increases with the informativeness of the report (although trading profits decrease). Therefore, condition (4.16) states that the demand of the uninformed investors must be above a threshold level relative to the informed demand. There is a similar condition in Gloston and Milgrom (1985), whose equilibrium also breaks down if the adverse selection problem is too extreme.

The remainder of this section examines the

- bias and precision of the report,
- demand of the uninformed investors and their utility
- informed demand and trading volume,
- price informativeness, and
- trading profits.

**The Bias and Precision of the Report**

The introduction of uninformed investors does not change the biasing decision of the manager, because their trading has no influence on the price reaction to the bias $b$. In other words, the marginal benefits of biasing do not change.

The defining polynomial (4.17) for $\alpha$ is equal to (3.14) for $N = 1$. In addition, the precision $\tau_r$ of the public report in (4.13) remains the same as in (3.13). The bias $b = \alpha u$ is not influenced by the new parameters $\rho$, $M$, and $\sigma_w^2$. Also, the utility of the manager remains unaffected (see figure 3.5).
Demand of the Uninformed Investors and their Utility

The uninformed investors trade linearly in their idiosyncratic utility $w_j$. According to (4.18.3), it is $y_j = \gamma w_j$ with

$$\gamma = \frac{4}{\rho} \frac{M}{2M-1} \left( \frac{1}{\text{Var}[\tilde{v}|r]} - \sqrt{\frac{1}{M \sigma_w^2 \text{Var}[\tilde{v}|r]}} \right).$$

Hence, what follows is

**Corollary 4.2.2.** The marginal influence of the parameters on the utility of the uninformed investors is shown in table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma_w^2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 4.1: The Marginal Effect of the Parameters on the Uninformed Demand $\gamma$."

The uninformed investors’ demand increases with the marginal costs $c$ of biasing, with the precisions $\tau_v$ and $\tau_u$, and the variance $\sigma_w^2$ of their liquidity preferences. It decreases with the uninformed investors’ risk-aversion $\rho$. The influence of the number $M$ of uninformed investors is indeterminate. The uninformed investors trade more pronounced (higher $\gamma$) if and only if their utility increases:

The utility of the uninformed investors in (4.4), which of course differs from their trading profits, grows monotonically in the certainty equivalent. Defining $B =$
Var[\tilde{S}_j|w_j], the certainty equivalent for traders $j$ is

$$CE_j = E[\tilde{S}_j|w_j] - \frac{1}{2}\rho \text{Var}[\tilde{S}_j|w_j]$$

(4.6)

$$= y_j(w_j - \delta_Q y_j) - \frac{1}{2}\rho B y_j^2$$

(4.18.3)

$$= \gamma w_j(w_j - \delta_Q \gamma w_j) - \frac{1}{2}\rho B (\gamma w_j)^2$$

$$= \frac{1}{2}\gamma w_j^2 [2(1 - \delta_Q \gamma) - \rho B \gamma]$$

(4.51.4)

$$= \frac{1}{2}\gamma w_j^2 \left[ 2(1 - \frac{\delta_Q}{2\delta_Q + \rho B}) - \frac{\rho B}{2\delta_Q + \rho B} \right]$$

$$= \frac{1}{2}\gamma w_j^2 \frac{1}{2\delta_Q + \rho B} \left[ 2(\delta_Q + \rho B - \delta_Q) - \rho B \right]$$

(4.51.4)

$$= \frac{1}{2}\gamma w_j^2 \text{ for } j = 1, \ldots, M.$$  

The utility of each uninformed investor $j$ thus increases with $\gamma$ and the square of the realized idiosyncratic value $w_j$ for a share. Note that the utility of the uninformed investors is always positive due to condition (4.16).

**Corollary 4.2.3.** The marginal influence of the parameters on the utility of the uninformed investor $j$, for $j = 1, \ldots, M$ is shown in table 4.2.

<table>
<thead>
<tr>
<th>$CE_j$</th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma_w^2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.2: The Marginal Effect of the Parameters on the Uninformed Investor's Certainty Equivalent $CE_j$.

An intuition for the influence of the parameters can be derived by considering a different representation of the coefficient $\gamma$ in (4.51.4) in the appendix. There, it
turns out that
\[
\gamma = \frac{1}{2\delta_Q + \rho B}.
\] (4.22)

Hence, the utility is influenced by two factors: the market depth and a risk discount. The market depth is given by
\[
\frac{1}{\delta_Q} = 2\beta \gamma = \frac{8}{\rho 2M - 1 \text{Var}[\tilde{v}|r]} \left( \sqrt{\frac{M \sigma_w^2}{\text{Var}[\tilde{v}|r]}} - 1 \right).
\] (4.23)

With table 4.3, it follows that the market depth is increasing in all parameters except for the risk-aversion \( \rho \). The risk discount measures the variance of the deviation of the market price from the terminal value, multiplied by the risk-aversion \( \rho \). It is given by
\[
\rho B = \rho \text{Var}(\tilde{v} - \tilde{P})
\] (4.58)
\[
= \frac{\rho}{4} \text{Var}[\tilde{v}|r] \frac{2M - 1}{M}.
\] (4.24)

Thus, the risk discount increases with the risk-aversion \( \rho \) and decreases with the informativeness of the report, i.e. decreases with \( \tau_v, \tau_u \) and \( c \). Combining the market depth and the risk discount, no countervailing effects can be observed between them with regard to the marginal influence of the parameters except for the number \( M \) of uninformed investors. For example, the marginal costs \( c \) of biasing increase the market depth and decrease the risk discount; together, the costs \( c \) also increase the utility of each uninformed investor. The number \( M \) of uninformed investors increases market liquidity, but also results in more asset riskiness as can be observed in (4.24). The overall effect is indeterminate.

**Informed Demand and Trading Volume**

As in chapter 3, the informed traders balance out any random influence stemming from the uninformed investors in their demand (4.18.2). The informed investors’
trading aggressiveness

\[
\beta \gamma = \frac{4}{\rho} \frac{M}{2M - 1} \frac{1}{\text{Var}[\tilde{v}|r]} \left( \sqrt{\frac{M \sigma_w^2}{\text{Var}[\tilde{v}|r]}} - 1 \right)
\] (4.25)

is now influenced by the equilibrium strategy \(\gamma w_j\) of the uninformed traders. It follows

**Corollary 4.2.4.** *The marginal influence of the parameters on the trading aggressiveness of the informed investor is shown in table 4.3.*

<table>
<thead>
<tr>
<th>(\beta \gamma)</th>
<th>(c)</th>
<th>(\tau_v)</th>
<th>(\tau_u)</th>
<th>(M)</th>
<th>(\sigma_w^2)</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 4.3: The Marginal Effect of the Parameters on the Trading Aggressiveness \(\beta \gamma\).*

Note that, due to condition (4.16), the trading aggressiveness is always positive. The informed investor scales his demand up with the uninformed demand (\(\gamma\)). In addition, he demands more if the informativeness of the report is higher (via \(\beta\)). Both effects reinforce each other, i.e. the same comparative statics are inherent in \(\beta\) and \(\gamma\).

The expected trading volume is\(^6\)

\[
E[\hat{V}] = \frac{1}{2} \left( E[\hat{x}] + \sum_j E[\hat{y}_j] + E[\hat{x} + \sum_j \hat{y}_j] \right)
\]

\[
= \frac{1}{\sqrt{2\pi}} \left( \sqrt{\text{Var}(\hat{x})} + \sum_j \sqrt{\text{Var}(\hat{y}_j)} + \sqrt{\text{Var}(\hat{x} + \sum_j \hat{y}_j)} \right)
\]

\[
= \gamma \sqrt{M \sigma_w^2} (1 + \sqrt{M} + \sqrt{M + M^2})
\] (4.26)

**Corollary 4.2.5.** *The marginal influence of the parameters on the trading volume is shown in table 4.4.*

\(^6\)Note that, similar to calculation (3.20) in chapter 3, it is \(\text{Var}[\hat{x}] = \beta^2 \gamma^2 \frac{\sigma^2_w}{\tau_u \tau_v} = M \sigma_w^2 \gamma^2\).
The expected trading volume is proportional to the coefficient of uninformed demand \(\gamma\). Therefore, in contrast to chapter 3, it is now influenced by all model parameters. Especially, the informativeness of the report increases the trading volume. Section 4.3 provides further comments on informed demand and trading volume.

### Price Informativeness

Ex post, the realized price is

\[
P = \frac{\tau_r}{\tau_v} r + \frac{1}{2} \frac{1}{\beta \gamma} Q
\]

\((4.18.2), (4.18.3)\)

\[
= \frac{1}{2} \left( \frac{\tau_r}{\tau_v} + 1 \right) v + \frac{1}{2} \frac{\tau_r}{\tau_v} b + \frac{1}{2} \frac{1}{\beta} \sum w_j.
\]

\(4.27\)

This is identical to (3.21) in chapter 3 (for \(N = 1\)), except that the influence of the exogenous liquidity demand \(y\) is now replaced by the sum of the idiosyncratic preferences \(\sum w_j\).

The price risk is

\[
Var[\tilde{P}] = \frac{1}{4} \left( \frac{\tau_r}{\tau_v} + 1 \right)^2 \frac{1}{\tau_v} + \frac{1}{4} \frac{\tau_r^2}{\tau_v^2} \alpha^2 + \frac{1}{4 \beta^2} M \sigma_w^2
\]

\((4.19)\)

\[
= \frac{1}{4 \tau_v} \left[ \left( \frac{\tau_r}{\tau_v} + 1 \right)^2 + \frac{\tau_r^2}{\tau_v} \alpha^2 + \tau_r \frac{\alpha^2}{\tau_v} \right]
\]

\[
= \frac{1}{2} \frac{\tau_r}{\tau_v} \left( \frac{\tau_r}{\tau_v} + 1 \right).
\]

\(4.28\)
Using this result, the remaining variance of terminal value after observing the price is

\[
\text{Var}[\tilde{v}|P] = \text{Var}[\tilde{v}] - \frac{\text{Cov}^2[\tilde{v}, \tilde{P}]}{\text{Var}[\tilde{P}]}
\]

\[
= \frac{1}{\tau_v} - \frac{1}{4} \left( \frac{\tau_v}{\tau_r} + 1 \right)^2 \frac{1}{\tau_v^2}
\]

\[
= \frac{1}{2\tau_v} \left( 1 - \frac{\tau_r}{\tau_v} \right)
\]

\[
= \frac{1}{2} \text{Var}[\tilde{v}|r].
\]

(4.28)

(4.14)

(4.29)

Defining the price efficiency \( \Psi \) as the inverse of this variance yields

**Corollary 4.2.6.** The marginal influence of the parameters on the price efficiency is shown in table 4.5.

<table>
<thead>
<tr>
<th>( \Psi )</th>
<th>( c )</th>
<th>( \tau_v )</th>
<th>( \tau_u )</th>
<th>( M )</th>
<th>( \sigma_w^2 )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.5: The Marginal Effect of the Parameters on the Price Efficiency \( \Psi \).*

The price efficiency increases only with the informativeness of the report. Thus, the introduction of rational uninformed traders does not change the price efficiency. As in chapter 3, the informed investor perfectly filters out their influence.
Trading profits

The informed investors’ expected trading profit $\Theta$ is

$$\Theta = E[(\tilde{v} - \tilde{P})\tilde{x}]$$

(4.27), (4.18.2)

$$= E\left[ v - \frac{1}{2} \left( \frac{\tau_r}{\tau_v} + 1 \right) v - \frac{1}{2} \frac{\tau_r b}{\tau_v} - \frac{1}{2} \beta \sum_j w_j \right]$$

$$\beta \gamma \left( \left( 1 - \frac{\tau_r}{\tau_v} \right) v - \frac{\tau_r b}{\tau_v} \right)$$

$$= \beta \gamma E\left[ \frac{1}{2} \left( 1 - \frac{\tau_r}{\tau_v} \right) v - \frac{1}{2} \frac{\tau_r \alpha u}{\tau_v} \right] \left( 1 - \frac{\tau_r}{\tau_v} \right) v - \frac{\tau_r \alpha u}{\tau_v} \right)$$

$$= \frac{\beta \gamma}{2} \left( 1 - \frac{\tau_r}{\tau_v} \right)^2 1 \left( \frac{\tau_r}{\tau_v} \alpha^2 \right)$$

$$= \frac{\beta \gamma}{2} \frac{\alpha^2 \tau_r}{\tau_u \tau_v} = \frac{1}{2} \gamma \sqrt{M \sigma_w^2} \sqrt{\text{Var}[\tilde{v}|r]}$$

(4.19), (4.20)

$$= \frac{2}{\rho} \frac{M}{2M - 1} \left( \sqrt{\frac{M \sigma_w^2}{\text{Var}[\tilde{v}|r]}} - 1 \right)$$

(4.30)

Note that the expected trading profits are always positive due to condition (4.16). They increase with the trading aggressiveness $\gamma$ of the uninformed investors, and decrease with the informativeness of the report. However, $\gamma$ increases with the informativeness of the report, and this countervailing effect is the dominant one.

**Corollary 4.2.7.** The marginal influence of the parameters on the trading profits of the informed investor is shown in table 4.6.

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma_w^2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 4.6: The Marginal Effect of the Parameters on the Informed Profit $\Theta$.*

In this setting, the zero sum condition

$$E[(\tilde{v} - \tilde{P})\tilde{x}] + \sum_{j=1}^{M} E[(\tilde{v} - \tilde{P})\tilde{y}_j] = 0$$

(4.31)
again holds per definition. Since the expected trading profits $\Theta_L$ of each uninformed investor are identical, it follows

$$
\Theta_L = E[(\tilde{v} - \tilde{P})\tilde{y}] = \frac{1}{M}E[(\tilde{v} - \tilde{P})\tilde{x}] = -\frac{2}{\rho 2M - 1} \left( \sqrt{\frac{M \sigma^2_w}{\text{Var}[\tilde{v}|r]}} - 1 \right).
$$

Except for the influence of the number $M$ of uninformed investors, the marginal effect of the parameters is exactly opposite to table 4.6. Thus, their trading loss decreases with the number of uninformed investors and with their risk-aversion $\rho$. However, their loss increases with the marginal costs $c$, with the precisions $\tau_v$ and $\tau_u$, and with the variance $\sigma^2_w$ of their liquidity preferences.

### 4.3 The Equilibrium with Hedging Motivations

The outcome with hedging motivations is only slightly different from the equilibrium in section 4.2. The only two differences pertain to the equilibrium set and the strategy of the uninformed investors $\gamma$. In so far as the strategy of the informed investor depends on $\gamma$, his demand and trading profits show a different behavior with respect to changes in the parameters, but the structure of his demand stays the same:

**Proposition 4.3.1.** Under hedging motivations, there is one unique equilibrium if and only if the condition

$$
\text{Var}[\tilde{v}|r] > \frac{4}{\rho^2 \frac{1}{M \sigma^2_w}}
$$

holds for the parameters $\tau_v$, $\tau_u$, $M$, $c$, and $\sigma^2_w$; thereby, $\alpha$ is the unique root of the polynomial

$$
c \alpha = \frac{1}{2} \frac{\tau_v}{\tau_v}.
$$

The equilibrium bias $b$, the demands $x$ and $y_j$, for $j = 1, \ldots, M$, and the price $P$
depend uniquely on $\alpha$ and are given by

$$b = \alpha u$$  \hspace{1cm} (4.34.1)

$$x = \beta \gamma \left( \left( 1 - \frac{\tau_r}{\tau_v} \right) r - b \right)$$  \hspace{1cm} (4.34.2)

$$y_j = -\gamma w_j \text{ for } j = 1, \ldots, M$$  \hspace{1cm} (4.34.3)

$$P = \frac{\tau_r}{\tau_v} r + \frac{1}{2} \frac{1}{\beta \gamma} Q,$$  \hspace{1cm} (4.34.4)

whereby

$$\beta = \sqrt{\frac{M \sigma_w^2}{\text{Var}[\hat{v}|r]}} \text{ and}$$  \hspace{1cm} (4.35)

$$\gamma = \frac{2M}{2M - 1} \left( 1 - \frac{2}{\rho} \sqrt{\frac{1}{\text{Var}[\hat{v}|r]} \frac{1}{M \sigma_w^2}} \right)$$  \hspace{1cm} (4.36)

Proof. See the appendix.

Because the informed investor perfectly backs out any influence of the uninformed trading on the price, the informativeness of the price and the bias in the report are not altered in comparison to the case with idiosyncratic liquidity preferences. Due to the similarities with section 4.2 this section only briefly states the results that change.

In contrast to section 4.2, the uninformed investors now sell shares instead of buying them. They sell the portion $\gamma$ of their endowment of the shares. Note that $\gamma$ may also be greater than one.

The equilibrium breaks down if the uninformed supply of shares is too small as can be observed in condition (4.32): The set of parameters for which an equilibrium exists reflects that the liquidity traders here trade to offset their risky position. Therefore, the equilibrium set turns out to be smaller if the report is more informative since this makes the position of the uninformed investors less risky. The risk aversion $\rho$, the number $M$ of uninformed investors and the variance $\sigma_w^2$ in their endowments increase the equilibrium set, because then the group of liquidity traders intend to hedge their initial position in the risky asset more intensively.

The remainder of this section examines the


- demand of the uninformed investors and their utility,
- informed demand and trading volume, and
- trading profits.

The bias and precision of the report and the price informativeness remain as in section 4.2.

**Demand of the Uninformed Investors and their utility**

The uninformed investors trade linearly in their endowment shock $w_j$. According to (4.34.3), it is $y_j = -\gamma w_j$, for $j = 1, \ldots, M$ with

$$\gamma = \frac{2M}{2M - 1} \left( 1 - \frac{2}{\rho} \sqrt{\frac{1}{\text{Var}[\tilde{v} | r]} \frac{1}{M \sigma^2_w}} \right).$$

Hence, what follows is

**Corollary 4.3.2.** The marginal influence of the parameters on the utility of the uninformed investors is shown in table 4.7.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma^2_w$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4.7: The Marginal Effect of the Parameters on the Uninformed Demand $\gamma$.

Note from condition (4.32) that $\gamma$ is always positive, i.e. the uninformed investors sell a part of their initial endowment $w_j$. The sold amount decreases with the informativeness of the report, because $\gamma$ is smaller for higher $c$, $\tau_v$ and $\tau_u$. They sell more, if they are more risk averse and if the ex ante variance $\sigma^2_w$ in their endowment increases. The influence of the number $M$ of uninformed investors is indeterminate.

Defining

$$B = (1 - \delta_r - \delta_Q \beta_r)^2 \frac{1}{\tau_v} + (\delta_r + \delta_Q \beta_r + \delta_Q \beta_b)^2 \frac{2 \sigma^2}{\tau_u} + \delta_Q^2 (M - 1) \gamma^2 \sigma^2_w$$

(4.37)
as in section 4.2, the uninformed investor $j$’s certainty equivalent now becomes

$$CE_j = E[\hat{S}_j^2 | w_j] - \frac{1}{2} \rho \text{Var}[\hat{S}_j^2 | w_j]$$

\begin{align*}
&\stackrel{(4.62)}{=} -\delta_Q y_j^2 - \frac{\rho}{2\tau_v} (2y_j (1 - \delta_r - \delta_Q \beta_r)w_j + w_j^2) - 0.5 \rho y_j^2 B \\
&\stackrel{(4.10)}{=} -\frac{1}{2} \gamma^2 w_j^2 (2\delta_Q + \rho B) - \frac{\rho}{2\tau_v} (-2\gamma (1 - \delta_r - \delta_Q \beta_r)w_j^2 + w_j^2) \\
&\stackrel{(4.65)}{=} -\frac{1}{2} \gamma w_j^2 \left( \frac{\rho}{\tau_v} (1 - \delta_r - \delta_Q \beta_r) \right) - \frac{\rho}{2\tau_v} (-2\gamma (1 - \delta_r - \delta_Q \beta_r)w_j^2 + w_j^2) \\
&\stackrel{(4.58)}{=} \frac{1}{2} \gamma^2 w_j^2 \rho - \frac{1}{2\tau_v} w_j^2 \\
&= \frac{1}{2} (\gamma \text{Var} [\hat{v} | r] - \frac{1}{\tau_v}) \rho w_j^2 \quad \text{for} \quad j = 1, \ldots, M \quad (4.38)
\end{align*}

Hence, the utility of the uninformed investors may be positive or negative depending on the constellation of the parameters. It is positive, if the precision $\tau_v$ of the firm value is high relative to the other parameters.

**Corollary 4.3.3.** The marginal influence of the parameters on the utility of the uninformed investor $j$, for $j = 1, \ldots, M$ is shown in table 4.8.

<table>
<thead>
<tr>
<th>$CE_j$</th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma_w^2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>$?$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

*Table 4.8: The Marginal Effect of the Parameters on the Uninformed Investor’s Certainty Equivalent $CE_j$."

The uninformed investors’ utility increases if they sell a higher portion of their endowment (higher $\gamma$) and if the report is less informative (higher $\text{Var} [\hat{v} | r]$). The latter point stems from the fact that, by selling $y_j$ shares, the uninformed investor reduces his risk position (and thus the discount $-\frac{1}{2} \rho \text{Var}[\hat{S}_j^2 | w_j]$) more effectively if the report is less informative, for any given level of $y_j$. This follows from the derivation of the certainty equivalent in (4.38). Therefore, the utility decreases with $c$, $\tau_u$ and $\tau_v$. The precision $\tau_v$ of terminal value, however, directly lessens the risk the uninformed investor is exposed to. This is the dominating effect for $\tau_v$. 
The positive effect of the variance $\sigma_w^2$ in the endowment stems from the pricing of the market maker. A higher $\sigma_w^2$ raises the level of noise in net demand, thus reducing his responsiveness $\delta_Q$ to net demand, which in turn increases the liquidity $1/\delta_Q$. This directly benefits the liquidity traders, while there is no countervailing effect.

If the uninformed investors are more risk averse, they engage in more hedging which increases their security equivalent directly. However, the ex ante risk exposure $-\frac{1}{\tau_v}\rho w_j^2$ is also higher. This second effect turns out to be stronger.

**Informed Demand and Trading Volume**

The informed investors’ trading aggressiveness is

$$\beta \gamma = \frac{2M}{2M - 1} \sqrt{\frac{M \sigma_w^2}{\text{Var}[\tilde{v}|r]}} \left(1 - \frac{2}{\rho} \sqrt{\text{Var}[\tilde{v}|r]} \sqrt{M \sigma_w^2} \right).$$  \hspace{1cm} (4.39)

Note that, due to condition (4.32), the trading aggressiveness is always positive. The informed investor scales his demand up with the uninformed demand ($\gamma$), more specifically, he buys more if the uninformed investors sell more shares. However, when the report is more informative, (i) the uninformed investor sells less while (ii) the informed investor wants to buy more because he benefits from the lower informativeness of the order flow for the market maker relative to the report directly. The combined effect of $c$, $\tau_v$, and $\tau_u$ is indeterminate. It follows

**Corollary 4.3.4.** The marginal influence of the parameters on the trading aggressiveness of the informed investor is shown in table 4.9.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma_w^2$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta \gamma$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

*Table 4.9: The Marginal Effect of the Parameters on the Trading Aggressiveness $\beta \gamma$.***

The expected trading volume is again

$$E[\tilde{V}] = \gamma \sqrt{M \sigma_w^2} (1 + \sqrt{M} + \sqrt{M + M^2})$$ \hspace{1cm} (4.40)
Corollary 4.3.5. The marginal influence of the parameters on the trading volume is shown in table 4.10.

\[
\begin{array}{c|cccccc}
E[V] & c & \tau_v & \tau_u & M & \sigma^2_w & \rho \\
\hline
- & - & - & + & + & +
\end{array}
\]

Table 4.10: The Marginal Effect of the Parameters on the Expected Trading Volume \(E[V]\).

The expected trading volume is proportional to the coefficient of the uninformed demand \(\gamma\), i.e. it increases if the shares are more risky for the uninformed investors.

Trading profits

The informed investors’ expected trading profit \(\Theta\) is

\[
\Theta = E[(\hat{v} - \hat{P})\hat{x}] = \frac{1}{2} \gamma \sqrt{M \sigma^2_w} \sqrt{\text{Var} \[\hat{v} | r]} \\
= \frac{M}{2M -1} \left( \sqrt{M \sigma^2_w} \sqrt{\text{Var} \[\hat{v} | r]} - \frac{2}{\rho} \right)
\]

(4.41)  \hspace{1cm} (4.42)

The expected trading profit of the informed investor are always positive. He benefits from more uninformed trading that is due to more uninformed investors \(M\), a higher variance \(\sigma^2_w\) in their endowment and a higher risk aversion \(\rho\); he also prefers a less informative public report \(r\) since the uninformed investors then trade more intensively.

Corollary 4.3.6. The marginal influence of the parameters on the trading profits of the informed investor is shown in table 4.11.

\[
\begin{array}{c|cccccc}
\Theta & c & \tau_v & \tau_u & M & \sigma^2_w & \rho \\
\hline
- & - & - & + & + & +
\end{array}
\]

Table 4.11: The Marginal Effect of the Parameters on the Informed Profit \(\Theta\).
Since the expected trading profits $\Theta_L$ of each uninformed investor are identical, it follows

$$\Theta_L = E[(\tilde{v} - \tilde{P})\tilde{y}_j] = -\frac{1}{M}E[(\tilde{v} - \tilde{P})\tilde{x}]
= -\frac{1}{2M - 1} \left( \sqrt{M\sigma^2_u} \sqrt{\text{Var}[\tilde{v}|r]} - \frac{2}{\rho} \right).$$

Except for the influence of the number $M$ of uninformed investors, the marginal effect of the parameters is exactly opposite to table 4.11. Thus, their trading loss decreases when the public report is more informative. However, a more risky endowment and a higher risk aversion $\rho$ harms their trading profits, since in this case their hedging activities cause them to lose money in trades with the informed investor. The influence of the number $M$ of uninformed investors is now indeterminate.

### 4.4 Discussion

This chapter introduced rational liquidity traders who either maximize idiosyncratic preferences for a share or who hedge the risks of their initial endowment. The following discussion first compares the outcome of chapter 3 with exogenously given liquidity demand with the present setting and then compares the two different motivations for the endogenous liquidity demand.

#### Effects of endogenous liquidity demand

The uninformed investors now act strategically, i.e. they take into account the effect of their own actions on the behavior of the other players and the price. The strategy of uninformed investor $j$ is given by $\gamma y_j$ or $-\gamma y_j$ depending on the motivation. These considerations are also anticipated by the manager, the market maker, and the informed investors. However, it is remarkable that a number of market characteristics is not altered in comparison to the setting with exogenous liquidity demand.

Specifically, the biasing decision $b = \alpha u$ of the manager, the precision $\tau_r$ of the
report, and also the price informativeness $\Psi$ (the informed investor doubles the informativeness of the price as in Kyle (1985)) remain unchanged as in chapter 3. The reason lies, of course, in the behavior of the informed investor as a monopsonist (see figure 2.1), i.e. he consumes all changes in the uninformed investors’ demand himself.

This fact is reflected in the corresponding rows of table 4.12 which compares the marginal influence of the parameters on the equilibrium properties between the setting of chapter 3 and the settings in sections 4.2 and 4.3. Note that the variance $\sigma_y^2$ of the exogenous uninformed demand and the number $N$ no longer exist in the current setting, while the variance $\sigma_w^2$ and the risk aversion $\rho$ are new. These new characteristics of the liquidity traders have no influence on the bias and the price because the informed investor perfectly balances out their influence.

\[
\begin{array}{c|cccccccc}
\alpha & c & N & \tau_v & \tau_u & \sigma_y^2 & \sigma_w^2 & M & \rho \\
\hline
\tau_{\gamma} & +++ & +.. & +++ & +++ & 0.. & .00 & 000 & .00 \\
E[V] & 0+- & +.. & 0+- & 0+- & +.. & .++ & .++ & .-+ \\
\Psi & +++ & +.. & +++ & +++ & 0.. & .00 & 000 & .00 \\
\Theta & -+- & -.. & -+- & -+- & +.. & .++ & .++ & .-+ \\
\Theta_L & -+- & -.. & -+- & -+- & -+- & -.. & -? & .+- \\
\gamma & -+- & -.. & -+- & -+- & -+- & -? & .++ & .-+ \\
CE & -+- & -.. & -+- & -+- & -+- & -? & .++ & .-+
\end{array}
\]

Table 4.12: The Marginal Effect of the Parameters on the Equilibrium Outcome in the Settings of Chapter 3 and Sections 4.2 and 4.3, respectively. (The symbols denote: "++" positive influence, "-" negative influence, "0" no influence, "?" influence is indeterminate, "." influence cannot be calculated, because parameter or characteristic is not included in the model.)

The trading aggressiveness\(^7\) $\beta\gamma$ of the informed investor now depends on the liquidity demand in addition to the old effect $\beta$. As well, the expected trading volume now depends on all model parameters because it increases proportionately with the

\(^7\)Note that, here, the trading aggressiveness is $\beta\gamma$ whereas in chapter 3, it is $\beta$ only. Also note that the strategy $\gamma$ is defined with a different sign in the two settings.
endogenous liquidity demand. However, the sign of the influences depends on the motivation for liquidity trading; therefore, its discussion is conferred to the next subsection.

The trading profits of informed and uninformed investors depends on the information advantage of the informed investors vis-a-vis the market maker as in chapter 3. In addition, now, the liquidity traders’ demand is influenced by the informativeness of the public report - although they do not observe the public report - because it alters the riskiness of holding shares. This influences the trading profits of both, the informed and the liquidity traders. The specific direction depends on the trading motivation of the liquidity traders. The influence of their risk aversion is completely new to the model.

The utility of the uninformed investors depends on the expected trading profits, but also on a risk discount because they are assumed to be risk averse. Due to their exponential utility combined with the normality of all random variables, the utility grows monotonically with their certainty equivalent. The risk of their position may in- or decrease with the informativeness of the report, depending on the trading motivation. This may reinforce the utility stemming from the trading profits or work in the opposite direction. The utility, in contrast to chapter 3, is also influenced by the variance $\sigma_w^2$ and the number $M$ of uninformed investors directly via their own trading strategy given by $\gamma$, and of course by their risk aversion $\rho$.

**Comparison of the two different liquidity trading motivations**

The equilibrium outcome critically depends on the specific motivation for liquidity trading: In the setting of section 4.2, each of the $M$ rational liquidity traders adds an idiosyncratic value to the 'objective' terminal firm value. This reflects their idiosyncratic opportunity cost of failing to trade at a given point in time, i.e. their personal liquidity preference. Thus, if a share is worth $v$, the uninformed trader $j$, for $j = 1, \ldots, M$, attributes the value $v + w_j$ to a share. In effect, liquidity traders
receive a positive expected utility; thus, they want to exchange shares with the informed investor. In section 4.3, uninformed investor $j$ faces a random endowment shock $w_j$, for $j = 1, \ldots, M$. Due to their risk aversion, they hedge this risky position by selling a portion $\gamma$ of this endowment. This leads them to exchange share with the informed investor, although he possesses superior information and is going to benefit from the trade on their costs.

These differing motivations determine the differences in the equilibrium set, their demands $\gamma$, their trading profits, and their expected utility. The two equilibrium sets, i.e. the combinations of parameters for which an equilibrium exists, are given by the restriction

$$\frac{M\sigma_w^2}{\text{Var}[\tilde{v}|r]} > 1 \quad (4.16)$$

in section 4.2 and

$$\frac{4}{\rho^2} \frac{1}{M\sigma_w^2 \text{Var}[\tilde{v}|r]} \leq 1 \quad (4.32)$$

in section 4.3. The market breaks down if there is not enough demand from the uninformed investors. Their trading increases the liquidity in the market and thus enlarges the equilibrium set. In section 4.2, the uninformed investors trade in spite of the riskiness of the shares, while they trade in order to offset the risk in section 4.3. This explains the influence of $\text{Var}[\tilde{v}|r]$ on the equilibrium set in both cases, because the informativeness of the report also determines the variance in the equilibrium price (see (4.28)). A higher cost $c$ of biasing for the manager, increases the equilibrium set in section 4.2 while it is reduced in section 4.3. Another difference pertains to the influence of the risk aversion coefficient $\rho$: under hedging motivations, more risk aversion increases the equilibrium set, while it is without influence in the case of idiosyncratic liquidity preferences.

These considerations also help to explain the influence of $c$, $\tau_v$, and $\tau_u$ on the expected trading profit of the informed and the expected trading loss of the uninformed investors. In section 4.2, the liquidity traders demand more if the shares are less risky, thus increasing their trading loss. In section 4.3, they are more comfortable with their endowment for higher $c$, $\tau_v$, and $\tau_u$, thus reducing their losses in trades with
the informed investor who makes use of his superior knowledge.

The most interesting results, however, pertain to the utility of the uninformed investors in the two scenarios. In section 4.2, the utility simply increases with their trading coefficient $\gamma$ and their idiosyncratic value $w_j$ of a share. The level of $\gamma$ was interpreted as the combined influence of a liquidity effect and a risk discount. A more informative report both increases liquidity and decreases the risk discount. In section 4.3, the situation is a bit more involved because their endowment $w_j$ also influences the riskiness $\text{Var}[S_j|w_j]$ of their trading profits. As explained in section 4.3 it follows that their utility actually decreases when the marginal costs $c$ of biasing for the manager are higher. Note that the effect on liquidity traders of a decision by a regulator to increase the punishment for earnings management therefore depend on the investors’ motivation to trade in the market!

**Proof of Propositions 4.2.1 and 4.3.1**

This section first looks at the idiosyncratic utility case. The hedging case is solved afterwards; it uses many of the interim solutions.

**Proof of proposition 4.2.1**

Similarly to the proof of Proposition 3.3.1, the derivation of an equilibrium proceeds in three steps. Firstly, the utility of the manager in (4.2), the expected trading profit of the informed trader in (4.3), and the utility of the $M$ uninformed investors in (4.4) are maximized, and the efficient price condition in (4.9) evaluated. Secondly, a comparison of the resulting choices with the conjectured strategies in (4.10) yields a system of equations. Thirdly, this system is solved for the equilibrium coefficients.
Step 1. The manager maximizes his utility function given by (3.5) with respect to the bias $b$. The utility is

$$u\hat{P} - \frac{1}{2}cb^2 = u(\delta_r r + \delta_Q Q) - \frac{1}{2}cb^2 $$

(4.10.3)

$$= u(\delta_r (v + b) + \delta_Q (\beta_r (v + b) + \beta_b b + \gamma \sum_j w_j)) - \frac{1}{2}cb^2 $$

(4.1), (4.8)

First and second order conditions are

$$u\delta_r + u\delta_Q \beta_r + u\delta_Q \beta_b - cb = 0$$

(4.43)

and $c > 0$. The first order condition yields

$$b = \frac{u}{c}(\delta_r + \delta_Q (\beta_r + \beta_b)).$$

(4.44)

The second order condition for a maximum holds by definition.

The expected profit of the informed trader is

$$E[(\tilde{v} - \hat{P}(x))x|r,b] = \left(E[\tilde{v}|r,b] - \delta_r r - \delta_Q \left(x + \gamma \sum_j E[\tilde{w}_j|r,b]\right)\right) x$$

First and second order conditions for a maximal profit with respect to $x$ are

$$r - b - \delta_r r - 2\delta_Q x = 0$$

(4.45)

and $\delta_Q > 0$. The first order condition yields

$$x = \frac{1}{2\delta_Q} (r - b - \delta_r r).$$

(4.46)

The market maker sets the price according to (4.9). He observes the report (4.1) and net demand (4.8). The observation of

$$(\tilde{v} + \alpha\tilde{u} = r, \tilde{x} + \sum_j \tilde{y}_j = Q)$$
is informationally equivalent (using (4.10)) to an observation of
\[
\left( \tilde{v} + \alpha \tilde{u} = r, \quad \tilde{v} - \frac{1}{\beta_b} \gamma \sum_j \tilde{w}_j = -\frac{1}{\beta_b} Q + \frac{\beta_r + \beta_b}{\beta_b} r \right).
\]
Thus, the price (4.9) is
\[
P = E[\tilde{v} | \tilde{v} + \alpha \tilde{u} = r, x + \sum_j \tilde{y}_j = Q].
\]
Lemma 2.4.1
\[
E[\tilde{v} | \tilde{v} + \alpha \tilde{u} = r, \tilde{v} - \frac{1}{\beta_b} \gamma \sum_j \tilde{w}_j = -\frac{1}{\beta_b} Q + \frac{\beta_r + \beta_b}{\beta_b} r] = \frac{\alpha^{-2} \tau_u r + \beta^2}{\alpha^{-2} \tau_u + \frac{\beta^2}{M \gamma^2 \sigma^2}}. \tag{4.47}
\]

The uninformed trader \( j \) has a utility of \( v + w_j \) for a share worth \( v \) and orders \( y_j \) shares. His utility depends on
\[
\tilde{S}_j^1(y_j) = y_j(\tilde{v} + \tilde{w}_j - \tilde{P}(y_j)) \tag{4.6}
\]
\[
y_j \left( \tilde{v} + \tilde{w}_j - \delta_r(\tilde{v} + \alpha \tilde{u}) - \delta_Q \left( \beta_r(\tilde{v} + \alpha \tilde{u}) + \beta_b \alpha \tilde{u} + y_j + \sum_{k \neq j} \gamma \tilde{w}_k \right) \right) \tag{4.10}
\]
\[
y_j \left( (1 - \delta_r - \delta_Q \beta_r) \tilde{v} + \tilde{w}_j + (-\delta_r - \delta_Q \beta_r - \delta_Q \beta_b) \alpha \tilde{u} \right.
\]
\[
- \delta_Q y_j - \delta_Q \sum_{k \neq j} \gamma \tilde{w}_k \right). \tag{4.48}
\]
Due to their negative exponential utility functions and since the payoff \( \tilde{S}_j(y_j) \) is normally distributed, the expected utility is maximized for a demand of \( y_j^* \) which maximizes the investor’s security equivalent
\[
CE_j = E[\tilde{S}_j^1 | w_j] - \frac{1}{2} \rho \text{Var}[\tilde{S}_j^1 | w_j]. \tag{4.48}
\]
\[
= y_j(w_j - \delta_Q y_j) - \frac{1}{2} \rho y_j^2 B,
\]
whereby \( B = Var[\hat{s}_1 | w_j] \). Note that \( B \) does not depend on \( y_j \), since the remaining \( y_j \) does not affect the variance as it is constant. The first order condition with respect to \( y_j \) is

\[
w_j - 2\delta_Q y_j - \rho y_j B = 0
\]

\[
\iff y_j = \frac{1}{2\delta_Q + \rho B} w_j.
\]

The second order condition demands

\[
-2\delta_Q - \rho B < 0.
\] (4.50)

**Step 2.** Comparing the coefficients of (4.44), (4.46), (4.47), and (4.49) with their conjectures in (4.10) yields the system of equations

\[
c\alpha = \delta_r + \delta_Q (\beta_r + \beta_b) \tag{4.51.1}
\]

\[
\beta_r = \frac{1}{2\delta_Q} (1 - \delta_r) \tag{4.51.2}
\]

\[
\beta_b = -\frac{1}{2\delta_Q} \tag{4.51.3}
\]

\[
\gamma = \frac{1}{2\delta_Q + \rho B} \tag{4.51.4}
\]

\[
\delta_r = \frac{\alpha^{-2} \tau_u + \beta_b}{\tau_v + \alpha^{-2} \tau_u + \frac{\beta_b^2}{M^2 \sigma_w^2}} (\beta_r + \beta_b) \tag{4.51.5}
\]

\[
\delta_Q = \frac{-\beta_b}{\tau_v + \alpha^{-2} \tau_u + \frac{\beta_b^2}{M^2 \sigma_w^2}} \tag{4.51.6}
\]

\[
B = Var[\hat{v} + \hat{w}_j - \hat{P}(y_j) | w_j] \tag{4.51.7}
\]

under the second order conditions

\[
\delta_Q > 0 \quad \text{and} \quad -2\delta_Q - \rho B < 0. \tag{4.52}
\]

\[
-2\delta_Q - \rho B < 0. \tag{4.53}
\]
Step 3. First, note that $\delta Q$ in (4.51.6) is

$$
\begin{align*}
\delta Q &= \frac{1}{\tau_v + \alpha^{-2}\tau_u + \frac{1}{2\delta_Q M\gamma^2\sigma_w^2}} \\
\Leftrightarrow \delta Q(\tau_v + \alpha^{-2}\tau_u) + \frac{1}{2\delta_Q M\gamma^2\sigma_w^2} &= \frac{1}{2\delta_Q M\gamma^2\sigma_w^2} \\
\Leftrightarrow \delta_Q^2(\tau_v + \alpha^{-2}\tau_u) &= \frac{1}{4M\gamma^2\sigma_w^2} \\
\Leftrightarrow \delta_Q^2 &= \frac{1}{\tau_v + \alpha^{-2}\tau_u} \frac{1}{4M\gamma^2\sigma_w^2}.
\end{align*}
$$

\(\delta Q\) must be the positive root due to the second order condition (4.52).

The denominator of (4.51.5) and (4.51.6) is

$$
\begin{align*}
\tau_v + \alpha^{-2}\tau_u + \frac{\beta_b^2}{M\gamma^2\sigma_w^2} &= \tau_v + \alpha^{-2}\tau_u + \frac{1}{4\delta_Q^2 M\gamma^2\sigma_w^2} \\
\begin{equation}
= \tau_v + \alpha^{-2}\tau_u + \frac{1}{4M\gamma^2\sigma_w^2} \frac{\alpha^2\tau_v + \tau_u}{\alpha^2}.
\end{equation}
\end{align*}
$$

So far, $\delta Q$ and $\beta_b$ have been shown to depend only on $\alpha$. Now, $\delta_r$ and $\beta_r$ are considered. $\delta_r$ in (4.51.5) is

$$
\begin{align*}
\delta_r &= \frac{\alpha^{-2}\tau_u + \frac{\beta_b}{M\gamma^2\sigma_w^2}(\beta_r + \beta_b)}{2\alpha^2\tau_v + \tau_u} \\
\begin{equation}
= \frac{\alpha^{-2}\tau_u + \frac{1}{2\delta_Q M\gamma^2\sigma_w^2} \left( \frac{1}{2\delta_Q} (1 - \delta_r) - \frac{1}{2\delta_Q} \right)}{2\alpha^2\tau_v + \tau_u}.
\end{equation}
\end{align*}
$$

$$
\begin{align*}
\Leftrightarrow 2\frac{\alpha^2\tau_v + \tau_u}{\alpha^2} \delta_r &= \alpha^{-2}\tau_u + \frac{1}{2\delta_Q M\gamma^2\sigma_w^2} \left( \frac{1}{2\delta_Q} \delta_r \right) \\
\Leftrightarrow 2\frac{\alpha^2\tau_v + \tau_u}{\alpha^2} \delta_r &= \alpha^{-2}\tau_u + \frac{\alpha^2\tau_v + \tau_u}{\alpha^2} \delta_r \\
\Leftrightarrow \delta_r &= \frac{\tau_u}{\tau_v + \alpha^{-2}\tau_u}.
\end{align*}
$$

This directly yields

$$
\beta_r = \frac{1}{2\delta_Q} \frac{\alpha^2\tau_v}{\alpha^2\tau_v + \tau_u}.
$$

(4.57)
The variance term $B$ can now be evaluated as

$$
B = \text{Var}[\hat{S}_j/y_j|w_j]
$$

(4.48)

$$
= (1 - \delta_r - \delta_Q \beta_r)^2 \frac{1}{\tau_v} + (\delta_r + \delta_Q \beta_r + \delta_Q \beta_b)^2 \frac{\alpha^2}{\tau_u} + \delta_Q^2 (M - 1) \gamma^2 \sigma_w^2
$$

(4.54), (4.56),

(4.51.3), (4.57)

$$
= \left( \frac{\alpha^2 \tau_v + \tau_u - \tau_u - \frac{1}{2} \alpha^2 \tau_v}{\alpha^2 \tau_v + \tau_u} \right)^2 \frac{1}{\tau_v} + \left( \frac{\tau_u + \frac{1}{2} \alpha^2 \tau_v - \frac{1}{2} \alpha^2 \tau_v - \frac{1}{2} \tau_u}{\alpha^2 \tau_v + \tau_u} \right)^2 \frac{\alpha^2}{\tau_u}
$$

$$
+ \frac{1}{4} \frac{\alpha^2}{\alpha^2 \tau_v + \tau_u} \frac{M - 1}{M}
$$

$$
= \left( \frac{\frac{1}{2} \alpha^2 \tau_v}{\alpha^2 \tau_v + \tau_u} \right)^2 \frac{1}{\tau_v} + \left( \frac{\frac{1}{2} \tau_u}{\alpha^2 \tau_v + \tau_u} \right)^2 \frac{\alpha^2}{\tau_u} + \frac{1}{4} \frac{\alpha^2}{\alpha^2 \tau_v + \tau_u} \frac{M - 1}{M}
$$

$$
= \frac{1}{4} \frac{\alpha^2}{\alpha^2 \tau_v + \tau_u} + \frac{1}{4} \frac{\alpha^2}{\alpha^2 \tau_v + \tau_u} \frac{M - 1}{M}
$$

$$
= \frac{1}{4} \frac{\alpha^2}{\alpha^2 \tau_v + \tau_u} \frac{2M - 1}{M}.
$$

(4.58)

This yields

$$
(4.51.4)
$$

$$
\gamma = \frac{1}{2 \delta_Q + \rho B}
$$

(4.54)

$$
= \frac{1}{2 \sqrt{\tau_v + \alpha^2 \tau_u} \sqrt{M \gamma^2 \sigma_w^2}} + \rho B
$$

$$
\gamma > 0 !
$$

$$
\iff \rho B \gamma = 1 - \frac{1}{\sqrt{\tau_v + \alpha^2 \tau_u} \sqrt{M \sigma_w^2}} \frac{1}{\sqrt{\alpha^2 \tau_v + \tau_u} \sqrt{M \sigma_w^2}}
$$

(4.58)

$$
\iff \gamma = \frac{4 \alpha^2 \tau_v + \tau_u}{\rho \alpha^2} \frac{M}{2M - 1} \left( 1 - \frac{1}{\sqrt{\tau_v + \alpha^2 \tau_u} \sqrt{M \sigma_w^2}} \frac{1}{\sqrt{\alpha^2 \tau_v + \tau_u} \sqrt{M \sigma_w^2}} \right)
$$

$$
= \frac{4 \alpha^2 \tau_v + \tau_u}{\rho \alpha^2} \frac{M}{2M - 1} \left( \frac{1}{\sqrt{\alpha^2 \tau_v + \tau_u} \sqrt{M \sigma_w^2}} \right)
$$

(4.59)
The parameter $\alpha$ is determined by

$$c\alpha = \delta_r + \delta_Q (\beta_r + \beta_b)$$

$$(4.51.1), (4.51.5), (4.51.6), (4.55)$$

$$\frac{1}{2} \tau_u + \alpha^2 \tau_v.$$ 

Thus, the solutions for $\alpha$ are given by the roots of the polynomial

$$2c\alpha^3 \tau_v + 2c\alpha \tau_u - \tau_u = 0. \quad (4.60)$$

This polynomial in $\alpha$ is identical to (3.41) for $N = 1$. It has one and only root, since the left hand side is monotonically increasing in $\alpha$, and is negative for $\alpha = 0$ and positive for $\alpha = \frac{1}{2c}$.

Now consider conditions (4.52) and (4.53). The second order condition $-2\delta_Q - \rho B < 0$ is always fulfilled for $\delta_Q > 0$. For $\gamma > 0$ in (4.59), however, it must hold that

$$\frac{\alpha^2 \tau_v + \tau_u}{\alpha^2} > \sqrt{\frac{\alpha^2 \tau_v + \tau_u}{\alpha^2}} \sqrt{\frac{1}{M^2 \sigma_w^2}}$$

$$\iff \sqrt{\tau_v + \alpha^{-2} \tau_u} > \sqrt{\frac{1}{M^2 \sigma_w^2}}$$

$$\iff \tau_v + \alpha^{-2} \tau_u > \frac{1}{M^2 \sigma_w^2}. \quad (4.61)$$

The coefficients $\beta_r, \beta_b, \delta_r, \delta_Q$, and $\gamma$ depend uniquely on $\alpha$, so that the overall equilibrium exists and is unique if condition (4.61) holds.

**Proof of proposition 4.3.1**

*Step 1.* Since the conjectured system (4.10) is identical in the two scenarios, the derivations for the manager, the informed investor, and the market maker are as in proposition 4.2.1.
Uninformed investor $j$ has an endowment of $w_j$ shares and his profit is

\[
\tilde{S}_j(y_j) = \tilde{v}(y_j + \tilde{w}_j) - y_j \tilde{P}(y_j)
\]

Due to the negative exponential utility and since the payoff $\tilde{S}_j(y_j)$ is normally distributed, the expected utility is maximized for a demand $y_j$ which maximizes the investor’s security equivalent

\[
CE_j = E[\tilde{S}_j|w_j] - \frac{1}{2} \rho Var[\tilde{S}_j|w_j]
\]

\[
= -\delta_Q y_j^2 - 0.5\rho \left( (y_j(1 - \delta_r - \delta_Q \beta_r) + w_j) \frac{1}{\tau_v} + y_j^2 \frac{2}{\tau_u} + y_j^2 \frac{2}{\tau_u} + y_j^2 \frac{2}{\tau_u} + \frac{2}{\tau_u} \right)
\]

\[
= -\delta_Q y_j^2 - \frac{\rho}{2\tau_v} (2y_j(1 - \delta_r - \delta_Q \beta_r)w_j + w_j^2) - 0.5\rho y_j^2 B
\]

whereby $B$ is calculated in (4.58). The first order condition with respect to $y_j$ is

\[-2\delta_Q y_j - \frac{\rho}{\tau_v} (1 - \delta_r - \delta_Q \beta_r)w_j - \rho B y_j = 0.\]  

(4.63)

The second order condition demands $-2\delta_Q - \rho B < 0$. Solving the first order condition for $y_j$ yields

\[y_j = -\frac{\rho}{2\tau_v} (1 - \delta_r - \delta_Q \beta_r)w_j.\]  

(4.64)

**Step 2.** All coefficients in (4.51) are still valid except for (4.51.4) which is replaced by

\[\gamma = \frac{\rho}{2\tau_v} (1 - \delta_r - \delta_Q \beta_r).\]  

(4.65)

**Step 3.** Insertion of $\delta_Q$ in (4.54) into (4.65) yields

\[\gamma \left( \frac{1}{\gamma} \sqrt{\frac{1}{\tau_v + \alpha^2\tau_u} \frac{1}{M\sigma_w^2} + \rho B} \right) = \frac{1}{\tau_v} (1 - \delta_r - \delta_Q \beta_r).\]
Solving for $\gamma$ yields

$$
\gamma = \frac{1}{\rho B} \left( \frac{1}{\tau_v} (1 - \delta_r - \delta_Q \beta_r) - \sqrt{\frac{1}{\tau_v + \alpha^{-2} \tau_u M\sigma_w^2}} \right)
$$

(4.58)

$$
= \frac{4M}{2M - 1} (\tau_v + \alpha^{-2} \tau_u) \left( \frac{1}{\tau_v} \frac{\tau_v}{2(\tau_v + \alpha^{-2} \tau_u)} - \frac{1}{\rho} \sqrt{\frac{1}{\tau_v + \alpha^{-2} \tau_u M\sigma_w^2}} \right)
$$

$$
= \frac{2M}{2M - 1} \left( 1 - \frac{2}{\rho} \sqrt{\frac{\tau_v + \alpha^{-2} \tau_u}{M\sigma_w^2}} \right)
$$

For $\gamma$ to be positive, it must hold that

$$
\frac{2}{\rho} \sqrt{\frac{\tau_v + \alpha^{-2} \tau_u}{M\sigma_w^2}} < 1
$$

$$
\iff \tau_v + \alpha^{-2} \tau_u < \frac{\rho^2}{4} M\sigma_w^2.
$$

All calculations for $\delta_r$, $\delta_Q$, $\beta_b$, $\beta_r$, and $\alpha$ in step 3 of proposition 4.2.1 do not use the formula for $\gamma$ in (4.51.4). Therefore, the results hold identically in proposition 4.3.1. □
Chapter 5

Disclosure to a Selected Group of Investors

"It is reportedly a common practice for corporate officials to discuss the future outlook of their companies and provide guidance on earnings forecast to selected groups of analysts and large shareholders like institutional investors through meetings, conference calls, and phone conversations in advance of the announcement to the public."\(^1\)

So far, it was assumed that the market maker observes a biased report while informed investors learn additional information about its bias. This chapter introduces a variant of the reporting game in which only the informed investors learn the report, yet without the bias; in addition, they interpret the report diversely. The market maker does not observe the report.

There are two alternative interpretations for the setting. Firstly, it fits to a situation where the financial statements are not read by the market maker and the single realized earnings number is of no use to him. Secondly, the model may represent an analyst conference in which the report is disclosed only to a selected group of investors (see the citation above). In the USA, the selected disclosure of information

\(^1\)See SEC (2000).
was recently restricted by Regulation Fair Disclosure.

Section 5.1 presents the current setting. The distribution of information is similar to Admati and Pfleiderer (1988b). The outcome is then analyzed in section 5.2. Some of the results change significantly in comparison with chapter 3. For example, now the informed investors benefit from higher marginal costs of biasing for the manager, since their signal is more precise. This has adverse consequences for market liquidity and the expected trading profits of the group of liquidity traders.

The discussion in 5.3 compares the results with the setting in chapter 3. Furthermore, it offers intuition for the diverse interpretation of the report by the informed investors and its effect on the equilibrium outcome. Finally, it discusses an application of the setting to public reports that are difficult to understand, to analyst conferences, and to analyst forecasts.

### 5.1 Economic Setting

The sequence of events is depicted in time line 5.1.

In comparison to chapter 3, there are only a few differences:

Firstly, the interest \( \hat{u} \) of the manager in the share price may now - while still normally distributed with precision \( \tau_u \) - have a mean value \( \mu_u \) different from zero. The manager still maximizes the utility function

\[
uP(b) - \frac{1}{2} cb^2 \quad (5.1)
\]

by choosing an optimal bias \( b \).

Secondly, the private signal of the informed investors takes the form of

\[
 r_i = v + b + e_i \quad \text{for} \quad i = 1, \ldots, N, \quad (5.2)
\]

---

\(^2\)Admati and Pfleiderer (1988b) develop a theory of intra-day trading of securities in which concentrated-trading patterns arise endogenously as a result of the strategic behavior of liquidity traders and informed traders. In an extension of Kyle (1985) to multiple periods and endogenous information acquisition, they can explain U-shaped intra-day patterns of volume and price changes. As in Kim and Verrecchia (1994), discretionary liquidity traders choose the time when they trade an exogenously given random amount.
whereby $e_i$ is the realization of a normally distributed random variable with mean zero and precision $\tau_e$ for all $i$. The investor specific error terms $e_i$, for $i = 1, \ldots, N$, are mutually independent. The first part of the signal $v + b$ represents a firm report which is subject to diverse interpretation. The investor’s private signal $r_i$ is then a noisy information about the report.\(^3\) A higher precision $\tau_e$ means that the informed investors are more sophisticated in interpreting the report.

There is evidence in the accounting literature which suggests that accounting earnings are interpreted heterogeneously. Holthausen and Verrecchia (1990) emphasize that it is not necessary that each investor perceives the same figure of earnings per share, but that investors reach varying conclusions about the revised value of the firm after an earnings announcement. Typically, one has to make use of other information to figure out the impact of such news on the firm’s value. Thus, traders with different background information might draw different conclusions from the same announcement.\(^4\)

Evidence on the divergence of analysts’ forecasts in Morse, Stephan and Stice

\(^3\)In Indjejikian (1991) and Bushman (1991), investors learn a similar signal $r_i = v + u + e_i$, yet in a competitive setting (see section 1.4). However, the manager chooses (and publicly announces) only the precision of the public report $v + u$ to be issued by his firm. Investors then choose the precision of their private signal with respect to the public report given the precision of the public report and a convex cost function depending on the precision of the private signal.

\(^4\)If the true firm value is unknown and investors differ in the interpretation of new information, then the statement that a firm is valued fairly or that it is over- or undervalued, becomes problematic.
(1991) is consistent with the notion that investors' assessments of the implications of earnings releases for the value of the firm are heterogeneous.

The $N$ informed investors identify their optimal demand by maximizing their trading profit

$$\Theta_i = E[(\tilde{v} - \tilde{P}(x_i))x_i | r_i] \quad \text{for } i = 1, \ldots, N$$

(5.3)

with respect to $x_i$, for $i = 1, \ldots, N$. The demand of the uninformed investors is now again exogenously given by the realization of a normal random variable $\tilde{y}$, which has a mean of zero and variance $\sigma_y^2$. The net demand is then

$$Q = \sum_i x_i + y.$$  

(5.4)

Thirdly, the market maker does not observe the report $v + b$. He sets the equilibrium price efficiently at the expected terminal value given only the net demand as

$$P = E[\tilde{v} | Q].$$

(5.5)

The $N + 3$ random variables $\tilde{u}, \tilde{v}, \tilde{y},$ and $\tilde{e}_1, \ldots, \tilde{e}_N$ are assumed to be independently distributed. All parameters of the distributions are common knowledge.

The linear equilibrium conjectures are now given by

$$\hat{b} = \alpha u$$

(5.6.1)

$$\hat{x}_i = \beta_0 + \beta_r r_i \quad \text{for } i = 1, \ldots, N$$

(5.6.2)

$$\hat{P} = \delta_0 + \delta_Q Q.$$  

(5.6.3)

Due to a mean value $\mu_u$ (possibly) different to zero, the conjectures about $x_i$ and $P$ now include the constant terms $\beta_0$ and $\delta_0$. As the derivation of the equilibrium in section 5.3 shows, the bias $b$ chosen by the manager contains neither a constant term nor does it depend on the realized firm value $v$. It will also turn out that $\delta_0$ is zero.

In the competitive rational expectations models of trading in section 1.4, the informed investors learned about other investors’ private information by observing

---

An asset may be undervalued from an optimistic point of view, while at the same time, it may be overvalued from a pessimistic point of view.

As in chapter 4, their demand could be made endogenous.
the price. In the Kyle (1985) setting, the informed investors do not observe the price before trading. Conversely, they draw conclusions about the private signals of the other informed investors from their own signal and how these other signals affect the price applying the above conjectures. Since there is no correlation between the error terms in the private signals, the conjecture about the other private signals is limited to the common part $v + b$.

5.2 The Equilibrium

**Proposition 5.2.1.** There is one and only one linear equilibrium for the setting described in section 5.1.

The equilibrium bias $b$, the demands $x_i$, for $i = 1, \ldots, N$, and the price $P$ depend uniquely on $\alpha$ and are given by

\begin{align}
    b &= \alpha u \\
    x_i &= \beta (r_i - \alpha \mu_u) \quad \text{for} \quad i = 1, \ldots, N \\
    P &= \frac{c\alpha}{N\beta} Q,
\end{align}

whereby

\begin{align}
    \beta &= \sqrt{\frac{\tau_r \sigma^2}{N}} \\
    \tau_r &= \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} + \frac{1}{\tau_e} \right)^{-1}.
\end{align}

The coefficient $\alpha$ of $u$ in the manager’s strategy is the unique root of the third-order polynomial

\begin{align}
    c\alpha &= \frac{N}{1} \left( \frac{N+1}{\tau_e} \right)^{-1}.
\end{align}

**Proof.** See the appendix.

The manager again trades off marginal benefits with the marginal costs of biasing. The marginal benefits correspond with the coefficient of the bias in the equilibrium
price. This coefficient increases if the informativeness of the aggregated private signals relative to the prior knowledge about the firm value is higher. Informed investors correct the expected bias, but cannot correct the unexpected bias. Thus, the latter finds its way directly into the price.

In contrast to chapter 3, the market maker now has to rely completely on the net demand $Q$. The net demand $Q$ aggregates the private signals of the informed investors; thereby, the individual errors $e_i$ tend to average out. However, the liquidity trading reduces the net demand’s informativeness. The market maker relies on the net demand the more, the higher its precision is relative to his prior information about the firm value. The marginal costs $c$ of biasing lead to a more precise net demand and equilibrium price. However, they also decrease the market depth and consequently harm the liquidity traders.

The remainder of this section calculates

- the bias and precision of the report,
- the demand and trading volume,
- the informativeness of the price,
- the welfare of the informed and uninformed investors, and
- the option of biasing for the manager.

**The Bias and Precision of the Report**

The bias $b$ in the report $r$ is given by a coefficient $\alpha$ times the interest $u$ of the manager in market price. The choice of the bias $b = \alpha u$ indirectly influences the precision $\tau_r$ of the private signals in (5.9). The effect of a change in the parameters on the bias $b$ and the precision $\tau_r$ is stated in

**Corollary 5.2.2.** The bias $b = \alpha u$ increases with the interest $u$ of the manager in the firm value. The marginal influence of the parameters on the coefficient $\alpha$ and the precision $\tau_r$ of the public report is shown in table 5.1.
An explanation for the observed effects rests on a comparison between the marginal costs and the benefits of biasing. The defining equation for $\alpha$ in (5.10) can be written as

$$c\alpha = \frac{N}{N + 1} \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} + \frac{2}{N+1} \frac{1}{\tau_e} \right)^{-1}.$$  

(5.11)

The marginal costs on the left hand side increase with $c$, which again reduces the bias; a corresponding countervailing effect which is due to a higher precision of the private signals is offset. A higher prior precision $\tau_v$ makes the market maker rely less on the informational content of the net demand, so that the marginal benefits decrease directly (a countervailing, indirect effect is subordinated). The term in the numerator reflects the informativeness of the net demand. Due to the aggregation of the individual errors $e_i$, the precision is higher than that of a single private signal; however, there is additional noise due to liquidity trading. The variance $\sigma_y^2$ of liquidity trading has no influence on the bias because the informed traders exactly offset its influence.

In contrast to chapter 3, the number $N$ of informed investors increases the coefficient $\alpha$. More informed investors make the net demand more informative for the market maker (since individual errors can be filtered out more effectively). The increased precision of the net demand in turn increases the marginal benefits of biasing because the market maker relies more on the net demand for the valuation of the shares. Therefore, the precision $\tau_r$ of the private signals indirectly decreases with $N$.

It follows from the discussion on page 150 in the appendix that, for a positive realization of $\tilde{u}$, the bias is bound by

$$0 < b < \frac{1}{c \frac{N}{N+1} \tilde{u}}.$$  

(5.12)
If the marginal costs $c$ of biasing increase, the bias converges to zero. The same is no longer true for the number $N$ of informed traders, however.

**Demand and Trading Volume**

The informed traders correct their private signal for the expected bias and trade proportionally to this unbiased signal of firm value as in

$$x_i = \beta(v + \alpha(u - \mu_u) + e_i) \quad \text{for} \quad i = 1, \ldots, N.$$  \hspace{1cm} (5.13)

They trade more aggressively on this corrected private signal when the coefficient $\beta = \sqrt{\frac{\tau_r \sigma_y^2}{N}}$ increases. The behavior of $\beta$ is stated in

**Corollary 5.2.3.** The marginal influence of the parameters on the trading aggressiveness $\beta$ is shown in table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$\tau_e$</th>
<th>$\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

*Table 5.2: The Marginal Effect of the Parameters on the Trading Aggressiveness $\beta$.*

The influence of the parameters is similar to chapter 3, but the trading aggressiveness now increases directly with the precision $\tau_r$ of the informed investor’s private signal. This explains the influence of $c$, $\tau_v$, $\tau_u$, and $\tau_e$ (see table 5.1). As in chapter 3, the aggressiveness decreases with the number $N$ and increases with the variance $\sigma_y^2$ of liquidity trading.
The expected trading volume $V$, as defined in section 2.2, turns out to be

$$E[\hat{V}] = \frac{1}{\sqrt{2\pi}} \left( \sum_i \sqrt{\text{Var}(x_i)} + \sqrt{\sigma_y^2} + \sqrt{\text{Var}(\sum_i \hat{x}_i + \hat{y})} \right)$$

The marginal influence of the parameters on the expected trading volume $E[\hat{V}]$ is shown in table 5.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$\tau_e$</th>
<th>$\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\hat{V}]$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 5.3: The Marginal Effect of the Parameters on the Expected Trading Volume $E[\hat{V}]$.

The expected trading volume clearly increases with $N$ and $\sigma_y^2$. The final term in (5.14) arises due to the idiosyncratic error terms $\tilde{e}_i$. For $N = 1$, this term vanishes since then, there is no diverse information. For $N > 1$, the fraction represents the precision of the aggregation of private signals (nominator) relative to one private signal (denominator). However, this effect is limited: the fraction remains smaller than 1 and tends to 1 for $\tau_e \to \infty$.

**Price Informativeness**

The realized equilibrium price is a weighted sum of the realized firm value $v$, the unexpected interest $(u - \mu_u)$ of the manager in the share price, and the liquidity
demand \( y \). It is

\[
P = \alpha \left( v + \alpha(u - \mu_u) + \frac{1}{N} \sum_i e_i + \frac{1}{\sqrt{N\tau_y\sigma_y^2}} y \right) .
\] (5.15)

The coefficients of terminal value and unexpected bias are equal because neither the informed investors nor the market maker can differentiate between them. The unexpected bias is not corrected for. However, if the number \( N \) of informed investors grows to infinity, the noise stemming from the investor-specific errors \( e_i \) and liquidity trading \( y \) vanish.

The informativeness \( \Psi \) of the price can be measured by the inverse of the remaining variance of firm value after having observed the price. It is

\[
\begin{align*}
V ar[\tilde{v}|P] & = \frac{1}{\tau_v} - \frac{Cov^2[\tilde{v}, \tilde{P}]}{V ar[P]} \\
& = \frac{1}{\tau_v} \left( 1 - \frac{1}{\tau_v} \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} + \frac{1}{N\tau_e} + \frac{1}{N\tau_r} \right) \quad (5.16) \\
& = \frac{1 - \alpha \tau_v}{\tau_v} \quad (5.17)
\end{align*}
\]

The price efficiency is then\(^6\)

\[
\Psi = V ar^{-1}[\tilde{v}|P] = \frac{\tau_v}{1 - \alpha \tau_v}. \quad (5.18)
\]

The influence of the parameters on this efficiency is summarized in

**Corollary 5.2.5.** The marginal influence of the parameters on the informational efficiency is shown in table 5.4.

The informativeness of the price depends on a trade-off between the informativeness of the private signals and the noise stemming from liquidity trading. The price becomes

---

\(^6\)For a relative definition of price efficiency as \( \frac{V ar[\tilde{v}] - V ar[\tilde{v}|P]}{V ar[\tilde{v}]} = 1 - \tau_v \frac{1 - \alpha \tau_v}{\tau_e} = \alpha \), the direct connection between price efficiency and \( \alpha \) would become apparent.
Table 5.4: The Marginal Effect of the Parameters on the Price Efficiency $\Psi$.

<table>
<thead>
<tr>
<th>$\Psi$</th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$\tau_e$</th>
<th>$\sigma_y^2$</th>
<th>$0$</th>
</tr>
</thead>
</table>

more informative relative to one private signal because the idiosyncratic error terms tend to average out; it gets less informative relative to one private signal because there is additional noise from liquidity demand (although the variance $\sigma_y^2$ does not influence price efficiency). This can be illustrated conveniently by considering the inverse of the informativeness of one private signal

$$Var[\tilde{v}|r_i] = \frac{1}{\tau_v} \left( 1 - \frac{\tau_v}{\tau_v} \right).$$

(5.19)

Using (5.16) and (5.19), the price is now more informative than a single private signal if and only if

$$Var[\tilde{v}|P] < Var[\tilde{v}|r_i]$$

$$\iff \frac{1}{N} \left( (N + 1) \frac{1}{\tau_v} + (N + 1) \frac{\alpha^2}{\tau_u} + \frac{2}{\tau_e} \right) < \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} + \frac{1}{\tau_e}$$

$$\iff \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} - \frac{N-2}{\tau_e} < 0.$$

In general, the relative informativeness depends on all parameters. For $N = 1$ and $N = 2$, the price is always less informative than one private signal. For $N > 2$, the same is true if $\tau_e$ is large enough relative to $N$, and if the precisions of $u$ and $v$ are small (see figure 5.2).

**Welfare**

While in chapter 3, the informational advantage of the informed investors decreased with the informativeness of the report, the situation is here exactly reversed. If the private signals of the informed investors are more precise, the market maker considers the net demand relatively more in his Bayesian update, i.e. he increases the coefficient $\delta_Q$. In effect, he thus lowers the liquidity $\frac{1}{\delta_Q}$ of the market (increasing the bid-ask
Figure 5.2: Graphs of $c\alpha$ (dashed) versus $\tau_r$ for $N = 1$, $N = 3$ and $N = 10$, given $c = 1$, $\tau_v = 1$, $\tau_u = 1$, $\sigma_y^2 = 1$. 
spread). The market maker is therefore protected against the informational advantage held by the processors of public information.

The liquidity (market depth) is

\[ \frac{1}{\delta_Q} = \frac{N\beta}{c\alpha} = \sqrt{N\tau_v \sigma_y^2} \]

(5.20)

**Corollary 5.2.6.** The marginal influence of the parameters on the market depth is shown in table 5.5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Marginal Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>-</td>
</tr>
<tr>
<td>( N )</td>
<td>+</td>
</tr>
<tr>
<td>( \tau_v )</td>
<td>+</td>
</tr>
<tr>
<td>( \tau_u )</td>
<td>-</td>
</tr>
<tr>
<td>( \tau_e )</td>
<td>?</td>
</tr>
<tr>
<td>( \sigma_y^2 )</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 5.5: The Marginal Effect of the Parameters on the Market Depth \( \delta_Q^{-1} \).

The market depth directly increases with the variance \( \sigma_y^2 \) of liquidity trading, but is indeterminate as to the influence of the number \( N \) of informed investors. It also increases with the trading aggressiveness \( \beta \) of the informed investors, but decreases with \( c\alpha \). The latter can be interpreted as the informativeness of the net demand relative to the prior information \( \tau_v \) of the market maker (see the subsection on price efficiency). Ultimately, the parameters influence the market depth as shown in table 5.5.

The influence of the precision \( \tau_e \) of the idiosyncratic error terms is indeterminate. On one hand, the liquidity increases with \( \tau_e \) because the informed trade more aggressive (higher precision \( \tau_r \)). On the other hand, net demand becomes more informative for the market maker (\( c\alpha \) increases), which makes the market less liquid because the market maker uses net demand more for valuation. The combined effect depends on the constellation of the parameters.

Consider the graphs of the liquidity in figures 5.3 and 5.4. For small \( N \) the liquidity strictly decreases in the precision \( \tau_e \), i.e. the informativeness effect dominates the increased informed trading, holding the other parameters at 1. This changes, when
the number \( N \) grows. For larger \( N \) \((N > 3 \text{ suffices})\), the liquidity first decreases in \( \tau_e \), and, from a threshold value on, increases in \( \tau_e \). The more informed traders there are, the lower is this threshold value. A higher marginal cost \( c \) of biasing reduces the liquidity (see figure 5.4). This also holds for \( N \leq 3 \).
The expected trading profits of the informed and uninformed investors are controlled by the market depth. For an informed trader $i$, for $i = 1, \ldots, N$, it is

$$
\Theta = E[(\hat{v} - \hat{P})x_i]
$$

\text{(5.7.2), (5.15)}

$$
= \beta E\left[(1 - c\alpha)\hat{v} - c\alpha(\hat{u} - \mu_u) + \frac{1}{N} \sum_j \hat{e}_j\right]
$$

\text{(5.21)}

$$
= \beta \left(\frac{1 - c\alpha}{\tau_v} - \frac{c\alpha^2}{\tau_u} - \frac{1}{N} c\alpha \frac{1}{\tau_e}\right)
$$

\text{(5.8)}

$$
= \frac{c\alpha \tau_r \sigma_y^2}{N \beta} \frac{1}{N} \left(\frac{1}{c\alpha \tau_v} - \frac{1}{\tau_v} - \frac{\alpha^2}{\tau_u} - \frac{1}{N \tau_e}\right)
$$

\text{(5.10)}

$$
= \frac{\sigma_y^2}{N} \delta_Q.
$$

\text{(5.22)}
The coefficient $\delta_Q$ measures the informativeness of the net demand. Hence, it also contains the informational advantage of the informed investors. In addition, their expected trading profits increase with the variance $\sigma_y^2$ of liquidity trading, and decrease with the number $N$ of informed investors.

The zero sum condition $N\Theta + \Theta_L = 0$ immediately yields

$$\Theta_L = -N\Theta = -\delta_Q\sigma_y^2$$

for the expected profit of the group of liquidity traders. In contrast to the informed investors, a higher market depth $\frac{1}{\delta_Q}$ lowers the trading losses of the uninformed investors (except for the influence of the variance $\sigma_y^2$).

**Corollary 5.2.7.** The marginal influence of the parameters on the expected trading profit is shown in table 5.6.

<table>
<thead>
<tr>
<th>$\Theta$</th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$\tau_e$</th>
<th>$\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>$\Theta_L$</td>
<td>?</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 5.6: The Marginal Effect of the Parameters on the Expected Profit $\Theta$ and $\Theta_L$.***

**The Value of the Option to Bias**

Similarly to the argument in chapter 3, the utility of the manager corresponds with the value of his option to bias (see page 77). However, the mean $\mu_u \neq 0$ of his interest $\tilde{u}$ allows for an additional interpretation of his utility. Ex-ante, the interest $\tilde{u}$ is uncertain for the manager, while ex-post, he knows his realized interest $u$. The former corresponds to the value of the option to bias in general, i.e. independent from the manager's specific interests at a certain point in time.
The ex-ante utility of the manager is
\[ E[\hat{u}\hat{P} - \frac{1}{2} \alpha^2 \hat{u}^2] = E[\frac{\alpha}{N\beta} \hat{Q} - \frac{1}{2} \alpha^2 \hat{u}^2] \]
\[ = E[\frac{\alpha}{N\beta} \hat{u}\beta \alpha (\hat{u} - \mu_u)] - \frac{1}{2} \alpha^2 \left( \frac{1}{\tau_u} - \mu_u^2 \right) \]
\[ = \frac{1}{2} \alpha^2 \left( \frac{1}{\tau_u} - \mu_u^2 \right). \quad (5.24) \]

Similarly, the ex-post utility turns out to be
\[ E[\hat{u}\hat{P} - \frac{1}{2} \alpha^2 \hat{u}^2 | \hat{u} = u] = \frac{1}{2} \alpha^2 \left( (u - \mu_u)^2 - \mu_u^2 \right). \quad (5.25) \]

This puts the results of chapter 3 in table 3.5 into perspective, where the utilities are always positive. In the more general case of a nonzero mean \( \mu_u \), the utility of the manager is not always positive:

- **Ex-ante**, the manager’s utility is high if the capital market assumes that he has little or no incentive to bias at all \( (\mu_u \approx 0) \), and if the incentives are very uncertain for all market participants \( (\tau_u \approx 0) \). This means that there is uncertainty regarding the type of the manager, i.e. whether he wants to over- or undervalue the price.

- **Ex-post**, the manager’s utility is more likely to be positive if his realized incentives \( u \) are far from its expected value \( \mu_u \).

By writing
\[ \frac{1}{2} \alpha^2 = \frac{(5.11)}{2c} \left( \frac{N}{N + 1} \left( \frac{1}{\tau_v} + \frac{\tau_u^2}{\tau_u} + \frac{2}{N + 1} \frac{1}{\tau_u} \right)^{-1} \right)^2, \]
the effect of the parameters becomes apparent as

**Corollary 5.2.8.** The marginal influence of the parameters on the expected trading profit is shown in table 5.7.

---

7These results are identical to those of Fischer and Verrecchia (2000).
In contrast to chapter 3, the manager benefits from a higher number of informed investors. The latter serve as the device to incorporate the bias into the price. It also follows from (5.24) and (5.25) that the manager prefers a small absolute expected interest \( \mu_u \) in the share price.

### 5.3 Discussion

In the current setting, the equilibrium mechanism has changed fundamentally in comparison to chapter 3. In chapter 3, the market maker observes the report directly and extracts additional information about the report’s bias from the net demand. The liquidity in the market increases if the informativeness of the report, i.e. the remaining precision of the firm value given the report, is higher.

In the current setting, the net demand is the device to transmit the report’s information, i.e. all information enters the price through the demand of the informed investors. If their private signal, i.e. the idiosyncratically interpreted report, becomes more precise, the market maker relies more on the net demand relative to his prior knowledge for the valuation of the shares. In effect, the market depth and the profit of the liquidity traders decrease in contrast to conventional wisdom if the manager’s report is more precise. Hence, the relation between report precision and liquidity is exactly opposite to the findings of chapter 3.

Bagehot (1971) offers an intuitive explanation as to why the market maker may not want to collect private information on his own in the presence of idiosyncratic errors \( e_i \): he would, just like the informed investors, obtain only noisy information about

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8For additional reasons, confer footnote 18 of chapter 3.
the report, while the individual errors in net demand average out. Bagehot (1971, p. 14) notes that while the resulting market price is a consensus of the investors in the market place, it is also “a consensus of their mistakes like errors in computation, errors of judgement, factual oversights and errors in the logic of analysis.” Accordingly, the mistakes of investors will offset each other in net demand and have no effect on the equilibrium price, provided that the errors are not systematic across investors.\textsuperscript{9} As Bagehot (1971, p. 14) observes, it is thus not surprising that "market makers of all kinds make surprisingly little use of fundamental information. Instead they observe the relative pressure of buy and sell orders and attempt to find a price that equilibrates these pressures.”

Table 5.8 below compares the results of this chapter with chapter 3. It summarizes the marginal influences of the parameters on the bias $b$, the precision $\tau_r$ of the private signals, the aggressiveness of trading $\beta$, the expected trading volume $E[\tilde{v}]$, the price informativeness $\Psi$, the trading profits of the informed and uninformed traders $\Theta$ and $\Theta_L$, and the ex post utility of the manager $U_p$.\textsuperscript{10}

The differences between chapter 3 and 5 relate to

- the influence of the number $N$ of informed investors,
- the trading profits $\Theta$ and $\Theta_L$ of the investors, and
- the new parameter $\tau_e$ and its influence on the expected trading volume.

Concerning the informed investors, they are now the device to transmit the bias to the price. If there are more informed investors $N$, the net demand is more informative for the market maker because their idiosyncratic errors tend to average out. In this case, the manager’s marginal benefits of biasing increase and, consequently, the conjectured

\textsuperscript{9}Note that here, there are systematic and unsystematic errors; the systematic error stems from the unexpected bias which the manager adds to the report. It does not cancel out and hence finds its way into the market price.

\textsuperscript{10}Note that the precision $\tau_r$ is now the precision of the private signals, not that of a public report. Note also that the utility $U_p$ of the manager can be negative in chapter 3, depending on the parameters $\tau_u$ and $\mu_u$. The marginal influences are stated for $U_p > 0$ and vice versa for a negative utility.
Table 5.8: The Marginal Effect of the Parameters on the Equilibrium Outcome in the Settings of Chapter 5 and 3, respectively. (The symbols denote: "+" positive influence, "+" negative influence, "0" no influence, "?" indeterminate influence, "." influence cannot be calculated because parameter or characteristic is not included in the model.)

<table>
<thead>
<tr>
<th></th>
<th>$c$</th>
<th>$N$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$\tau_e$</th>
<th>$\sigma_y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>- -</td>
<td>+ -</td>
<td>- -</td>
<td>+ +</td>
<td>+ -</td>
<td>0 0</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>+ +</td>
<td>- ?</td>
<td>+ +</td>
<td>+ +</td>
<td>+ -</td>
<td>0 0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>+ +</td>
<td>- -</td>
<td>+ +</td>
<td>+ +</td>
<td>+ -</td>
<td>+ +</td>
</tr>
<tr>
<td>$E[V]$</td>
<td>- 0</td>
<td>+ +</td>
<td>- 0</td>
<td>- 0</td>
<td>+ -</td>
<td>+ +</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>+ +</td>
<td>+ +</td>
<td>+ +</td>
<td>+ +</td>
<td>+ -</td>
<td>0 0</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>+ -</td>
<td>? -</td>
<td>- -</td>
<td>+ -</td>
<td>? -</td>
<td>+ +</td>
</tr>
<tr>
<td>$U_p$</td>
<td>??</td>
<td>+ ?</td>
<td>+ ?</td>
<td>+ ?</td>
<td>+ ?</td>
<td>0 0</td>
</tr>
</tbody>
</table>

equilibrium bias is higher. This implies a higher managerial utility only if there is sufficient uncertainty about the type of the manager ex-ante ($U_a > 0$), or if his realized interest $u$ diverges by far from the expected value $\mu_u$ ($U_p > 0$).

The behavior of the trading profits now changes for $c$, $\tau_u$ and $N$. The marginal costs $c$ of biasing and the precision $\tau_u$ of the manager’s interest in the share price increase the precision $\tau_r$ of the informed investors’ private signal. Hence, $c$ and $\tau_u$ naturally increase the trading profits of the informed investors as well. The number $N$, in contrast, decreases the precision $\tau_r$. A countervailing effect is present for $\tau_v$. If the prior knowledge about the firm value is higher, the market maker does not consider the net demand to the same degree, to the detriment of the informed investors. The sign of the parameter’s marginal influence on liquidity traders is exactly opposite due to the zero-sum condition of the expected trading profits. The influence of $N$ is now indeterminate; however, if all informed investors receive the same private signal ($\tau_e \to \infty$), the behavior of $\Theta$ and $\Theta_L$ with respect to the number $N$ of informed investors is as in chapter 3.

New to the model (in comparison to chapters 3 and 4) are the investor-specific
noise terms $\tilde{e}_i$, who have equal precision $\tau_e$ and a mean of zero. The marginal influence of the noise terms’ precision $\tau_e$ on the model’s properties largely corresponds to those of $\tau_u$. It increases the precision $\tau_r$ of the private signal of the informed investors, and hence their trading aggressiveness $\beta$, the coefficient of the bias $\alpha$, the utility $U_p$ of the manager, and the price efficiency $\Psi$.

The effect of $\tau_e$ on the expected trading profits depends on the number $N$ of informed investors. For $N < 3$, the liquidity always decreases if the precision $\tau_e$ of individual errors increases. However, if there are many informed investors $N$, the liquidity increases for a higher precision $\tau_e$. In this case, the informed investors trade more aggressive, while the informativeness of the order flow for the market maker increases less strongly.

The precision $\tau_e$ also introduces a new influence of the other model parameters on the expected trading volume. Now, the trading volume increases with the precision of the aggregate of all private signals relative to a single private signal. In the benchmark model of section 2.2, the expected trading volume similarly increased with $\tau_e$ and decreased with $\tau_u$ and $\tau_v$. However, here, the precision $\tau_e$ also alters the bias in the report as well as its precision.

Fischer and Verrecchia’s (2000) model also includes an error term $\tilde{n}$, yet it represents an error in the measurement of the firm value on the side of the manager. He learns $v + n$ instead of the firm value $v$ and discloses a report $r = v + n + b$. The role of Fischer and Verrecchia’s (2000) measurement error $\tilde{n}$ is similar to the idiosyncratic errors $e_i$, for $i = 1, \ldots, N$, provided they are all equal. However, here, the precision of $\text{Var} \left[ \sum_i \tilde{e}_i \right] = N^2 \frac{1}{\tau_e}$ is lower than that of $\text{Var} [Ne] = N^2 \frac{1}{\tau_e}$ because the individual errors are uncorrelated.

This chapter’s setting may be applied to three real world situations:

1. The report continues to represent a public financial disclosure by the manager.
   In contrast to chapter 3, however, the report is now not received by the market maker, while the informed investors do now interpret the report diversely, instead of knowing the bias. The uninformedness of the market maker can be
motivated either in the sense of Bagehot (1971) as cited above, or else can the market maker’s chosen price be interpreted as the limit of a large number of uninformed investors who submit limit orders.\textsuperscript{11} Uninformed investors might not benefit from new information if substantial knowledge is prerequisite to understanding it.\textsuperscript{12} In this scenario, one therefore obtains the adverse result that increasing the costs $c$ of biasing for the manager serves only the informed investors while harming the liquidity traders.

2. In analyst conferences, information is disclosed only to a select group of investors, as in the model. One motivation for biasing not covered by the current setting is that managers may seek to provoke favorable ratings and recommendations by disclosing material information prior to the full disclosure of the same information to the general public. Analysts could then trade on this information or exchange it with large clients for brokerage business. This adds to the existing information asymmetry and prohibits a level playing field. In addition, the model shows that in this case, the manager is better able to use the analysts as a device to realize his incentives regarding the share price.\textsuperscript{13}

3. The demand of the informed investors can also be interpreted as an earnings forecast by analysts with is influenced by the disclosure policy of the manager. The market then values the shares on the basis of $Q$, i.e. the aggregated forecasts plus noise. Abarbanell and Lehavy (2003, p. 107) argue that the biases of analysts’ forecasts might be due to earnings manipulation: "Thus, it is possible that

\textsuperscript{11}See Hirth (2000, p. 102).
\textsuperscript{12}IAS 39, for example, involves the usage of option pricing models for the valuation of financial instruments.
\textsuperscript{13}The disclosure of material information to selected groups of market participants was prohibited in the USA by the controversial Regulation Fair Disclosure of August 2000. The rule went into effect on October 23rd, 2000 and requires that "when an issuer, or person acting on its behalf discloses material non-public information to certain enumerated persons (in general, securities market professionals and holders of the issuer's securities who may trade on the basis of the information), the issuers must make public disclosure of that same information simultaneously (for intentional disclosures) or promptly (for non-intentional disclosures)", see SEC (2000). Moreover, see Bushee, Matsumoto and Miller (2004) for an empirical study on the effectiveness of this regulation.
some evidence previously deemed to reflect the impact of analysts’ incentives and cognitive tendencies on forecasts is, after all, attributable to the fact that analysts do not have the motivation or ability to completely anticipate earnings management by firms in their forecasts.” They provide evidence that managers’ manipulation of earnings is associated with analysts’ stock recommendations. The model formalizes this explanation for the apparent under- and overreactions to public information.\textsuperscript{14} It is furthermore argued that analysts encourage earnings management by setting targets for the managers which are impossible to meet, except by manipulating firm performance.\textsuperscript{15} In the current setting, analysts also encourage earnings management in the sense that their expectations about the manager’s behavior forces him to manage earnings even though he may be worse off by doing so (prisoners dilemma situation).\textsuperscript{16}

\section*{Proof of Proposition 5.2.1}

Similarly to the proof of Proposition 3.3.1, the derivation of an equilibrium proceeds in three steps. Firstly, the utility of the manager in (5.1) and the expected trading profit of the informed trader in (5.3) are maximized, and the efficient price condition in (5.5) is evaluated. Secondly, a comparison of the resulting choices with the conjectured strategies in (5.6) yields a system of equations. Thirdly, this system is solved for the equilibrium coefficients.

\textsuperscript{14}On empirical studies of analyst behavior, confer Fuller and Jensen (2002), Degeorge, Patel and Zeckhauser (1999), Graham, Harvey and Rajgopal (2004).

\textsuperscript{15}Analysts have also been accused of utilizing earnings management in order to prevent loosing other types of businesses. For example, in October 2001, one week before Enron collapsed, a Goldman Sachs analyst published a recommendation on Enron starting with the phrase "Still the best of the best."

\textsuperscript{16}This is contrary to chapter 3, where the presence of informed investors always curbed earnings management.
Step 1. The manager maximizes the utility function (5.1) with respect to $b$

\[
\begin{align*}
(5.6.3) \\
u P - \frac{1}{2} c b^2 &= u(\delta_0 + \delta Q) - \frac{1}{2} c b^2 \\
(5.6.2)(5.2) \\
&= u \left( \delta_0 + \delta Q \left( N \beta_0 + N \beta_r (v + b) + \beta_r \sum_i e_i + y \right) \right) - \frac{1}{2} c b^2.
\end{align*}
\]

First and second order conditions for a maximum with respect to $b$ are

\[
u \delta Q N \beta_r - cb = 0
\]

and $c > 0$.

The first order condition yields

\[
b = \frac{u}{c} N \beta_r \delta Q.
\]  

(5.26)

Informed trader $i$ conjectures the market maker and the other investors to trade according to (5.6.3) and (5.6.2). His expected profit in (5.3) is

\[
E[(\tilde{v} - \tilde{P}(x_i))x_i | r_i] = \left( E[\tilde{v} - \delta_0 - \delta Q \left( x_i + \sum_{j \neq i} (\beta_0 + \beta_r \tilde{r}_j) + \tilde{y} \right) | r_i] \right) x_i \\
= \left( E[\tilde{v}|r_i] - \delta_0 - \delta Q \left( x_i + (N - 1) \beta_0 + \beta_r \sum_{j \neq i} E[\tilde{r}_j | r_i] \right) \right) x_i.
\]  

(5.6.3)(5.4)

The first order condition is

\[
x_i = \frac{1}{2 \delta Q} \left( E[\tilde{v}|r_i] - \delta_0 - \delta Q \left( (N - 1) \beta_0 + \beta_r \sum_{j \neq i} E[\tilde{r}_j | r_i] \right) \right)
\]

and the second order condition demands $\delta Q > 0$. Defining

\[
\tau_r = \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} + \frac{1}{\tau_e} \right)^{-1}
\]  

(5.27)

and taking the expectation conditional on $r_i$, it is\textsuperscript{17}

\[
x_i = \frac{1}{2 \delta Q} \left( \frac{\tau_r}{\tau_v} (r_i - \alpha \mu_u) - \delta_0 - \delta Q (N - 1) \left( \beta_0 + \beta_r \alpha \mu_u + \beta_r \frac{\tau_r}{\tau_v} + \alpha^2 \frac{\tau_r}{\tau_u} (r_i - \alpha \mu_u) \right) \right)
\]  

(5.28)

\textsuperscript{17}Here, lemma 2.4.2 cannot be applied because the means are not zero.
The market maker sets the price (5.5) at

\[ P = E[\tilde{v} | \tilde{Q} = Q] \]
\[ = E[\tilde{v} | N\beta_0 + \beta_r \sum_{i} \tilde{e}_i + \tilde{y} = Q] \]
\[ = E[\tilde{v} | N\beta_r(\tilde{v} + \alpha \tilde{u}) + \beta_r \sum_{i} \tilde{e}_i + \tilde{y} = Q - N\beta_0] \]
\[ = \frac{N\beta_r}{\tau_v} \frac{1}{N^2\beta_r^2 \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} \right) + \frac{N\beta_r^2}{\tau_e} + \sigma_y^2} (Q - N\beta_0 - N\beta_r\alpha \mu_u.) \quad (5.29) \]

**Step 2.** Comparing the coefficients yields the system of equations

\[ \alpha = \frac{N}{c} \beta_r \delta_Q \quad (5.30.1) \]
\[ \beta_0 = -\beta_r \alpha \mu_u - \frac{1}{2\delta_Q} (\delta_0 + (N - 1)\delta_Q \beta_0) \quad (5.30.2) \]
\[ \beta_r = \frac{1}{2\delta_Q} \left( \frac{\tau_r}{\tau_v} - \delta_Q(N - 1)\beta_r \left( \frac{\tau_r}{\tau_v} + \alpha^2 \frac{\tau_r}{\tau_u} \right) \right) \quad (5.30.3) \]
\[ \delta_0 = -\delta_Q(N\beta_0 + N\beta_r \alpha \mu_u) \quad (5.30.4) \]
\[ \delta_Q = \frac{N\beta_r}{\tau_v} \frac{1}{N^2\beta_r^2 \left( \frac{1}{\tau_v} + \frac{\alpha^2}{\tau_u} \right) + \frac{N\beta_r^2}{\tau_e} + \sigma_y^2} \quad (5.30.5) \]

under the second order condition

\[ \delta_Q > 0. \quad (5.31) \]

**Step 3.** At first, only the equations for \( \alpha, \beta_r \) and \( \delta_Q \) are considered. (5.30.3) can be solved for \( \beta_r \delta_Q \)

\[ \beta_r \delta_Q = \frac{1}{2\frac{\tau_r}{\tau_v} + (N - 1) \left( 1 + \alpha^2 \frac{\tau_r}{\tau_u} \right)}. \quad (5.32) \]

Using this result for (5.30.1), it is

\[ c\alpha = \frac{N\beta_r \delta_Q}{\tau_v} \]
\[ = \frac{N}{(N + 1) \left( 1 + \alpha^2 \frac{\tau_r}{\tau_u} \right) + 2 \frac{\tau_r}{\tau_v}}. \quad (5.33) \]

Hence, \( \alpha \) is given by the root of the polynomial

\[ c\alpha^3 \frac{\tau_v}{\tau_u} + c\alpha \left( 1 + \frac{2}{N + 1} \frac{\tau_v}{\tau_e} \right) - \frac{N}{N + 1} = 0. \quad (5.34) \]
This polynomial has one and only one solution for $\alpha$. It suffices to note that the derivative with respect to $\alpha$ is always positive, and that the left hand side is negative for $\alpha = 0$ and positive for $\alpha = \frac{1}{c} \frac{N}{N+1} \frac{1}{\tau_v}$. Thus, the solution to $\alpha$ lies in the interval

$$0 < \alpha < \frac{1}{c} \frac{N}{N+1}.$$  \hfill (5.35)

Inserting (5.30.5) into (5.32) yields

$$\beta^2 \frac{N}{\tau_v} \beta^2 \left( \frac{1}{\tau_v^2} + \frac{\tau_e}{\tau_v} \right) = \frac{1}{2 \tau_v + (N+1)(1+\alpha^2 \tau_v^2 \tau_u^2)}$$

$$\iff 2 \beta^2 \frac{N}{\tau_v} + \beta^2 \frac{N}{\tau_v} (N+1) + \beta^2 N (N+1) \alpha^2 \tau_u = N^2 \beta^2 \left( \frac{1}{\tau_v} + \alpha^2 \tau_u \right) + \frac{N}{\tau_v} \tau_e + \sigma_y^2$$

$$\iff \beta_r^2 = \frac{1}{N \tau_r \sigma_y^2}.$$ 

$\beta_r$ must be the positive solution because $\alpha$ and $\delta_Q$ (see (5.31)) are positive. Furthermore, due to (5.30.1), it is $\beta_r = \frac{\alpha}{N \delta_Q}$. This equation can also be solved for

$$\delta_Q = \frac{c \alpha}{N \beta_r} = \frac{c \alpha}{\sqrt{N \tau_r \sigma_y^2}}.$$ \hfill (5.36)

Now, only the parameters $\beta_0$ and $\delta_0$ are left. Inserting (5.30.4) into (5.30.2) yields

$$\beta_0 = -\beta_r \alpha \mu_u.$$ \hfill (5.37)

$\delta_0$ is given by inserting this solution into (5.30.4) as

$$\delta_0 = -\delta_Q N (-\beta_r \alpha \mu_u + \beta_r \alpha \mu_u) = 0.$$ \hfill (5.38)
Chapter 6

Endogenous Uninformed Trading without Market Maker

In chapters 3 to 5, the market maker played a key role: whether he observed the report or not determined the behavior of the utility of the uninformed investors. This chapter checks the robustness of the results in a setting without market maker.

Along the lines of Kyle (1989), section 6.1 presents the setting which also involves a new equilibrium concept, namely a Nash equilibrium in demand functions. Section 6.2 states and analyzes the resulting equilibrium. The discussion in section 6.3 compares the model with the Kyle (1989) setting.

The resulting equilibrium exists and is unique if there is enough uncertainty in the firm value and if there is enough liquidity demand. The equilibrium is similar to those in chapters 3 to 5; however, only few comparative statics are determinate. One innovation is that the price efficiency now depends on the characteristics of the liquidity traders.

6.1 Economic Setting

The setting remains similar to that in section 4.1. The sequence of events (see figure 6.1) changes only insofar as now, there is no market maker; instead, the equilibrium
price is given as the price that clears demand and supply.

\[ r = v + b \]  \hspace{1cm} (6.1)

on firm value \( v \). He chooses the bias \( b \) so as to maximize his utility

\[ uP(b) - \frac{1}{2}cb^2. \]  \hspace{1cm} (6.2)

The informed investor\(^1\) maximizes the trading profit

\[ \Theta = E[(\hat{v} - \hat{P}(x))x|r, b] \]  \hspace{1cm} (6.3)

with respect to his demand \( x \). He observes the report \( r \) and the bias \( b \).

As in chapter 4, there are \( M \) rational, risk-averse liquidity traders, who hedge a random endowment shock. Uninformed investor \( j \) has an initial endowment of \( w_j \) shares for \( j = 1, \ldots, M \), whereby \( w_j \) is the realization of a normal random variable with mean zero and variance \( \sigma^2_w \). They have a negative exponential utility with risk-aversion coefficient \( \rho \). The variance \( \sigma^2_w \), the mean of zero, the risk aversion \( \rho \), and the number \( M \) of uninformed investors are common knowledge. Uninformed investor

\(^1\)As in chapter 4, the equilibrium would also exist for \( N > 1 \) investors.
maximizes his certainty equivalent with respect to his demand \( y_j \). In contrast to chapter 4, he now observes the public report \( r \), but still not the bias \( b \).

The equilibrium strategies are now conjectured to be

\[
\begin{align*}
b & = \alpha u \\
x & = \beta r + \beta_b b - \beta_P P \\
\gamma_j & = \gamma_r r - \gamma_w w_j - \gamma_P P \\
& \text{for } j = 1, \ldots, M.
\end{align*}
\]

While the investors in chapters 3 to 5 did not observe the price before trading, here, they are allowed to condition their demand on the price, i.e. they submit a whole demand function. The market clearing price is calculated by an auctioneer who aggregates the demand curves. The equilibrium price \( P \) equalizes supply and demand as

\[
x(P) + \sum_j y_j(P) = 0. \tag{6.5}
\]

It follows

\[
\begin{align*}
P & = \frac{\beta r + \beta_b b + M \gamma r + \gamma w \sum_j w_j}{\beta_P + M \gamma_P}.
\end{align*}
\]

\[
\begin{align*}
\text{Var}^{-1}[\hat{v}|r] & = \tau_v + \alpha^{-2} \tau_u \tag{6.7}
\end{align*}
\]

measures the informativeness of the report.

**Proposition 6.2.1.** There is one unique equilibrium in the setting of section 6.1 if the condition

\[
(M - 1) \rho^2 \sigma_w^2 \text{Var}[\hat{v}|r] > 1 \tag{6.8}
\]
holds for the parameters $c, \tau_v, \tau_u, M, \sigma_w^2$ and $\rho$. The equilibrium bias $b$, demands $x$ and $y_j$, $j = 1, \ldots, M$, and the price $P$ are given by

\begin{align*}
b &= \alpha u \\
x &= M\gamma_P (r - b - P) \\
y_j &= \gamma_P \left( \frac{\tau_r}{\tau_v} r - \frac{1}{A} w_j - P \right) \text{ for } j = 1, \ldots, M \\
P &= \frac{1}{2} (r - b) + \frac{1}{2\tau_v} r - \frac{1}{2M} A \sum_j w_j,
\end{align*}

whereby

\begin{align*}
\gamma_P &= \frac{2M}{2M - 1} \left( A - \frac{\tau_v + \alpha^{-2} \tau_u}{M \rho} \right)^\dagger > 0, \\
A &= \frac{\rho \sigma_w^2}{2M} \left( \sqrt{1 + \frac{4M}{\rho^2 \sigma_w^2} (\tau_v + \alpha^{-2} \tau_u) - 1} \right)^\dagger > 0, \\
c\alpha &= \frac{1}{2} \frac{\tau_r}{\tau_v}.
\end{align*}

Proof. See the appendix.

Condition (6.8) demands that there has to be enough uninformed trading activity, which grows with $(M - 1) \rho^2 \sigma_w^2$, or that the report has to be sufficiently uninformative. This resembles condition (4.32) in chapter 4: the uninformed investors want to hedge their risky endowment only if the report itself is not already informative enough. It follows immediately that the equilibrium exists only if there are more than one uninformed investors. It must be

\[ M > 1. \]

The remainder of this section examines the

- bias and precision of the report,
- informed and uninformed demand,
- price informativeness, and
- trading profits.
Bias and Precision of the Report

The bias \( b = \alpha u \) is given by

\[
\alpha = \frac{1}{2} \frac{\tau_v}{\tau_v}
\]  

(6.14)

and hence remains as in chapter 3 (for \( N = 1 \)). The bias decreases in \( c \) and \( \tau_v \)
but increases in \( \tau_u \). The parameters \( M, \sigma_w^2 \) and \( \rho \) are without influence. The three
parameters \( c, \tau_v \) and \( \tau_u \) increase the precision \( \tau_r \) of the report, confer proposition
3.3.2.

Informed and Uninformed Demand

The informed investor trades on the difference between the true per share firm value \( v \)
and the market price \( P \) for the shares. Note that the informed investor traded on the
difference between the expected firm value and the price in prior models, although he
knew the true firm value too.

The trading aggressiveness is

\[
M \gamma_P = \frac{2M}{2M-1} \left( MA - \frac{\tau_v + \alpha^{-2} \tau_u}{\rho} \right).
\]

Corollary 6.2.2. The marginal influence of the parameters on the trading aggressiveness of the informed investors is shown in table 6.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( c )</th>
<th>( \tau_v )</th>
<th>( \tau_u )</th>
<th>( M )</th>
<th>( \sigma_w^2 )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M \gamma_P )</td>
<td>( ? )</td>
<td>( ? )</td>
<td>( ? )</td>
<td>( + )</td>
<td>( + )</td>
<td>( + )</td>
</tr>
</tbody>
</table>

Table 6.1: The Marginal Effect of the Parameters on the Trading Aggressiveness \( M \gamma_P \).

The uninformed investors trade on the difference between the expected terminal
value conditional on the report \( E[\tilde{v}|r] \) and the price \( P \). They trade more aggressive on
this difference if \( \gamma_P \) grows. In addition, they sell a portion of their random endowment
$w_j$ to hedge the risk of their initial position. The coefficients of $r$, $w_j$ and $P$ are

$$
\gamma_r = \frac{\tau_v}{\tau_v 2M - 1} \left( A - \frac{\tau_v + \alpha^{-2} \tau_u}{M \rho} \right) > 0
$$

$$
\gamma_w = \frac{2M}{2M - 1} \left( \frac{1 - \tau_v + \alpha^{-2} \tau_u}{M \rho A} \right) > 0
$$

$$
\gamma_P = \frac{2M}{2M - 1} \left( A - \frac{\tau_v + \alpha^{-2} \tau_u}{M \rho} \right) > 0
$$

respectively. Only for $\gamma_w$ is the influence of the parameters unambiguous. As in section 4.3, the uninformed investors hedge less ($\gamma_w$ decreases) if the report is more informative, i.e. $c$, $\tau_v$ and $\tau_u$ increase, and if the number $M$ of uninformed investors, the variance $\sigma_w^2$ in their endowment and their risk aversion $\rho$ increases.\(^2\)

### Price Informativeness

The price informativeness $\Psi$ is here defined as the precision in the firm value conditional on the price and the report because all market participants observe the report as well:

$$
\Psi = \text{Var}^{-1} \left[ \tilde{v} | r, P \right]
$$

$$
\doteq \tau_v + \alpha^{-2} \tau_u + \frac{M \gamma_P^2}{\sigma_w^2 \gamma_w}
$$

$$
\doteq \tau_v + \alpha^{-2} \tau_u + \frac{M A^2}{\sigma_w^2}
$$

$$
= 2(\tau_v + \alpha^{-2} \tau_u) - \rho A
$$

since

$$
A^2 = \frac{\rho^2 \sigma_w^4}{4M^2} \left( 1 - 2 \sqrt{1 + \frac{4M}{\rho^2 \sigma_w^4} (\tau_v + \alpha^{-2} \tau_u)} + 1 + \frac{4M}{\rho^2 \sigma_w^4} (\tau_v + \alpha^{-2} \tau_u) \right)
$$

$$
= \frac{\sigma_w^2}{M} (\tau_v + \alpha^{-2} \tau_u) + (2 - 2A \frac{2M}{\rho \sigma_w^2}) \frac{\rho^2 \sigma_w^4}{4M^2}
$$

$$
= \frac{\sigma_w^2}{M} (\tau_v + \alpha^{-2} \tau_u - \rho A).
$$

\(^2\)The influence of $M$ is indeterminate in section 4.3.
Here, the price informativeness, for the first time in this thesis, also depends on $M$, $\sigma^2_w$ and $\rho$. This difference is not due to the different definition of $\Psi$: note that in the other chapters $Var^{-1}[\tilde{v}|r,P]$ does also not depend on $M$, $\sigma^2_w$ and $\rho$.

**Corollary 6.2.3.** The marginal influence of the parameters on the price informativeness is shown in table 6.2.

<table>
<thead>
<tr>
<th>$\Psi$</th>
<th>$c$</th>
<th>$\tau_v$</th>
<th>$\tau_u$</th>
<th>$M$</th>
<th>$\sigma^2_w$</th>
<th>$\rho$</th>
</tr>
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</tr>
</tbody>
</table>

Table 6.2: The Marginal Effect of the Parameters on the Price Informativeness $\Psi$.

An increase in $M$, $\rho$ and $\sigma^2_w$ flattens the residual supply curve against which each informed trader trades ($\lambda_I = \frac{1}{M\gamma_P}$ is lower, confer proposition 6.1). As a result, each informed trader makes the quantity he trades more sensitive to his private information, and this tends to make the equilibrium price more informative.

A counter effect here is that a higher risk aversion $\rho$ or a more variant endowment $\sigma^2_w$ increase the hedging activities $\gamma_w$ of the uninformed investors. This effect tends to make the price less informative.

**Trading Profits**

The informed investor gains on average

$$\Theta = E[(\tilde{v} - \tilde{P})\tilde{x}] \overset{(6.9.2)}{=} E[(\tilde{v} - \tilde{P})M\gamma_P(\tilde{v} - \tilde{P})]$$

$$= M\gamma_P Var[\tilde{v} - \tilde{P}]$$

$$\overset{(6.9.4)}{=} M\gamma_P \left( \frac{1}{\tau_v} \left( \frac{1}{2} - \frac{1 \tau_r}{2 \tau_v} \right)^2 + \frac{\alpha^2}{\tau_u} \left( \frac{1 \tau_r}{2 \tau_v} \right)^2 + \frac{1}{4M^2 A^2} \frac{1}{\sigma^2_w} \right)$$

$$= M\gamma_P \frac{1}{4} \left( \frac{1}{\tau_v + \alpha^{-2} \tau_u} + \frac{1}{\tau_v + \alpha^{-2} \tau_u - \rho A} \right).$$

The expected trading profit consist of the product of the inverse of the slope of the informed’s supply curve (“his market depth”) and the variance of the bias in the share price. Expressions for the marginal influences are involved.
Due to the zero-sum condition, the uninformed investors’ expected trading loss is

\[ E[(\tilde{v} - \tilde{P})y_j] = -\gamma_P Var[\tilde{v} - \tilde{P}] \quad \text{for} \quad j = 1, \ldots, M. \] (6.15)

Therefore, they benefit from a steeper (!) demand curve \( \lambda_U = \frac{1}{2M-1} \frac{1}{\gamma_P} \) and from less variance in the valuation error \( \tilde{v} - \tilde{P} \). The ultimate influence of the parameters on their trading profits is involved and likely to be indeterminate.

Unfortunately, the expected certainty equivalent \( E[CE_j(y_j|w_j)] \) of the uninformed investors \( j = 1, \ldots, M \) does not have a nice representation.

### 6.3 Comparison with Kyle (1989)

This chapter uses the concept of an equilibrium in supply curves developed in Kyle (1989). The objective of Kyle (1989) is to compare the competitive rational expectations equilibria with an imperfect setting. Kyle (1989) states two reasons for modeling traders as imperfect competitors:

1. Arbitrageurs with private information buy and sell significant percentages of the outstanding equity so that the price-taking assumption does not seem to be reasonable.

2. The competitive equilibria do not have reasonable properties, e.g. if the investors are almost risk-neutral and if there is only few variance in noise trading.

He shows that, in the imperfect equilibrium, even if the informed investors become risk-neutral the incentives to acquire private information do not disappear. As well, if noise trading vanishes the price does not become fully informative. Instead, the profits based on private information are driven to zero because the market becomes illiquid. If the number of informed investors grows, the equilibrium converges to the competitive outcome of Hellwig (1980).

---

3These equilibria are surveyed in section 1.4.
The model presented in section 6.1 makes a number of simplifying assumptions in comparison to Kyle (1989). This allows to derive a closed form solution\(^4\) whose comparative statics can be analyzed in some cases. However, in other respects it extends Kyle’s setting.

In Kyle (1989), there are \(N\) informed investors who learn diverse signals \(\hat{v} + \hat{e}_i\) with the same precision. The setting in section 6.1 involves only one informed investor who observes the report \(r = v + b\), the bias \(b\) and, consequently, the firm value \(v\). In Kyle (1989), the informed investors are risk-averse with equal risk aversion coefficient, while the informed investor is risk-neutral in the current setting.

In both models, the uninformed investors are risk averse with equal risk aversion coefficient. However, in Kyle (1989), they do not observe any information and are not endowed with shares. Additional noise stems from noise traders in Kyle (1989) who purchase a random, exogenous quantity of shares. Here, the uninformed investors are endowed with a random amount of shares at the beginning and try to hedge this initial position. There are no noise traders in the setting of section 6.1.\(^5\)

The final difference is, of course, that the private signals are given exogenously in Kyle (1989) while the current setting includes a manager who has an interest in the share price.

**Proof of Proposition 6.2.1**

Kyle (1989) introduces an equilibrium concept in demand curves which is also employed here. The proof proceeds in three steps: Firstly, it expresses the optimal strategy of the informed investor in terms of the coefficients of the uninformed investors’ equilibrium strategy. Secondly, a system of equations for the coefficients in the uninformed investors’ strategy is derived and solved. The last step includes the manager’s behavior.

\(^4\)The solution were in closed form if the manager did not participate. Kyle’s equilibrium without manager is given by a nonlinear system of equations which prevents an analysis of marginal influences of the parameters.

\(^5\)A similar equilibrium with a manager can be derived without the endowment shock of the uninformed investor, but with additional noisy demand. In the resulting equilibrium, which gets by without \(\gamma_w\), there are still no simpler formulas for price efficiency and trading profits.
Step 1. The informed investor maximizes against the residual supply curve $\tilde{P}_I$ given by

$$\tilde{P} = \tilde{P}_I + \lambda_I x \quad (6.6)$$

whereby $\lambda_I$ is given by

$$\lambda_I = \frac{1}{M \gamma_P}. \quad (6.17)$$

A key insight in this Kyle (1989) setting is that the residual demand curves $\tilde{P}_I$ happen to be linear with non-random slopes and are thus informationally equivalent to the market clearing price $\tilde{P}$. The informed investor’s trading profits are

$$E[(\tilde{v} - \tilde{P}_I - \lambda_I x)|r, b, P_I] = (r - b - P_I - \lambda_I x) x$$

First and second order conditions are

$$r - b - P_I - 2\lambda_I x = 0 \quad (6.16)$$

and $\lambda_I > 0$. The first order condition yields

$$x = \frac{1}{\lambda_I} (r - b - P).$$

Comparing the first order condition with the informed investors’ conjecture in (6.4.2) yields

$$\beta_r = \frac{1}{\lambda_I} = M \gamma_P \quad (6.18.1)$$

$$\beta_b = \frac{-1}{\lambda_I} = -M \gamma_P \quad (6.18.2)$$

$$\beta_P = \frac{1}{\lambda_I} = M \gamma_P. \quad (6.18.3)$$

under the second order condition

$$\gamma_P > 0. \quad (6.19)$$

Step 2. Uninformed investor $j$, for $j = 1, \ldots, M$, maximizes against the residual supply curve $\tilde{P}_U$ given by

$$\tilde{P} = \tilde{P}_U + \lambda_U y_j \quad (6.6)$$

whereby $\lambda_I$ is given by

$$\lambda_I = \frac{1}{M \gamma_P}. \quad (6.17)$$

A key insight in this Kyle (1989) setting is that the residual demand curves $\tilde{P}_U$ happen to be linear with non-random slopes and are thus informationally equivalent to the market clearing price $\tilde{P}$. The uninformed investor’s trading profits are

$$E[(\tilde{v} - \tilde{P}_U - \lambda_U y_j)|r, b, P_I] = (r - b - P_I - \lambda_U y_j) x$$

First and second order conditions are

$$r - b - P_I - 2\lambda_U y_j = 0 \quad (6.16)$$

and $\lambda_I > 0$. The first order condition yields

$$x = \frac{1}{\lambda_I} (r - b - P).$$

Comparing the first order condition with the uninformed investors’ conjecture in (6.4.2) yields

$$\beta_r = \frac{1}{\lambda_I} = M \gamma_P \quad (6.18.1)$$

$$\beta_b = \frac{-1}{\lambda_I} = -M \gamma_P \quad (6.18.2)$$

$$\beta_P = \frac{1}{\lambda_I} = M \gamma_P. \quad (6.18.3)$$

under the second order condition

$$\gamma_P > 0. \quad (6.19)$$
whereby $\lambda_U$ is given by

$$\begin{align*}
\lambda_U &= \frac{1}{\beta P + (M-1)\gamma P} \\
&= \frac{1}{2(M-1)\gamma P}.
\end{align*}$$

(6.17) (6.21)

Uninformed trader $j$’s trading profit given his endowment $w_j$ is

$$\begin{align*}
(y_j + w_j)\tilde{v} - (\tilde{P}_U + \lambda_U y_j)y_j,
\end{align*}$$

(6.22)

which yields a certainty equivalent of

$$CE_j(y_j, r, w_j, P_U) = E[\tilde{v}|r, P_U](y_j + w_j) - (P_U + \lambda_U y_j)y_j - 0.5\rho Var[\tilde{v}|r, P_U](y_j + w_j)^2.$$

First and second order conditions with respect to the demand $y_j$ are

$$
E[\tilde{v}|r, P_U] - P - 2\lambda_U y_j - \rho Var[\tilde{v}|r, P_U](y_j + w_j) = 0
$$

(6.20)

and $2\lambda_U + \rho Var[\tilde{v}|r, P_U] > 0$. The second order condition is always fulfilled since $\lambda_U > 0$ in (6.21) due to (6.19) and a variance is always non-negative. The first order condition yields

$$y_j = \frac{E[\tilde{v}|r, P_U] - P - \rho Var[\tilde{v}|r, P_U]w_j}{\lambda_U + \rho Var[\tilde{v}|r, P_U]}$$

(6.23)

The following calculates the conditional expectation and variance in (6.23): An observation of $(P_U, r, w_j)$ and $(P, r, w_j)$ is informationally equivalent which follows with (6.6) and (6.20); hence, $E[\tilde{v}|r, P_U] = E[\tilde{v}|r, P]$ and $Var[\tilde{v}|r, P_U] = Var[\tilde{v}|r, P]$.

The price in (6.6) is

$$\hat{P} (6.18) \equiv \frac{M\gamma_P(r - \tilde{b}) + M\gamma_r \bar{r} + \gamma_w \sum_j \tilde{w}_j}{M\gamma_P + M\gamma_P}$$

$$= \frac{1}{2} \tilde{v} + \frac{1}{2} \gamma_P \bar{r} + \frac{1}{2M} \gamma_w \sum_j \tilde{w}_j.$$

It follows that

$$2P - \frac{\gamma_r}{\gamma_P} r = \hat{v} + \frac{1}{M} \gamma_P \sum_j \tilde{w}_j.$$
Therefore, it is
\[ E[\tilde{v}|r, P_U] = E[\tilde{v}|\tilde{v} + \alpha \tilde{u} = r, \tilde{P} = P] \]
\[ = E[\tilde{v}|r, \bar{\tilde{v}} + \frac{1}{M} \sum_j \tilde{w}_j = 2P - \gamma_r r] \]
\[ \text{Lemma (2.4.2)} \]
\[ = \frac{\alpha^{-2} \tau_u}{\tau^*} r + \frac{1}{\tau^* \sigma_w^2 \gamma^2_w} \]
\[ (2P - \gamma_r r). \]

with
\[ \tau^* = \text{Var}^{-1}[\tilde{v}|r, P] = \tau_v + \alpha^{-2} \tau_u + \frac{M \gamma^2_P}{\sigma^2_w \gamma^2_w}. \] (6.24)

Thus, it is \( E[\tilde{v}|r, P] = k_1 r + k_2 P \) with
\[ k_1 = \frac{\alpha^{-2} \tau_u}{\tau^*} - \frac{1}{\tau^* \sigma_w^2 \gamma^2_w} \]
\[ k_2 = \frac{2}{\tau^* \sigma_w^2 \gamma^2_w}. \]

Comparing the demand \( y_j \) of uninformed investor \( j \) in (6.23) with the conjecture (6.4.3) yields
\[ \gamma_r \left( \frac{1}{(2M-1)\gamma_P} + \frac{\rho}{\tau^*} \right) = \frac{\alpha^{-2} \tau_u}{\tau^*} - \frac{1}{\tau^* \sigma_w^2 \gamma^2_w} \] \[ \gamma_P \left( \frac{1}{(2M-1)\gamma_P} + \frac{\rho}{\tau^*} \right) = 1 - \frac{2}{\tau^* \sigma_w^2 \gamma^2_w} \] \[ \gamma_w \left( \frac{1}{(2M-1)\gamma_P} + \frac{\rho}{\tau^*} \right) = \rho \frac{1}{\tau^*}. \]

Note that \( \frac{1}{(2M-1)\gamma_P} + \frac{\rho}{\tau^*} \neq 0 \) since \( \gamma_P > 0 \) (confer (6.19)) and \( \tau^* > 0 \). Dividing (6.25.1) and (6.25.2) by (6.25.3) yields
\[ \frac{\gamma_r}{\gamma_w} = \frac{1}{\rho} \left( \alpha^{-2} \tau_u - \frac{1}{\rho} \frac{\gamma_r \gamma_P}{\gamma_w} \frac{M}{\sigma_w} \right) \] \[ \frac{\gamma_P}{\gamma_w} = \frac{1}{\rho} \left( \alpha^{-2} \tau_u - \frac{2}{\rho} \frac{\gamma_P^2}{\gamma_w} \frac{M}{\sigma_w} \right) \]
\[ \frac{(6.24)}{= \frac{1}{\rho} (\tau_v + \alpha^{-2} \tau_u) - \frac{1}{\rho} \frac{M \gamma_P^2}{\sigma_w^2 \gamma_w}} \] \[ \text{The second equation (6.27) determines the fraction } \frac{\gamma_P^2}{\gamma_w} \text{ as} \]
\[ \frac{\gamma_P^2}{\gamma_w} + \frac{\sigma_w^2 \gamma_P}{M \gamma_w} - \frac{\sigma_w^2}{M} (\tau_v + \alpha^{-2} \tau_u) = 0. \]
Since $\gamma_P > 0$ (confer (6.19)) and $\gamma_w > 0$ (confer (6.25.3)), it follows that $\frac{2\rho}{\gamma_w}$ is the positive root given by

$$\gamma_P = \frac{\rho \sigma_w^2}{2M} + \sqrt{\frac{\rho^2 \sigma_w^4}{4M^2} + \frac{\sigma_w^2}{M} (\tau_v + \alpha^{-2} \tau_u)}$$

$$= \frac{\rho \sigma_w^2}{2M} \left(1 + \frac{4M}{\rho^2 \sigma_w^2} (\tau_v + \alpha^{-2} \tau_u) - 1\right)$$

$$= A. \quad (6.28)$$

Multiplying (6.26) with $\frac{2\rho}{\gamma_r}$ and subtracting (6.27) yields

$$0 = \frac{\alpha^{-2} \tau_u}{\rho} - \frac{1}{\rho} (\tau_v + \alpha^{-2} \tau_u)$$

$$\iff \quad \frac{\gamma_P}{\gamma_r} = \frac{\tau_v + \alpha^{-2} \tau_u}{\alpha^{-2} \tau_u}.$$   \quad (6.29)

A solution for $\gamma_P$ - of course depending on $\alpha$ - can be obtained from (6.25.2), which is

$$\frac{1}{2M-1} \tau^* + \rho \gamma_P = \tau^* - 2 \frac{M \gamma_P^2}{\sigma^2 \gamma_w}$$

$$\iff \quad \frac{\gamma_P}{\gamma_r} = \frac{\tau_v + \alpha^{-2} \tau_u + M \gamma_P^2}{\alpha^{-2} \tau_u} + \rho \gamma_P = \tau_v + \alpha^{-2} \tau_u - \frac{M \gamma_P^2}{\sigma^2 \gamma_w}$$

Thus, $\gamma_P$ is

$$\gamma_P = \frac{1}{\rho \sigma_w^2} \left(2M - 2 (\tau_v + \alpha^{-2} \tau_u) - \frac{1}{\rho \sigma_w^2} \gamma_w^2\right)$$

$$= \frac{1}{\rho \sigma_w^2} \left(2M - 1 \tau_v + \alpha^{-2} \tau_u - \frac{1}{\rho \sigma_w^2} A^2\right)$$

$$= \frac{1}{\rho \sigma_w^2} \left(2M - 1 \tau_v + \alpha^{-2} \tau_u - \frac{1}{\rho \sigma_w^2} A^2\right)$$

$$= \frac{1}{\rho \sigma_w^2} \left(2M - 1 \tau_v + \alpha^{-2} \tau_u - \frac{1}{\rho \sigma_w^2} A^2\right)$$

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$$= \frac{1}{\rho \sigma_w^2} \left(2M - 1 \tau_v + \alpha^{-2} \tau_u - \frac{1}{\rho \sigma_w^2} A^2\right)$$

Equations for $\gamma_r$ and $\gamma_w$ simply follow from (6.29) and (6.28) using this $\gamma_P$. All coefficients uniquely depend on $\alpha$.

**Step 3.** The manager maximizes his utility function in (6.2) with respect to the bias $b$.

The utility is

$$u_P = \frac{1}{2} cb^2 \quad \text{(6.6)}$$

$$\iff \quad \frac{\beta_r (v + b) + \beta_u b + M \gamma_r (v + b) + \gamma_w \sum_j w_j}{\beta_P + M \gamma_P} = \frac{1}{2} cb^2$$
First and second order conditions are
\[ u_\beta r + \beta b + M_\gamma r - cb = 0 \]
and \( c > 0 \) which holds per definition. The first order condition yields
\[ b = \frac{u_\beta r + \beta b + M_\gamma r}{c \beta p + M_\gamma p}. \]
Comparing the optimal bias of the manager with the conjecture in (6.4.1) yields
\[ c_\alpha = \frac{\beta r + \beta b + M_\gamma r}{\beta p + M_\gamma p} = \frac{M_\gamma p - M_\gamma p + M_\gamma r}{2M_\gamma p} = (6.29) \frac{1}{\frac{\alpha^{-2} \tau_u}{2 \tau_v + \alpha^{-2} \tau_u}}. \]
Hence, the bias is given as the root of the polynomial
\[ \alpha^3 + \frac{\alpha \tau_u}{\tau_v} - \frac{1}{2c \tau_v} \tau_u = 0, \quad (6.31) \]
which is monotonically increasing in \( \alpha \) and has only one root lying in the interval
\[ \alpha \in \left( 0, \frac{1}{2c} \right). \quad (6.32) \]
The overall equilibrium therefore exists and is unique if and only if \( \gamma_p > 0 \) in (6.30).
Thus, it must be true that
\[
\begin{align*}
2(\tau_v + \alpha^{-2} \tau_u) - 2M_\rho &< 0 \\
\iff& 2 \frac{\tau_v + \alpha^{-2} \tau_u}{\rho^2 \sigma^2_v} + 1 < \sqrt{1 + \frac{4M_\rho}{\rho^2 \sigma^2_v} (\tau_v + \alpha^{-2} \tau_u)} \\
\iff& \left(2 \frac{\tau_v + \alpha^{-2} \tau_u}{\rho^2 \sigma^2_v} + 1\right)^2 < 1 + \frac{4M_\rho}{\rho^2 \sigma^2_v} (\tau_v + \alpha^{-2} \tau_u) \\
\iff& 4 \frac{\tau_v + \alpha^{-2} \tau_u}{\rho^2 \sigma^2_v} \left(M - 1 - \frac{\tau_v + \alpha^{-2} \tau_u}{\rho^2 \sigma^2_v}\right) > 0 \\
\iff& M - 1 > \frac{\tau_v + \alpha^{-2} \tau_u}{\rho^2 \sigma^2_v}. 
\end{align*}
\]
Chapter 7

The Model of Fischer and Stocken (2004)

In the December 2004 issue of the Journal of Accounting Research, Fischer and Stocken published a model which has similarities with the setting of chapters 3 to 6. It also combines Fischer and Verrecchia (2000) with Kyle (1985), but considers the effect of private information in advance of the public report. This section describes their setting and results, and compares their main conclusions with the results of this thesis.

Setting

Fischer and Stocken (2004) assume three strategic risk-neutral players: a manager, a speculator, and a market maker. There are three periods whose events are depicted in time line 7.1. In the first period, the speculator is endowed with private information about the firm, and trades on this information in a security market intermediated by the market maker. In the second period, the manager obtains private information about the firm and issues an earnings report $r$. The security market then reopens and the speculator liquidates his first-period position. In the third period, the terminal value of the firm is realized.
The speculator has information about a forthcoming earnings report which is divided into two components: information about earnings management on the one hand and information about fundamental earnings on the other hand. The fundamental earnings \( \tilde{v} \) and the interest \( \tilde{u} \) of the manager in the share price are split up into two independent normal random variables as
\[
\tilde{v} = \tilde{v}_1 + \tilde{v}_2 \quad \text{and} \quad \tilde{u} = \tilde{u}_1 + \tilde{u}_2. \tag{7.1}
\]
The variance of \( \tilde{v}_1 \) is \( q_v \sigma_v^2 \) and \( \tilde{v}_2 \) has the variance \( (1 - q_v) \sigma_v^2 \) with \( q_v \in [0, 1) \). Similarly, \( \tilde{u}_1 \) has a variance of \( q_u \sigma_u^2 \) and \( \tilde{u}_2 \) has a variance of \( (1 - q_u) \sigma_u^2 \). Prior to the trading of period 1, the speculator observes the share \( q_v \) of the terminal value (i.e., the realization of \( v_1 \)) and the share \( q_u \) of the manager’s interest in the share price (i.e., the realization of \( u_1 \)). Since he liquidates his first period position in period 2 (short-term speculation), his objective is to maximize
\[
\max_x E[x(P_2 - P_1)|v_1, u_1]. \tag{7.2}
\]
The market maker efficiently sets the prices at dates 1 and 2, i.e. at the expected terminal value given his observed information:
\[
P_1 = E[\tilde{v}|x + y_1] \\
P_2 = E[\tilde{v}|r, x + y_1, -x + y_2],
\]
whereby \( y_1 \) and \( y_2 \) signifies independent, normally-distributed noise trading. The security’s price \( P_1 \) in the first period is the expected terminal value \( v \) given the total
demand \( x + y_1 \) of the speculator and the liquidity-motivated traders. The price \( P_2 \) in the second period equals the expected terminal value \( v \) given the total demands of periods one \( x + y_1 \) and two \( -x + y_2 \), and the manager’s earnings report \( r \).

The report \( r = v + b \) is set by the manager as the sum of the terminal value and a bias which is chosen so as to maximize

\[
\max_b u P_2 - \frac{1}{2} b^2. \tag{7.3}
\]

**Results**

Fischer and Stocken (2004) are able to demonstrate that a unique linear solution exists if a convex objective function of the speculator is ruled out.\(^1\) However, the equilibrium is stated in a complex form:\(^2\) the solution is given depending on two parameters. The two parameters are given implicitly as the roots of two interdependent polynomials of degree four. This is also the reason why the equilibrium analysis remains very limited with regard to the marginal influence of the parameters.

Having shown the existence of an equilibrium, Fischer and Stocken (2004) consider the earnings quality and price efficiency, and compare the results with a setting without the speculator. All results critically depend on the difference

\[ q_u - q_v. \]

If the speculator’s information about earnings management dominates \((q_u > q_v)\), the speculator’s presence increases earnings management. If the speculator’s information is primarily about fundamental earnings \((q_u < q_v)\), his presence lessens earnings management. The intuition goes as follows: if the speculator has more information about fundamental earnings, the order flow at date 1 is more informative and the market maker’s responsiveness to the reported earnings is thus reduced at date 2.

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\(^1\)This convex case would lead to an infinitely long or short position in the firm’s security. Fischer and Stocken (2004) argue in footnote 13 that such a strategy is not supportable in a linear equilibrium.

\(^2\)Note that the calculation of the price \( P_2 \) in period two involves the inversion of a three-dimensional matrix.
This, in effect, decreases the manager’s incentive to manipulate earnings. Fischer and Stocken (2004) call the private information a substitute for the report in this case. In contrast, more complementary information derived from the order demand ($q_u > q_v$) increases earnings management. The direction of the influence of the presence of the speculator on the earnings quality in equilibrium depends only on the sign of $q_u - q_v$.

Next, Fischer and Stocken (2004) ask how the informativeness of prices is affected by the presence of the speculator. They establish that the speculator’s influence on the informativeness of the firm’s stock price depends on the nature of the speculator’s information. If the speculator is better informed about the firm’s fundamental earnings ($q_u < q_v$), the speculator’s trade causes the earnings quality to increase. The increase in earnings quality, coupled with the private information about fundamental earnings conveyed through the speculator’s demands, leads to more efficient second-period prices. In contrast, if the speculator is better informed about the manager’s reporting incentives ($q_u > q_v$), the speculator’s effect on price efficiency in the second period is ambiguous. On the one hand, the speculator’s trade leads to a reduction in earnings quality, which in turn reduces price efficiency. On the other hand, the speculator’s trade augments the firm’s reported earnings as a source of information relative to the net demand, which in turn increases the price efficiency. Fischer and Stocken (2004) find that the decline in earnings quality dominates the additional information conveyed through the speculator’s trade if the information about the manager’s reporting incentive is sufficiently noisy. Therefore, the presence of privately informed speculators might lead to less efficient prices.

In a further section, Fischer and Stocken (2004) discuss testable hypotheses which follow from their equilibrium for the interpretation of long-window earnings association studies and short-window event studies: the earnings-association coefficient can be lower although the incremental value-relevance of earnings is higher (a finding which contradicts basic accounting insights). The earnings-response coefficient (i.e., the coefficient on earnings surprise, which Fischer and Stocken (2004) define as the difference between the earnings realization and the market maker’s expectation of
earnings) may rise if the quality of the earnings drops.

**Comparison with the Models of Chapters 3 to 6**

Fischer and Stocken (2004) follow the same basic idea as this thesis, which is to combine the setting of Fischer and Verrecchia (2000) with that of Kyle (1985). However, the two models display some major modeling differences and consider different research questions.

First of all, Fischer and Stocken (2004) consider private information about a forthcoming public report. Consequently, the speculator does not observe the bias as the informed investors do in chapter 3 of this thesis, whereas they do observe a part of the managers’ preferences regarding the share price. The order of events is realized invulnerably in a two-period setting, while in chapters 3 to 6 of this thesis, there is no additional round of trading in between the release of public information and the reception of the private signal. However, this makes the equilibrium of Fischer and Stocken (2004) very complex; in effect, it becomes inaccessible to an analysis of the marginal influence of the model parameters.

The models used in this thesis do not include separate information on the fundamental value of the firm. In fact, the informed investors only learn the interest $u$ of the manager in the share price, but they learn it completely. This seems to be the special case of Fischer and Stocken’s (2004) analysis for $q_u = 1$. However, in chapter 3 of this thesis, the informed investors can perfectly deduce the firm value as well also implying $q_v = 1$. It is shown on page 78 of chapter 3 that the presence of the informed investors always decreases the bias while increasing the price efficiency. In chapter 5 of this thesis, if there is no informed investor, there is also no additional information available beyond the initial information on the firm value $\hat{v}$. In this context, the question of the influence of the presence of an informed investor on the equilibrium does not make sense.

The models of chapters 3 to 6 explicitly examine the utility of the liquidity traders. One important parameter linking the manager’s decision problem with the welfare
of the investors are the marginal costs $c$ of biasing. Fischer and Stocken (2004) set $c$ equal to one. They consider neither its influence on the equilibrium, nor do they calculate the utility of either the noise traders or the speculator.
Chapter 8

Conclusion

Financial reports are prepared to serve the informational needs of a set of diverse users.\(^1\) The International Financial Reporting Standards focus on the needs of investors:

"As investors are providers of risk capital to the enterprise, the provision of financial statements that meet their needs will also meet most of the needs of other users that financial statements can satisfy."\(^2\)

Thus, the provision of decision useful information for investors is not the only, yet the main purpose of financial reports according to IASB (2003). Investors are "concerned with the risk inherent in, and return provided by, their investments. They need information to help them determine whether they should buy, hold or sell."\(^3\) The remainder of this section assumes that the provision of decision useful information were the only purpose of financial reports.

Although the financial reporting of firms is highly regulated, the necessity for such a regulation is difficult to prove:

"Surprisingly little is known about why financial reporting and disclosure is regulated in the capital market. Is there a significant market

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\(^1\)See section 1.1.

\(^2\)See IASB (2003), Framework 10.

\(^3\)See IASB (2003), Framework 9.
imperfection or externality that regulation attempts to resolve? If so, how effective is disclosure regulation in resolving this problem?”

Why should an efficient information production not be accomplished by market forces alone? One quite robust capital market incentive for managers to produce information voluntarily is the disclosure principle. The argument here is that if investors know that the manager possesses relevant information, but are unaware of its content, they will assume that if it were favorable, the manager would release it. Thus, if investors do not observe any information release on the side of the manager, they will assume the worst and bid down the market value of the firm’s shares accordingly. Another incentive for truthful information disclosure is the reputation of managers: releasing false or incomplete information may damage the manager’s chances in the labor market, decrease the liquidity and market value of his firm, and finally increase the probability of a takeover. Hence, in the absence of market imperfections or externalities, firms have incentives to optimally trade-off the costs and benefits of voluntary disclosure, and to produce the efficient level of information for investors in the economy. A justification for disclosure regulation therefore has to rely on the presence of market imperfections.

One potential market imperfection may arise from the information’s characteristic as a public good; thus, the producers of information cannot exclude others from receiving benefits or costs. Already established stockholders implicitly pay for the production of information but cannot charge potential investors for their usage of such information. Prospective investors, therefore, free-ride in information which is paid for by existing shareholders, leading to a potential underproduction of information.

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5Information is also necessary to monitor contracts such as debt contracts. Since contracts are negotiated, it can be argued that the right amount of information to monitor them should be agreed upon automatically.
8The disclosure principle assumes that the information is truthful or verifiable. Otherwise, the application of a finite penalty would only lead to a partial revelation of information; see Korn (2004).
in the economy.\textsuperscript{9} The same externality evolves because information released by one firm will simultaneously convey information about other firms.\textsuperscript{10}

Another justification for the requirement of mandatory disclosures is the adverse selection problem.\textsuperscript{11} Here, adverse selection denotes the information asymmetry between informed and uninformed investors in capital markets. By creating minimum disclosure requirements, regulators intend to reduce the information gap between informed and uninformed investors. The American Institute of Certified Public Accountants’ (AICPA) Study Group on the Objectives of Financial Statements (1973, p.17) argues that "financial statements should meet the needs of those with the least ability to obtain information." The regulation of public disclosure therefore aims at providing a 'level playing field' among diversely informed investors.\textsuperscript{12} Here, a level playing field denotes a situation where each investor has equal access to the information relevant for asset allocation.\textsuperscript{13} The concept of a level playing field advocates the equality of ex-ante opportunities. According to Lev (1988), the equity orientation is not a call for fairness or for the protection of small and defenseless investors against fraud or exploitation by insiders. Instead, he emphasizes the beneficial effects of symmetric information on transaction costs and liquidity in the market which follow from formal microstructure models.

Market microstructure models, such as the basic Kyle (1985) model used throughout this thesis,\textsuperscript{14} establish that a higher information asymmetry implies higher bid-ask spreads, and consequently higher transaction costs. Kyle (1989b) compares the adverse-selection component of trading costs with a tax on noise traders which subsidizes the acquisition of private information and its subsequent dissemination through the price system. Those noise traders who use the marketplace for exchanging assets

\textsuperscript{10}Even less feasible is the internalization of external effects on the public in general, as noted by Neus (2005, p. 365).
\textsuperscript{11}See Leftwich (1980), Watts and Zimmerman (1986), and Beaver (1998).
\textsuperscript{12}See Hakansson (1977).
\textsuperscript{13}See Lev (1988).
\textsuperscript{14}See also Copeland and Galai (1983) or Glosten and Milgrom (1985).
thus pay in effect for this information, although they may not use it.\textsuperscript{15} As trading costs increase because a higher volume of informed trading erodes market liquidity, noise traders will decrease their trading activity or withdraw entirely from the market. This leads to low volumes of trade and a lower number of traders. Ultimately, a massive withdrawal of uninformed investors from the market will strip the informed investors of the benefits of their costly-acquired information (therefore decreasing incentive for information production), and will furthermore deprive the economy of the allocation and risk sharing benefits of large and efficient capital markets.\textsuperscript{16}

The underlying assumption of this adverse selection argument is that public disclosure reduces the information asymmetry among investors. This assumption is generally accepted among accounting practitioners. According to Verrecchia (2004, p. 149), ”allusions to the fact that more disclosure improves liquidity, reduces costs of capital, and makes markets more efficient are commonplace among regulators and standard setters.” For example, Levitt (1998b, p. 82), the former president of the SEC, put it simply by stating that ”the truth is, high quality standards lower the cost of capital.” However, accounting research has produced discordant results so far.

A strand of theoretical research posits that public announcements can increase idiosyncratic beliefs among market participants. For example, public announcements are seen as creating idiosyncratic beliefs in Harris and Raviv (1993), and Kandel and Pearson (1995) because market participants use different valuation functions and hence interpret public information diversely. Holthausen and Verrecchia (1990) and Indjejikian (1991) model public disclosures as signals with both common and idiosyncratic error components, which comes as the equivalent to the assumption that public announcements can create idiosyncratic beliefs. In Kim and Verrecchia (1994, 1997), a public information release triggers the generation of new idiosyncratic information

\textsuperscript{15}One assumption is that unsophisticated investors do not reduce the information gap by investing in financial knowledge or hiring the services of sophisticated intermediaries.

\textsuperscript{16}According to Beck, Levine and Loayza (2000) and Levine (1997), investor protection accelerates economic growth in three ways. Firstly, it can enhance savings. Secondly, it can channel these savings into real investment and thereby foster capital accumulation. Thirdly, to the extent that financiers exercise some control over the investment decisions of the entrepreneurs, financial development allows capital to flow towards more productive uses, and thus improves the efficiency of resource allocation.
from the public announcement by agents with diverse information-processing skills.

Empirical evidence on the relationship between disclosures and liquidity has produced mixed evidence so far. Barron, Byard and Kim (2002) examine changes in the precision and commonality of information contained in individual analysts’ earnings forecasts, focusing on changes around the time of earnings announcements. They conclude that the idiosyncratic information contained in these individual analysts’ forecasts increases immediately after earnings announcements. Kandel and Pearson (1995) observe that the relation between the volume of trade and stock returns around earnings announcements is inconsistent with the assumption that investors interpret public information identically.

To summarize, it is neither theoretically nor empirically clear whether more disclosure per se increases liquidity or not. This is particularly unsatisfactory since the question of whether or not public accounting disclosures trigger significant idiosyncratic information is central to the understanding of the role of accounting disclosures.\(^{17}\)

This thesis, with the exception of chapter 5, is not concerned with diverse information interpretation, but with the inaccessibility of information to a subset of the investors. It is supposed that a financial report creates information asymmetries

- because some investors do not use the report (they may not understand its implications, have other, e.g. liquidity motivated, reasons for trading or have higher costs of information procession)

and

- because the preparer of the financial report engaged in earnings management and only market insiders see through these manipulations.

\(^{17}\)See also Bamber, Barron and Stober (1999).

\(^{18}\)McNichols and Trueman (1994, pp. 69-70) point at another issue regarding the period prior to the disclosure: "To fully understand how mandatory public disclosures affect the extent to which there is equal access to information, however, it is necessary to examine how these disclosures affect investment in pre-announcement private information collection. If mandatory disclosures actually stimulate such investment by one set of investors relative to another, then they decrease the extent to which the 'playing field' is level prior to the disclosures."
Market participants, who are capable to process a report into private information about a firm’s performance, can be thought of as market experts who follow a firm closely (e.g., large shareholders or financial analysts). Through their activities, price impounds opinions of a firm’s performance. In the absence of the report there are no opportunities for traders capable of informed judgements to exploit their ability to process public information.

The models do not directly analyze the effect of the introduction of a public signal, but the influence of a change in the precision or informativeness of the public signal. However, it can be assumed that the quantity and precision of information are positively correlated, because a higher quantity of information renders it also more credible, since it becomes more difficult to hide a bias. Leuz and Verrecchia (2000, p. 92) note that "the theory [alluding also to the models surveyed in chapter 2] is sufficiently broad as to allow the notion of 'increased levels of disclosure' to be interpreted as either an increase in the quantity of disclosure or an increase in the quality of disclosure (or both).”

If a public signal is only observed by a subset of the investors, then it suggests itself that these informed investors benefit from the information at the cost of the less informed investors. Similarly, if the precision of one player’s information is increased, he is expected to fare better. This is a basic result in Kyle (1985) type of models. However, in the models of chapters 3 to 6, the benefit of the informed investors is also limited by the biasing decision of the manager. The presence of the informed investors decreases the extent of the bias in the report as was shown on page 78 and thus reduces their informational advantage in comparison to observers of the report alone. Thus, the overall effect depends on the interaction among all participants: the diverse investors and the market maker.

In such a scenario, it becomes important to know who generates information (and with which incentives), how that information is disseminated in capital markets and incorporated into market prices and also why the uninformed investors participate. All of these aspects influence whether the overall effect of such releases of information
benefits the uninformed investors. The formal analysis in this thesis is comprehensive in the sense that it simultaneously includes the information releaser and the price formation process (see figure 8.1). Since the manager has an interest in the share price, he will conjecture the influence of his biasing decision on the share price and take this into account in his choice. Correspondingly, by trading on the information in a financial report, investors will include the possibility that the report may be biased into their considerations. The framework allows to predict the effects of parameters of the manager’s biasing decision on the utility of the differently informed investors and to a lesser extent (because the informed investor balances out changes in market parameters) the price formation on the manager’s decision.

The most interesting effect therefore pertains to the influence of the cost of biasing of the manager and the informativeness of the report on the welfare of the informed and the liquidity traders. The marginal cost of biasing can be interpreted in a number of ways, e.g. as search costs, psychic costs, or reputation losses. It may also be under the control of a regulator if it denoted the tightness of accounting standards which

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19 Hartmann-Wendels (1991, p. 18) argues that rational investors protect themselves against the danger of being mislead.
may be reduced by less discretion, or the expected litigation costs in the case of a detection of a bias. The marginal costs make the report more informative about the terminal firm value.

Chapter 3 supposes that the market maker observes the biased report of the manager and the net order flow. The informed investor in addition learns the bias while the liquidity traders demand an exogenously given amount of shares. In this scenario the manager biases the report less if the marginal costs of biasing are higher and if there are more informed investors. The $N$ informed traders reduce the uncertainty in the firm value given net demand and the report versus the report alone by the factor $N + 1$. The informed’s trading profits decrease with the informativeness of the report and increase with the variance in liquidity demand. An analysis of the utility of the liquidity traders must remain unsatisfactory because their preferences are not explicitly modeled.

Therefore, chapter 4 includes explicit trading motivations of the liquidity traders. The biasing decision and the price informativeness remain as in chapter 3 since the informed investor balances out any influence of the liquidity demand on the price (see figure 2.1). However, the biasing decision influences the welfare of the informed versus the liquidity traders. A key insight here is that the effects depend on the trading motivation. If the uninformed investors denote an idiosyncratic value to a share that may stem from a subjective willingness to pay for the immediate execution of the trade, then their utility directly increases with the informativeness of the report. The reason is that a more informative report both increases the liquidity in the market, because the market maker uses the information out of the report directly relatively more to net demand, and reduces the risk inherent in the price. These mechanisms change when the liquidity traders are endowed with a random number of shares and try to hedge the risk in this initial position by selling shares at the beginning. Here, the liquidity traders’ utility increases if their hedging is more effective which is the case if the report is less informative. Hence, the overall effect of a more informative report is to reduce the utility of the uninformed investors. Only the ex ante information on
the firm value, which also increases the informativeness of the report, increases their utility.

The model in chapter 5 takes a more pessimistic view on the reporting system. It supposes that only the \( N \) informed investors observe the "public" information, but do not learn the inherent bias. In addition, they interpret the report diversely. The market maker only observes net demand while the uninformed investors trade exogenously. Opposite to chapters 3 and 4, here, the informed investors are the device to incorporate the bias into the price. Their number now increases the bias and the utility of the manager. The price averages out the idiosyncratic errors of the informed investors and is therefore still more informative if there are more informed investors. As before, price efficiency increases if the report is more informative. The liquidity traders, whose demand is only given exogenously which makes an analysis of their profits questionable, benefit from a more liquid market. The market liquidity, however, decreases for a higher marginal cost of biasing for the manager since in this case the net demand is more relevant for the market maker.

The final setting in chapter 6 omits the market maker. The information he observed played an important role for the equilibrium outcome in chapters 3 to 5. A new equilibrium concept in demand curves is employed, in which the traders condition their demand on the price and the price is determined by balancing supply and demand. In this setting, the biasing decision of the manager is still independent of the characteristics of the uninformed investors. However, the price informativeness is now influenced by their activities: more uninformed investors make the price more informative, while a higher risk aversion and a higher variance in their random endowment makes the price less informative. The uninformed investors benefit if the market is more liquid and if the valuation error (the difference between the terminal value and the price) is less variant. Although, here the influence of a higher marginal cost of biasing on the liquidity traders' utility is left open, it seems to be reasonable that, if the uninformed did not observe the report, the informed investors would benefit from a more precise report since only they observe it (this would also correspond
So far, the thesis deduced only positive relationships between market parameters, like the marginal cost of biasing or the number of uninformed investors, and market characteristics, like the bias in the report or the informativeness of the market price. However, positive and normative results are closely related insofar as it suggests itself to proscribe a certain change in a parameter if it has a desired positive effect. However, explicit recommendations for standard setters are not intended because the analysis looks only at one particular part of the entire disclosure environment (see figure 1.1 in section 1.1). It concentrates on equity valuation as the dominant function of accounting procedures.

In general, the "right" amount of information production\(^{20}\) equalizes the marginal social benefits with the marginal social costs. However, both costs and benefits are hard to measure. Direct costs include the bureaucracy needed to establish and administer existing regulations,\(^{21}\) but may also include proprietary costs if firms are forced to disclose, for example, technical information about valuable patents or plans for strategic initiatives. Of possibly even greater magnitude are indirect costs, arising if the socially optimal amount of information production is not reached. Since information regulations affect firms’ financing, investment, and production decisions, the indirect costs of any ‘wrong’ amount of information production can be extensive indeed.\(^{22}\)

However, even if the only user group of accounting information considered are the current investors, no definite normative conclusions can be drawn. In a setting where decision relevant information necessarily involves future-oriented, manipulative information and a public disclosure inherently creates information asymmetries instead of

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\(^{20}\)The situation is further complicated by the fact that information is a complex good; it can be more or less detailed, credible, relevant, timely, understandable and so forth. For the sake of simplicity, suppose that information were a one-dimensional good and the purpose were to determine the optimal quantity (or equivalently the precision) produced.

\(^{21}\)These costs can be considerable indeed; the costs of compliance with the Sarbanes-Oxley act, enacted in the USA in 2002, are frequently considered as exceeding its benefits; see The Economist (2005).

\(^{22}\)See Scott (2003).
relieving them as information processing skills differ among investors, the information no longer levels the playing field between market insiders and the ‘naive investors’. The effects of a more informative report depend on how the information is disseminated in the market and on the specific trading motivations of liquidity traders. In such a situation, it becomes unclear what the economic purpose of a ‘full and fair disclosure’, which calls for a disclosure of all material facts relating to securities publicly offered and sold, actually is.

In reality, equity valuation is only one of multiple financial reporting roles such as the stewardship role, taxes, regulation, or litigation.\(^{23}\) Due to this multipurpose role, normative conclusions are difficult to draw; the regulation of information disclosure has an inherently social character, as Christensen and Demski (2003) note, it needs to trade off a variety of interests. According to the public interest theory of regulation, a standard setter has the best interests of society at heart, i.e. he acts so as to maximize social welfare. However, it is impossible to please everyone. Interest groups such as security analysts, public accountants, firms in an industry, customers, environmentalists and others, will try to push through their own interests and lobby for legislature and various amounts and types of regulation. Political authorities are also an interest group who want to retain power and avoid public scandals.\(^{24}\) They will supply regulation to those constituencies which they believe will be most effective and useful in helping them to retain their power. At least according to interest group theory, it is the ‘power’ of the various groups in the political process which determines accounting standards.

\(^{23}\)For example, contracting theory suggests that the performance measure be associated with management effort and actions. This does not imply that the performance measure should be necessarily that with the highest association with stock prices; see Lambert and Larcker (1987).

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