Information Exchange with Cost Uncertainty: An Alternative Approach and New Results

Stephan O. Hornig[a)]

This version: February 26, 2004

Abstract

This paper further develops the standard modelling of information exchange between firms in the presence of cost uncertainty. In order to avoid consistency problems, we replace the normal distribution of the random variables, commonly used because of its convenient mathematical properties, by an alternative one, namely a non-symmetrically distributed random variable with a binomial positive outcome. This leads to new results concerning firms' information-disclosure policy: Confirming the empirical evidence and in contrast to the existing literature, we show that in Cournot markets firms never exchange their private information and in Bertrand markets only for very steep demand functions.

Keywords: information sharing, cost uncertainty, oligopoly

JEL classification: L13, D43, D82, C72, C73

[a)] fhs KufsteinTirol, University of Applied Sciences, International Business Studies, Andreas Hofer Straße 7, A-6330 Kufstein, Austria, Tel. +43-5372-71819210, Fax +43-5372-71819104, E-mail: stephan.hornig@fh-kufstein.ac.at, Internet: http://www.fh-kufstein.ac.at

1 Introduction

The role of uncertainty and the function of information in oligopolistic markets have been fields of major interest in recent years in Industrial Organisation research. In addition to strategic parameters as capacity, location, advertising or R&D investments which all decisively influence competition because of the commitment-effect of sunk costs (see e.g. *Shy* 1995 or *Tirole* 1988) one also has to consider the more subtle entrepreneurial strategies concerning the information-disclosure policy. Talking to managers of industrial firms or studying company press releases and annual reports, one finds a broad unanimity that there is a great readiness to reveal information on sales or demand data, but a persevering silence about the technologies applied or firms' cost structures. The same information policy pattern can be observed in the activities of existing trade associations. It is therefore an important issue to analyse the basic rationale hidden behind these decisions.

For this reason, ever since the 1970s, numerous articles on information sharing in oligopoly have been written. Pioneers in this field are *Basar*, *Ho* (1974), *Ponssard* (1979) and *Novshek*, *Sonnenschein* (1982). Two main directions of research have evolved: Models analysing demand uncertainty (cf. for example *Clarke* 1983, *Vives* 1984, *Gal-Or* 1985, *Sakai* 1986, *Kirby* 1988, *Sakai*, *Yamato* 1989, *Hviid* 1989) and models analysing unit-costs uncertainty (cf. for example *Fried* 1984, *Li* 1985, *Gal-Or* 1986, *Shapiro* 1986, *Hornig*, *Stadler* 2000). The articles of *Sakai* (1990, 1991), *Jin* (1992) and especially *Raith* (1996) present general models that contain most of the results obtained in the cited papers as special cases.[1]

In general, the authors dealing with demand uncertainty show that Cournot firms producing very close substitutes do not exchange their information, whereas for not very close substitutes and for the whole range of complementary goods, disclosure is always favourable. In contrast, under Bertrand competition, firms producing substitutes and not very close complements, generally disclose their private information, otherwise they do not. All these models which analyse demand uncertainty have in common that there is uncertainty regarding the stochastic intercept of linear demand functions. This uncertainty is modelled by assuming that the random variable is normally distributed. This is done mainly for technical reasons concerning

[1] Recently, *Vives* (1999, ch. 8) provides a non-formal and comprehensive overview and *Stadler*, *Hornig* (2000) show the effects of information sharing in a simple general model.

the mathematics of the models. Although the normal distribution is defined over the range $-\infty$ to $+\infty$, the authors implicitly assume the realisations of the random variable to be "very close" to the expected intercept in order to guarantee that the non-negativity constraint of the intercept of the demand function is fulfilled. However, this obviously contradicts the properties of the normal distribution function, creating consistency problems and a logical break in the analysis.[2] Using a distribution function that guarantees non-negativity *Hornig* (2000b, 2003, p. 111 ff.) solves this drawback. Further, as the random variables do not necessarily have to be symmetrically distributed - as is implicitly assumed by using the normal distribution -, by assuming a random variable with two possible (positive) realisations that do not need to be equally likely, the results of the existing literature can be confirmed (for the case of a symmetric distribution). However, for a non-symmetric distribution of the random variable, the firms will disclose their information for a much wider range of parameter constellations in the Bayesian Nash equilibrium than they would do in situations with a symmetric probability distribution.

In contrast to the demand uncertainty setup, in the normal distribution case with cost uncertainty and substitutive goods, Cournot firms exchange the private information about their own unit costs in the production process (cf. *Fried* 1984, *Li* 1985, *Gal-Or* 1986, *Shapiro* 1986), while Bertrand firms are better off by keeping it secret (cf. *Gal-Or* 1986, *Hornig*, *Stadler* 2000, *Jin* 2000). Firms producing complementary goods do not exchange private cost information with Cournot competition, while Bertrand firms do (cf. *Raith* 1996).

With this knowledge, the aim of this paper is to investigate the effects of changing the distributional properties of the unit-costs random variable. As in the setup with demand uncertainty, nearly all the models in the existing literature analysing unit-costs uncertainty have in common that this uncertainty is modelled by

[2] If the authors at all mention this potential consistency problem their typical justifications are: The probability of the existence of negative realisations may be reduced by an appropriate choice of the variances of the random variables (*Vives* 1984, p. 77), which however not really reduces the problem. Other autors like *Li* (1985, p. 523), *Kirby* (1988, p. 140) or *Cason* (1994, p. 7) argue that non-negativity could technically be imposed by assuming combinations of beta- and Binomial distribution or gamma- and Poisson distribution - however only with a poorer information and signal structure -, or make reference to *Ericson* (1969) for further distributional combinations.

assuming a normally distributed random variable.[3] Therefore, again the mentioned non-negativity and consistency problems arise. In this paper, however, we will model a more general information and signal structure and assume unit costs as a random variable with two possible (positive) realisations that do not have to be equally probable. Thus, we will use a distributional form as in *Hornig* (2000a, 2000b) and therefore will be able to look for parallels or differences, on the one side to the existing literature using the normal distribution and on the other side, to the effects of the distributional choice in the demand uncertainty setup.

The following section of the paper presents the assumptions and the information structure of the model. Section 3 analyses the output-setting and information-exchange decisions as well as the pricing and information-exchange decisions, respectively. This is done for both duopolists in the Bayesian Nash equilibrium and the results are compared with the ones obtained in models which assume a normal distribution. Section 4 concludes.

2 The Model

The market structure is comprised of two risk-neutral and profit-maximising firms $i, j = 1, 2$, producing differentiated goods. The (inverse) demand functions for the two products are given by

$$p_i\left(q_i, q_j\right) = \alpha - \beta\left(q_i - g q_j\right), \qquad i, j = 1, 2,\ i \neq j \tag{1}$$

with $\alpha, \beta > 0$, $0 < |g| < 1$ as parameters, p_i as the price of firm i and q_i, q_j the respective outputs of both duopolists.[4] For $g > 0$ the goods are characterised as substitutes, whereas for negative values of the substitutability parameter g, the firms produce complements and for $g = 0$ independent goods.

[3] *Shapiro* (1986) uses more general assumptions concerning the probability distributions so that in contrast to most authors not only the normal distribution is included. Within a less general information and signal structure *Stadler* (2001) substitutes the normal with a uniform distribution of unit costs and confirms the established results.

[4] This demand function results from an appropriate quadratic utility function of the type
$U\left(q_0, q_1, q_2\right) = q_0 + \alpha\left(q_1 + q_2\right) - \frac{\beta}{2}\left(q_1^2 + 2g q_1 q_2 + q_2^2\right)$
of a representative consumer with q_0 indicating the consumed quantity of the numéraire good.

In order to model cost uncertainty, the unit costs c are assumed to be stochastic. In contrast to the existing literature which uses the normal distribution (see the articles listed in the introduction), this paper analyses the case of a non-symmetric distribution. Specifically, there are two states unit costs may take on: They can be high or low represented by the index $k = H, L$. Thus, high unit costs are denoted by the parameter value $c_H \in [0, \alpha)$ and low unit costs by $c_L = hc_H$, with $0 \leq h \leq 1$. As the parameter h indicates the ratio between the two possible unit-costs levels, it may also be labelled as the unit-costs variability parameter.[5] Both firms know that low or high unit costs occur with the probabilities $\mathrm{P}(c_L) = \kappa$ and $\mathrm{P}(c_H) = 1 - \kappa$, respectively.

Before the duopolists start competing, they independently observe a private signal s_{il} about the stochastic common unit costs $c_k \in \{c_L; c_H\}$ with the index $l = H, L$ representing the the level of unit costs the signal is indicating. Thus, the signal may indicate high (s_H) or low unit costs (s_L), i.e. $s_{il} \in \{s_L, s_H\}$.[6] The relationship between the private signal s_{il} and the realised unit-costs level (represented by c_k) is assumed to be determined by the following conditional probabilities $\mathrm{P}(s_{il}|\, c_k)$ which are common knowledge to both firms:

$\mathrm{P}(s_{il}\|\, c_k)$		s_{il}	
		s_L	s_H
c_k	c_L	ξ	$1-\xi$
	c_H	$1-\xi$	ξ

Table 1: Conditional probabilities $\mathrm{P}(s_{il}|\, c_k)$.

Consequently, the quality of the signal improves with an increasing probability ξ. The private signals s_{il} can also be viewed as the firms' a priori beliefs about the unit-costs level. These are different because in the modelled uncertainty situation, the information source or interpretation method may differ. To ensure that the firms will actually consider c_L (c_H) most probable after having received the signal s_L (s_H), we assume $0.5 \leq \xi \leq 1$.[7] From the conditional probabilities in Table 1, it

[5] The special case of a deterministic scenario is given by $h = 1$.

[6] In this context, "independently" means that in spite of an identical value of the realised unit costs c_k for both duopolists, one firm may observe a private signal indicating high and the other a signal for low unit costs.

follows that the problem of incomplete information becomes less severe due to the additional signal s_{il}, but does not completely disappear.

With incomplete information, the firms have the possibility of mutually exchanging their private unit-costs information. They can do this before they start engaging in competition in the goods market. To exclude the possibility of strategic information exchange as modeled for example by *Crawford*, *Sobel* (1982) or *Okuno-Fujiwara et al.* (1990) and in order to be better able to compare the results with the mainstream literature, we stay as close as possible to the assumptions made there. For this reason, the firms are assumed to choose their exchange strategies before receiving their private signals. For this purpose, they enter into a binding agreement of either disclosing their private unit-costs information or keeping it to themselves. As is standard in the literature, a trustee or a trade association will guarantee this information exchange agreement.

If the two competitors commit themselves to complete disclosure, the amount of information concerning the expected unit-costs level both firms possess increases from only containing their own private information ($\mathbf{z_i} = \{s_i\}$, $i = 1, 2$) before exchange, to containing both signals ($\mathbf{z_i} = \{s_i, s_j\}$, $\mathbf{z_i} = \mathbf{z_j}$, $i \neq j$) afterwards. When subsequently competing on the commodity market, they can then make use of this larger information set. Of course, for the case of no disclosure, the information level remains unchanged: $\mathbf{z_i} = \{s_i\}$, $i = 1, 2$. Basically, the firms will always exchange their private information if they expect higher profits as a result of less intensive competition.[8]

To summarise, in this two stage game of incomplete information, the time and information structure of the firms results as (cf. also Figure 1):

I. On the first stage the competitors simultaneously decide about their information exchange policy:

[7] This assumption implies no loss of generality because the probability ξ is exogenous and common knowledge to both firms. For a value $\xi < 0.5$ they just would presume the opposite unit-costs level more probable, i.e. for the signal s_L (s_H) they would expect high (low) unit costs.

[8] However, it should be noted that even when firms mutually exchange information, they do not collude in the classical sense, as they maximise their profits and set their output levels or prices independently.

1. First of all, the firms commit to disclose their private unit costs information completely or not at all.
2. Player "nature" determines the unit-costs realisations c_H and c_L, while the firms only know the corresponding probabilities $\mathrm{P}(c_L) = \kappa$ and $\mathrm{P}(c_H) = 1 - \kappa$.
3. Every firm observes a private signal s_i about the unit-costs level, given the conditional probabilities $\mathrm{P}(s_L | c_L) = \mathrm{P}(s_H | c_H) = \xi$ and $\mathrm{P}(s_L | c_H) = \mathrm{P}(s_H | c_L) = 1 - \xi$ as common knowledge.
4. Firms disclose their private information or not, depending on the commitment of stage I.1.

II. On the second stage, competing in the commodity market, firms set their output quantities q_i or commodity prices p_i depending on their information sets $\mathbf{z_i}$ which are $\mathbf{z_i} = \{s_i\}$ in case of complete disclosure or $\mathbf{z_i} = \{s_i, s_j\}$ in case of no disclosure.

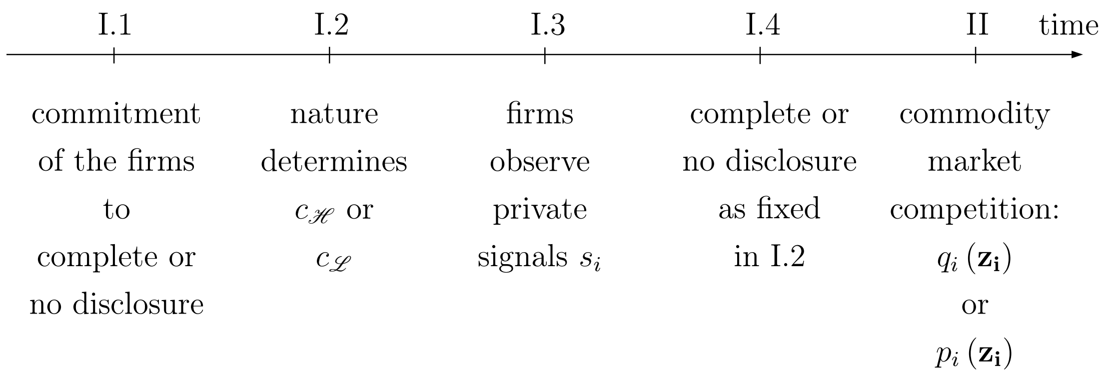

Figure 1: Time and information structure of the model.

3 Bayesian Nash Equilibria

In this section we will analyse the two basic market equilibria of Industrial Organisation: the Cournot equilibria with quantity competition and the Bertrand equilibria with price competition. In the course of this analysis, these equilibria will be derived by the standard backward induction method, depending on the existing information set available to the firms.

3.1 Bayesian Nash Equilibria of Quantities

In this two-stage model, the firms have two decision parameters by which they maximise their expected profits: Formally, the strategies of the two Cournot firms in the Bayesian Nash equilibrium consist of the output levels they produce and the decision on whether to exchange information or not. Therefore, the firms choose their respective output levels depending on their information sets $\mathbf{z_i}$, $i = 1, 2$, in order to maximise expected profits. With the demand function (1) and given the information set $\mathbf{z_i}$, the expected profit of firm i is

$$\mathrm{E}\left[\pi_i^C(q_i, q_j)\middle|\,\mathbf{z_i}\right] = \mathrm{E}\left\{\{[\alpha - \beta(q_i + gq_j) - c]\, q_i\}\,|\mathbf{z_i}\right\} \tag{2}$$

with the index C indicating Cournot competition and E as the expected value operator. Maximising this expected profit (2) leads to the reaction function of firm i given by:[9]

$$q_i^*(\mathbf{z_i}) = \frac{1}{2\beta}\left[\alpha - g\beta\mathrm{E}\left(q_j^*\,|\mathbf{z_i}\right) - \mathrm{E}\left(c\,|\mathbf{z_i}\right)\right] \tag{3}$$

The firms are symmetric in all aspects with the exception of their information set. Consequently, if the information set is identical for both, they also behave symmetrically in the equilibrium. This means that they choose an identical output $q_{1k}^* = q_{2k}^* = q_k^*$ for the signal s_{ik}, indicating the state of unit costs k ($k = L, H$).

Inserting the reaction function (3) into the profit function (2) leads to the following expected profit of firm i in reduced form which depends on the available information set $\mathbf{z_i}$ determined by both firms' exchange behaviour of the first stage:

$$\mathrm{E}\left(\pi_i^C\middle|\,\mathbf{z_i}\right) = \beta\left[q_i^*(\mathbf{z_i})\right]^2 \tag{4}$$

[9] The sufficient condition for profit maximising is globally met: $\frac{\partial^2 \mathrm{E}\left[\pi_i^C(q_i,q_j)\middle|\mathbf{z_i}\right]}{\partial q_i^2} = -2\beta < 0$. An asterisk always symbolises equilibrium values.

If instead of only one given information set $\mathbf{z_i}$ we consider all possible information sets and if we weight the corresponding expected profits of firm i in reduced form (4) with their probabilities $\mathrm{P}(\mathbf{z_i})$ there results the following ex ante expected profit of firm i in reduced form:

$$\mathrm{E}\left(\pi_i^C\right) = \beta\left\{\sum_{\mathbf{z_i}} \mathrm{P}(\mathbf{z_i})\left[q_i^*(\mathbf{z_i})\right]^2\right\} \tag{5}$$

Using (5), the expected equilibrium profits can be determined depending on the exchange behaviour. As is standard in the existing information-exchange literature, we will derive the incentives to share the private information in a comparative static manner, analysing the two extreme cases of “no information exchange” and “complete information exchange”. Note, that due to the symmetry between the firms, if it is optimal for one firm to reveal (conceal) its private signal, it is optimal for the other firm to also do so. Therefore, we can exclude asymmetric information-exchange behaviour of the firms in equilibrium.

3.1.1 No Information Exchange

If the competitors do not exchange their private information, the information set of firm i only consists of the own private signal about the unit-costs level ($\mathbf{z_i} = \{s_i\}$). Because of the assumption $\xi \geq 0.5$ and no additional information from the competitor, firm i will infer c_l $(l = L, H)$ from s_{il} and will choose the equilibrium output q_l.

Using the respective probabilities and considering that firm j may observe a private signal indicating a high (s_{jH}) or a low unit-costs level (s_{jL}), from the reaction function (3) of firm i, we obtain for the private signal s_{iL} which indicates low unit costs:

$$\begin{aligned}\mathrm{P}(s_L)\, q_L = \frac{1}{2\beta}\big\{&\alpha - g\beta\left[\mathrm{P}(s_L \wedge s_L)\, q_L + \mathrm{P}(s_L \wedge s_H)\, q_H\right] \\ &- \mathrm{P}(c_L \wedge s_L)\, c_L - \mathrm{P}(c_H \wedge s_L)\, c_H\big\}\end{aligned} \tag{6}$$

In an identical way, for the private signal s_{iH} indicating a high unit-costs level results:

$$\begin{aligned}
\mathrm{P}(s_H)\,q_H &= \frac{1}{2\beta}\big\{\alpha - g\beta\left[\mathrm{P}(s_H \wedge s_L)\,q_L + \mathrm{P}(s_H \wedge s_H)\,q_H\right] \\
&\quad -\mathrm{P}(c_L \wedge s_H)\,c_L - \mathrm{P}(c_H \wedge s_H)\,c_H\big\} \qquad (7)
\end{aligned}$$

Taking into account that $c_L = hc_H$, these equations (6) and (7) can be combined to the following equation system:

$$\begin{aligned}
&\frac{1}{\beta}\begin{pmatrix} \alpha - c_H\left[h\mathrm{P}(c_L \wedge s_L) + \mathrm{P}(c_H \wedge s_L)\right] \\ \alpha - c_H\left[h\mathrm{P}(c_L \wedge s_H) + \mathrm{P}(c_H \wedge s_H)\right] \end{pmatrix} \\
&= \begin{pmatrix} 2\mathrm{P}(s_L) + g\mathrm{P}(s_L \wedge s_L) & g\mathrm{P}(s_L \wedge s_H) \\ g\mathrm{P}(s_H \wedge s_L) & 2\mathrm{P}(s_H) + g\mathrm{P}(s_H \wedge s_H) \end{pmatrix}\begin{pmatrix} q_L \\ q_H \end{pmatrix} \qquad (8)
\end{aligned}$$

Using the probabilities, as derived in the Appendix, the equation system (8) solves for the equilibrium outputs for the respective signals indicating low or high unit costs as follows:[10]

$$\begin{aligned}
q_L &= \frac{1}{(2+g)\,\beta\left[2\xi(1-\xi) + (2+g)\,\kappa(1-\kappa)(1-2\xi)^2\right]} \\
&\quad \cdot\Big\{\alpha\left\{2\xi - (1-2\xi)\left[g\xi - (2+g)\,\kappa\right]\right\} \\
&\quad -c_H\left\{h\kappa\xi\left[2(\kappa+\xi-2\kappa\xi) + g(\kappa+2\xi-2\kappa\xi-1)\right]\right. \\
&\quad \left.+(1-\kappa)(1-\xi)\left[2\xi + (2+g)\,\kappa(1-2\xi)\right]\right\}\Big\} \qquad (9) \\
q_H &= \frac{1}{(2+g)\,\beta\left[2\xi(1-\xi) + (2+g)\,\kappa(1-\kappa)(1-2\xi)^2\right]} \\
&\quad \cdot\Big\{\alpha\left\{4\kappa\xi - (\kappa+\xi-1)\left[2+g(1-2\xi)\right]\right\} \\
&\quad -c_H\Big\{h\kappa(1-\xi)\left\{2\left[1-\xi-\kappa(1-2\xi)\right] + g(1-\kappa)(1-2\xi)\right\} \\
&\quad +(1-\kappa)\,\xi\left[2(1-\xi) - (2+g)\,\kappa(1-2\xi)\right]\Big\}\Big\} \qquad (10)
\end{aligned}$$

With the ex ante expected profit in reduced form (5), the equilibrium outputs (9), (10) and the corresponding probabilities given in the Appendix, in the no-

[10] For interested readers, an extensive mathematical appendix with derivations of all the results stated in the text is available from the author upon request.

information-sharing Cournot equilibrium, the ex ante expected profit $\mathrm{E}\left(\pi_i^{C,NN}\right)$ of firm i is:[11]

$$\begin{aligned}
\mathrm{E}\left(\pi_i^{C,NN}\right) &= \frac{1}{(2+g)^2\beta\left[(2+g)\kappa(1-\kappa)(1-2\xi)^2+2\xi(1-\xi)\right]^2} \\
&\cdot\Bigg\{[1-\xi-\kappa(1-2\xi)]\Big\{\alpha\{2\xi-(1-2\xi)[g\xi-(2+g)\kappa]\} \\
&-c_H\{h\kappa\xi[2(\kappa+\xi-2\kappa\xi)+g(\kappa+2\xi-2\kappa\xi-1)] \\
&+(1-\kappa)(1-\xi)[2\xi+(2+g)\kappa(1-2\xi)]\}\Big\}^2 \\
&+(\kappa+\xi-2\kappa\xi)\Big\{\alpha\{4\kappa\xi-(\kappa+\xi-1)[2+g(1-2\xi)]\} \\
&-c_H\Big\{h\kappa(1-\xi)\{2[1-\xi-\kappa(1-2\xi)]+g(1-\kappa)(1-2\xi)\} \\
&+(1-\kappa)\xi[2(1-\xi)-(2+g)\kappa(1-2\xi)]\Big\}\Big\}^2\Bigg\} \qquad (11)
\end{aligned}$$

As can be seen from equation (11), the expected profit in the no-information-sharing Cournot equilibrium depends on the demand parameters α, β and g, on the unit-costs variability h, on the unit-costs level c_H as well as on the probabilities κ and ξ.

3.1.2 Complete Information Exchange

If the firms disclose their information completely, the information set of both is identical and consists of the two private unit-costs signals: $\mathbf{z_i} = \{s_i, s_j\}$, $i = 1, 2$, $i \neq j$. With it, the optimality condition (3) of firm i can be expressed as:

$$q_i(s_i, s_j) = \frac{1}{2\beta}\{\alpha - g\beta q_j(s_i, s_j) - \mathrm{E}[c\,|(s_i \wedge s_j)]\} \qquad (12)$$

As the firms are symmetric, with identical information sets they consequently produce the identical equilibrium output $q_i(s_i, s_j) = q_j(s_i, s_j) =: q(s_i, s_j)$. This implies for equation (12):

[11] The index NN characterises the situation when neither of the two firms discloses any information.

$$q\left(s_i, s_j\right) = \frac{\alpha - \mathrm{E}\left[c\left|\left(s_i \wedge s_j\right)\right.\right]}{(2+g)\,\beta} \tag{13}$$

With the probabilities κ and ξ from Table 1 and Bayes' theorem, the three possible signal combinations (s_L, s_L), (s_L, s_H) and (s_H, s_H) lead to the three corresponding output levels q_{LL}, q_{LH} and q_{HH}:

$$q_{LL} = \frac{\alpha\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right] - c_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]}{(2+g)\,\beta\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]} \tag{14}$$

$$q_{LH} = \frac{\alpha - c_H\left[1-(1-h)\,\kappa\right]}{(2+g)\,\beta} \tag{15}$$

$$q_{HH} = \frac{\alpha\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] - c_H\left[h\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]}{(2+g)\,\beta\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]} \tag{16}$$

Using the ex ante expected profit in reduced form (5), the equilibrium output levels (14) to (16) and the corresponding probabilities derived in the Appendix, the expected profit of firm i in the Cournot equilibrium with complete information exchange is:[12]

$$\begin{aligned}\mathrm{E}\left(\pi_i^{C,RR}\right) = {} & \frac{1}{(2+g)^2\,\beta\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]} \\ & \cdot\Big\{\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] \\ & \cdot\left\{\alpha\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right] - c_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\right\}^2 \\ & + 2\xi(1-\xi)\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] \\ & \cdot\left\{\alpha - c_H\left[1-(1-h)\,\kappa\right]\right\}^2 + \left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right] \\ & \cdot\left\{\alpha\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] - c_H\left[h\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]\right\}^2\Big\}\end{aligned} \tag{17}$$

Thus, the expected profit in the complete-information-sharing Cournot equilibrium depends on the same parameters as in the no-exchange case.

12 The index RR characterises the situation when both firms completely reveal their private information.

3.1.3 Which Information-Exchange Strategy Do Cournot Firms Choose?

The decision criterion for the profit-maximising firms is the difference in the respective expected profits $\Delta \mathrm{E}\left(\pi_i^{C,RR/NN}\right) := \mathrm{E}\left(\pi_i^{C,RR}\right) - \mathrm{E}\left(\pi_i^{C,NN}\right)$. This reflects the firm's rationale that it wants to choose the strategy that bears the highest possible expected profit. For a positive profit difference, it will exchange its private information, for a negative it will not. In order to obtain the profit-difference function we define the relative importance of unit costs in the Cournot case as $f^C := \frac{c_H}{\alpha}$. Then, using the complete-exchange profit (17) and the no-exchange profit (11), we obtain:

$$\begin{aligned} \Delta \mathrm{E}\left(\pi_i^{C,RR/NN}\right) &= \Delta \mathrm{E}\left[\pi_i^{C,RR/NN}\left(\alpha, \beta, f^C, g, h, \kappa, \xi\right)\right] \\ &\sim \Delta \mathrm{E}\left[\pi_i^{C,RR/NN}\left(f^C, g, h, \kappa, \xi\right)\right] \end{aligned} \quad (18)$$

As the sign of the expected profit difference (18) cannot be analytically identified directly, instead of the explicit equational form we use this abbreviated functional form. Both expected profits (11) and (17) depend on the exogenous demand parameters α, β and g, on the given cost parameters h and f^C as well as on the (equally exogenous) probabilities κ and ξ. This also holds for the difference in expected profits (18). However, the sign of the profit difference only depends on the substitution parameter g, on the unit-costs variability h, on the relative unit-costs importance f^C, and on the probabilities κ and ξ. By contrast, the other demand parameters, i.e. the absolute demand level α and the slope parameter β, have no influence on the decision as they only function as shift parameters.

In addition to these common statements about the relevance of the various parameters of the model, numerical analysis of this Cournot situation leads to the general conclusion that in an environment of unit-costs uncertainty, quantity-setting firms will never disclose their private information. Figure 2 shows a graphical visualisation of the expected profit differences $\Delta \mathrm{E}\left(\pi_i^{C,RR/NN}\right)$ depending on the probabilities κ and ξ. The relief of the expected profit difference "landscape", as shown in Figure 2, is the typical one. The shape of this landscape does not differ fundamentally if any of the parameters f^C, g or h changes. As one would expect, under the highest decision uncertainty ($\kappa = 0.5$ and $\xi = 0.5$) the expected-profit difference shows up the lowest losses. This means that under these conditions, the firms' willingness

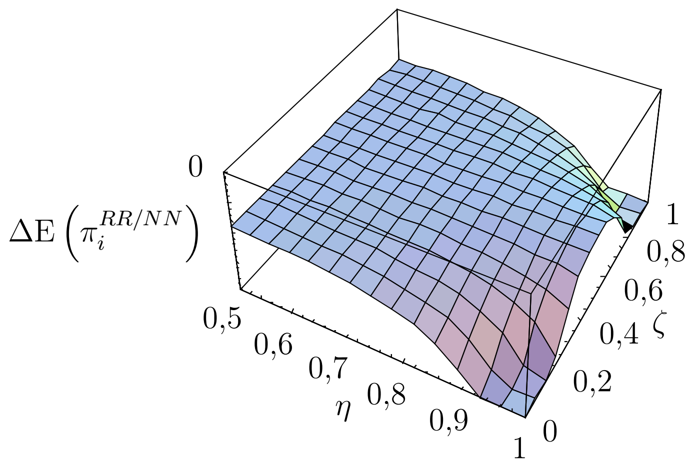

Figure 2: Expected profit differences $\Delta \mathrm{E}\left(\pi_i^{C,RR/NN}\right)$ with Cournot competition.

to exchange information is highest, although the incentive does not suffice as the no-information-sharing expected profits $\mathrm{E}\left(\pi_i^{C,NN}\right)$ are still above the information-sharing expected profits $\mathrm{E}\left(\pi_i^{C,RR}\right)$. The highest expected losses can be observed when ξ is highest, and simultaneously κ is close to one or zero. The reason is that a high probability ξ represents a high signal quality. In this case of very good own information, additional information by the rival is less attractive. Further, κ close to one or zero means that the realisation of unit costs is almost certainly known. Hence, with these parameter constellations, information exchange becomes even more unattractive.

Additionally, there is a monotonous relationship between the substitutability parameter g and the expected profit difference: The closer substitutes the goods become, i.e. the higher g is, the less desirable is information exchange. Besides, it can be seen that the higher the unit-costs variability h is, the more the expected-profit difference landscape is flattened. Finally, the higher the relative unit-costs importance f^C is, the more the landscape tilts down towards $\kappa = 1$.

In models using the normal distribution (cf. the articles cited above), Cournot firms producing substitutes deliberately exchange their private cost information, whereas for complementary goods, disclosure is never favourable. Thus, in contrast to the existing literature, with unit costs modelled as a random variable with a binomial outcome, not even firms producing substitutive goods will be willing to disclose private information.

3.2 Bayesian Nash Equilibria of Prices

The case of Bertrand competition between the two firms will be analysed in a similar way. Again, there are two decision parameters of the firms: The Bertrand equilibria consist of the prices the competitors demand and the decision on whether to exchange the private unit-costs information or not. Therefore, on the second stage of the game the firms choose their respective prices depending on their information sets $\mathbf{z_i}$, $i = 1, 2$, (determined on the first stage) in order to maximise expected profits. With the (inverse) demand functions (1), the demand functions for the price-setting firms are

$$q_i\left(p_i, p_j\right) \quad = \quad a - b\left(p_i - gp_j\right) \ , \qquad i, j = 1, 2 \tag{19}$$

with $a := \frac{\alpha}{(1+g)\beta}$ and $b := \frac{1}{(1-g^2)\beta}$ as positive parameters. Given the information set $\mathbf{z_i}$, and with the demand function (19), the expected profit of firm i is

$$\mathrm{E}\left[\pi_i^B\left(p_i, p_j\right)\middle|\, \mathbf{z_i}\right] \quad = \quad \mathrm{E}\left\{\left\{\left(p_i - c\right)\left[a - b\left(p_i - gp_j\right)\right]\right\}\left|\mathbf{z_i}\right.\right\} \qquad , \tag{20}$$

where the index B indicates Bertrand competition. Maximising the expected profit (20), leads to the reaction function of firm i as:[13]

$$p_i^*\left(\mathbf{z_i}\right) \quad = \quad \frac{1}{2b}\left\{a + b\left[g\mathrm{E}\left(p_j\left|\mathbf{z_i}\right.\right) + \mathrm{E}\left(c\left|\mathbf{z_i}\right.\right)\right]\right\} \tag{21}$$

The firms are symmetric in all aspects with the exception of their respective private signals s_i. Consequently, if these private signals imply for both competitors identical information sets $\mathbf{z_i}$, they also behave symmetrically in the equilibrium. This means that in the case of no disclosure after having observed a private signal s_{il} which inticates the state of unit costs $l \in \{L, H\}$ they choose an identical price $p_{1l}^* = p_{2l}^* = p_l^*$. Accordingly, in the case of complete information exchange corresponding to identical information sets $\mathbf{z_1} = \mathbf{z_2} = \{s_{1l}, s_{2k}\}$, $l, k \in \{L, H\}$ both firms choose $p_{1lk}^* = p_{2lk}^* = p_{lk}^*$.

Inserting the reaction function (21) into the expected profit function (20), leads to the following reduced form expected profit of firm i, which depends on the available

[13] The sufficient condition for profit maximizing is globally met: $\frac{\partial^2 \mathrm{E}\left[\pi_i^B(p_i,p_j)|\mathbf{z_i}\right]}{\partial p_i^2} = -2b < 0$.

information set $\mathbf{z_i}$ determined by the exchange behaviour decision in the first stage of the game:

$$\mathrm{E}\left(\pi_i^B \middle| \mathbf{z_i}\right) = b\left[p_i^*\left(\mathbf{z_i}\right) - \mathrm{E}\left(c\left|\mathbf{z_i}\right.\right)\right]^2 \tag{22}$$

Weighting these ex ante expected profits (22) by their probabilities and aggregating for all possible information sets we obtain the ex ante expected profit of firm i:

$$\mathrm{E}\left(\pi_i^B\right) = b\left\{\sum_{\mathbf{z_i}} \mathrm{P}\left(\mathbf{z_i}\right)\left[p_i^*\left(\mathbf{z_i}\right) - \mathrm{E}\left(c\left|\mathbf{z_i}\right.\right)\right]^2\right\} \tag{23}$$

Using this profit function, the ex ante expected equilibrium profits can be determined depending on the exchange behaviour. Again and as in the case of Cournot competition, we will derive the incentives to share the private information in a comparative static manner, analysing only the two extreme cases of "no information exchange" and "complete information exchange".

3.2.1 No Information Exchange

If the firms do not exchange their private information, each information set only consists of the own private signal about the level of unit costs ($\mathbf{z_i} = \{s_i\}$). As $\xi \geq 0.5$, with no additional information from the competitor, firm i will infer c_l, $l = H, L$ from s_{il} and will choose the equilibrium price p_k.

Using the corresponding probabilities and the reaction function (21) of firm i, we respectively obtain for the private signal s_{iL}, indicating a low, and for s_{iH}, indicating a high unit-costs level:

$$\begin{aligned}\mathrm{P}\left(s_L\right)p_L &= \frac{1}{2b}\Big\{a + b\big\{g\left[\mathrm{P}\left(s_L \wedge s_L\right)p_L + \mathrm{P}\left(s_L \wedge s_H\right)p_H\right] \\ &\quad + \mathrm{P}\left(c_L \wedge s_L\right)c_L + \mathrm{P}\left(c_H \wedge s_L\right)c_H\big\}\Big\} \qquad (24)\\ \mathrm{P}\left(s_H\right)p_H &= \frac{1}{2b}\Big\{a + b\big\{g\left[\mathrm{P}\left(s_H \wedge s_L\right)p_L + \mathrm{P}\left(s_H \wedge s_H\right)p_H\right] \\ &\quad + \mathrm{P}\left(c_L \wedge s_H\right)c_L + \mathrm{P}\left(c_H \wedge s_H\right)c_H\big\}\Big\} \qquad (25)\end{aligned}$$

Equations (24) and (25) can be combined to the equation system:

$$\begin{pmatrix} \frac{a}{b} + c_H \left[h\mathrm{P}\left(c_L \wedge s_L\right) + \mathrm{P}\left(c_H \wedge s_L\right)\right] \\ \frac{a}{b} + c_H \left[h\mathrm{P}\left(c_L \wedge s_H\right) + \mathrm{P}\left(c_H \wedge s_H\right)\right] \end{pmatrix}$$
$$= \begin{pmatrix} 2\mathrm{P}\left(s_L\right) - g\mathrm{P}\left(s_L \wedge s_L\right) & -g\mathrm{P}\left(s_L \wedge s_H\right) \\ -g\mathrm{P}\left(s_H \wedge s_L\right) & 2\mathrm{P}\left(s_H\right) - g\mathrm{P}\left(s_H \wedge s_H\right) \end{pmatrix} \begin{pmatrix} p_L \\ p_H \end{pmatrix} \tag{26}$$

Inserting the probabilities, as derived in the Appendix, allows us to solve for the equilibrium prices for the respective signals indicating either low or high unit-costs levels:

$$\begin{aligned} p_L \;=\;& \frac{1}{(2-g)\, b \left[2\xi\left(1-\xi\right) + \left(2-g\right)\kappa\left(1-\kappa\right)\left(1-2\xi\right)^2\right]} \\ & \cdot \Big\{ g\xi\left(1-\xi\right)\left\{a + bc_H\left[h\kappa\left(1-\xi\right) + \left(1-\kappa\right)\xi\right]\right\} \\ & + \left[\left(2-g\right)\kappa\left(1-2\xi\right) + \xi\left(2-g\xi\right)\right]\Big\{a + bc_H\left\{1-\xi+\kappa\left[\left(1+h\right)\xi-1\right]\right\}\Big\}\Big\} \end{aligned} \tag{27}$$

$$\begin{aligned} p_H \;=\;& \frac{1}{(2-g)\, b \left[2\xi\left(1-\xi\right) + \left(2-g\right)\kappa\left(1-\kappa\right)\left(1-2\xi\right)^2\right]} \\ & \cdot \Big\{ g\xi\left(1-\xi\right)\Big\{a + bc_H\left\{1-\xi+\kappa\left[\left(1+h\right)\xi-1\right]\right\}\Big\} \\ & - \left\{2\left(\kappa+\xi-2\kappa\xi-1\right) + g\left[\left(1-\xi\right)^2 - \kappa\left(1-2\xi\right)\right]\right\} \\ & \cdot \Big\{a + bc_H\left\{\xi + \kappa\left[h - \left(1+h\right)\xi\right]\right\}\Big\}\Big\} \end{aligned} \tag{28}$$

Using the ex ante expected equilibrium profit in reduced form (23), the equilibrium prices (27) and (28) and the corresponding probabilities given in the Appendix, the ex ante expected profit of firm i in the no-information-sharing Bertrand equilibrium is:

$$\begin{aligned} \mathrm{E}\left(\pi_i^{B,NN}\right) \;=\;& b\Big\{\left[\kappa\xi + \left(1-\kappa\right)\left(1-\xi\right)\right] \\ & \cdot\Big\{\frac{1}{(2-g)\, b \left[2\xi\left(1-\xi\right) + \left(2-g\right)\kappa\left(1-\kappa\right)\left(1-2\xi\right)^2\right]} \\ & \cdot\Big\{ g\xi\left(1-\xi\right)\left\{a + bc_H\left[h\kappa\left(1-\xi\right) + \left(1-\kappa\right)\xi\right]\right\} \end{aligned}$$

$$
\begin{aligned}
&+\left[(2-g)\kappa(1-2\xi)+\xi(2-g\xi)\right] \\
&\cdot\Big\{a+bc_H\left\{1-\xi+\kappa\left[(1+h)\xi-1\right]\right\}\Big\}\Big\} \\
&-\frac{h\kappa\xi+(1-\kappa)(1-\xi)}{\kappa\xi+(1-\kappa)(1-\xi)}c_H\Big\}^2+\left[\kappa(1-\xi)+(1-\kappa)\xi\right] \\
&\cdot\Bigg\{\frac{1}{(2-g)b\left[2\xi(1-\xi)+(2-g)\kappa(1-\kappa)(1-2\xi)^2\right]} \\
&\cdot\Big\{g\xi(1-\xi)\Big\{a+bc_H\left\{1-\xi+\kappa\left[(1+h)\xi-1\right]\right\}\Big\} \\
&-\left\{2(\kappa+\xi-2\kappa\xi-1)+g\left[(1-\xi)^2-\kappa(1-2\xi)\right]\right\} \\
&\cdot\Big\{a+bc_H\left\{\xi+\kappa\left[h-(1+h)\xi\right]\right\}\Big\}\Big\} \\
&-\frac{h\kappa(1-\xi)+(1-\kappa)\xi}{\kappa(1-\xi)+(1-\kappa)\xi}c_H\Big\}^2\Bigg\} \qquad (29)
\end{aligned}
$$

As can be seen from equation (29), the expected profit depends on the demand parameters a, b and g, on the unit-costs variability h, on the unit-costs level c_H, as well as on the probabilities κ and ξ.

3.2.2 Complete Information Exchange

Just as in the Cournot case, if the firms disclose their information completely, the information sets of both are identical and consist of the two private unit-costs signals: $\mathbf{z_i} = \{s_i, s_j\}$, $i = 1, 2$, $i \neq j$. The optimality condition (21) of firm i can now be written as:

$$
p_i(s_i, s_j) = \frac{1}{2b}\Big\{a + b\left\{g p_j(s_i, s_j) + \mathrm{E}\left[c\,|(s_i \wedge s_j)\right]\right\}\Big\} \qquad (30)
$$

With symmetric firms, they consequently choose the identical equilibrium price $p_i(s_i, s_j) = p_j(s_i, s_j) =: p(s_i, s_j)$. From equation (30) this implies:

$$
p(s_i, s_j) = \frac{a + b\mathrm{E}\left[c\,|(s_i \wedge s_j)\right]}{(2-g)b} \qquad (31)
$$

Using the probabilities in Table 1 and Bayes' theorem, the three possible signal combinations (s_L, s_L), (s_L, s_H) and (s_H, s_H) lead to the corresponding prices p_{LL}, p_{LH} and p_{HH}:

$$p_{LL} = \frac{a\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right] + bc_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]}{(2-g)\,b\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]} \quad (32)$$

$$p_{LH} = \frac{a + bc_H\left[1-(1-h)\kappa\right]}{(2-g)\,b} \quad (33)$$

$$p_{HH} = \frac{a\left[\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right] + bc_H\left[h\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right]}{(2-g)\,b\left[\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right]} \quad (34)$$

With the ex ante expected equilibrium profit in reduced form (23), the equilibrium prices (32) to (34) and the corresponding probabilities derived in the Appendix, the expected profit of firm i in the Bertrand equilibrium with complete information exchange is:

$$\begin{aligned}
\mathrm{E}\left(\pi_i^{B,RR}\right) = {} & b\Bigg\{\frac{1}{\kappa\xi^2 + (1-\kappa)(1-\xi)^2} \\
& \cdot\Bigg\{\frac{a\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right] + bc_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]}{(2-g)\,b} \\
& - \left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]c_H\Bigg\}^2 \\
& +2\xi(1-\xi)\left\{\frac{a + bc_H\left[1-(1-h)\kappa\right]}{(2-g)\,b} - \left[1-(1-h\kappa)\right]c_H\right\}^2 \\
& +\frac{1}{\kappa(1-\xi)^2 + (1-\kappa)\xi^2} \\
& \cdot\Bigg\{\frac{a\left[\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right] + bc_H\left[h\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right]}{(2-g)\,b} \\
& - \left[h\kappa(1-\xi)^2 + (1-\kappa)\xi^2\right]c_H\Bigg\}^2\Bigg\}
\end{aligned} \quad (35)$$

Thus, the ex ante expected profit (35) depends on the same parameters as in the no-information-exchange case.

3.2.3 Which Information-Exchange Strategy Do Bertrand Firms Choose?

Whether profit-maximising Bertrand firms disclose their information or not depends on the difference of the respective expected profits $\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right) := \mathrm{E}\left(\pi_i^{B,RR}\right) - \mathrm{E}\left(\pi_i^{B,NN}\right)$. Of course, as in the Cournot case, the rivals will exchange their private unit-costs information for a positive profit difference, for a negative difference they will not. With the no-exchange profit equation (29) and the complete-exchange profit equation (35), we obtain:

$$\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right) = \Delta\mathrm{E}\left[\pi_i^{B,RR/NN}\left(a, b, g, h, c_H, \kappa, \xi\right)\right] \tag{36}$$

As the sign of this ex ante expected profit difference (36) cannot be unambiguously determined analytically again we will argue on the base of numerical analysis in order to determine - depending on the market conditions - which information-exchange decisions both competitors will make.

In order to facilitate the interpretation of the numerical results we define (analogically to the Cournot case above) a parameter $f^B := \frac{c_H}{a}$ of the relative importance of unit costs in the Bertrand case with $f^B \in [0, 1)$ and B indicating Bertrand competition. Considering this and the ex ante expected firm profits (29) and (35) the ex ante expected profit difference in the Betrand case may also be represented by the following function:

$$\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right) = \Delta\mathrm{E}\left[\pi_i^{B,RR/NN}\left(a, b, f^B, g, h, \kappa, \xi\right)\right] \tag{37}$$

As both expected profits (29) and (35) depend on the absolute demand level a, on the demand slope parameter b, on the substitutability parameter g, on the unit-costs variability h, on the unit-costs importance f^B, and on the probabilities κ and ξ which are all exogenous, the same also holds for the difference in expected profits (37). In contrast to the Cournot case treated above, the sign of the profit difference depends on all these parameters, too.

Numerical simulations of the ex ante expected profit difference (37) lead to the following decision rules, shown graphically in Figures 3 to 5. These figures represent the expected-profit differences $\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right)$ depending on the probabilities κ and

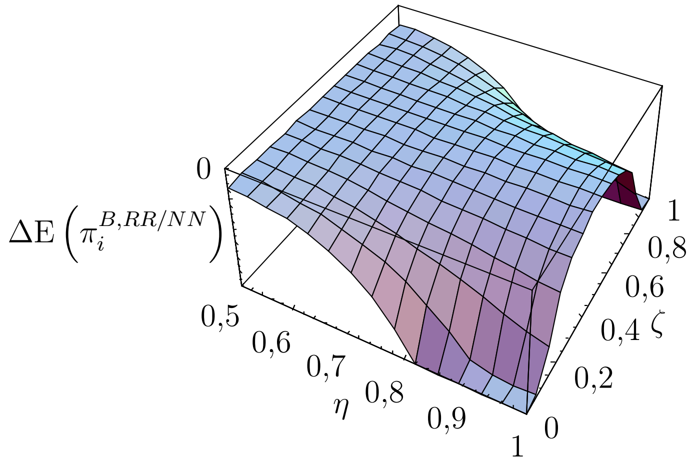

Figure 3: Ex ante expected profit difference $\Delta E\left(\pi_i^{B,RR/NN}\right)$ with Bertrand competition and a not very high demand slope parameter b.

ξ. Figure 3 shows the whole plot of the expected profit difference which is negative for the entire range. In Figures 4 and 5 we cut the plot at the zero expected-profit difference level $\left(\Delta E\left(\pi_i^{B,RR/NN}\right) = 0\right)$ and explicitly show the negative regions only. Consequently, the flat areas represent parameter combinations with positive, the deepenings combinations with negative profit differences.

An overall view on the results of the numerical analysis of the information-exchange situation with price competition leads to the general conclusion that the firms scarcely ever reveal their private unit-costs information. The ex ante expected profit difference (37) is negative for nearly all possible parameter combinations (cf. Figure 3 for a typical visualisation of the ex ante expected profit differences). Like in the case of quantity competition above a decision uncertainty effect is observed: As can be seen in Figure 3 the relative losses of the competitors from information exchange again are lowest in the case of highest decision uncertainty ($\kappa = 0,5,\ \xi = 0,5$).

However, there may be identified parameter combinations that lead to positive ex ante expected profit differences (cf. Figure 4 in the case of complete substitutes and Figure 5 in the case of complete complements), which implies complete information exchange by the competitors. In general, these regions only occur for very high values of the demand parameter b, i.e. for a very steep slope of the underlying demand function. This reflects the fact that with a steep demand function, small price variations cause heavy profit changes. This leads to a powerful incentive to re-

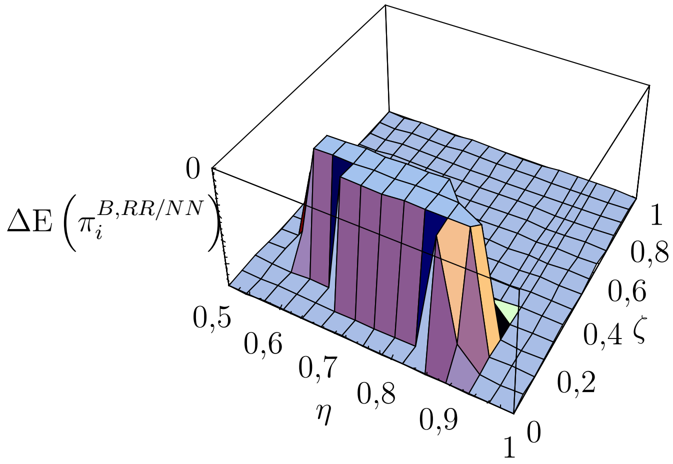

Figure 4: Ex ante expected profit difference $\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right)$ with Bertrand competition in the case of perfect substitutes, a very high demand slope parameter b and high variabiliy of unit costs (i.e. low h).

duce unit-costs uncertainty via information exchange in order to avoid price settings which prove to be suboptiomal ex post. This demand-slope effect even dominates the decision-uncertainty effect. For lower values of b, the expected profit difference landscape exhibits relief shapes as in Figure 3 (independent of the values of the parameters a, f^B, g and h).

In the case of a very high parameter value b, i.e. in a situation which may be characterised by complete information exchange, it appears that independent of the substitutability of the goods an increasing relative importance of unit costs (i.e. an increasing value of f^B) induces a shrinking region of complete information exchange. This effect particularly occurs in presence of a low unit costs variability, i.e. a high value of h. Additionally, even on its own a decreasing unit costs variability is able to drastically reduce the parameter region of complete information exchange. In this context, there can be shown for the case of high values of h and close substitutes no parameter combination can exist that leads to complete information exchange as an optimal firm strategy.

In the existing literature which uses the normal distribution, Bertrand firms always disclose their private unit-costs information. In the setup of unit costs as a random variable with a binomial outcome as modelled here the firms only consider information sharing for very steep demand functions. Thus, apart from this exceptional

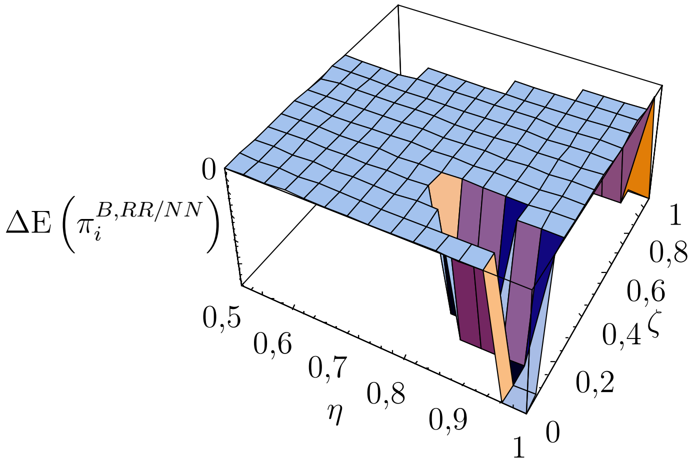

Figure 5: Ex ante expected profit difference $\Delta\mathrm{E}\left(\pi_i^{B,RR/NN}\right)$ with Bertrand competition in the case of complete complements, a very high demand slope parameter b and high variabiliy of unit costs (i.e. low h).

case, we again observe a concordance between the setup using the normal distribution and the setup of unit costs as a random variable with a binomial outcome with respect to the optimal information exchange decisions of the firms.

4 Concluding Remarks

Starting from the non-negativity and consistency problems of the normal-distribution assumption in the standard modelling of the existing literature which deals with information exchange between firms in the presence of unit-costs uncertainty, this paper provides an alternative approach. We have replaced the normal distribution of the random variables and signals, commonly used because of its convenient mathematical properties, by a non-symmetrically distributed random variable with a binomial positive outcome. Hence, the model here solves two drawbacks of the existing literature concerning information exchange by assuming a non-symmetric distribution in which only positive realisations of the random variable are allowed.

In contrast to the existing literature using the normal distribution, we found that in Cournot markets firms never exchange their private unit-costs information and in Bertrand markets only for a very steep demand. Thus, in a world with price competition and substitutive goods, the observed behaviour of managers and trade associations of not disclosing unit-costs information can be explained for almost all theoretically thinkable situations. Therefore, we can further reinforce *Stadler*'s (2001) message: "*Talk is Silver, Silence is Golden.*" These new results are driven by the alternative distributional approach presented here. The observed decision-uncertainty effect is generally not strong enough so as to induce information revelation by the firms. The only pro-disclosure force in our model that is powerful enough to prevail stems from the demand-slope effect.

While the inverse symmetric results of Cournot relative to Bertrand competition information-exchange behaviour - a well established fact in the information-exchange literature - can also be observed in our distributional setup for the case of demand uncertainty (cf. *Hornig* 2000a, 2000b), it vanishes for the case of cost uncertainty here.

Nevertheless, for the practically relevant situation of price competition with substitutive goods, the presented model provides further support for the robustness of the theoretical equilibrium result as well as the empirical observation, both showing no information sharing. With this in mind, our non-symmetric distribution setup fills another gap in the research program *Novshek* (1996, p. 14 f.) propagates in saying: "*Since there can be no hope of finding a general model that provides unambiguous policy implications, the alternative is to expand the set of 'boxes' covered so as to create a better fit with the real markets of concern to practitioners.*" Our result of no disclosure under the empirically relevant market conditions also may weaken the apprehensions of authors like *Neumann* (2000, p. 128 ff.), who criticise the collusion-encouraging effects of information-sharing agreements from the point of view of anti-trust policy.

Appendix

From Table 1 and κ, indicating the probability of low unit-costs realisations, it is possible to derive the following probabilities:

$$\mathrm{P}\left(c_L \wedge s_L\right) = \kappa\xi \tag{A.1}$$
$$\mathrm{P}\left(c_H \wedge s_L\right) = \left(1-\kappa\right)\left(1-\xi\right) \tag{A.2}$$
$$\mathrm{P}\left(c_L \wedge s_H\right) = \kappa\left(1-\xi\right) \tag{A.3}$$
$$\mathrm{P}\left(c_H \wedge s_H\right) = \left(1-\kappa\right)\xi \tag{A.4}$$
$$\mathrm{P}\left(s_L\right) = \kappa\xi + \left(1-\kappa\right)\left(1-\xi\right) \tag{A.5}$$
$$\mathrm{P}\left(s_H\right) = \kappa\left(1-\xi\right) + \left(1-\kappa\right)\xi \tag{A.6}$$
$$\mathrm{P}\left(s_L \wedge s_L\right) = \kappa\xi^2 + \left(1-\kappa\right)\left(1-\xi\right)^2 \tag{A.7}$$
$$\mathrm{P}\left(s_L \wedge s_H\right) = \xi\left(1-\xi\right) \tag{A.8}$$
$$\mathrm{P}\left(s_H \wedge s_L\right) = \xi\left(1-\xi\right) \tag{A.9}$$
$$\mathrm{P}\left(s_H \wedge s_H\right) = \kappa\left(1-\xi\right)^2 + \left(1-\kappa\right)\xi^2 \tag{A.10}$$

References

Basar, T., Ho, Y.-C. (1974): Informational Properties of the Nash Solutions of two Stochastic Nonzero-sum Games. *Journal of Economic Theory* 7, 370–387.

Cason, T. N. (1994): The Strategic Value of Asymmetric Information Access for Cournot Competitors. *Information Economics and Policy* 6, 3–24.

Clarke, R. N. (1983): Collusion and the Incentives for Information Sharing. *Bell Journal of Economics* 14, 383–394.

Crawford, V. P., Sobel, J. (1982): Strategic Information Transmission. *Econometrica* 50, 1431–1451.

Ericson, W. A. (1969): A Note on the Posterior Mean of a Population Mean. *Journal of the Royal Statistical Society* 31, 332–334.

Fried, D. (1984): Incentives for Information Production and Disclosure in a Duopolistic Environment. *Quarterly Journal of Economics* 99, 367–381.

Gal-Or, E. (1985): Information Sharing in Oligopoly. *Econometrica* 53, 329–343.

Gal-Or, E. (1986): Information Transmission - Cournot and Bertrand Equilibria. *Review of Economic Studies* 53, 85–92.

Hornig, S. O. (2000a): When Do Firms Exchange Demand Information? *ifo Studien* 46, 381–394.

Hornig, S. O. (2000b): When Do Firms Exchange Information? Tübinger Diskussionsbeitrag Nr. 181.

Hornig, S. O. (2003): *Informationsaustausch im Oligopol? Zur Informationspolitik von Unternehmen in ihren Branchenverbänden*. Wiesbaden.

Hornig, S. O., Stadler, M. (2000): No Information Sharing in Oligopoly: The Case of Price Competition with Cost Uncertainty. Tübinger Diskussionsbeitrag Nr. 194.

Hviid, M. (1989): Risk-averse Duopolists and Voluntary Information Transmission. *Journal of Industrial Economics* 38, 49–64.

Jin, J. Y. (1992): Information Sharing in Oligopoly: A General Model. Discussion Paper No. 1026, Krannert Graduate School of Management, Purdue University, West Lafayette.

Jin, J. Y. (2000): A Comment on “A General Model of Information Sharing in Oligopoly”, Vol. 71 (1996), 260-288, by Michael Raith. *Journal of Economic Theory* 93, 144–145.

Kirby, A. J. (1988): Trade Associations as Information Exchange Mechanisms. *Rand Journal of Economics* 19, 138–146.

Li, L. (1985): Cournot Oligopoly with Information Sharing. *Rand Journal of Economics* 16, 521–536.

Neumann, M. (2000): *Wettbewerbspolitik. Geschichte, Theorie und Praxis*. Wiesbaden.

Novshek, W. (1996): Directions for Research in Information Sharing. In: Albach, H., Jin, J. Y., Schenk, C. (eds.), *Collusion through Information Sharing? New Trends in Competition Policy*, Berlin, 13–26.

Novshek, W., Sonnenschein, H. (1982): Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release. *Bell Journal of Economics* 13, 214–218.

Okuno-Fujiwara, M., Postlewaite, A., Suzumura, K. (1990): Strategic Information Revelation. *Review of Economic Studies* 57, 25–47.

Ponssard, J. P. (1979): The Strategic Role of Information on the Demand Function in an Oligopolistic Market. *Management Science* 25, 25–47.

Raith, M. (1996): A General Model of Information Sharing in Oligopoly. *Journal of Economic Theory* 71, 260–288.

Sakai, Y. (1986): Cournot and Bertrand Equilibria under Imperfect Information. *Journal of Economics* 46, 213–232.

Sakai, Y. (1990): Information Sharing in Oligopoly: Overview and Evaluation. Part I. Alternative Models with a Common Risk. *Keio Economic Studies* 27, 17–42.

Sakai, Y. (1991): Information Sharing in Oligopoly: Overview and Evaluation. Part II. Private Risks and Oligopoly Models. *Keio Economic Studies* 28, 51–71.

Sakai, Y., Yamato, T. (1989): Oligopoly, Information and Welfare. *Journal of Economics* 49, 3–24.

Shapiro, C. (1986): Exchange of Cost Information in Oligopoly. *Review of Economic Studies* 53, 433–446.

Shy, O. (1995): *Industrial Organization. Theory and Applications*. Cambridge, Mass., London.

Stadler, M. (2001): Reden ist Silber, Schweigen ist Gold. Zur Relevanz asymmetrischer Kosteninformation im Preiswettbewerb. *WiSt* 30, 219–222.

Stadler, M., Hornig, S. O. (2000): Wettbewerb bei unvollständiger Information: Informationsaustausch oder stillschweigende Kollusion? Tübinger Diskussionsbeitrag Nr. 177.

Tirole, J. (1988): *The Theory of Industrial Organization*. Cambridge, Mass., London.

Vives, X. (1984): Duopoly Information Equilibrium: Cournot and Bertrand. *Journal of Economic Theory* 34, 71–94.

Vives, X. (1999): *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, Mass.