

Modals in Comparatives

von

SVETLANA KRASIKOVA

Philosophische Dissertation

angenommen von der Philosophischen Fakultät

der Universität Tübingen

am 13. Mai 2011

Tübingen, 2011

Gedruckt mit Genehmigung der Philosophischen Fakultät
der Universität Tübingen

Hauptberichterstatter: Prof. Dr. Sigrid Beck
Mitberichterstatter: Prof. Dr. Arnim von Stechow
Mitberichterstatter: Prof. Dr. Wolfgang Sternefeld
Mitberichterstatter: Prof. Dr. Gerhard Jäger
Dekan: Prof. Dr. Jürgen Leonhardt

Table of Contents

Chapter 1. Introduction	8
1.1. SEMANTIC SYSTEM	11
1.2. BUILDING BLOCKS.....	13
1.2.1. DEGREES.....	13
1.2.2. ADJECTIVES AND COMPARISON	14
1.2.3. EXCURSUS: MODALS AND CONDITIONALS IN KRATZER’S PREMISE SEMANTICS	20
1.3. THE PROBLEM OF INTENSIONAL COMPARATIVES	24
1.4. A NEW PERSPECTIVE	28
1.4.1. SUFFICIENCY READING	28
1.4.2. MAXIMALITY WITH POSSIBILITY MODALS	31
1.4.3. THE PLOT	34
1.5. STRUCTURE OF THE THESIS.....	36
Chapter 2. Landscape	38
2.1. STRUCTURE OF THE CHAPTER	38
2.2. AMBIGUITY IN MAIN CLAUSES	42
2.2.1. COMPARATIVES WITH ‘LESS’ AND ‘EXACTLY’ DIFFERENTIALS.....	42
2.2.2. HEIM’S PROPOSAL.....	43
2.2.3. PROPOSAL IN ODA (2008).....	47
2.2.4. DISCUSSION	50
2.2.4.1. TWO CLASSES OF MODALS	50
2.2.4.2. ‘BEFORE’ CLAUSES	53
2.2.5. CONCLUSION.....	56
2.3. QUANTIFIERS IN COMPARATIVE COMPLEMENTS	58
2.3.1. VON STECHOW’S DESIDERATA.....	58
2.3.2. WIDE SCOPE QUANTIFIERS.....	63
2.3.3. NARROW SCOPE QUANTIFIERS.....	69
2.3.4. DISCUSSION	77
2.3.4.1. CONSTRAINING SCOPE	77
2.3.4.2. IS THERE AMBIGUITY?	79
2.3.4.3. NEGATIVE POLE ADJECTIVES	83
2.3.4.4. DATA RE-EVALUATED	86
2.4. RULLMANN AMBIGUITY	91
2.4.1. RULLMANN’S PUZZLE.....	91
2.4.2. DECOMPOSITION	92
2.4.3. POLARITY?	95
2.5. CONCLUSION	100

Chapter 3. Comparing to What is Necessary	102
3.1. STRUCTURE	102
3.2. BACKGROUND	103
3.3. PINNING DOWN THE KIND OF MODALITY	105
3.3.1. ANANKASTIC MODALS	105
3.3.2. WEAK AND STRONG ANANKASTIC MODALS	109
3.3.2.1. SCOPE OF NEGATION	110
3.3.2.2. MINIMAL REQUIREMENT READING	113
3.4. ACCOUNTING FOR THE MINIMAL REQUIREMENT READING	118
3.4.1. PLOT	118
3.4.2. ANANKASTIC CONDITIONALS AS COUNTERFACTUALS	123
3.4.2.1. ANTECEDENT	123
3.4.2.2. ACCESSIBILITY	126
3.4.3. INTRODUCING ALTERNATIVES	130
3.4.3.1. ANALOGY TO DESIRE REPORTS	131
3.4.3.2. CONTEXTUAL RESTRICTION	134
3.4.4. SUFFICIENCY INFERENCE	137
3.4.4.1. MINR WITH A NUMERAL	138
3.4.4.2. MINR UNDER 'ONLY'	141
3.4.5. MINR IN COMPARATIVES	149
3.4.5.1. MINR IN COMPARATIVE COMPLEMENTS	149
3.4.5.2. MINR IN MAIN CLAUSES OF COMPARATIVES	151
3.4.6. ALTERNATIVES	158
3.4.7. CONCLUSION	161
3.5. WEAK ANANKASTIC MODALS	163
3.5.1. TWO MODAL LAYERS	163
3.5.2. THE ROLE OF SUBJUNCTIVE	168
3.5.3. NEGATION	169
3.5.4. ABSENCE OF MINR	172
3.6. DISTRIBUTION OF MINR	175
3.7. CONCLUSION	178
 Chapter 4. Comparing to What is Possible	 181
4.1. AGAINST STRUCTURAL AMBIGUITY	184
4.1.1. HEIM (2007)	184
4.1.2. CHALLENGES TO A SCOPE APPROACH	190
4.2. PRAGMATIC ENRICHMENT	195
4.2.1. HEIM AMBIGUITY IN 'LESS' COMPARATIVES	195
4.2.2. DEGREE TERMS UNDER POSSIBILITY	201
4.3. CONSEQUENCES OF A PRAGMATIC APPROACH	209
4.3.1. CONSTRAINTS ON RULLMANN AMBIGUITY	209
4.3.2. CANCELLATION	214

4.4. CONCLUSION	217
Chapter 5. Conclusion	218
Bibliography	222

Acknowledgements

I am very grateful to my supervisors Arnim von Stechow and Sigrid Beck who guided me through my PhD years with their wisdom, intelligence and passion for semantics.

Arnim has taught me everything I know, influenced my way of thinking and made my life worth living. Three years of study and work with him and his team were the happiest and most rewarding in my life. I love and esteem him as a semanticist, a teacher, a friend and a person.

I am thankful to Sigrid for letting me join her project and bringing me into the realm of comparatives. If it were not for her wit and intuition, her sharp criticism, her ingenuity, clarity of thought and her belief in me and my success, I would not be where I am now.

I am indebted to Wolfgang Sternefeld who influenced me and my work immensely without knowing it. I admire his profoundness and wisdom and thank him for his kind-heartedness and warmth to me during my years in Tübingen. It is to him and his deep insights that I owe the core idea of this work.

I am grateful to Gerhard Jäger, who expressed interest in my work and agreed to join committee. His review confirmed me in my views and boosted my self-confidence.

I am also thankful to Frank Richter, Fritz Hamm, Dale Gerdemann who have been excellent teachers and advisors.

With all my heart I thank my colleagues and friends who have always been willing to talk about semantics with me and made my life in Tübingen so enjoyable: Hedde Zeijlstra, Doris Penka, Vera Hohaus, Lucas Champollion, Penka Stateva, John Vanderelst, Remus Gergel, Nadine Bade, Sonja

Tiemann, Stefan Hoffstetter, Daniel Fleischer, Kim Fechner, Irina Piir, Christiane Savelsberg, Yana Panchenko, Elisabeth Villalta.

I am much indebted to our secretaries who have taken care of me as if I were their own child: Christl Glauder, Beate Starke and Sonja Haas-Gruber.

Many thanks to my dear friends Lena and Clement Kabagema-Bilan who have been my second family all these years and shared the best moments with me.

I am grateful to my relatives in Russia and Bulgaria for their love and belief in me, for having come to Tübingen when I needed their help most.

Finally, I want to thank Ventzi for being my inspiration, my friend and my love, for sharing all his knowledge with me and teaching me to be patient in search for the truth. I dedicate this disseration to him and our son Yassen.

Chapter 1. Introduction

Since von Stechow’s seminal work “Comparing semantic theories of comparison”, (von Stechow, 1984), the focus of the semantic literature on comparison has been on the analysis of comparative sentences with quantificational expressions in ‘than’-clauses. In a degree-based approach, a comparative complement is traditionally treated as a property of degrees. For example, the embedded clause of (1)a is said to denote a set of degrees that equal Bill’s height. By analogy, the embedded clauses of (1)b and (1)c denote sets of heights that Mary is allowed and required to have, respectively.

- (1) a. Mary is taller than Bill is.
- b. Mary is taller than allowed.
- c. Mary is taller than required.

While the degree of height to which Mary’s height is compared can be uniquely reconstructed from a comparative complement like that in (1)a, it is not always the case with the ‘than’-clauses in (1)b and (1)c. The degrees of height Mary is allowed or required to have may well correspond to proper intervals. Suppose, Mary is required to be between 1.75 m and 1.80 m tall to qualify for a basketball team. If we adopt a classical method, going back to (Russell, 1905), to reconstruct the object of comparison by coercing the comparative complement into a degree description we immediately face a problem in such cases. There may be several degrees meeting the relevant criteria; in the case at hand, any height between 1.75 m and 1.80 m could be selected. Intuitively, if we deal with a proper interval we tend to pick the minimal or the maximal bound for the standard of comparison. Implementing this simple idea remains one of the major challenges in the semantics of comparison for at least two decades because this task requires making a number of decisions with far-reaching consequences for the

degree semantics.

First of all, one needs to answer the question whether the way a point is selected from an interval in such cases hinges on the syntax and semantics of comparative constructions. All existing analyses give a positive answer to this question. Numerous revisions motivated by the need to handle quantificational ‘than’-clauses targeted various aspects of a standard degree-based analysis of comparative sentences; such as the notion of degree involved in the analysis, cf. (Schwarzschild and Wilkinson, 2002); the lexical meanings of the comparative morpheme and gradable predicates, cf. (Heim, 2006); the structure of ‘than’-clauses, cf. (Schwarzschild, 2008). Therefore, the treatment of quantificational ‘than’-clauses is considered the touchstone of any analysis of comparatives.

Once there is a well-motivated and efficient mechanism at hand that allows one to pick a point out of an interval another crucial question arises. What constrains the choice? Consider (1)b and (1)c again. If a proper height interval corresponding to some rule is at issue, we obviously go for the maximum allowed height in (1)b, while (1)c expresses a comparison with the minimal requirement. Most existing theories agree that the choice is sensitive to the kind of quantificational expression inside the comparative clause. Clearly, without a well-motivated system of constraints blocking unavailable readings, there is a severe over-generation problem. Nonetheless, constraining the choice is considered a secondary task and has seldom been addressed, the few exceptions are (Schwarzschild, 2008), (van Rooij, 2008) and (Beck, 2010b).

The main questions this thesis addresses are related to the problem of choice. It is, however, not the problem of constraining the choice in the above sense that will be the main focus of this work but rather that of determining the choice. I propose to change the perspective and decouple the problem of quantifiers in comparative clauses from the analysis of comparative constructions. Where can the problem be rooted, if not in the syntax and

semantics of comparatives? The answer I give is that in most cases the choice is determined by the semantics and pragmatics of underlying quantificational expressions. The main motivation for this view comes from the fact that the problem of choice arises with modals outside comparative complements, too. To give two examples, in (2)a and (3)a the measure phrase ‘1.70 m’ may refer to the required minimum and the allowed maximum, respectively; and sentences (2)b and (3)b have readings saying that the required minimum and the allowed maximum are under 1.85 m, respectively.

- (2) a. Mary is required to be 1.70 m tall.
b. Ann is 1.85 m tall. Mary is required to be less tall than that.
- (3) a. Mary is allowed to be 1.70 m tall.
b. Ann is 1.85 m tall. Mary is allowed to be less tall than that.

The main strategy I follow in this work is to focus on the interpretation of the relevant modal expressions, i.e. ‘be required’ and ‘be allowed’ in the examples above, and by doing this, account for the effects they trigger in scalar contexts in general. The results are then extended to derive adequate semantics for comparatives with intensional complements.

The rest of this introduction sets the stage for the subsequent chapters, by discussing some technical preliminaries. I start with a description of the semantic framework adopted for the present purposes and lay out my assumptions concerning the treatment of comparative sentences and modal expressions. Then I discuss the main aspects of the problem that intensional comparatives present, give a preview of my position and formulate specific research questions this work aims to answer.

1.1 SEMANTIC SYSTEM

In this work I shall adhere to a view of the syntax-semantics interface under which there is a level of syntactic representation, called logical form (LF), that serves as an input to the semantic interpretation. An LF has the form of a phrase structure tree consisting of semantically interpretable material. My syntactic assumptions are minimal and non-committing. For the most part, I shall present simple clausal structures without functional projections adopting the assumptions of the authors under discussion.

The core of the interpretative component is the type-driven interpretation function $\llbracket \cdot \rrbracket$, relativized to an assignment function g . Every node in an LF tree is assigned a logical type which corresponds to its semantic denotation.

(4) Logical Types

- a. Let e be the type of individuals, t the type of truth values, s the type of possible worlds.
- b. If a and b are logical types, then (ab) is the type of functions from meanings of type a into meanings of type b .

One may distinguish various semantic domains containing expressions with the meaning of the same logical type.

(5) Semantic Domains

D_e = the set of individuals

D_t = $\{0,1\}$

D_s = the set of possible worlds

D_{ab} = the set of functions from D_a in D_b

Interpretation function $\llbracket \cdot \rrbracket^g$ applies in a type-driven fashion, i.e. the logical type of its argument determines the interpretation rule that is called on at a particular step of composition. I adopt the inventory of interpretation rules from chapter 12 of (Heim and Kratzer, 1998). Heim and Kratzer's semantic framework is based on an intensional language. In an intensional language meanings of expressions are intensions, i.e. functions from

possible worlds into their extensions; logical types correspond to extensions.

(6) Interpretation Principles

- a. Lexicon:
If α is a lexical item, then $\llbracket \alpha \rrbracket^g$ is the lexical meaning of α .
- b. Variable Rule:
If α is a variable with an index x , then $\llbracket \alpha_x \rrbracket^g = \lambda w g(x)$
- c. Extensional Functional Application (EFA):
If γ is a tree with daughters α and β of types (ab) and a , respectively, then $\llbracket \gamma \rrbracket^g = \lambda w \llbracket \alpha \rrbracket^g(w)(\llbracket \beta \rrbracket^g(w))$
- d. Intensional Functional Application (IFA):
If γ is a tree with daughters α and β of types $((sa)b)$ and a , respectively, then $\llbracket \gamma \rrbracket^g = \lambda w \llbracket \alpha \rrbracket^g(w)(\llbracket \beta \rrbracket^g)$
- e. Predicate Abstraction (PA):
If γ is a tree with daughters λx and α , where x is a variable of type a and α is of type t , then $\llbracket \gamma \rrbracket^g = \lambda w \lambda u \in D_a. \llbracket \gamma \rrbracket^{g[x/u]}(w)$

In the following section, I shall lay out my assumptions on the treatment of gradable predicates and sketch an analysis of simple comparative sentences, closing up with a short excursus into the standard treatment of modals and conditionals in a possible world semantics.

1.2 BUILDING BLOCKS

1.2.1 DEGREES

Any theory of comparatives employs some notion of degree. Following the work of (Cresswell, 1976), most contemporary theories of comparison handle degrees as abstract entities forming part of the object language. A different tradition going back to (Lewis, 1970) is to treat degrees as contextual coordinates. (Klein, 1991), who gives a background on this division and discusses their strengths, calls the former approach degree-based. In this work I shall take such an approach for granted, for a comparison see (Klein, 1980), (Kennedy, 1997), (Sassoon, 2007), (van Rooij, 2008).

The hallmark of a degree-based analysis is the presence of objects of type *degree* in its underlying semantic ontology. A classical method is to construct degrees as equivalence classes of individuals indistinguishable relative to a relevant gradable property. It is based on the Frege-Russell treatment of cardinal numbers and was first proposed and spelled out in detail in (Cresswell, 1976). As an example, let me show how a degree of cleverness is defined in such approach. I rely on a recent comprehensive exposition in (von Stechow, 2008).

We start with the relation ‘cleverer than’ holding of individuals of type e , which is empirically given and has the properties of being irreflexive, asymmetric and transitive. By assumption, each gradable predicate is associated with at least one such relation. Let us denote this relation as $>_{\text{clever}}$ and its field as $F(>_{\text{clever}})$. Using ‘cleverer than’ we may define equivalence relation ‘exactly as clever as’ with the same field.

$$(7) \quad \forall x_e, y_e \in F(>_{\text{clever}}): y \approx_{\text{clever}} x \text{ iff } \forall z_e \in F(>_{\text{clever}}): [y >_{\text{clever}} z \text{ iff } x >_{\text{clever}} z] \\ \wedge [z >_{\text{clever}} y \text{ iff } z >_{\text{clever}} x]$$

The cleverness degree of individual u , written as $[u]_{\text{clever}}$, can now be defined as a subset of $F(>_{\text{clever}})$ corresponding to the equivalence class of u relative to \approx_{clever} , more precisely:

$$(8) \quad \forall u \in F(>_{\text{clever}}): [u]_{\text{clever}} = \{x: x \approx_{\text{clever}} u\}$$

Cleverness degrees are thus predicates of individuals, i.e. have semantic type *et*, and denote sets of individuals which are indistinguishable with respect to how clever they are.

$$(9) \quad D_{\text{clever}} = \{[u]_{\text{clever}}: u \in F(>_{\text{clever}})\}$$

It is common practice to introduce type *d* into the semantic ontology representing objects of this kind.

- (10) a. Let *d* be the semantic type of degrees.
- b. Let D_d consist of disjoint sets of degrees of various sorts.

Constructed as equivalence classes, cleverness degrees can be related to each other by a second order relation based on $>_{\text{clever}}$, as defined below:

$$(11) \quad \forall d, d' \in D_{\text{clever}} : d >_{\text{clever}} d' \text{ iff } \forall x \in d, \forall y \in d': x >_{\text{clever}} y$$

Like their domains, degrees constitute a scale structure. I shall call a scale a tuple consisting of a set of degrees of a particular sort and '>' ordering on this set.

- (12) Call each pair $\langle D_a, >_a \rangle$, such that $D_a \subseteq D_d$ and $>_a$ is the ordering on D_a , a scale of degrees in D_a .

Degree scales like $\langle D_{\text{clever}}, >_{\text{clever}} \rangle$ constitute an ordinal system of measurement, which implies that degrees of cleverness can be compared by 'greater than' or 'less than' relation, but operations of addition or subtraction cannot be meaningfully applied to them. Thus, degrees as equivalence classes represent only the order of entities they correspond to, they do not encode the distance between them.

1.2.2 ADJECTIVES AND COMPARISON

(Cresswell, 1976) suggests that degrees are introduced into a semantic representation by lexical entries of gradable predicates that take them as their arguments. More specifically, he proposes to analyse gradable adjecti-

ves as expressing relations between individuals and degrees of the relevant sort. For example, adjective ‘tall’ is treated as a function that maps a degree of tallness to a set of individuals whose height is represented by that degree.

$$(13) \quad \llbracket \text{tall} \rrbracket = \lambda w \lambda d \in D_{\text{tall}} \lambda x \text{ x is tall to degree d in w}$$

Assignment of an individual to its unique degree of some sort is called a measure function. A gradable predicate is therefore associated with a certain kind of measure function. Let us call a measure function that relates an entity to its height in world w HEIGHT_w .

$$(14) \quad \text{HEIGHT}_w := \lambda x \iota d (d \in D_{\text{tall}} \wedge x \in d)$$

We may now redefine the lexical rule in (13) by making use of measure function HEIGHT .^{1,2}

$$(15) \quad \llbracket \text{tall} \rrbracket = \lambda w \lambda d \in D_{\text{tall}} \lambda x \text{ HEIGHT}_w(x) = d$$

A simple predicative sentence like (16)a can be analysed as relating Mary’s height to a degree of height considered large in a given context. I assume that the degree argument is bound by an existential closure at the sentential or some higher discourse level. The information that the value of the degree argument is contextually significant may be seen as requirement that has to be met for the sentence to be informative; without it the sentence would express the trivial thought that Mary has a height degree.³

-
1. Another established way to define adjectives in a degree-based approach, which goes back to (Bartsch and Vennemann, 1972), is to treat them as measure functions, i.e. as functions of type *ed*.
 2. For simplicity, I assume at this stage that adjectives are not monotone, which is a non-standard move. Usually, adjectives are defined as monotone functions in their degree argument, which requires that for any x, w, d', d : $d' > d \ \& \ A(w)(d')(x) \rightarrow A(w)(d)(x)$. At different points in the course of the thesis, I will be introducing the monotonicity assumption into the meaning of adjectives and discussing its consequences.
 3. An alternative analysis that goes back to (Cresswell, 1976) is to introduce a covert morpheme, the so called positive operator, that relates the subject’s height to a contextually supplied degree or degree interval. As the present discussion does not

- (16) a. Mary is tall.
 b. $\exists d$ [Mary [d tall]]
 c. $\text{HEIGHT}_w(\text{Mary}) = d$, where d is a significant height in the given context.

An analysis of the comparative sentence in (17) in a degree-based approach should intuitively involve a comparison of the degree representing Mary's height to the degree representing Bill's height by means of $>_{\text{tall}}$ relation, as reflected in the truth conditions given in (18).

(17) Mary is taller than Bill is.

- (18) $\text{HEIGHT}_w(\text{Mary}) >_{\text{tall}} \text{HEIGHT}_w(\text{Bill})$
 'The degree d , such that Mary's height corresponds to d in w , is greater than the degree d' , such that Bill's height corresponds to d' in w .'

There is a variety of analyses trying to capture this intuition, for a comparison see (von Stechow, 1984). I adopt the standard line to treat the comparative ending '-er' as a separate semantic unit that contributes the comparative relation and the comparative complement as the degree-denoting argument of '-er' corresponding to the object of comparison. Following an analysis in (Heim, 2006), I define '-er' as a two-place relation between degrees, as the following lexical entry shows.

- (19) $\llbracket \text{-er} \rrbracket = \lambda w \lambda d \in D_R \lambda d' \in D_R d' >_R d$

'Than'-clauses are assumed to undergo ellipsis, known as the so called comparative deletion since the work of (Bresnan, 1973), see (20).

- (20) than Bill is ~~how tall~~

After reconstruction, the question word is moved from the degree argument position of the adjective to the edge of the clause and interpreted as a lambda abstractor. As a result, the comparative complement denotes a

hinge on the presence of the positive morpheme, I assume a simpler structure.

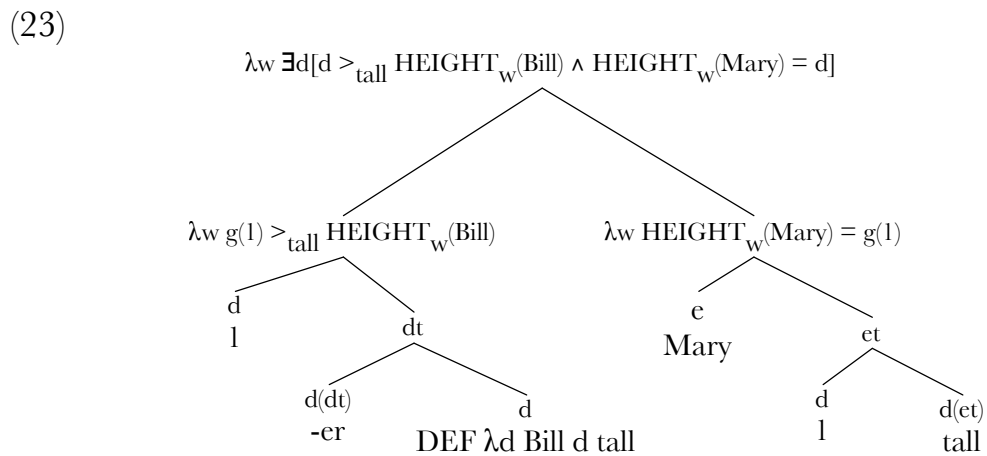
property of degrees. This property is coerced into a degree description by an application of a covert definite term.⁴

$$(21) \quad \llbracket \text{DEF} [\lambda d \text{ Bill } d \text{ tall}] \rrbracket = \lambda w \text{ td } (\text{HEIGHT}_w(\text{Bill}) = d)$$

The comparative relation ‘-er’ maps this degree description to a property that applies to a degree if it exceeds Bill’s height.

$$(22) \quad \llbracket \text{-er} \rrbracket(w)(\llbracket \text{DEF} [\lambda d \text{ Bill } d \text{ tall}] \rrbracket(w)) = \lambda w \lambda d' d' >_{\text{tall}} \text{td } (\text{HEIGHT}_w(\text{Bill}) = d)$$

I assume that an ‘-er’ phrase is a restriction of an indefinite term in the spirit of (Heim, 1988), which introduces a new degree variable and is treated as an open proposition combining conjunctively with the rest of the clause. A degree indefinite is scoped out of its base position as shown in the tree in (23) and leaves a definite trace co-indexed with the variable it contains. All degree variables are bound by a global existential closure. At the top node we obtain the desired proposition that there is a degree that corresponds to Mary’s height and is greater than Bill’s height.



Let us look at an example with the negative pole adjective ‘short’.

4. In some languages, e.g. Bulgarian, the definite term is realised overtly.

(i) Ти си по-висок, отколкото би искал да бъдеш.
 you are taller from_what_DEF SUBJ want to be
 ‘You are taller than you wished you were.’

(24) Bill is shorter than Mary is.

The lexical entry for ‘short’ is given in (25). I assume that ‘short’ employs the same measure function as its positive pole antonym ‘tall’. This amounts to postulating that the equivalence classes formed by \approx_{short} relation are identical to those formed by \approx_{tall} relation. Shortness degrees are therefore indistinguishable from tallness degrees. They are, however, associated with a different ordering. This ordering is based on the basic relation $>_{\text{short}}$ which is the inverse of the $>_{\text{tall}}$.

(25) $\llbracket \text{short} \rrbracket = \lambda w \lambda d \in D_{\text{short}} \lambda x \text{HEIGHT}_w(x) = d$

Sentence (24) is predicted to be true iff Bill’s height is greater than Mary’s on the shortness scale $\langle D_{\text{short}}, >_{\text{short}} \rangle$. Since $>_{\text{short}}$ is the inverse of $>_{\text{tall}}$, by virtue of $>_{\text{short}}$ being the inverse of $>_{\text{tall}}$, we arrive at the validity of the equivalence in (26).

(26) Mary is taller than Bill is. \Leftrightarrow Bill is shorter than Mary is.

Note that degrees as construed here are not crucial for the formulation of the truth conditions of the sentences in (26). The extensions of these examples can be reduced to more simple ones. The clause on the left is true iff Mary belongs to a group of individuals who are judged taller than a group of individuals that Bill belongs to. The clause on the right conveys that Bill belongs to a group of individuals that are considered shorter than the group that Mary belongs to. Since there is no notion of distance involved, we are not yet equipped to treat comparatives like (27).

(27) Mary is 5 cm taller than Bill is.

The standard method to handle comparatives with measures of distance, called differential comparatives, is to replace an ordinal system of measurement induced by degrees by an interval system of measurement, see (Klein, 1991) and (von Stechow, 2008). For this purpose, an operation of

addition is defined on degrees via some operation of fusion on individuals in their domain. For degrees of length such operation is concatenation.

- (28) Assume that \circ is an operation of concatenation on the domain of individuals. Let $d, d' \in D_{\text{tall}}$.
 $d + d' = \{u: u \in F(>_{\text{tall}}) \ \& \ \exists x \in d, \exists y \in d': x \circ y = u\}$

Given addition, it is possible to define multiplication of degrees.

- (29) For any degree d the following holds:
 $2d = d + d$ &
 $nd = (n - 1)d + d$

The sequence $d, 2d, 3d \dots$ is called a standard sequence based on d in (Krantz *et al.*, 1971). The notion of standard sequence is used in semantics to define measure expressions. A standard sequence can be based on a unit of measurement. First, we construct a degree corresponding to the equivalence class of entities that are exactly as tall as a meter unit object. Call this degree $[m]_{\text{tall}}$. The standard sequence that is based on $[m]_{\text{tall}}$ looks as follows: $[m]_{\text{tall}}, 2[m]_{\text{tall}}, 3[m]_{\text{tall}} \dots$. Expressions, like ‘2 meters’ are normally assumed to refer to members of that sequence. Once fractions are defined, the phrase ‘1.80 m’ in (30)a can be analysed as referring to the degree $1.8[m]_{\text{tall}}$. Thus, a measure phrase, by analogy to the comparative indefinite, can saturate the degree argument of a gradable adjective.

- (30) a. Mary is 1.80 m tall.
 b. $\text{HEIGHT}_w(\text{Mary}) = 1.8[m]_{\text{tall}}$

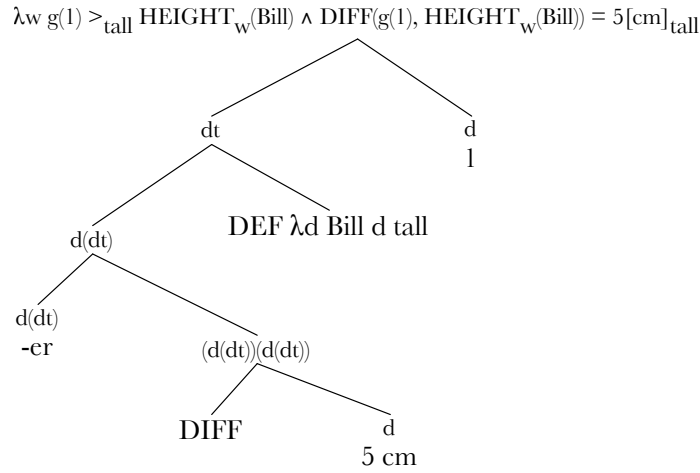
Differentials are usually treated as measures of distance between two degrees. The distance between two degrees can be defined as follows.

- (31) $\forall d, d' \text{ DISTANCE}(d, d') = \iota d''$, such that $d + d'' = d' \vee d' + d'' = d$

To handle differential phrases, I assume with (von Stechow, 2008) that the differential ‘(by) 5 cm’ is composed with the comparative morpheme and adds the measure of distance. I introduce a covert operator DIFF with the semantics in (32).

$$(32) \quad \llbracket \text{DIFF} \rrbracket = \lambda_w \lambda d'' \lambda R \lambda d \in F(>_R) \lambda d' \in F(>_R) R(d)(d') \wedge \text{DISTANCE}(d, d') = d''$$

(33)



I close up this section with the main steps in the analysis of (27).

$$(34) \quad \exists d [\llbracket \text{DIFF} \rrbracket(w)(5[\text{cm}]_{\text{tall}})(\llbracket -\text{er} \rrbracket(w))(\text{HEIGHT}_w(\text{Bill}))(d) \wedge \llbracket \text{tall} \rrbracket(w)(d)(\text{Mary})] =$$

$$\exists d [\llbracket -\text{er} \rrbracket(w)(\text{HEIGHT}_w(\text{Bill}))(d) \wedge \text{DISTANCE}(\text{HEIGHT}_w(\text{Bill}), d) = 5[\text{cm}]_{\text{tall}} \wedge \llbracket \text{tall} \rrbracket(w)(d)(\text{Mary})] =$$

$$\exists d [d >_{\text{tall}} \text{HEIGHT}_w(\text{Bill}) \wedge \text{DISTANCE}(\text{HEIGHT}_w(\text{Bill}), d) = 5[\text{cm}]_{\text{tall}} \wedge \text{HEIGHT}_w(\text{Mary}) = d]$$

1.2.3 EXCURSUS: MODALS AND CONDITIONALS IN KRATZER'S PREMISE SEMANTICS

This subsection summarises the main points of the premise semantics of modals and conditionals developed in (Kratzer, 1977), (Kratzer, 1981), (Kratzer, 1991). Modals are assumed to quantify over a set of worlds determined by two contextual parameters called modal base and ordering source. A modal base is a function that assigns a world a set of facts in that world against which a modal statement is evaluated. What kind of facts are important can be made explicit by ‘in view of ...’ phrases. For instance, in (35) ‘must’ quantifies over the worlds that make true what the speaker knows in the actual world.

(35) In view of what I know, David must be at home.

The role of the second contextual parameter is to induce an ordering on the set of worlds restricted by the modal base. It represents a kind of ideal that the worlds accessible from the modal base must come close to. Formally, an ordering source g is a function that assigns a world w a set of propositions that induce a partial ordering on the set of possible worlds W in the following way:

$$(36) \quad \forall w, w' \in W: w \leq_{g(w)} w' \text{ iff} \\ \{p: p \in g(w) \wedge w' \in p\} \subseteq \{p: p \in g(w) \wedge w \in p\}$$

According to (36), a world w is at least as close to the ideal provided by $g(w)$ as w' if and only if all propositions from $g(w)$ that are true in w' are true in w as well. Example (35) can be evaluated with respect to what Kratzer calls a stereotypical ordering source (“in view of the normal course of events”). The worlds that are selected on the basis of the speaker’s knowledge are restricted by the ordering source to those where nothing extraordinary happens. Informally, when uttering (35), we do not only rely on the available factual evidence but on other more or less reliable sources of information as well, namely, the facts concerning the normal course of events. Thus, the set of worlds that the modal quantifies over, does not have to include the actual one and therefore (35) does not imply that David is at home, which it would if it were not for the contribution of the ordering source.

Simplifying a bit⁵, a proposition is a necessity in Kratzer’s theory if and only if it is true in all worlds compatible with the modal base and closest to the ideal provided by the ordering source.

5. Kratzer does not make the so called Limit Assumption, i.e. she does not assume that closest worlds exist. The definition of necessity under the Limit Assumption is more complex. For the purpose of this brief introduction, the simplified formulation I provide would do.

- (37) p is a necessity in w with respect to the modal base f and the ordering source g iff
 $\forall w' \in \cap f(w): (\neg \exists w'' \in \cap f(w): w'' <_{g(w)} w') \rightarrow w' \in p$

I shall abbreviate the maximality condition imposed by the ordering source on the accessible worlds w' by $\text{Max}_{g(w)}(w')$ and represent (37) as (38).

- (38) p is a necessity in w with respect to the modal base f and the ordering source g iff
 $\forall w' \in \cap f(w): \text{Max}_{g(w)}(w') \rightarrow w' \in p$

Under this analysis, (35) has the following truth conditions in w :

- (39) $\forall w' \in \cap f(w): \text{Max}_{g(w)}(w') \rightarrow w' \in [\lambda w \text{ David is at home in } w]$,
 where $f(w)$ is a set of propositions that the speaker knows in w and
 $g(w)$ is a set of propositions that represent the normal course of
 events in w .

Informally, the sentence is true in w iff in every world in which what the speaker knows in w is true and in which nothing unexpected from the perspective of w happens David is at home.

Kratzer proposes that conditionals impose an additional restriction on accessible worlds, namely, the requirement that accessible worlds make true the antecedent expressed by the ‘if’ clause, which is captured by the following rule:

- (40) $\llbracket \text{if } A \text{ must } C \rrbracket^{f,g} = \llbracket \text{must } C \rrbracket^{f^*,g}$, where $\forall w: f^*(w) = f(w) \cup \{\llbracket A \rrbracket^{f,g}\}$

The definition in (40) states that for each world the meaning of the ‘if’ clause is added to the set of propositions assigned to that world by the modal base. It is relative to this modified set that the main clause is evaluated. As an example consider the truth conditions of sentence (41) given in (42).

- (41) If my watch is correct, David must be at home.

- (42) $\forall w' \in \cap (f(w) \cup \{\lambda w \text{ the speaker's watch is correct in } w\})$:
 $\text{Max}_{g(w)}(w') \rightarrow w' \in [\lambda w \text{ David is at home in } w]$,
 where $f(w)$ is a set of propositions that the speaker knows in w and
 $g(w)$ is a set of propositions that represent the normal course of
 events in w .

This condition is met iff the proposition that David is at home is true in every world from the modal base closest to the ideal in which the speaker's watch is correct.

To conclude, a modal is analysed along two dimensions. First, one specifies the content of its contextual backgrounds, i.e. pins down the kind of *modal flavour*. Second, depending on what kind of quantification over possible worlds a modal expresses it is associated with a strong or a weak *modal force*. A necessity modal is characterised by a stronger force than a possibility modal: the former introduces universal quantification over worlds and the latter is an existential quantifier.

1.3 THE PROBLEM OF INTENSIONAL COMPARATIVES

One of the much debated issues in the recent semantic literature on degree constructions has been the treatment of intensional comparatives (see (Heim, 2001), (Heim, 2006), (Schwarzschild and Wilkinson, 2002), (Schwarzschild, 2008), (Beck, 2009) (Beck, 2010b)). Under an intensional comparative I understand a comparative of inequality featuring an intensional predicate in the main or the embedded clause. There are at least three facts about this kind of comparatives that caused a keen interest in them. First, some of them appear to be ambiguous. Second, sometimes the ambiguity seems to be linked to the polarity of the predicate. Third, in some cases embedded modals appear to be interpreted outside of the scope of the comparative relation. Let us look at the pertinent examples from the literature.

Consider first a comparative sentence from (Heim, 2001) featuring a necessity modal in the main clause.

- (43) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

Heim observes that sentence (43) is ambiguous between a reading that says that the paper is exactly 15 pages in the worlds meeting the requirement and a reading on which it is not allowed to be shorter than 15 pages. She tentatively proposes to treat the availability of two readings as an instance of structural ambiguity stemming from the ability of the comparative morpheme to take scope above or below the modal.

The second example of ambiguity, this time with a possibility modal in the embedded clause, is due to (Rullmann, 1995).

- (44) The helicopter was flying lower than a plane can fly.

This sentence is reported to convey two kinds of comparison depending on the context of utterance. On the first reading, it means that the altitude of the helicopter was smaller than the maximal altitude of a plane. On the

other reading, the altitude of the helicopter is compared to the minimal altitude of a plane. The availability of distinct interpretations in such comparatives has been argued to be sensitive to the polarity of the gradable predicate and consequently presuppose a special treatment of antonyms. (Rullmann, 1995) and some more recent authors, (Heim, 2007), (Büring, 2007a), suggest a negative pole adjective should be decomposed into a positive pole antonym and the negation. The presence of negation gives them a handle on the ambiguity, which is analysed as a structural one. For example, under Rullmann's analysis the following two paraphrases capture the above-mentioned readings.

- (45) a. The helicopter's altitude was smaller on the scale of highness degrees than the maximum of a plane can reach on the scale of highness degrees.
- b. The helicopter's altitude was greater on the scale of lowness degrees than the maximum a plane can reach on the scale of lowness degrees.

These paraphrases reflect the position of negation with respect to the comparative and the gradable predicate: it either attaches to the comparative directly ('less high') or realised on the adjective below the comparative ('more low').

Third, look at an example from (Schwarzschild and Wilkinson, 2002) with the embedded intensional predicate 'predict'.

- (46) Bill did better than John predicted most of his students would do.

Schwarzschild and Wilkinson observe that sentence (46) can be uttered in a situation in which John predicted for the majority of his students to get a score within a certain span. Bill could have expressed himself in the following way: "Most of my students will get between 50 and 70 points." In this situation, the sentence intuitively conveys that Bill's actual score exceeds 70 points. To capture this intuition, Schwarzschild and Wilkinson propose a paraphrase along the following lines.

- (47) For any world w compatible with John's prediction in the actual world, Bill's score in the actual world exceeds the score reached by most students in w .

This paraphrase suggests that 'predict' is interpreted outside the comparison. The property of some quantifiers to be interpreted above the comparative is known as the problem of quantifiers in comparative complements. Treatment of cases like this inspired a new analysis of comparative complements involving a type-shift from degrees to sets of degrees.

Finally, the following example from (Heim, 2006), which also features an embedded universal quantifier over worlds, unlike (46), does not seem to have the problematic wide-scope-of-the-quantifier reading.

- (48) He was more cautious than he needed to be.

This sentence is understood to convey that he achieved his goal and that he could have been less cautious and still have achieved it. If the worlds that 'need' quantifies over are taken to be the ones in which his goal is achieved a paraphrase in the spirit of (47), see (49), does not correspond to the intuitive meaning, since it implies that his goal has not been achieved because he was too cautious.

- (49) For any world w compatible with his goals in the actual world, his degree of caution in the actual world exceeds his degree of caution in w .

The existence of examples like (48) vis-à-vis (46) has led some researchers to the conclusion that comparatives with embedded intensional predicates are ambiguous between the so called more-than-maximum reading, prominent in (46), and the more-than-minimum reading, prominent in (48).

To summarise, I touched upon three aspects in the interpretation of intensional comparatives that led to important revisions in recent theories of comparison. First, ambiguous comparatives with intensional predicates

in the main clause present a major argument for the degree operator semantics of the comparative morpheme in (Heim, 2001). Less-than-minimum and less-than-maximum readings in comparatives with embedded possibility modals, discussed in (Rullmann, 1995), are considered to present a crucial empirical base for an adequate analysis of negative pole adjectives. Finally, intensional ‘than’ clauses motivated a type-shift from degrees to sets of degrees or intervals in a composition of a comparative sentence.

This brief discussion has shown that intensional comparatives present a challenge for a theory of comparatives. More concretely, the interpretation of these cases is argued to hinge on the analysis of the comparative morpheme and the comparative complement, and the treatment of antonyms.

1.4 A NEW PERSPECTIVE

The goal of this thesis is to show that ambiguities observed in intensional comparatives do not motivate a revision of the analysis of comparatives adopted above. The discussion to follow will centre around two empirical aspects: the availability of a reading corresponding to a comparison with the required minimum, which we have encountered in (43) and (48); and the availability of readings corresponding to a comparison with the permitted or possible maximum and minimum, discussed in connection with Rullmann's example in (44). In the following, I will explain why I want to consider the data under these two rubrics and give a preview of my analysis.

1.4.1 SUFFICIENCY READING

Chapter 3 of the thesis is concerned with the source of the reading that can be described as a comparison with the minimal requirement. It is a striking fact, not yet captured in the literature on intensional comparatives, that only a restricted class of necessity modals triggers comparison with the minimally required amount. Consider a pair of examples that differ in the kind of modal occurring in the matrix clause.

- (50) a. The paper is required to be exactly five pages longer than that.
b. The paper should be exactly five pages longer than that.

Whereas (50)a can mean that the required minimum for the length of the paper exceeds the contextually salient amount by exactly five pages, (50)b has no such interpretation. The same pattern is replicated when 'be required' and 'should' are embedded in a 'than' clause. Sentence (51)a below conveys that the length of the paper exceeds the minimally required number of pages. For (51)b this interpretation is impossible.

- (51) a. The paper is longer than required.
b. The paper is longer than it should be.

The property of 'be required' and such predicates as 'need', 'have to', 'be

necessary' to trigger a comparison with an amount minimally sufficient for achieving a relevant goal was pointed out in (Heim, 2001) and (Heim, 2006). However, comparatives like (50) and (51) have not received a uniform analysis although the importance of the modal for the relevant reading to obtain is hard to overlook.

To the two environments in (50) and (51) one can add another one where the same contrast between 'be required' and 'should' is observed. Consider a context in which a journal imposes a strict requirement on the length of submissions specifying the minimal and the maximal page length. In this context, only (52)a from the pair of sentences below can be used to convey that the length of ten pages is minimally sufficient to comply with the relevant requirement. If appropriate at all, (52)b cannot report on the minimally sufficient page length. With 'should', the required range has to be introduced explicitly, for example, by means of 'between ... and ...' as shown in (53).

- (52) a. To be accepted, the paper is required to be ten pages long.
- b. To be accepted, the paper should be ten pages long.
- (53) To be accepted, the paper should be between 10 and 15 pages long.

Under the adopted analysis of measure phrases as degree-denoting expressions (section 1.2.2), the complements of the modals in (52) refer to definite degrees and an explanation of this contrast in terms of structural differences between (52)a and (52)b is hard to maintain. I suggest that comparison with the minimum characteristic of comparatives involving modals of 'be required' type should be derived from the properties of these modals. Therefore, the first series of questions that I set out to answer in this thesis can be formulated as follows:

- (54) What kind of modals give rise to the minimal requirement reading (MinR)? Why does MinR obtain with these modals and not with others?

I tackle the task in (54) by investigating the type of accessibility that MinR and non-MinR modals are associated with.⁶ The first conclusion is that MinR is possible with anankastic necessity modals used in sentences with ‘in order to’ complements, the so-called anankastic conditionals.

The second conclusion will be that MinR is the result of a totally realistic ordering source used by anankastic modals for restricting their accessible worlds. Consider an example of an anankastic conditional.

(55) Sean has to score 90 points in order to win.

Scoring exactly 90 points is intuitively not the only possibility for Sean to win. In situations in which he scores more, he is likely to win, too. However, such alternatives are more remote in view of what is the case, taking into account that one usually picks the easiest means to achieve one’s goal. This reasoning enables us to arrive at MinR.

The third conclusion is be that the so called weak anankastic conditionals formed by augmenting a modal predicate with the subjunctive morphology do not have MinR. I argue that a weak anankastic conditional is an anankastic conditional embedded under a deontic or bouletic modal. For evaluation of such modal statements facts about the effort made for the achievement of the relevant goal do not play a role whatsoever. Therefore, MinR does not obtain.

6. The conclusion that the availability of different interpretations is linked to the modal semantics and, particularly, to the kind of accessibility, is reached in (Meier, 2002), too. However, there are important motivational and strategic differences between Meier’s approach and the one pursued in this work. First, Meier takes the inevitability of a modal approach to be as a result of the failure of degree-based mechanisms to account for the full range of data; she does not consider the characteristic behaviour of specific lexical groups of modals. Second, Meier proposes a revision of the modal semantics, while I propose that the existing theories of modality have the potential to account for the presence or absence of MinR.

1.4.2 MAXIMALITY WITH POSSIBILITY MODALS

Chapter 4 of the thesis focuses on intensional comparatives with possibility modals. I have mentioned the relevance of such comparatives when discussing Rullmann's ambiguous example repeated below.

(44) The helicopter was flying lower than a plane can fly.

This sentence has been argued to make a comparison with the minimal or the maximal altitude of a plane depending on the context of utterance. The possibility to express a comparison with an extreme degree, i.e. the permitted minimum or maximum, in such sentences has been first pointed out in (von Stechow, 1984), although von Stechow discusses comparison with the maximum without addressing the ambiguity issue. Let me call this type of comparison 'extreme' reading (ExR).

ExR may surface in comparatives featuring a possibility modal in the main clause. Consider an example from (Heim, 2001).

(56) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

This sentence is usually judged to express a rather weak thought, namely, that the paper is allowed to be 15 pages long, leaving it open whether it can be longer or shorter. Heim notes that it can also mean that the maximally allowed length of the paper is 15 pages. Under this interpretation, corresponding to ExR, the maximally allowed length of the paper is compared to the length of the draft. This is strikingly parallel to a less-than-maximum reading of (44), expressing a comparison to the maximal altitude of a plane.

Next, consider an example from (Boguslavski, 2001), who argues that the availability of ExR is not restricted to comparatives.

(57) This plane can fly 100 m high.

This sentence can be used to make a weak claim that the plane can fly at

100 m without implicating anything about its maximal or minimal altitude. For example, the weak reading would be prominent if (57) were an answer to the question “Can this plane fly 100 m high?” On a different reading, 100 m refers to the minimal or the maximal altitude depending on the context of utterance. Suppose, (57) answers the question “How high can this plane fly?” in a situation in which this plane can well sustain an altitude of 120 m. Under normal circumstances, i.e. if the question is understood as a request for the maximal altitude, the sentence is judged false. This shows that (57) may express a stronger claim than the one it does as an answer to an alternative question.

Turning back to the ambiguity of (44), Rullmann observes that, in contrast to it, (58) containing the positive pole ‘higher’ has only one ExR, viz. a more-than-maximum reading.

(58) The helicopter was flying higher than a plane can fly.

Note, however, that embedding a counterfactual possibility modal in a ‘than’ clause may result in a more-than-minimum interpretation. The following example may convey that Mary’s actual expense exceeded the minimum she could have spent, say, had she been less extravagant, see (59)a. On another reading it says that Mary spent even more than the maximum she could have spent on a different occasion, as highlighted by the continuation in (59)b.⁷

(59) Mary spent more than she could have.

a. ... if she had not been so extravagant.

7. This discussion presupposes that the contribution of the so-called Q-words ‘much’ and ‘little’ and their comparative realisations is parallel to that of other gradable predicates, an issue on which there is no consensus in the literature, partly due to the facts I am concerned with in this thesis. I do not distinguish between adverbial or NP-internal Q-words and other gradable predicates, however, I exclude the pre-adjectival uses, like ‘more/less tall’, from the discussion as they are associated with additional implications.

- b. ... if she had had to organise a wedding party.

These data suggest that the availability of distinct readings depends on the modal flavour or some other pragmatic factors rather than on the polarity of the predicate involved.

This discussion illuminates two crucial points. First, ExR has its roots in the interpretation of the modal. If the modal occurs in the comparative complement, this reading is obligatory. Thus (44), (58) and (59) express a comparison with the maximum and the minimum or under some circumstances with one extreme only. Otherwise, ExR is optional, as we have seen in (56) and (57). Second, the availability of this or that extreme reading depends on the modal flavour, as the data (58) and (59) suggest.

The second series of questions that this thesis aims to answer can be formulated as follows:

- (60) What is the source of ‘extreme’ readings (ExR) in comparatives with possibility modals? Under which conditions is the extreme set to the minimum and the maximum?

I argue that ExR is the result of enriching the semantic meaning of a modal statement by a pragmatic implicature in all three cases in (61).

- (61) a. You are allowed to drive 100 km/h.
- b. You drive faster than allowed.
- c. You are allowed to drive exactly 10 km/h faster than that.

My second claim will be that the presence of ambiguity in cases like (61)b is not restricted to comparatives with negative pole adjectives as has been suggested in the literature. Rather, ambiguity is not observed due to independent pragmatic factors, e.g. if the modal flavour renders the resulting comparative statement uninformative under one of ExR. For instance, consider the following pair of sentences.

- (62) a. John drives slower than allowed.
b. John drives faster than allowed.

‘Allowed’ has deontic modal flavour, which means that the worlds it quantifies over are restricted to those that comply with certain rules or regulations. Embedding a deontic statement in a comparative in (62) can convey a violation of a rule. If the speaker intends to report on a violation, the addressee is bound to interpret (62)a as expressing a comparison with the minimal speed limit and (62)b as expressing a comparison with the maximal speed limit. One can however come up with different interpretations of these examples if the intention of the speaker happens to be different.

In the remainder of the introductory section I will present the plot and elaborate on the structure of the thesis.

1.4.3 THE PLOT

The general strategy I pursue in this work is to provide a semantic analysis of modal sentences containing degree-denoting expressions in the scope of a modal operator and apply it to comparatives, which are subsumed under this class.

The first type of modal sentences to be considered are anankastic conditionals, e.g. (63).

- (63) In order to join the team Mary *should/has to be 1.70 m* tall.

I focus on the interpretation of ‘have to’ and ‘should’ in this kind of sentences and show why MinR obtains in some cases and is blocked in the others. Examples in (64)a and (64)b are structurally more complex cases containing the same constellation of an anankastic modal and a degree-denoting expression at the conceptual level, which correspond to a bound degree variable at the LF. The ‘have to’ version of (64)a conveys that Mary is taller than minimally required for achieving the relevant goal. Likewise,

the ‘have to’ version of (64)b conveys that Mary’s minimally required height is smaller than the amount referred to by the demonstrative.

- (64) a. Mary is taller than she should/has to be.
Mary is taller than [she *should/has to be d* tall]
- b. Mary *should/has to be less* tall than that.

The second type of modal sentences to be considered feature degree-denoting expressions in the scope of a possibility modal, e.g. (65).

- (65) Mary is *allowed* to be *1.70 m* tall.

I show that the ExR of such sentences is derived as a pragmatic implicature. To be concrete, (65) conveys that Mary’s maximally allowed height equals 1.70 m as a result of pragmatic strengthening of the basic semantic meaning of this sentence, according to which 1.70 m is understood to be just a permitted height. This analysis will be extended to the comparatives in (66).

- (66) a. Mary is taller than allowed.
Mary is taller than [*allowed* Mary *d* tall]
- b. Mary is *allowed* to be *less* tall than that.

Thus, my central claim is that the observed readings of intensional comparatives do not hinge on the comparative environment but depend exclusively on the interpretation of the modal predicates involved.

1.5 STRUCTURE OF THE THESIS

The thesis is organised in the following way.

In chapter 2 I present the existing approaches to the analysis of intensional comparatives. The focus will be on two aspects: on comparing the behaviour of intensional predicates with that of nominal quantifiers, for this is the empirical background of the accounts to be discussed; and on highlighting the role of underlying modals in determining the available interpretation(s) of comparatives. This chapter should also clarify how the questions that I want to address in this thesis, repeated in (54) and (60) below, are related to the much debated problem of quantifiers in comparative complements.

The main goal of chapter 3 is to answer the questions in (54).

- (54) What kind of modals give rise to the minimal requirement reading (MinR)? Why does MinR obtain with these modals and not with others?

I start by giving a characterisation of modals that lead to SuffR and those that do not trigger this interpretation. A number of empirical tests is introduced to distinguish between the two classes of modals. In the next step, I develop an analysis for the two classes of modals and show that it accounts for the observed empirical differences.

The goal of chapter 4 is to answer the series of questions in (60) repeated below:

- (60) What is the source of ‘extreme’ readings (ExR) in comparatives with possibility modals? Under which conditions is the extreme set to the minimum and the maximum?

I first argue against a scope solution to the derivation of strong readings, i.e. ExR, in comparatives with modalised main clauses. I then present an alternative derivation of ExR by claiming that they result from a pragmatic enrichment of the plain meanings of modal statements involved. In the remaining part of the chapter I shall address some challenges to a

pragmatic approach.

Chapter 2. Landscape

2.1 STRUCTURE OF THE CHAPTER

In this chapter I discuss existing approaches to the analysis of intensional comparatives. I present the data following the logic of the recent literature on this topic. In a subsequent discussion I motivate a new perspective on the data. The chapter consists of three parts focusing on the data clusters to be discussed shortly. In this section I briefly introduce the empirical facts and the mainstream analyses, which are elaborated in the following discussion.

The first data cluster to be discussed are comparatives with intensional matrix clauses. Section 2.2 is concerned with the cases of ambiguity in such comparatives, first discussed in (Heim, 2001). Heim considers the following examples:

- (67) (The draft is ten pages long.)
- a. The paper is required/allowed to be less long than that.
 - b. The paper is required/allowed to be exactly five pages longer than that.

The variant of (67)a with ‘allowed’ means either that it is permitted for the paper to be less than ten pages or that the paper should be less than ten pages. Likewise, (67)b with ‘allowed’ may simply convey that 15 pages is a permissible length or additionally imply that the paper is not allowed to be longer than 15 pages. Other examples also permit two interpretations, see section 2.2.1 for a detailed discussion. This kind of ambiguity is argued to be a result of scope interactions between the comparative morpheme and the modal involved, see (Heim, 2001). I shall highlight some already known complications of such position and bring forth additional arguments contra a scope theory.

The second set of data, to be discussed in section 2.3, are comparatives with intensional ‘than’-clauses. It is a common practice to consider intensional comparative complements in parallel with those featuring DP quantifiers. Indeed, embedded indefinites often behave like possibility modals. In (68) both examples express a comparison with the maximal heights from sets of degrees provided by their restrictors. In (68)a it is a set of heights of salient individuals and in (68)b it is a set of heights that John reaches in law-abiding worlds.

- (68) a. John is taller than anyone else.
b. John is taller than allowed.

Similarly, in the pair of examples in (69) below, both the indefinite DP and the possibility modal appear to be interpreted outside the comparative, as the paraphrases below the examples suggest.

- (69) a. John drove faster than somebody else did.
‘For some x different from John: John drove faster than x .’
b. John drove faster yesterday than he might drive tomorrow.
‘It is possible that John will drive faster tomorrow than he drove yesterday.’

Likewise, universal DPs seem to pattern with intensional predicates expressing universal quantification over worlds, as illustrated by the pair in (70). Both sentences have a more-than-maximum reading.

- (70) a. John is taller than everyone else.
b. John is taller than we expected.

However, some necessity modals systematically effect a comparison with the minimal requirement, e.g. ‘be required’, ‘need’, ‘have to’. As such they do not have a counterpart in the nominal domain. Compare (71)a and (71)b.

- (71) a. John is taller than required/he needs to be/he has to be.
 ‘John’s height exceeds the minimum from the heights he has in the accessible worlds.’
- b. John is taller than he should be/ought to be.
 ‘John’s height exceeds the maximum from the heights he has in the accessible worlds.’

A popular approach to intensional ‘than’-clauses maintains that embedded quantificational expressions interact scopally with some comparative specific operators to produce different readings, e.g. (Heim, 2006), (Schwarzschild, 2008). The main question that such an approach raises is why some intensional predicates seem to take wide scope with respect to the comparative relation and others are obligatorily interpreted in the scope of the comparative. Some answers to it and new puzzles that those answers lead to are discussed later in the chapter. I motivate a new clustering of the data, which is only partially based on the parallel to DP quantifiers.

Finally, section 2.4 addresses some problems in the analysis of comparatives with embedded possibility modals, viz. examples of Rullmann ambiguity, which can be illustrated by the following example⁸:

- (72) John was driving slower than allowed.

Sentence (72) can mean that John was driving under the minimal or the maximal speed limit. The availability of two distinct readings in such sentences arguably depends on the polarity of the gradable predicate. I discuss some proposals maintaining that the ambiguity arises with negative pole adjectives only. I then argue that it need not be the polarity that determines which readings obtain when.

8. Rullmann’s paradigmatic example, viz. (i), involves a combination of ‘less’ and a positive pole adjective. However, Rullmann does not distinguish such cases from those involving negative antonyms, viz. (ii), as far as ambiguity is concerned.

- (i) The helicopter was flying less high than a plane can fly.
 (ii) The helicopter was flying lower than a plane can fly.

In section 2.5 I point out the links between the three data clusters data discussed here and give a preview of my position on the relevant phenomena.

2.2 AMBIGUITY IN MAIN CLAUSES

2.2.1 COMPARATIVES WITH ‘LESS’ AND ‘EXACTLY’ DIFFERENTIALS

(Heim, 2001) points out two kinds of ambiguous intensional comparatives. The first one are comparatives with ‘less’. The following two examples are reported to have two distinct readings.

(73) (The draft is ten pages.) The paper is required to be less long than that.

(74) (The draft is ten pages.) The paper is allowed to be less long than that.

Sentence (73) can mean that the paper is shorter than ten pages in every accessible world, implying that it is not allowed to be as long as the draft, see (75)a. On the other reading, the minimal required length of the paper is said to be less than ten pages, which is compatible with the paper being as long as ten pages and complying with the rules, see (75)b.

- (75) a. The paper is not allowed to be as long as the draft.
b. The paper is not required to be as long the draft.

Sentence (74) can mean that there is a permissible state of affairs in which the paper is less than ten pages long, see (76)a. It can also express a much stronger claim conveying that the maximal permitted length lies under ten pages, see (76)b.

- (76) a. A length of the paper under ten pages is permitted.
b. The paper is only allowed to be shorter than ten pages.

Heim notes that this kind of ambiguity is impossible to detect in plain comparatives without ‘less’. For example, consider (77).

(77) (The draft is ten pages long.) The paper is allowed to be longer than that.

This sentence conveys that a paper with a length exceeding ten pages

conforms with the relevant requirement, which is equivalent to the claim that the maximal compliant length exceeds ten pages, if the maximum is defined. So the two kinds of readings available in (74) are indistinguishable in (77). However, if we insert an exact measure of difference, two interpretations become detectable again. Examples like (78) and (79) constitute the second ambiguity environment discussed by Heim.

- (78) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.
- (79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

Sentence (78) either states that the paper is exactly 15 pages long in every world compatible with the relevant rules or that the minimal required length of the paper is exactly 15 pages, see (80).

- (80) a. The paper is allowed to be only 15 pages long.
- b. The paper is only required to be 15 pages long.

(79) either conveys that the paper can be 15 pages long leaving it open whether it can be longer or shorter; or it means that the maximal permitted length of the paper is exactly 15 pages, see (81).

- (81) a. The length of 15 pages is permitted.
- b. The paper is only allowed to be 15 pages long.

2.2.2 HEIM'S PROPOSAL

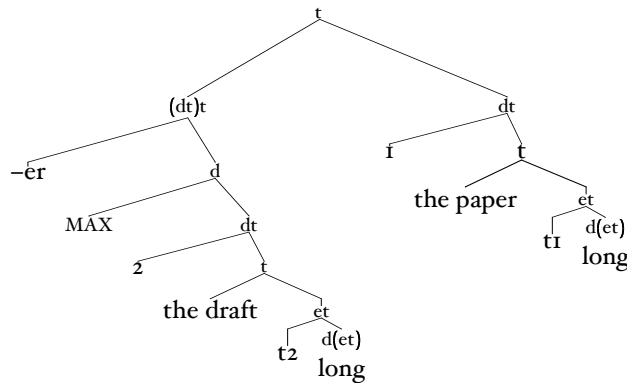
Heim suggests that this kind of ambiguity is a matter of the relative scope of the modal and the comparative morpheme. If 'be required' and 'be allowed' are interpreted at different sites with respect to the position of the comparative morpheme, the desired readings can be derived. The crucial prerequisite for such an account is the treatment of the comparative morpheme as a degree operator. (Heim, 2001) defines it in the following way:

$$(82) \quad \llbracket \text{-er} \rrbracket = \lambda d \lambda D' \text{MAX}(D') > d,$$

where for any D_{dt} : $\text{MAX}(D) = \text{td}$: $d \in D \wedge \forall d' \in D: d' \leq d$.

The first argument of ‘-er’ is a definite description of the maximal degree from the set denoted by the comparative complement, or a referential expression of type d as in the examples above. So the comparative morpheme and the ‘than’-phrase form a constituent that is interpreted as a generalised quantifier over degrees. The scope of the comparative quantifier is the property of degrees expressed by the main clause. ‘-er’ ultimately expresses the greater-than relation between its restriction and the maximal degree in its scope. Like other quantifiers, the comparative quantifier is assumed to be subject to QR. The following example features an LF of a simple comparative sentence.

(83) The paper is longer than the draft is long.



Another important assumption is that gradable adjectives express monotone functions. Heim’s gives the following lexical entry of ‘long’:

$$(84) \quad \llbracket \text{long} \rrbracket = \lambda w \lambda d \lambda x \text{LENGTH}_w(x) \geq d$$

According to (84), ‘long’ relates an individual to all degrees that are smaller than or equal its length. Thus, the extension of the comparative complement in (83) at index w is the set defined in (85).

$$(85) \quad \lambda d \text{LENGTH}_w(\text{the draft}) \geq d$$

= the set of degrees that are smaller than or equal the length of the draft in w

Turning to ambiguous examples (78) and (79) repeated below, they involve a more complex comparative quantifier than example (83). Abstracting away from its internal structure and composition, the meaning of ‘exactly five pages -er than 10 pages’ is given in (86).

(78) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

(79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

(86) $\llbracket \text{exactly 5 pages -er than ten pages} \rrbracket = \lambda D' \max(D') = 10 \text{ pp} + 5 \text{ pp}$

In general, intensional comparatives offer two landing sites for QR. If ‘-er’ lands above the modal, this leads to the ‘minimal requirement’ reading of (78), paraphrased in (80)b, and the ‘maximal permission’ reading of (79), paraphrased in (81)b, see below. The corresponding LFs and their interpretations are given in (87) and (88).

(80) b. The paper is only required to be 15 pages long.

(87) a. $\llbracket \text{exactly 5 pp -er than that} \rrbracket \lambda d \text{ required} \llbracket \text{the paper } d \text{ long} \rrbracket$

b. $\text{MAX}(\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) \geq d) = 15 \text{ pages}$

‘The length of the paper in the accessible world in which it is the shortest is exactly 15 pages.’

‘The minimally required length of the paper equals 15 pages.’

(81) b. The paper is only allowed to be 15 pages long.

(88) a. $\llbracket \text{exactly 5 pp -er than that} \rrbracket \lambda d \text{ allowed} \llbracket \text{the paper } d \text{ long} \rrbracket$

b. $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) \geq d) = 15 \text{ pages}$

‘The length of the paper in the accessible world in which it is the longest is exactly 15 pages.’

‘The maximally permissible length of the paper equals 15 pages.’

It should be noted that the minimal requirement reading in (87)b hinges on the monotonicity assumption. Due to monotonicity, the set of degrees that MAX applies to in (87)b contains only the lengths that the paper reaches in every accessible world, i.e. the degrees of length that it reaches in the world

in which it is the shortest. Without monotonicity, the set in (87)b would be empty if the paper had different lengths across accessible worlds or a singleton, otherwise. In contrast, in (88) monotonicity is not crucial. The reader may verify that the following equation holds.

$$(89) \quad \text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) \geq d) = \\ \text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) = d)$$

If the comparative morpheme undergoes local movement, without crossing the modal, we obtain the second set of readings for examples (78) and (79).

(80) a. The paper is allowed to be only 15 pages long.

(90) a. required [exactly 5 pp -er than that] λd [the paper d long]

b. $\forall w' \in \text{Acc}_w: \text{MAX}(\lambda d \text{LENGTH}_{w'}(p) \geq d) = 15$ pages
 ‘The length of the paper is exactly 15 pages in every accessible world.’

(81) a. The length of 15 pages is permitted.

(91) a. allowed [exactly 5 pp -er than that] λd [the paper d long]

b. $\exists w' \in \text{Acc}_w: \text{MAX}(\lambda d \text{LENGTH}_{w'}(p) \geq d) = 15$ pages
 ‘There is an accessible world in which the paper is exactly 15 pages long.’

Ambiguity in the examples with ‘less’ receives the same account. Consider first two LFs for example (73) and the truth conditions they correspond to. The paraphrases for the derived readings are repeated in (75)a and (75)b, respectively.

(73) (The draft is ten pages.) The paper is required to be less long than that.

(75) a. The paper is not allowed to be as long as the draft.

(92) a. required [[less than that] λd [the paper d long]]

b. $\forall w' \in \text{Acc}_w: \text{MAX}(\lambda d \text{LENGTH}_{w'}(p) \geq d) < 10$ pages

- (75) b. The paper is not required to be as long the draft.
- (93) a. [less than that] [required λd [the paper d long]]
 b. $\text{MAX}(\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_w(p) \geq d) < 10$ pages

In (92)a, the comparative morpheme ‘less’ moves locally and the resulting interpretation is that the paper is less than ten pages in every accessible world, cf. (92)b. In (93), ‘less’ moves to the highest landing site possible, which results in the claim that the minimal requirement is under ten pages. Next, compare (94) and (95), which are Heim’s derivations of (74) repeated below. The local movement LF in (94) corresponds to the weak reading which states that ten pages is a permitted length. The non-local movement LF in (95) represents the strong reading under which the maximal permitted length is under ten pages.

- (74) (The draft is ten pages.) The paper is allowed to be less long than that.
- (76) a. A length of the paper under ten pages is permitted.
- (94) a. allowed [[less than that] λd [the paper d long]]
 b. $\exists w' \in \text{Acc}_w: \text{MAX}(\lambda d \text{LENGTH}_w(p) \geq d) < 10$ pages
- (76) b. The paper is only allowed to be shorter than ten pages.
- (95) a. [less than that] [allowed λd [the paper d long]]
 b. $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) \geq d) < 10$ pages

Heim takes the availability of distinct readings with ‘be required’ and ‘be allowed’ in the two environments discussed here to indicate that a degree phrase can take scope over intensional predicates. This is a weighty argument in favour of the degree operator analysis of the comparative.

2.2.3 PROPOSAL IN ODA (2008)

(Oda, 2008) proposes an alternative way to handle some cases of ambiguity discussed in (Heim, 2001). She observes that in intensional comparatives with ‘exactly’ differentials the two readings can be derived by mani-

pulating the scope of the differential measure phrase with respect to the modal. Let us see how this works concretely.

Oda assumes that differential measure phrases have quantifier denotations. For example, ‘exactly five pages’ in Heim’s example (78), for convenience repeated below, denotes a set of degree properties whose maximal degree measures five pages, see (96).

(78) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

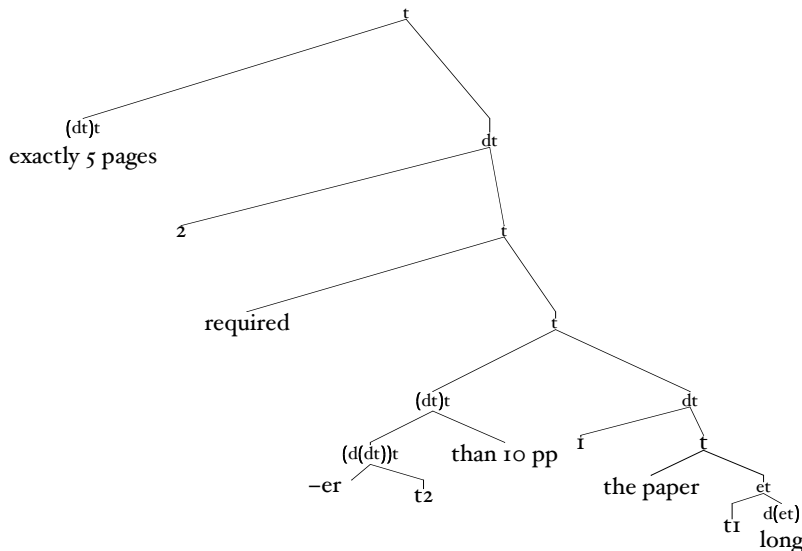
(96) $\llbracket \text{exactly 5 pages} \rrbracket = \lambda D \text{ MAX}(D) = 5 \text{ pages}$

Differential phrases undergo movement leaving a trace of type d that saturates the first argument of the comparative morpheme. The latter is assumed to express the relation of addition on degrees.

(97) $\llbracket \text{-er} \rrbracket = \lambda d \lambda d' \lambda D \text{ MAX}(D) \geq d + d'$

The following LF is derived by raising ‘exactly five pages’ outside the scope of the modal. In (99) I give the main steps in the computation of the extension of (78) at index w .

(98)



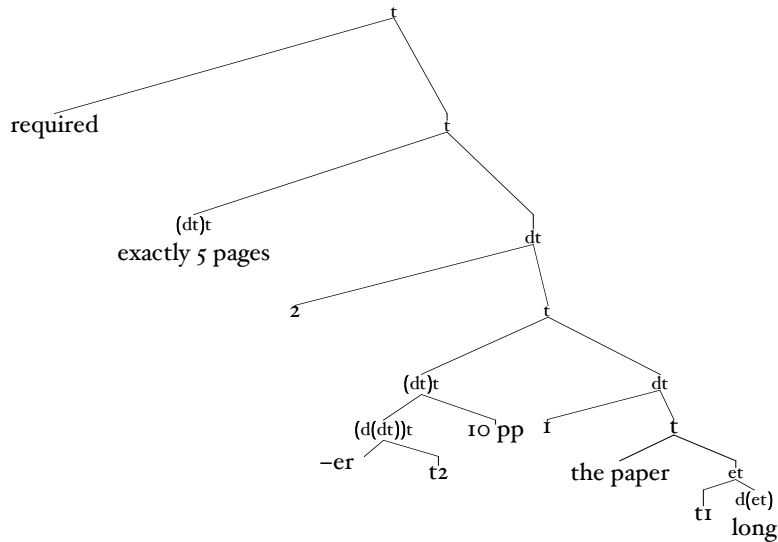
(99) $\llbracket \text{exactly 5 pages} \rrbracket (\lambda d \forall w' \in \text{Acc}_w : \text{LENGTH}_{w'}(p) \geq d + 10 \text{ pp})$
 $= [\lambda D \text{ MAX}(D) = 5 \text{ pp}] (\lambda d \forall w' \in \text{Acc}_w : \text{LENGTH}_{w'}(p) \geq d + 10 \text{ pp})$
 $= \text{MAX}(\lambda d \forall w' \in \text{Acc}_w : \text{LENGTH}_{w'}(p) \geq d + 10 \text{ pp}) = 5 \text{ pp}$

MAX returns the difference between 10 pages, i.e. the length of the draft, and the length the paper reaches in the accessible world in which it is the shortest. So the sentence is predicted true iff the required minimum is five pages longer than the draft. The resulting truth conditions correspond to Heim’s ‘wide scope of the comparative morpheme’ reading and the reader may verify that the following holds:

$$(100) \text{ MAX}(\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) \geq d + 10 \text{ pp}) = 5 \text{ pp} \Leftrightarrow \\ \text{MAX}(\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) \geq d) = 15 \text{ pp}$$

The second reading of (78) is derived by moving the differential within the scope of the modal, see (101) – (102).

(101)



$$(102) \forall w' \in \text{Acc}_w : \text{MAX}(\lambda d \text{ LENGTH}_{w'}(p) \geq d + 10 \text{ pp}) = 5 \text{ pp}$$

‘In every permissible state of affairs the difference between the length of the paper and the actual length of the draft is five pages.’

Example (79), repeated below, can also be assigned two LFs differing in the position on the measure phrase.

(79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

Interpreting ‘exactly five pages’ above ‘allowed’ results in Heim’s ‘wide scope of the comparative morpheme’ reading:

- (103) $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w \text{LENGTH}_w(p) \geq d + 10 \text{ pp}) = 5 \text{ pp}$
 ‘The difference between the maximally allowed length of the paper and the actual length of the draft is five pages.’

Interpreting ‘exactly five pages’ in the scope of ‘allowed’ leads to a weaker reading, corresponding to Heim’s ‘narrow scope of the comparative morpheme’ reading:

- (104) $\exists w' \in \text{Acc}_w \text{MAX}(\lambda d \text{LENGTH}_w(p) \geq d + 10 \text{ pp}) = 5 \text{ pp}$
 ‘There is a permissible state of affairs in which the length of the paper is five pages longer than the actual length of the draft.’

I assumed the Heim-style treatment of the comparative morpheme throughout, but obviously Oda’s analysis does not hinge on the movement of the comparative quantifier. It only presupposes the mobility of differential measure phrases. However, since not all ambiguous comparatives discussed in (Heim, 2001) feature differentials, Oda’s proposal does not cover the full range of the relevant data. If one adopts Oda’s solution for comparatives with ‘exactly’ differentials, one has to look for a different explanation of ambiguity in comparatives with ‘less’, viz. examples (73) – (74) repeated below. So far Heim’s movement analysis is the only candidate available.

- (73) (The draft is ten pages.) The paper is required to be less long than that.
- (74) (The draft is ten pages.) The paper is allowed to be less long than that.

In the following subsection I shall discuss some challenges for the structural ambiguity analyses presented above and argue against a scope approach.

2.2.4 DISCUSSION

2.2.4.1. TWO CLASSES OF MODALS

As Heim observes, not all expressions that the comparative quantifier could potentially interact with behave as ‘be allowed’ and ‘required’. In

fact, ambiguity obtains with a restricted class of modals. In the following examples ambiguity is not attested as can be seen on the infelicity of paraphrases corresponding to the ‘wide scope of the comparative’ readings.

- (105) The paper might be less long than that.
#‘It not possible for it to be as long as that.’
- (106) The paper should be less long than that.
#‘It is not required for it to be as long as that.’
- (107) The paper is supposed to be less long than that.
#‘It is not required for it to be as long as that.’
- (108) I want the paper to be less long than that.
#‘I don’t require it to be as long as that.’

Heim does not provide an explanation for the fact that the modals in (105) – (108) do not interact with the comparative morpheme. She suggests that there can be something in the semantics of these predicates that masks the relevant reading. For example, all predicates in (106) – (108) are known as Neg-Raising, i.e. their outer negations are systematically understood as their inner negations. This property could be responsible for the absence of relevant readings.

Furthermore, Heim points out that only a few modals lead to ambiguity, patterning with ‘be required’ and ‘be allowed.’ Among them are ‘need’ and ‘be able.’ Interestingly, necessity modals permitting wide scope readings result in special interpretations in other scalar contexts, too. Compare the pairs of examples in (109) and (110). This contrast suggests that the problematic reading is not restricted to comparative sentences.

- (109) a. John needs/is required to/has to score less than that.
‘The minimal required score is less than that.’
b. John needs/is required/has to score ten points to win.
‘The minimal required score is ten points.’

- (110) a. John should/is supposed to score less than that.
 #‘The minimal required score is less than that.’
- b. John should/is supposed to score ten points to win.
 #‘The minimal required score is ten points.’

Both sentences in (109) refer to the minimal requirement. This is not the case in (110). Consider the context in (111), which favours a ‘minimal requirement’ interpretation. Sentence (109)b is felicitous in this context, whereas (110)b appears somewhat odd.⁹

- (111) Bill holds a 9 points lead over John. For John to win, he has to outscore Bill.

The same point can be made with possibility modals. Only those modals that give rise to wide scope readings allow for a strong interpretation in a non-comparative environment. Compare (112) and (113).

- (112) a. John is able/is allowed to run less fast than that.
 ‘The maximal speed that John reaches/is allowed to reach is less than that.’
- b. John is able/is allowed to run 10 km/h.
 ‘The maximal speed that John reaches/is allowed to reach is 10 km/h.’
- (113) a. John might be running less fast than that.
 #‘John’s maximal possible speed is less than that.’
- b. John might be running 10 km/h.
 #‘John’s maximal possible speed is 10 km/h.’

The parallel interpretations of (109)a and (109)b on the one hand, and (112)a and (112)b on the other, call for a unified account and suggest that the ambiguity in comparatives need not result from the mobility of the comparative morpheme.

9. A few speakers found (110)b acceptable in the given context; most speakers, however, agree that an ‘at least’ interpretation of the numeral renders the sentence infelicitous without more context.

Although all examples of comparatives in this subsection involve ‘less’, one can make the same point with ‘exactly’ comparatives. As shown in (114) – (115), a comparative with an ‘exactly’ differential can have a wide scope reading if it features a necessity modal that has a ‘minimal requirement’ reading outside comparatives. The same holds for comparatives with possibility modals, modulo a ‘permitted maximum’ reading. By a wide scope reading I mean the reading derived by the long movement of the comparative morpheme in (Heim, 2001) or by the long movement of the differential in (Oda, 2008). This strongly suggests that ambiguities in comparatives with ‘exactly’ differentials and those in ‘less’ comparatives need to be treated in parallel and are unlikely to be rooted in the semantics of the comparative.

- (114) a. John needs/is required to/has to score exactly five points more than that.
 ‘The minimal required score is exactly five points more than that.’
- b. John needs/is required/has to score ten points to win.
 ‘The minimal required score is ten points.’
- (115) a. John should/is supposed to score exactly five points more than that.
 #‘The minimal required score is exactly five points more than that.’
- b. John should/is supposed to score ten points to win.
 #‘The minimal required score is ten points.’

To conclude, a scope analysis is not able to predict whether a particular modal leads to an ambiguity, nor does it account for the parallels in the interpretation of the two groups of modals in comparative and non-comparative environments.

2.2.4.2. ‘BEFORE’ CLAUSES

It does not come as a surprise that the kind of ambiguity Heim’s paper focuses on surfaces in other contexts as well. Sentences with temporal ‘befo-

re' phrases behave exactly like comparatives with 'less'. (Fox and Hackl, 2006) discuss a reading of (116) that corresponds to the strong 'maximum' readings in (112).

(116) You are allowed to arrive before 10 p.m.

Fox and Hackl consider (116) as addressed to a person who is staying at a Youth Hostel that is locked up over night. In this context the sentence is a way to convey the time at which the doors are locked. One can use (116) to communicate that the latest time at which one can enter the building precedes 10 p.m. This reading is analogous to the 'wide scope of the comparative' reading of intensional comparatives with 'be allowed'. Compare Fox and Hackl's example in (116) to Heim's example in (74) repeated below.

(74) (The draft is ten pages.) The paper is allowed to be less long than that.

It would be plausible to assume that the ambiguity in (116) and (74) is of the same nature. If we agree with Heim that in comparatives it is structural, i.e. that it results from the non-local movement of the comparative morpheme, we have to come up with a similar analysis for sentences with 'before' PPs. To see if such a project can have a success, it is useful to look at the standard assumptions about the semantics of temporal prepositions.

(Beaver and Condoravdi, 2004), following an earlier work by (von Stechow, 2002), argue convincingly that 'before' expresses a relation of temporal precedence between two times. In example (117) below, it combines directly with the time expressed by the measure phrase. The 'before' phrase expressing a property of times, is composed conjunctively with the rest of the clause, as shown in (118). Alternatively, it can be attached to the complement of the modal, see (119). Then, the tense, here Past, applies and instantiates the conjoined temporal property.

(117) John was allowed to arrive before 10 p.m.

(118) Past $[\lambda t \text{ allowed}(t)(\lambda t' \text{ John arrive}(t'))][\lambda t t \text{ before } 10 \text{ p.m.}]$

(119) Past $[\lambda t \text{ allowed}(t)([\lambda t' \text{ you arrive}(t')][\lambda t t \text{ before } 10 \text{ p.m.}])]$

The first attachment site gives us a reading saying that the permission to arrive was given to John before 10 p.m. The other scope configuration results in a more natural reading, under which the permitted time of John's arrival precedes 10 p.m. To see that this is indeed what (118) and (119) express we need to consider the temporal orientation of the modal. Crucially, as argued in (Condoravdi, 2001) and (von Stechow, 2005), modals do not only control the instantiation time of their complements, they also shift that time into the future. Details aside¹⁰, the truth conditions derived are the following:

(120) $\exists t < \text{now}: t \text{ precedes } 10 \text{ p.m.} \ \& \ \exists w' \in \text{Acc}(w,t) (\exists t' > t) \text{ John arrives in } w' \text{ at } t'$
 'There is a past time t before 10 p.m., s.t. John was permitted at t to arrive a time following t .'

(121) $\exists t < \text{now}: \exists w' \in \text{Acc}(w,t) (\exists t' > t) \text{ John arrives in } w' \text{ at } t' \ \& \ t' \text{ precedes } 10 \text{ p.m.}$
 'There is a past time t , s.t. John was allowed at t to arrive at a time t' following t and preceding 10 p.m.'

Thus, depending on whether the 'before' PP sits above the modal or below it at the LF, it contributes to the temporal perspective of the modal or that of its complement. One may conclude that the wide scope of 'before' relative to the modal cannot be responsible for the strong reading reported by Fox and Hackl.

Fox and Hackl themselves do not even consider the possibility of a scope account for their example in (116). They suggest that there are two pragmatic mechanisms responsible for the strong reading of this sentence. The first one produces the so called free choice implicature, which turns the

10. Von Stechow, for instance, proposes to handle the future orientation of a modal by embedding a covert future operator in its complement.

plain assertion that you can arrive at a time t before 10 p.m. to the stronger statement that you can arrive at any time t before 10 p.m. The second pragmatic enrichment component is a strengthening by a scalar implicature which is possible due to the existence of the free choice interpretation. The fact that you are allowed to arrive any time before 10 p.m. is inferred to be the most informative true possibility among alternative assertions of the kind ‘you are allowed to arrive any time before t ’. If the plain meaning of (116) is strengthened by these implicatures the addressee gets informed about the latest time before which she is allowed to arrive. I elaborate Fox and Hackl’s proposal in chapter 4 and consider whether it can be applied to comparatives.

2.2.5 CONCLUSION

The foregoing discussion highlighted two problems for Heim’s scope approach to ambiguity in intensional comparatives. First, this kind of ambiguity is not comparative specific. If one is committed to a scope solution a unified account becomes a challenge, as shown by our discussion of ‘before clauses’. Second, most intensional predicates do not lead to the problematic wide scope readings in comparatives. Those that do receive analogous interpretations, i.e. ‘minimal requirement’ and ‘maximal permission’ readings, in non-comparative contexts as well.

The discussion envisaged some alternative ways to handle Heim ambiguities. I mentioned the possibility informally discussed in (Fox and Hackl, 2006) that the ambiguity in sentences with ‘be allowed’ might be a pragmatic effect. The cases with necessity modals call for a different account. To identify the source of ambiguity with ‘be required’, it is necessary to pin down semantic differences between modals that trigger ambiguities and those that behave like ‘should’ and other Neg-Raising predicates. To conclude, in what follows I do not attempt a unified account of all ambiguous intensional comparatives discussed in (Heim, 2001). Instead, the cases with

possibility modals and necessity modals will be handled as two independent tasks.

2.3 QUANTIFIERS IN COMPARATIVE COMPLEMENTS

Traditionally, intensional comparative complements, i.e. ‘than’-clauses containing intensional operators, are treated by analogy to ‘than’-clauses containing DP quantifiers, cf. (Schwarzschild and Wilkinson, 2002), (Heim, 2006), (Beck, 2010b), (van Rooij, 2008), (Schwarzschild, 2008). I follow in the footsteps of these theories and argue that, though plausible in some cases, the parallel between embedded nominal quantifiers and intensional predicates should be given up in others.

2.3.1 VON STECHOW’S DESIDERATA

(von Stechow, 1984) points out some interesting aspects in the interpretation of conjunction and disjunction in comparative complements. He notes that his observations concerning these connectives can be extended to DP quantifiers. The following two facts are of interest in the light of the present discussion. First, in many cases disjunction inside a ‘than’ clause is interpreted as conjunction. For example, (122)a has a reading paraphrased by (122)b.

- (122) a. Ede is wiser than Pericles or Socrates.
b. Ede is wiser than Pericles and he is wiser than Socrates.

This reading can be derived in one way or another by most theories of comparatives. Let us consider the analysis adopted at the beginning of this thesis. As observed in (von Stechow, 1984), the definite analysis of ‘than’-clauses that was laid out in section 1.2 is *prima facie* problematic for such cases. The denotation of the embedded clause does not always correspond to a singleton set from which a unique degree can be selected. (123) is undefined unless Pericles is as wise as Socrates.

- (123) $\text{DEF}\{\lambda d \text{ WISDOM}_w(\text{Pericles}) = d \vee \text{WISDOM}_w(\text{Socrates}) = d\}$
‘the degree d , such that Pericles is d wise or Socrates is d wise’

As a remedy, von Stechow proposes to constrain the restriction set of the

definite operator by a prior application of MAX operator, i.e. turn it into the a singleton containing the maximum from the original set, as shown in (124). This step leads to the desired more-than-maximum interpretation of (122)a, see (125).

(124) $\text{DEF}\{\text{MAX}\{\lambda d \text{ WISDOM}_w(\text{Pericles}) = d \vee \text{WISDOM}_w(\text{Socrates}) = d\}\}$
‘the maximal degree d , such that Pericles is d wise or Socrates is d wise’

(125) $\text{WISDOM}_w(\text{Ede}) >_{\text{wise}}$ the maximal d , s.t.
 $\text{WISDOM}_w(\text{Pericles}) = d \vee \text{WISDOM}_w(\text{Socrates}) = d$

Second, von Stechow observes that conjunction is interpreted as one would expect, i.e. (126) can also be paraphrased by (122)b. However, even if a theory is equipped to deal with disjunction it still faces problems with conjunction. For instance, von Stechow’s theory wrongly predicts that (126) is undefined if Pericles and Socrates are not equally wise, cf. (127).

(126) Ede is wiser than Pericles and Socrates.

(127) $\text{WISDOM}_w(\text{Ede}) >_{\text{wise}}$ the maximal d , s.t. $\text{WISDOM}_w(\text{Pericles}) = d \wedge \text{WISDOM}_w(\text{Socrates}) = d$
‘Ede’s wisdom exceeds the maximal d , s.t. Pericles is d wise and Socrates is d wise.’

Von Stechow notes that the correct truth condition for (126) can be derived by interpreting ‘Pericles and Socrates’ outside the scope of the comparative, as outlined in (128).

(128) Pericles and Socrates $\lambda x \text{ WISDOM}_w(\text{Ede}) >_{\text{wise}}$ the maximal d ,
s.t. $\text{WISDOM}_w(x) = d$
‘Ede is wiser than Pericles and he is wiser than Socrates.’

Since von Stechow’s work it has been a challenge to develop a theory that is able to account for the strong reading of disjunction and \exists quantifiers and derives adequate truth conditions for comparatives with embedded conjunction and \forall quantifiers.

As one would expect, comparatives with intensional operators in ‘than’-clauses have always been considered against this background. Prima

facie, it seems that intensional predicates are not different from nominal quantifiers with respect to the interpretation they give rise to in comparative complements. Comparatives with embedded possibility modals have more-than-maximum readings that can be described as a comparison with the permitted/possible maximum. For example, (129) compares the amount of food that Pat ate to what was maximally permitted. This meaning is correctly captured by the formula à la von Stechow in (130). The parallel to the example with disjunction, viz. (125), is striking.

(129) Pat ate more than he was allowed to.

(130) $\iota d(\text{Pat ate } d \text{ much in } w) > \text{the maximal } d'$,
s.t. $\exists w' \in \text{Acc}_w$: Pat ate d' much in w'
‘Pat ate more than maximally permitted.’

Similarly, intensional predicates with universal force seem to pattern with conjunction in that they also trigger more-than-maximum readings. Consider an example with ‘predict’. Suppose that my prediction is that Pat’s height lies within the interval from 1.65 m to 1.70 m.

(131) Pat is taller than I predicted.

In the given situation sentence (131) is true iff Pat is taller than 1.70 m, which is what the formula in (132) correctly predicts. Again the analogy to conjunction is hard to overlook, compare (132) with (128).

(132) $\forall w' \in \text{Acc}_w$: $\text{HEIGHT}_w(\text{Pat}) >_{\text{tall}} \text{HEIGHT}_{w'}(\text{Pat})$
‘For all worlds w' accessible from w : Pat’s height in w exceeds Pat’s height in w' .’

This parallelism between intensional verbs and other \forall quantifiers lead most researchers to the conclusion that they need a unified account, and since intensional predicates are not likely to QR, scoping of a conjunction or a \forall quantifier from a comparative complement is not considered a viable option to derive correct interpretations of such sentences. (Schwarzschild and Wilkinson, 2002) discuss cases like (131) to argue

against von Stechow’s ‘quantifying in’ solution. QR normally does not apply to expressions like ‘predict’ which are assumed to be immobile at the LF.¹¹ Even if one gives up a unified account of universal DPs and intensional predicates and applies QR to nominal quantifiers only, this still has unattractive consequences. For example, allowing QR out of a comparative clause is not in accord with the well-known parallels between constraints on QR and the *wh*-movement, see (133).

- (133) a. *Who₁ did John ran faster than t₁ rode the bicycle?
 b. John ran faster than everyone else rode the bicycle.

Though most intensional predicates behave like DP quantifiers, some of them have been noticed to pattern quite differently. First, (Heim, 2006) shows that epistemic ‘might’ is not interpreted as deontic ‘be allowed’, and for that matter as ‘or’. Heim’s example in (134) does not have a more-than-maximum reading, which we have seen with ‘be allowed’ in (129). More concretely, today’s temperature is not compared to tomorrow’s maximal temperature. Rather, the sentence is judged true iff today’s temperature is higher than at least the minimal temperature predicted for tomorrow. I will call this interpretation more-than-minimum reading. As the paraphrase under the example suggests, ‘might’ is interpreted higher than the comparative.

- (134) It is warmer today than it might be tomorrow.
 ‘It is possible that it will be colder tomorrow that it is today.’

Second, (Schwarzschild, 2004) and (Heim, 2006) observe that not all universal modals require quantifying in. Some seem to take narrow scope with respect to comparison. In the terminology adopted here, some modals

11. Technically, this is possible, as Arnim von Stechow (p.c.) remarks. Assuming an extensional framework, with world variables in the object language, one can treat intensional predicates as occupying the world argument slot of their complements and subject to QR.

lead to more-than-minimum rather than more-than-maximum readings. Examples of such modals are ‘need’, ‘have to’, ‘be required.’ Consider a comparative with an embedded ‘have to’.

(135) Pat is taller than he has to be in order to become a pilot.

The argument in favour of a narrow scope interpretation goes as follows. Suppose that there is a defined interval into which one’s height has to fall, for one to become a pilot. Assume that the accessible worlds of ‘have to’ are the ones in which the goal of becoming a pilot is achieved. The wide scope reading of ‘have to’ can be represented by the formula in (132), corresponding to the wide scope of ‘predict’. The difference in the interpretation of ‘predict’ and ‘have to’ boils down to the difference in the accessibility relation. Crucially, (135) does not imply that Pat’s height exceeds the maximally permitted height as (132) reads. Rather, Pat’s height is said to exceed the minimally required height, which cannot be captured by von Stechow’s analysis as it stands. Interpreting ‘have to’ in situ does not produce a comparison with the minimal requirement, cf. (136).

(136) $\text{HEIGHT}_w(\text{Pat}) >_{\text{tall}}$ the maximal d' , s.t.
 $\forall w' \in \text{Acc}_w: \text{HEIGHT}_{w'}(\text{Pat}) = d'$
 ‘Pat’s height in w exceeds the unique required height.’

However, as (Heim, 2006) shows, building monotonicity into the meaning of ‘tall’ results in the correct truth conditions for this example, cf (136).

(137) $\text{HEIGHT}_w(\text{Pat}) >_{\text{tall}}$ the maximal d' , s.t.
 $\forall w' \in \text{Acc}_w: \text{HEIGHT}_{w'}(\text{Pat}) \geq d'$
 Pat’s height in w exceeds the height Pat reaches in every accessible world, i.e. the minimally required height.’

Summing up, assigning ‘might’ a wide scope and ‘have to’ a narrow scope with respect to the comparative captures the intuitively available interpretations of the relevant examples. A lesson learned from this is that scope might be the right mechanism to account for the interpretational variability of quantifiers in comparative complements, see (Schwarzschild,

2004), (Heim, 2006), (Schwarzschild, 2008), (Gajewski, 2008).

Let us recapitulate. Von Stechow put forth two desiderata concerning the treatment of comparatives with embedded quantifiers. The first one is the derivation of more-than-maximum readings of disjunction and \exists quantifiers. The second one is the derivation of more-than-maximum readings of conjunction and \forall quantifiers. The pattern of intensional comparative complements is to a large extent parallel to that of ‘than’ clauses with DP quantifiers. Can this parallelism be our guide in tackling von Stechow’s tasks? On the one hand, it suggests that a scope solution is not on the right track. I have already commented on the implausibility of deriving a more-than-maximum reading by QRing a DP quantifier out the comparative complement. With intensional predicates the QR solution appears to be even less plausible. On the other hand, the availability of more-than-minimum readings with some intensional predicates, i.e. in ‘than’ clauses with ‘might’ and ‘have to’, might be argued to speak in favour of a scope strategy.

In the following, I present the existing analyses of quantifiers that apparently take wide scope with respect to the comparative. These are mostly attempts to solve the problem of conjunction and \forall quantifiers. I then explore what these analyses propose for the apparent narrow scope quantifiers and conclude by a discussion.

2.3.2 WIDE SCOPE QUANTIFIERS

Von Stechow’s observation that conjunction has to be quantified in the comparative complement in the analysis of sentences like (126), repeated below, was debated a lot in the subsequent literature on comparatives. We have seen that an in-situ scope of conjunction leads to a wrong interpretation of (126) in a definite-style analysis like von Stechow’s – (126) is only defined under such an analysis if Pericles and Socrates have the same degree of wisdom, a possible but not the only reading of (126), cf. (138).

(126) Ede is wiser than Pericles and Socrates.

(138) $\iota d(\text{Ede is } d \text{ wise}) >_{\text{wise}}$ the maximal d , s.t. Pericles is d wise and Socrates is d wise

It can be easily shown that Heim’s modification of von Stechow’s analysis based on a monotone meaning of gradable adjectives does not fare better. It predicts that (126) compares Ede’s wisdom with whoever is the least wise among Pericles and Socrates, cf. (139).

(139) $\iota d(\text{Ede is } d \text{ wise}) >_{\text{wise}}$ the maximal d , s.t. Pericles is at least d wise and Socrates is at least d wise

In general, the difference in the predictions of the approaches that von Stechow’s analysis draws on and those of the successors of his theory stems from the treatment of gradable predicates. In a ‘non-monotone’ theory a gradable predicate relates an individual x to a unique degree to which x possesses the relevant property, cf. (140)a. In a ‘monotone’ theory a gradable predicates relates an individual x to a set of degrees d such that x possesses the relevant property to at least d , cf. (140)b.

(140) a. $\llbracket \text{wise}_{\text{non-mon}} \rrbracket = \lambda_w \lambda d \lambda x \text{ WISDOM}_w(x) = d$

b. $\llbracket \text{wise}_{\text{mon}} \rrbracket = \lambda_w \lambda d \lambda x \text{ WISDOM}_w(x) \geq d$

Skipping over the particulars of the analyses of (126) in other frameworks, it suffices to say that ‘monotone’ theories predict the unavailable weak interpretation of (126) given in (139) (e.g. (Seuren, 1973), (Kennedy, 2001), (Heim, 2001)); and those that do not treat adjectives as monotone relations impose the ‘equality’ condition, which need not be met (e.g. (Cresswell, 1976), (von Stechow, 1984)).

It does not come as a surprise that universal DP show exactly the same pattern as conjunction. Similar to (126), the comparative sentence in (141) does not have a reading paraphrasable by (141)a. It can only be understood

as (141)b or (141)c. The latter reading is problematic for any theory of comparatives mentioned so far.

- (141) Hubert is taller than everyone else is.
- a. Hubert's height exceeds the height of the shortest other person.
 - b. Hubert's height exceeds the height that every other person shares.
 - c. Hubert's height exceeds the height of the tallest other person.

(Schwarzschild and Wilkinson, 2002) point out a wide range of expressions that seem to outscope the comparative in the same fashion as conjunction but for which the derivation of the relevant readings in terms of QR appears problematic, see (142) – (147).

- (142) Hubert is taller than exactly five of the others are.
'For exactly five people x different from Hubert: Hubert's height exceeds x 's height.'
- (143) Hubert is taller than only one of the others is.
'For only one x different from Hubert: Hubert's height exceeds x 's height.'
- (144) Alice is richer than George was and than most of his children will ever be.
'For most of George's children x : Alice is richer than George was and Alice is richer than x will be.'
- (145) Lucy paid more for her suit than they both paid in taxes last year.
'For both x : Lucy paid more for the suit than x paid in taxes last year.'
- (146) It is colder in Stony Brook today than it usually is in New Brunswick.
'For most times t : the temperature in Stony Brook at t is exceeded by the temperature in New Brunswick at t .'
- (147) Bill did better than John predicted most of his students would do.
'For all worlds w compatible with John's prediction in the actual world: Bill's score in the actual world exceeds the score reached by most students in w .'

To this list I add Heim’s example with ‘might’ repeated below:

- (134) It is warmer today than it might be tomorrow.
‘It is possible that it will be colder tomorrow than it is today.’

There are several proposals as to how to tackle the problem, i.e. to derive the problematic reading without raising the quantifier out of the comparative complement. They can be divided into two groups. The first group, which I shall refer to as ‘scope-over-negation’ approaches, adopts a Seuren-style analysis of comparatives, which will be introduced shortly, and follows the strategy first pursued by (Larson, 1988) to interpret conjunction above the comparative internal negation, cf. (Marques, 2004), (Schwarzschild, 2008), (van Rooij, 2008), (Gajewski, 2008). The second group can be called ‘interval’ approaches. ‘Interval’ analyses share an assumption that a semantic shift from degrees to intervals is necessary for an adequate treatment of embedded quantifiers, cf. (Schwarzschild and Wilkinson, 2002), (Heim, 2006), (van Rooij, 2008), (Beck, 2010b). In the rest of the section I elaborate a representative approach from each group.

‘Scope-over-negation’ theories assume an indefinite analysis of the comparative first proposed in (Seuren, 1973). Seuren’s semantics for a simple comparative sentence ‘*A* is taller than *B*’ is outlined in (148). *A* is attributed a degree of height which is not reached by *B*.

- (148) ‘*A* is taller than *B*’ iff $\exists d$: *A* is at least *d* tall and \neg (*B* is at least *d* tall)

Such an analysis presupposes a monotone meaning of ‘tall’ and, consequently, it faces a problem, shared by ‘monotone’ theories, of predicting an unavailable weak interpretation in the case of embedded conjunction and \forall quantifiers, cf. (149).

- (126) Ede is wiser than Pericles and Socrates.
(149) $\exists d$: Ede is at least *d* wise \wedge \neg (Pericles and Socrates is at least *d* wise)
‘Ede is wiser than Pericles or he is wiser than Socrates.’
‘Ede is wiser than the least wise of Pericles and Socrates.’

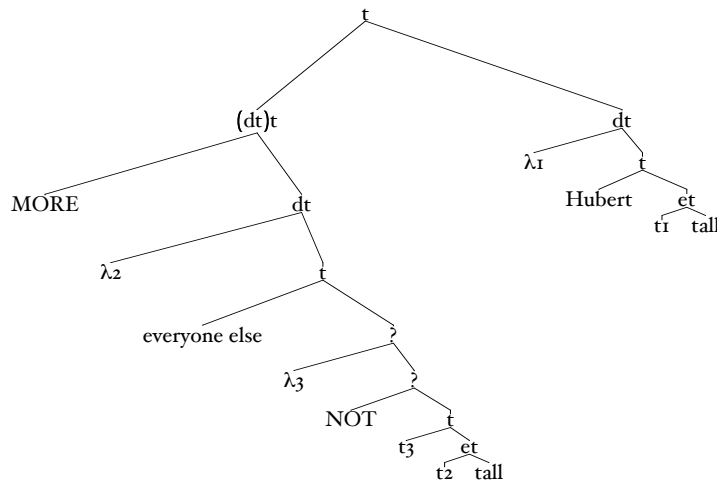
The main goal of ‘scope-over-negation’ approaches is to get a quantifier out of the scope of negation. (150) demonstrates that this results in a correct interpretation.

- (150) $\exists d$: Ede is at least d wise \wedge
 $\forall x \in X$: $X = \{\text{Pericles, Socrates}\} \rightarrow \neg(x \text{ is at least } d \text{ wise})$
 ‘Ede is wiser than the wisest of Pericles and Socrates.’

Technically, there are different possibilities to arrive at the interpretation in (150). (Schwarzschild, 2008) assumes that a comparative complement contains an abstract operator NOT with the meaning of sentential negation. An LF for (141) that implements his idea is given in (151).

- (141) Hubert is taller than everyone else is.

(151)



MORE combines with the sets of degrees gained by abstracting over the degree arguments of the adjectives in the main and the embedded clause and existentially binds the degree variables, see (152). NOT simply switches a truth value to the opposite, see (153).¹²

$$(152) \llbracket \text{MORE} \rrbracket = \lambda D_{dt} \lambda D'_{dt} \exists d: D(d) \wedge D'(d)$$

$$(153) \llbracket \text{NOT } \alpha \rrbracket = 1 \text{ iff } \llbracket \alpha \rrbracket = 0.$$

12. To simplify exposition, I omit reference to worlds whenever possible.

Assuming that ‘tall’ is defined as a monotone function that relates an individual x to the degrees of height that x reaches, one can easily show that the LF in (151) corresponds to the desired more-than-maximum reading of (141), see (154).

$$(154) \quad \llbracket \text{MORE} \rrbracket (\lambda d \text{ HEIGHT}(\text{Hubert}) \geq d) \\
(\lambda d \forall x \neq \text{Hubert}: \neg \text{HEIGHT}(x) \geq d) = \\
\exists d': \text{HEIGHT}(\text{Hubert}) \geq d \wedge \forall x \neq \text{Hubert}: \neg \text{HEIGHT}(x) \geq d$$

Two remarks are on order concerning the compositionality aspect. First, under this analysis MORE does not express a comparative relation. It has to be defined as an existential quantifier over degrees. Moreover, such a quantifier has to apply to an open proposition that contains NOT, otherwise no comparison can be expressed. This restriction has to be stipulated in the lexical meaning of MORE, e.g. as a definedness condition. This renders the analysis uncompositional. The second stipulation necessary to obtain the correct truth conditions for (141) is that NOT is interpreted under the quantifier. This assumption does not appear to have any independent motivation, but see next section for a discussion of the scope possibilities of NOT.

‘Interval’ approaches build on an idea first discussed in (Schwarzschild and Wilkinson, 2002) that a comparative complement provides an interval on the relevant scale, not a degree. (Heim, 2006) implements this by type-shifting the meaning of gradable predicates, which are assumed to express relations between degrees and individuals. A sample type-shifted lexical entry for ‘tall’ is given in (155). According to it, ‘tall’ relates an individual x to a set of degrees that contains x ’s height.

$$(155) \quad \llbracket \text{tall}_{\text{Int}} \rrbracket = \lambda D_{\text{dt}} \lambda x_e \text{ HEIGHT}(x) \in D$$

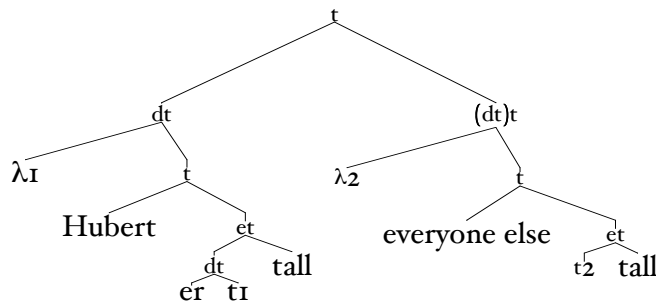
Let me apply Heim’s analysis to (141). The embedded clause expresses a generalised degree quantifier. This meaning is the result of abstracting over the first argument of ‘tall’. The embedded clause originates in the argument

position of the comparative morpheme. Since the latter denotes the ‘>’ relation between two degrees, see (156), the embedded clause has to undergo movement to fix a type mismatch. The resulting LF is sketched in (157). In (158) I give the main steps in the computation and the resulting extension, which corresponds to the reading we were after, viz. (141)c repeated below.

(141) Hubert is taller than everyone else is.

(156) $\llbracket \text{er} \rrbracket = \lambda d \lambda d' d' > d$

(157)



(158) $[\lambda D \forall x \neq H: \text{HEIGHT}(x) \in D](\lambda d \text{HEIGHT}(H) \in \llbracket \text{er} \rrbracket(d)) =$
 $[\lambda D \forall x \neq H: \text{HEIGHT}(x) \in D](\lambda d \text{HEIGHT}(H) \in [\lambda d' d' > d]) =$
 $[\lambda D \forall x \neq H: \text{HEIGHT}(x) \in D](\lambda d \text{HEIGHT}(H) > d) =$
 $\forall x: x \neq H: \text{HEIGHT}(x) \in [\lambda d \text{HEIGHT}(H) > d] =$
 $\forall x: x \neq H: \text{HEIGHT}(H) > \text{HEIGHT}(x)$
 ‘Hubert is taller than the tallest other person.’

(141) c. Hubert’s height exceeds the height of the tallest other person.

Both kinds of approaches offer straightforward solutions to the problem of embedded universal quantifiers. The analyses presented here can be successfully extended to examples in (142) – (147). Before comparing their strengths let us look at what they predict for the apparent narrow scope quantifiers.

2.3.3 NARROW SCOPE QUANTIFIERS

Disjunction, \exists quantifiers and most possibility modals do not create an unsurmountable problem for the analyses of comparatives I introduced above. Von Stechow shows that a definite-style analysis can handle the more-

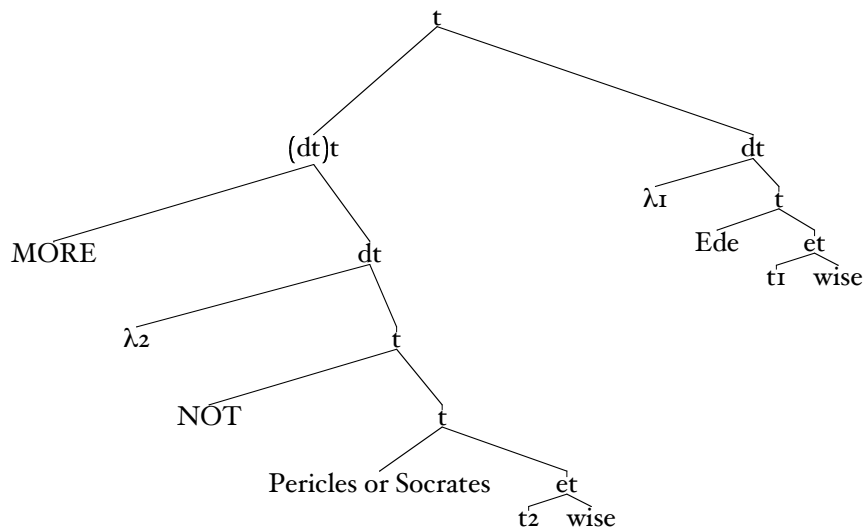
than-maximum reading of (122)a paraphrased in (122)b by maximizing the set of degrees expressed by the comparative complement. In an indefinite Seuren-style analysis the more-than-maximum reading of (122)a is the result of interpreting ‘Pericles or Socrates’ in the scope of negation, cf. (159).

- (122) a. Ede is wiser than Pericles or Socrates.
 b. Ede is wiser than Pericles and he is wiser than Socrates.

(159) $\exists d$: Ede is at least d wise \wedge \neg (Pericles or Socrates is at least d wise)

In order to derive this reading (Schwarzschild, 2008) stipulates that NOT takes wide scope with respect to disjunction. He proposes to treat NOT as an operator that can undergo movement at the logical form. His analysis of (122)a, which is sketched in (160), derives the desired truth conditions in (159).

(160)



Schwarzschild claims that all DP quantifiers take scope above NOT unless they are polarity sensitive. In the latter case they are forced to stay in the downward entailing context created by negation, a plausible explanation for the apparent narrow scope of ‘anyone else’ in (161), cf. (162). Disjunction has to be subsumed under the class of polarity sensitive expressions as well.

(161) Hubert is taller than anyone else is.

(162) $\exists d$: Hubert is at least d tall $\wedge \neg \exists x \neq \text{Hubert}$: x is at least d tall

Further on, (Schwarzschild, 2008) observes that the scope of many embedded intensional predicates with respect to NOT is a reflex of their behaviour in non-comparative sentences with negation. Consider the a. and c. pairs in (163) – (166). The reader may check that the paraphrases provided for the c. examples can be derived in Schwarzschild’s analysis by assigning NOT the same scope that negation has in the a. variants. In (163)d – (166)d I sketch Schwarzschild-style LFs for the c. examples.

(163) should > NOT

- a. The tower should not be that high.
‘It is required that the tower is not that high.’
- b. should [NOT the tower d high]
- c. The tower is higher than it should be.
‘For all worlds w compatible with the relevant rules in the actual world: the actual height of the tower exceeds its height in w .’
- d. MORE [λd should [NOT the tower d high]] [λd the tower d high]

(164) might > NOT

- a. The tower might not have been that high.
‘It is possible that the tower was not that high.’
- b. might [NOT the tower have been d high]
- c. The tower is higher than it might have been.
‘There is a world w compatible with what the speaker knows in the actual world: the actual height of the tower at the time of speech exceeds the height it had at some past time in w .’
- d. MORE [λd might [NOT the tower have been d high]] [λd the tower d high]

(165) NOT > have to

- a. The tower does not have to be that high.
‘It is not required that the tower is that high.’

- b. NOT [have to [the tower d high]]
- c. The tower is higher than it has to be.
‘There is a world w compatible with the relevant rules in the actual world: the actual height of the tower exceeds its height in w .’
- d. MORE [λd NOT [have to [the tower d high]]] [λd the tower d high]

(166) NOT > be allowed

- a. The tower is not allowed to be that high.
‘It is not permitted that the tower is that high.’
- b. NOT [allowed [the tower d high]]
- c. The tower is higher than it is allowed to be.
‘For all worlds w compatible with the relevant rules in the actual world: the actual height of the tower exceeds its height in w .’
- d. MORE [λd NOT [allowed [the tower d high]]] [λd the tower d high]

An ‘interval’ theory, like the one developed in (Heim, 2006), is *prima facie* not as well equipped to deal with the narrow scope quantifiers as ‘scope-over-negation’ theories. Heim remarks that removing the problem of wide scope quantifiers through a type-shift leads to difficulties with expressions that pattern with disjunction and are not a problem for classical analyses, like (Seuren, 1973) or (von Stechow, 1984). To be able to account for both kinds of quantifiers Heim modifies her original proposal. She suggests that the type-shifting operation can apply at the LF directly instead of lexically. Depending on the place in the derivation of the comparative complement at which it applies a narrow or a wide scope reading results.

Heim assumes that the argument slot of a gradable adjective hosts a phrase headed by the type-shifting operator which she calls PI (‘point to interval’), see (167).

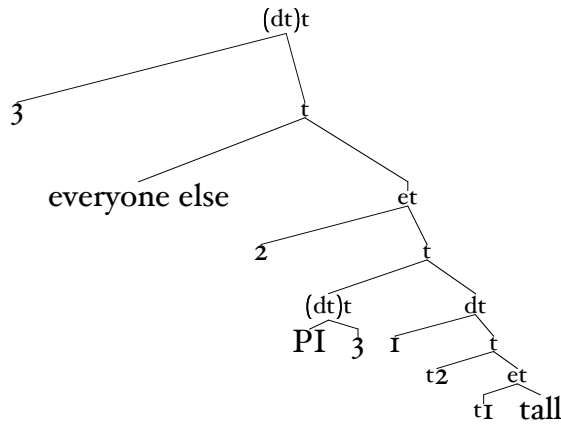
$$(167) \llbracket \text{PI} \rrbracket = \lambda D_{\text{dt}} \lambda D'_{\text{dt}} \text{MAX}(D') \in D,$$

where for any D_{dt} : $\text{MAX}(D) = \text{td}$: $d \in D \wedge \forall d' \in D: d' \leq d$.

Heim's new architecture for the 'than' clause of (141) repeated below is given in (168). The new features are the degree-based monotone meaning of 'tall', which is provided in (169), and the operator PI . In (170) I demonstrate that the extension of the entire clause has not changed, cf. (158).

(141) Hubert is taller than everyone else is.

(168)



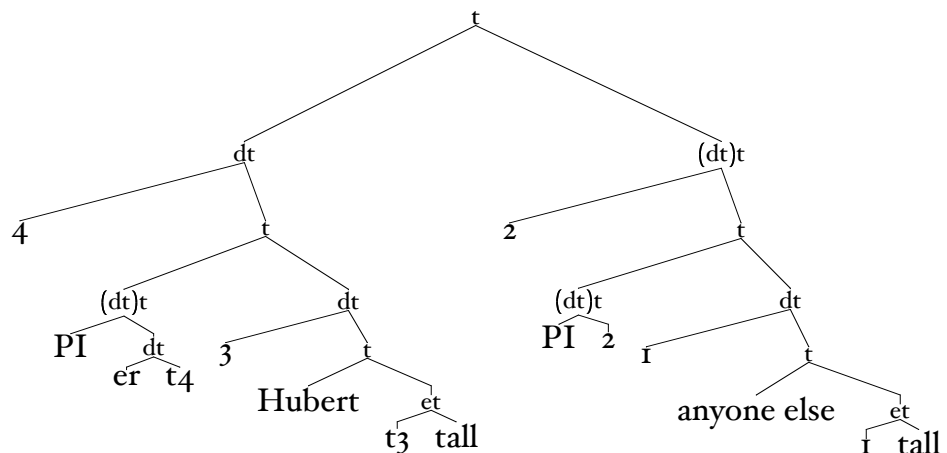
(169) $\llbracket \text{tall} \rrbracket = \lambda d \lambda x \text{HEIGHT}(x) \geq d$

(170) $\lambda D \forall x \neq \text{Hubert}: \llbracket \text{PI} \rrbracket(D)(\lambda d \text{HEIGHT}(x) \geq d) =$
 $\lambda D \forall x \neq \text{Hubert}: \text{MAX}(\lambda d \text{HEIGHT}(x) \geq d) \in D =$
 $\lambda D \forall x \neq \text{Hubert}: \text{HEIGHT}(x) \in D$

To appreciate the role of PI consider now the derivation of an example with 'anyone else'.

(161) Hubert is taller than anyone else is.

(171)



Note that the PI constituents originate in the argument positions of ‘tall’ and then move to avoid a type mismatch. The entire ‘than’ clause is also QRed out of the argument position of the comparative for type reasons. Crucially, after QR the embedded PI ends up above ‘anyone else’. As the computation below shows, this LF corresponds to the narrow scope reading of ‘anyone else’.

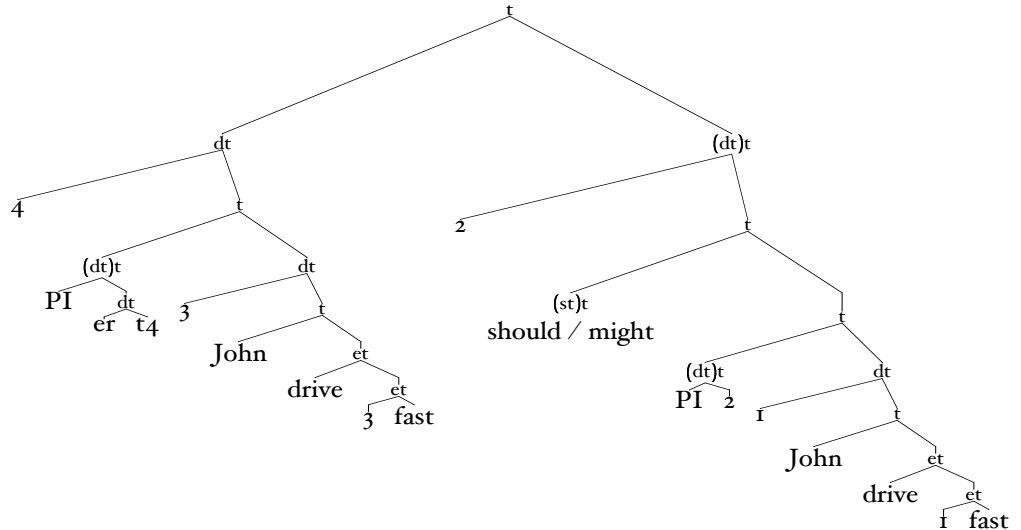
$$\begin{aligned}
 (172) \quad & [\lambda D \text{ [[PI]]}(D)(\lambda d \exists x \neq H: \text{HEIGHT}(x) \geq d)](\lambda d \text{ [[PI]]}(\lambda d' d' > d)(\lambda d \\
 & \text{HEIGHT}(H) \geq d)) = \\
 & [\lambda D \text{ MAX}(\lambda d \exists x \neq H: \text{HEIGHT}(x) \geq d) \in D](\lambda d \text{ MAX}(\lambda d \text{ HEIGHT}(H) \geq \\
 & d) \in (\lambda d' d' > d)) = \\
 & [\lambda D \text{ MAX}(\lambda d \exists x \neq H: \text{HEIGHT}(x) \geq d) \in D](\lambda d \text{ HEIGHT}(H) > d) = \\
 & \text{MAX}(\lambda d \exists x \neq H: \text{HEIGHT}(x) \geq d) \in \lambda d \text{ HEIGHT}(H) > d = \\
 & \text{HEIGHT}(H) > \text{MAX}(\lambda d \exists x \neq H: \text{HEIGHT}(x) \geq d) \\
 & \text{‘Hubert is taller than the tallest other person.’}
 \end{aligned}$$

In the same vein, Heim derives correct truth conditions for comparatives with embedded disjunction. Differences in the interpretation of intensional comparative complements are also explained as a structural ambiguity. Quantifiers like ‘have to’ and ‘be allowed’ are assumed to be interpreted under PI , whereas ‘might’, ‘should’ and numerous other intensional predicates do not apply until the type-shift has taken place. Let us consider the analysis of the minimal pairs in (173) and (174).

- (173) a. John drove faster than he should have driven.
 b. John drove faster than he had to.
- (174) a. John drove faster than he might drive tomorrow.
 b. John drove faster than he was allowed to drive.

It has to be stipulated that in (173)a and (174)a PI moves locally without crossing the modal. Skipping over the contribution of tense and aspect, the resulting LFs are outlined below:

(175)



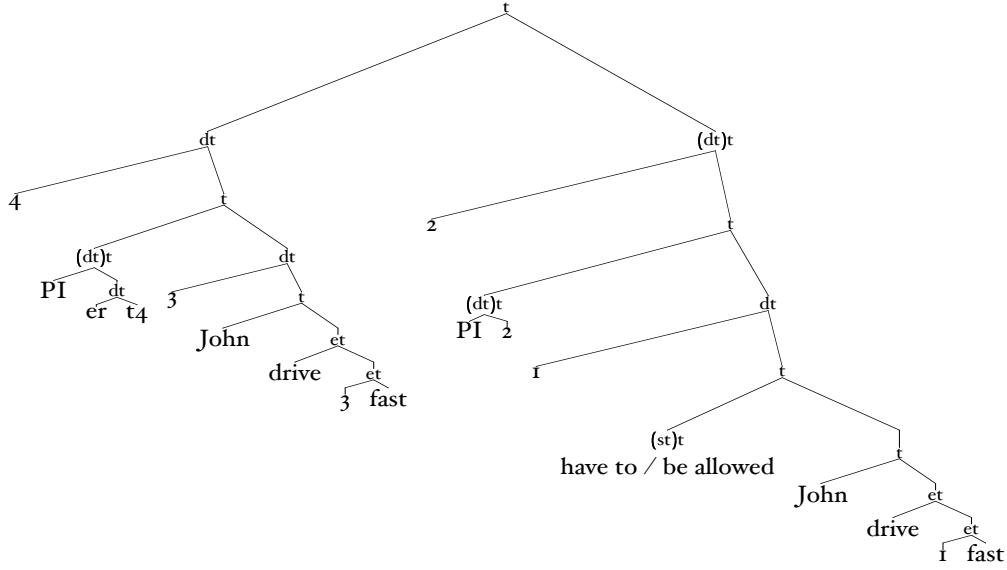
From these LFs we compute the following truth conditions at index w :

(176) $[\lambda D \forall w' \in Acc_w SPEED_{w'}(John) \in D](\lambda d SPEED_w(John) > d) =$
 $\forall w' \in Acc_w : SPEED_{w'}(John) \in [\lambda d SPEED_w(John) > d] =$
 $\forall w' \in Acc_w : SPEED_w(John) > SPEED_{w'}(John)$
 'For any world w' in which the relevant rules hold, John's actual speed is greater than the speed he reaches in w' .'
 'John's actual speed exceeded the posted maximum.'

(177) $[\lambda D \exists w' \in Acc_w SPEED_{w'}(John) \in D](\lambda d SPEED_w(John) > d) =$
 $\exists w' \in Acc_w : SPEED_{w'}(John) \in [\lambda d SPEED_w(John) > d] =$
 $\exists w' \in Acc_w : SPEED_w(John) > SPEED_{w'}(John)$
 'There is a epistemically accessible world w' in which John's speed is smaller than John's actual speed.'
 'It is possible that tomorrow John will drive faster than he drives today.'

To get (173)b and (174)b variants right, Heim has to stipulate that PI obligatorily undergoes long movement in those cases. The resulting LFs and their extensions look as follows:

(178)



(179) $[\lambda D \text{ MAX}(\lambda d \forall w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d) \in D]$
 $(\lambda d \text{ SPEED}_w(\text{John}) > d) =$
 $\text{MAX}(\lambda d \forall w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d) \in [\lambda d \text{ SPEED}_w(\text{John}) > d] =$
 $\text{SPEED}_w(\text{John}) > \text{MAX}(\lambda d \forall w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d)$
 ‘John’s actual speed is greater than the maximum speed d , s.t. he drives at least d fast in every accessible world.’
 ‘John’s actual speed exceeded the posted minimum.’

(180) $[\lambda D \text{ MAX}(\lambda d \exists w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d) \in D]$
 $(\lambda d \text{ SPEED}_w(\text{John}) > d) =$
 $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d) \in [\lambda d \text{ SPEED}_w(\text{John}) > d] =$
 $\text{SPEED}_w(\text{John}) > \text{MAX}(\lambda d \exists w' \in \text{Acc}_w \text{ SPEED}_{w'}(\text{John}) \geq d)$
 ‘John’s actual speed is greater than the maximal speed d , s.t. there is a law-abiding world w' in which he drives d fast.’
 ‘John drove faster than maximally allowed.’

Since neither of the examples in (173) – (174) is ambiguous, it is crucial that the movement of PI is constrained accordingly. Only with a restricted number of quantifiers and intensional predicates can it undergo long movement. This is a major challenge for Heim’s proposal. As it stands, it does not predict any difference in the interpretation of the pairs in (173) – (174). Two distinct readings are wrongly predicted to be available for each example.

To sum up, both ‘scope-over-negation’ and ‘interval’ approaches can be adapted to handle narrow scope quantifiers. We have seen that the former may assume that negation undergoes movement and the latter may perform a shift to intervals directly in the syntax.¹³ In both cases a narrow scope quantifier occupies a position different from that occupied by a wide scope quantifier. It is interpreted either in the scope of negation or in the scope of a type-shifting operator.

2.3.4 DISCUSSION

2.3.4.1. CONSTRAINING SCOPE

In the previous two sections I have elaborated two approaches to the analysis of comparative complements, treating DP quantifiers and intensional predicates in parallel. Despite their *prima facie* differences, they have the same core. They agree that quantifiers can occupy different positions at the LF of comparative sentences. Since all examples under discussion are not ambiguous, these theories need to take care that each quantificational expression is interpreted at a rigid position and no structural ambiguity is possible. Schwarzschild’s analysis seems to fare better with respect to this criterion, for it turns out that the behaviour of intensional predicates in comparative complements reflects their behaviour with respect to negation outside comparative sentences. However, it is unclear what mechanism constrains the scope of universal DP quantifiers. Heim’s modified analysis cannot rely on any parallels outside the realm of comparatives whatsoever.

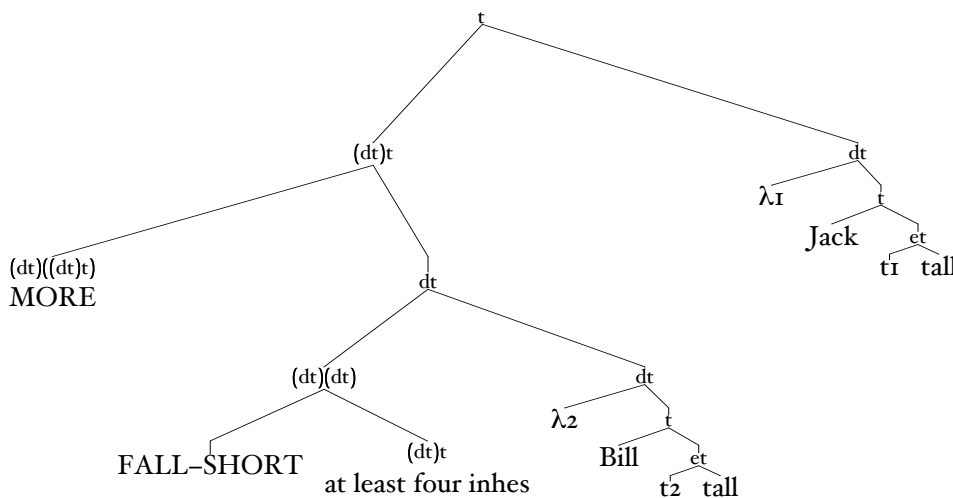
As (Beck, 2010b) points out, Schwarzschild’s analysis loses its advantage over Heim’s modified analysis in its treatment of differential comparatives like (181).

13. (Beck, 2010b) and (van Rooij, 2008), though interval-based, cannot be subsumed under movement theories. They provide a different non-scope mechanism to derive wide- and narrow-scope readings of quantifiers. Neither of them addresses the case of narrow scope intensional predicates.

(181) Jack is four inches taller than Bill is.

To incorporate measures of difference in a Seuren-style analysis, Schwarzschild replaces NOT with a negative operator FALL-SHORT, defined in (183). The LF of (181) is given in (182). FALL-SHORT combines with a differential measure phrase and returns a function that maps the set of degrees D' expressed by the comparative complement to the set of degrees D which abut on D' and whose size is measured by the differential.

(182)



(183) $\llbracket \text{FALL-SHORT} \rrbracket = \lambda Q \lambda D \lambda d Q \llbracket \lambda d'' d'' \leq d \wedge D(d'') = 0 \rrbracket$

Differential phrases like ‘(by) at least four inches’ name particular measures. According to the following semantic rule, ‘at least four inches’ names a measure that holds, for example, of the set of height degrees reached by a 65 inch pole but not by 61 inch pole, i.e. the size of the set $\{65, 64, 63, 62\}$.

(184) $\llbracket \text{at least four inches} \rrbracket = \lambda D$ the size of D is at least four inches

The input to the comparative is thus the following set:

(185) $\llbracket \text{FALL-SHORT} \rrbracket(\llbracket \text{at least four inches} \rrbracket)(\lambda d \text{ Bill is at least } d \text{ tall}) =$
 $\lambda d \llbracket \text{at least four inches} \rrbracket(\lambda d'' d'' \leq d \wedge \neg \text{Bill is at least } d'' \text{ tall}) =$
 the set of degrees that abuts on Bill’s height, i.e. is right on top of it, and measures at least four inches.

The remaining steps in the composition are straightforward. MORE applies

to a four inch set of degrees ‘missed’ by Bill and locates them below Jack’s height.

- (186) $\llbracket \text{MORE} \rrbracket (\lambda d \llbracket \text{at least four inches} \rrbracket (\lambda d'' d'' \leq d \wedge \neg \text{Bill is at least } d'' \text{ tall})) (\lambda d \text{ Jack is at least } d \text{ tall}) =$
 $\exists d: \text{Jack is at least } d \text{ tall} \wedge \text{Bill's height falls short of } d \text{ by at least four inches.}$

It has to be stipulated that FALL-SHORT takes scope just like NOT. For example, to derive the correct interpretation of (187) FALL-SHORT has to be interpreted in the scope of ‘should’, as one can see on the paraphrase in (188).

- (187) Jack is four inches taller than he should be.
 (188) For all worlds w in which the relevant rules in the actual world hold: $\exists d: \text{Jack is at least } d \text{ tall in the actual world} \wedge \text{Jack's height in } w \text{ falls short of } d \text{ by at least four inches.}$

I agree with (Beck, 2010b) that this aspect of Schwarzschild’s analysis is not an improvement over Heim’s PI theory, as far as constraining the scope of embedded quantifiers and intensional predicates is concerned. Without independently motivated restrictions on the scope of quantifiers both approaches share an overgeneration problem.

2.3.4.2. IS THERE AMBIGUITY?

The examples we have considered so far do not seem ambiguous. There are however claims to the contrary in the literature on the topic. In this subsection I shall assess the position that embedded modals do not lead to genuine ambiguity.

(Heim, 2006) remarks that given an adequate pragmatic setting more-than-minimum modals like ‘have to’ can receive a more-than-maximum reading, which is derived by interpreting PI in the scope of the modal. Her example of such a reading is the following sentence with ‘need’:

- (189) He was older than he needed to be (to get a kid’s ticket).

This sentence is understood to convey that his actual age exceeds his age in every law-abiding world, i.e. that he is too old to get a kid's ticket.

(Krasikova, 2008) provides some naturally occurring examples of more-than-maximum readings with 'have to'. The following example, similarly to the previous one, says that he arrived later than the latest time permitting to retain the overall lead.

- (190) He was coming through later than he had to if he were going to retain the overall lead. ... The time flashed up above the finish line: 36:53. Almost a minute back.¹⁴

Such examples are much less frequent than those with a more-than-minimum reading. A different picture emerges when we consider sentences with negative pole adjectives. Normally, comparison with the maximum rather than with the minimum on a relevant adjective scale is expressed in such cases, cf. (191). In the next subsection I shall demonstrate that 'maximum' readings are a problem for Heim's analysis.

- (191) John is shorter than he has to be (to become a pilot).

This sentence describes a situation in which John's height does not reach the required minimum of height. Since the comparison is between two shortness degrees, John's shortness degree is claimed to exceed the maximum from the admissible interval on the shortness scale, which corresponds to a less-than-maximum reading. A less-than-maximum reading is almost exclusive in such cases. However, I came across examples with negative pole adjectives that make a comparison with the minimum, see (192) – (193).¹⁵

14. <http://velonews.com/article/9533>

15. (Büring, 2007a) provides examples from British National Corpus that also demonstrate that 'have to' and other modals of its type are ambiguous.

- (192) As for errors of omission, the series makes them inevitable, for it restricts the text to a maximum of 200 pages in all, but Buck's text is much shorter than necessary, running only 153 pages in all. He could easily have added 30 pages under the rules of the series. He would then have been in a position to fill in some gaps that are the principal weakness of this volume.¹⁶
- (193) Shelf-life requirements for processed foods are far shorter than necessary to preserve freshness and result in processed U.S. goods being non-competitive ...¹⁷

Comparatives with embedded possibility modals can also be ambiguous irrespective of the polarity of the gradable predicate. In this case, the availability of this or that reading depends to a large extent on the modal flavour. Examples with a more-than-minimum reading of 'allowed' are difficult to contrive and they hardly occur naturally. However, they are not completely impossible. The following Russian sentence can have but such an interpretation. The actual marriage age is compared to the minimal law-abiding age.

- (194) У большинства народов браки заключаются в подавляющем числе случаев позже, чем разрешено по закону, причем для мужчин это характернее, чем для женщин.
 'In most cultures people predominantly get married later than allowed by the law; and it is more typical of men than women.'¹⁸

There are plenty of naturally occurring examples with 'could' where a more-than-minimum reading of the modal is more prominent. For instance, in (195) our actual expenses are compared to the minimal expenses in the hypothetical worlds. Compare this example to the one in (196) where our expenses are compared to the maximum we afford in the hypothetical worlds.

16. <http://www.jstor.org/pss/436484>

17. <http://commerce.nic.in/trade/GCC%20NTMS%20final/Kuwait.doc>

18. <http://referats.5-ka.ru/bibliotek/Sociolog/Plat/12.html>

- (195) Many of us spend more than we could each month. Here are seven ways to detox your finances, cut your outgoings and make sure you're richer this month!¹⁹
- (196) However, the pressure to buy presents is so much that we often spend more than we could afford.²⁰

What about wide scope modals like ‘should’ and ‘might’? They do not display the kind of variability observed with the modals discussed above. Consider the following pair of sentences in a context in which John is requested to be punctual and encouraged to arrive beforehand. Only sentence (197)a appears acceptable in this case. Sentence (197)b claims that John arrives too early to meet the requirement. Therefore it is infelicitous in a situation in which there are no restrictions on how early one is allowed to arrive.

- (197) a. John always arrives earlier than required.
 b. John always arrives earlier than he should.

In Heim’s terms, the earliest compliant time reading of (197)b, which is not compatible with the described situation, corresponds to the wide scope of the modal with respect to PI, see (198). The interpretation of (197)a that is prominent in the context at hand can be obtained by scoping PI over the modal, see (199). This strongly suggests that (197)b cannot be used interchangeably with (197)a to express a ‘narrow scope of the modal’ reading.

- (198) $\forall w' \in \text{Acc}_w$: John arrives at t in $w \wedge t \in \{t'' : \text{John arrives at } t' \text{ in } w' \text{ and } t'' < t'\}$
 ‘For all worlds w' accessible from w : the time of Fritz’s arrival in w is included in a time interval that precedes the time of Fritz’s arrival in w' .’

19. <http://www.lovemoney.com/news/manage-your-finances/seven-easy-ways-to-cut-your-living-costs-3751.aspx>

20. <http://ariel.sagum.net/blog/milestones.php?p=16>

(199) John arrives at t in $w \wedge t \in \{t'' : \forall w' \in \text{Acc}_w: \text{Fritz arrives at } t' \text{ or earlier in } w' \text{ and } t'' < t'\}$

The time of Fritz's arrival in w is included in a time interval that precedes the latest time among the times of Fritz's arrival in the worlds accessible from w .

To sum up, we have seen that intensional predicates that usually take narrow scope with respect to PI in Heim's analysis or the abstract negation in Schwarzschild's analysis allow wide scope readings as well. This is a bonus to scope theories, which predict an ambiguity. However, the problem of constraining the scope intensional predicates and quantifiers that take only wide scope in those theories remains open.

2.3.4.3. NEGATIVE POLE ADJECTIVES

In this subsection I shall compare the scope approaches by Heim and Schwarzschild along another dimension, namely the treatment of quantificational comparative complements with negative pole adjectives. To this end, let us consider how example (191) repeated below is analysed in both cases.

(191) John is shorter than he has to be (to become a pilot).

Both Schwarzschild and Heim treat positive adjectives as monotone functions that relate an individual to the positive extent on a degree scale, i.e. to an initial region on a scale, see (200)a. Though neither is concerned with defining antonyms and therefore neither is explicit about the meaning of negative adjectives, it is plausible to assume that an extent-based analysis would treat a negative antonym as relating an entity to its negative extent, i.e. a final segment on a scale, as, for example, is done in (Heim, 2007).

(200) a. $\llbracket \text{tall} \rrbracket = \lambda_w \lambda_d \lambda_x \text{HEIGHT}_w(x) \geq d$

b. $\llbracket \text{short} \rrbracket = \lambda_w \lambda_d \lambda_x \text{HEIGHT}_w(x) < d$

With this interpretation of antonyms we arrive at the following truth conditions for (191) in Schwarzschild's analysis. Recall that 'have to' is

interpreted below NOT.

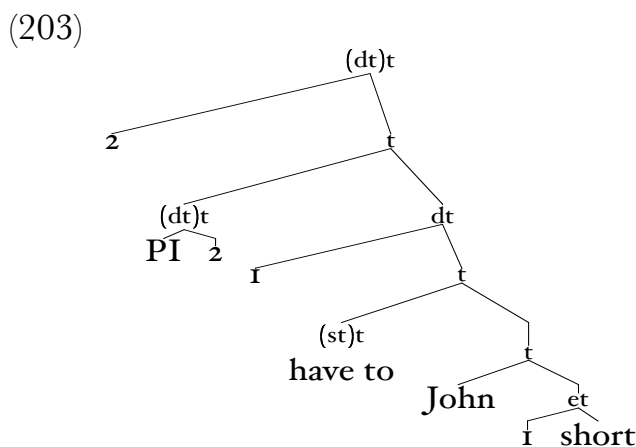
- (201) $\exists d: \text{HEIGHT}_w(\text{John}) < d \wedge \neg \forall w' \in \text{Acc}_w : \text{HEIGHT}_{w'}(\text{John}) < d =$
 $\exists d: \text{HEIGHT}_w(\text{John}) < d \wedge \exists w' \in \text{Acc}_w : \text{HEIGHT}_{w'}(\text{John}) \geq d$
 ‘John’s actual height is smaller than the height John reaches in an accessible world.’

According to the formula in (201), (191) means that John could be taller than he is and still become a pilot. If the context specifies the minimal height requirement this is by no means the prominent reading of this sentence. If we assign ‘have to’ wide scope relative to NOT we derive the following truth conditions:

- (202) $\exists d: \text{HEIGHT}_w(\text{John}) < d \wedge \forall w' \in \text{Acc}_w : \neg \text{HEIGHT}_{w'}(\text{John}) < d =$
 $\exists d: \text{HEIGHT}_w(\text{John}) < d \wedge \forall w' \in \text{Acc}_w : \text{HEIGHT}_{w'}(\text{John}) \geq d$
 ‘John’s actual height is smaller than the height John reaches in the accessible world in which he is the shortest.’

This is a welcome prediction, for the sentence is understood to convey that John is too short to become a pilot. Taking into account that comparatives with ‘have to’ are ambiguous and require two scope configurations, we conclude that Schwarzschild’s analysis can cope with negative antonyms.

This is different in Heim’s type-shifting approach. Recall that ‘have to’ normally takes narrow scope with respect to the type shifter. The following LF of the embedded clause of (191) reflects this pattern:



From this LF we compute the following extension at w :

$$(204) \lambda D \text{ MAX}(\lambda d \forall w' \in \text{Acc}_w : \text{HEIGHT}_w(\text{John}) < d) \in D \quad \text{undef!}$$

The set that MAX applies to contains degrees that are greater than John's height in every accessible world, i.e. it corresponds to a height interval spanning from above the maximally permitted height upwards. This set does not have a maximum, therefore the entire set in (204) is undefined.

Interpreting PI below the modal does not help either. If PI applies before the modal, the maximality operator ends up undefined on John's degrees of shortness, cf. (205).

$$(205) \lambda D \forall w' \in \text{Acc}_w : \text{MAX}(\lambda d \text{ HEIGHT}_w(\text{John}) < d) \in D \quad \text{undef!}$$

An ad hoc solution to fix the problem is to define two different type-shifting operators that go with positive or negative adjectives.²¹ PI+ matching positive antonyms makes use of MAX operator and PI- matching negative antonyms employs MIN operator, cf (206).

$$(206) \text{ a. } \llbracket \text{PI+} \rrbracket = \lambda D \lambda D' \text{ MAX}(D') \in D$$

$$\text{ b. } \llbracket \text{PI-} \rrbracket = \lambda D \lambda D' \text{ MIN}(D') \in D$$

The comparative morpheme has to be made polar flexible as well:

$$(207) \text{ a. } \llbracket \text{er+} \rrbracket = \lambda d \lambda d' d' > d$$

$$\text{ b. } \llbracket \text{er-} \rrbracket = \lambda d \lambda d' d' < d$$

If we use the entries in (206)b we can derive the following two denotations of the embedded clause, by interpreting PI- above and below 'have to' respectively.

21. This solution is due to Arnim von Stechow, p.c.

- (208) a. $\lambda D \text{ MIN}(\lambda d \forall w' \in \text{Acc}_w : \text{HEIGHT}_{w'}(\text{John}) < d) \in D$
 = the set of degree sets that include John's maximally permitted height
- b. $\lambda D \forall w' \in \text{Acc}_w : \text{MIN}(\lambda d \text{ HEIGHT}_{w'}(\text{John}) < d) \in D$
 = the set of degree sets that include all heights that John is allowed to have

From the denotation in (208)b and the meaning of the comparative in (207)b we can arrive at the intuitive meaning of (191):

- (209) $[\lambda D \forall w' \in \text{Acc}_w : \text{MIN}(\lambda d \text{ HEIGHT}_{w'}(\text{John}) < d) \in D]$
 $(\lambda d \text{ HEIGHT}_w(\text{John}) < d)$
 = $\forall w' \in \text{Acc}_w : \text{HEIGHT}_w(\text{John}) < \text{HEIGHT}_{w'}(\text{John})$

To conclude, scope-over-negation analyses fare better in their treatment of comparatives with negative pole adjectives than interval approaches. Heim's analysis has to be adapted to handle antonyms.

2.3.4.4. DATA RE-EVALUATED

I have discussed some analyses that strive to develop a unified account of embedded DP quantifiers and intensional predicates. Let us look at the data from a slightly different perspective. We deal with three empirical patterns. The first one is a more-than-maximum interpretation of comparative complements with possibility modals, polarity sensitive indefinites like 'anyone' and disjunction. The second one is a more-than-minimum interpretation of some necessity modals, viz. 'need', 'have to,' etc. The third pattern are more-than-maximum readings of all remaining quantificational expressions. Note that the parallel between DP quantifiers and intensional predicates holds in the first data cluster only. Narrow scope necessity modals do not have counterparts in the DP domain. Nor can we say that more-than-maximum expressions form a homogeneous class. Along with \forall quantifiers, we find there expressions such as 'most,' 'exactly five', 'only,' etc., see (Schwarzschild and Wilkinson, 2002) for an overview. In the following, I shall sketch an approach to the treatment of comparative complements that

handles the three data clusters separately and argue that it has advantages over the analyses presented above.

Some classical theories of comparatives such as (Seuren, 1973) straightforwardly derive more-than-maximum readings of disjunction and other expressions with existential quantificational force. (von Stechow, 1984) argues that definite theories like his also make correct predictions. Such theories rely on a pragmatic theory that specifies with respect to which implicit parameters definite terms are evaluated. For instance, (von Stechow, 1984)'s example in (210)a is analysed along the lines of the paraphrase in (210)b, which corresponds to the intended sense of this sentence.

- (210) a. A polar bear could be bigger than a grizzly bear could be.
b. The maximal size a polar bear can have exceeds the maximal size a grizzly bear can have.

If we want to adopt a definite analysis, the task is to spell out the pragmatic mechanism that is responsible for more-than-maximum readings of such comparative complements.

The second data cluster are the more-than-minimum necessity modals. The problem they present can be summarised as follows. Unlike other species of necessity modals, e.g. 'should', they give rise to a comparison with the minimally required degree. Consider the following minimal pair:

- (211) a. The abstract is longer than it has to be.
b. The abstract is longer than it should be.

The sentence in (211)a expresses comparison with the minimal compliance length, i.e. the minimal sufficient degree of length. In (211)b such interpretation is ruled out. The second sentence instead conveys that the length of the abstract exceeds the maximally allowed length. This difference in interpretation of 'have to' and 'should' is not confined to comparative complements. In the following pair only the (a) variant can convey a comparison with the minimal compliance length.

- (212) a. The abstract has to be two pages long.
b. The abstract should be two pages long.

The challenge this data cluster presents is to explain why comparing with the minimal sufficient degree is only possible with a restricted class of necessity modals and find out the source of this reading. Obviously, the strategy should be to investigate the semantics of the modals involved, in general, rather than only look at comparative complements, for the problematic reading surfaces in other scalar context as well.

We are left with more-than-maximum readings of the remaining quantificational expressions. I submit that this is the only problem that needs a comparative specific solution. If the first two data clusters are successfully handled by an independently motivated mechanism, one may conclude with von Stechow that all embedded quantifiers, intensional predicates and other complex expressions are interpreted above the comparative. I have shown in section 2.4.2 that there are two rivalling theories that are equally successful in deriving more-than-maximum readings. One of them is the proposal in (Heim, 2006) to treat gradable adjectives as relations between individuals and degree properties. The second one is a neo-Seuren theory developed in (Schwarzschild, 2008) under which a quantifier is interpreted outside the scope of the ‘than’ clause internal negation.

Schwarzschild’s analysis is tailored to accommodate all three data clusters under discussion. It is similar to the PI analysis in (Heim, 2006) in that both pursue scope strategies. This is precisely the aspect that makes it different from the original non-PI analysis in (Heim, 2006). If the view that only more-than-maximum readings are to be accounted for by comparative specific mechanisms is correct, (Schwarzschild, 2008) has to ban more-than-minimum readings. We have seen that a parallel to non-comparative negated sentences is exploited for this purpose. It remains to be seen whe-

ther each expression leading to a more-than-maximum reading consistently avoids the scope of negation.

Heim's non-PI type-shifting approach predicts only a more-than-maximum reading, which makes it look more attractive under the above-mentioned view that this reading is a default. Recall that Heim's analysis of (141) amounts to (213).

(141) Hubert is taller than everyone else is.

(213) For each individual x different from Hubert: x 's height is contained in a set of degrees that are smaller than Hubert's height.

The shift from degrees to intervals, or degree sets, is driven solely by the need to interpret complex comparative complements. We do not find any other comparative specific phenomena beyond the third data cluster that would require a treatment with intervals. Nonetheless, there are several promising proposals in the literature as to how to motivate or rather conceptualise a shift to intervals in comparative complements.

One of them is due to (van Rooij, 2008) and can be called a granularity switch solution. Van Rooij entertains the idea that what appears to be a set of degrees can be conceived as a point in a model with a coarse granularity setting, where the expansion of a point to an interval is tolerated. He thus develops an analysis that does not use intervals but makes reference to coarse grained models in which a degree may correspond to a set of points in a finer grained model.

Another proposal is outlined in (Beck, 2010b) who credits the idea to an unpublished manuscript by Irene Heim. Beck suggests that intervals can be derived from plural predication. She points out to the parallel between the cumulative interpretation of 'drink' in (214)a and the relation expressed by 'tall' in example (215)a. In one case a plurality is related to a mass noun, in the other case it is related to a degree set, compare (214)b and (215)b.

- (214) a. Our children drank the milk.
 b. $\forall x \leq C: \exists y \leq M: \text{drank}(y)(x) \ \& \ \forall y \leq M: \exists x \leq C: \text{drank}(y)(x)$
 ‘All children participated in drinking the milk, and all parts of the milk were drunk by one of the children.’
- (215) a. (You have to be five feet tall to enter.) Our children are that tall.
 b. $\forall x \leq C: \exists d \in D: \text{tall}(d)(x) \ \& \ \forall d \in D: \exists x \leq C: \text{tall}(d)(x)$
 ‘All the children's heights fall into D, and all parts of D contain the height of a child.’

This appears to be an interesting direction to pursue. It remains open, however, how such an analysis can be extended to cases like (216) – (217) which do not contain plural individuals.

- (216) (You have to be five feet tall to enter.) I predicted that John would be that tall.
- (217) (You have to be five feet tall to enter.) Everyone is that tall.

The discussion of approaches to the analysis of intensional comparative complements has shown that the relevant data need to be restructured. The established clustering into the wide and narrow scope quantifiers is not only less empirically motivated than the three clusters suggested here but also leads to a serious theoretical problem, namely predicting correct scope in each case.

2.4 RULLMANN AMBIGUITY

2.4.1 RULLMANN'S PUZZLE

(Rullmann, 1995) observes that the following comparative sentence with an embedded possibility modal is ambiguous.

(218) The helicopter was flying less high than a plane can fly.

He calls two readings of this sentence a less-than-maximum and a less-than-minimum reading. They are prominent in the following contexts.

(219) Less-than-maximum reading

Because the helicopter was flying less high than a plane can fly, the jet fighter could easily fire at it from above.

(220) Less-than-minimum reading

The jet fighter was trying to chase the helicopter, but because the helicopter was flying less high than a plane can fly, the helicopter crashed into a building.

According to Rullmann, this kind of ambiguity can also surface with negative pole adjectives, i.e. (221) has a less-than-maximum and a less-than-minimum reading as well.

(221) The helicopter was flying lower than a plane can fly.

Comparatives with positive pole adjectives and without 'less' are claimed to express comparison with the maximum of the admissible interval only. Thus, (222) can only mean that the helicopter was flying at an altitude exceeding the maximal altitude of a plane.

(222) The helicopter was flying higher than a plane can fly.

The ambiguity in (218) and (221) is therefore expected to have its source in the meaning of 'less'/'lower'. Indeed, Rullmann suggests that the semantics of negative pole adjectives may be responsible for the availability of two distinct readings. In the next section I shall elaborate that position.

2.4.2 DECOMPOSITION

Rullmann proposes an analysis based on the assumption that the surface string ‘less high’ in (218) corresponds to three morphemes, ‘little’, ‘-er’ and ‘high’. Let us look at a compositional variant of Rullmann’s proposal in (Heim, 2007) which is worked out in detail.

(218) The helicopter was flying less high than a plane can fly.

Heim assumes that ‘less’ spells out ‘little’ + ‘-er’ and the former is treated as an degree negation, see (223).

(223) $\llbracket \text{little} \rrbracket = \lambda d \lambda P \neg P(d)$

The comparative morpheme is analysed as a degree quantifier and is assumed to form a constituent with the comparative complement. The lexical entry is given in (224). Heim generalises the greater-than relation between degrees to the inclusion relation between degree sets in order to be able to compare initial as well as final segments on a scale.

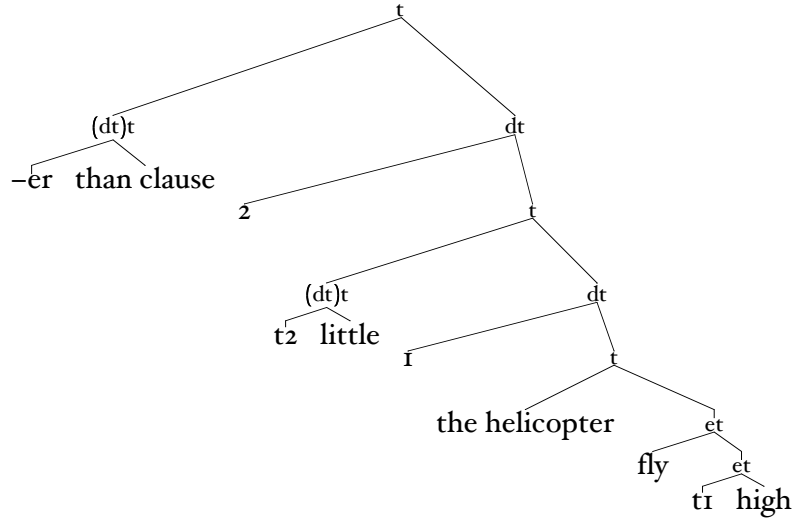
(224) $\llbracket \text{-er} \rrbracket = \lambda Q \lambda P Q \subset P$

Adjectives are assumed to express monotone relations between degrees and individuals.

(225) $\llbracket \text{high} \rrbracket = \lambda w \lambda d \lambda x \text{ALTITUDE}_w(x) \geq d$

By successively QRing the ‘little’ phrase out of the degree argument position of the adjective and then the comparative quantifier out of the ‘little’ phrase the following LF of example (218) is derived.

(226)



The extension of (226) in a world w amounts to the following truth conditions:

$$(227) \quad [\lambda d \neg \text{ALTITUDE}_w(\text{the helicopter}) \geq d] \subset \llbracket \text{than clause} \rrbracket = \\ [\lambda d \text{ALTITUDE}_w(\text{the helicopter}) < d] \subset \llbracket \text{than clause} \rrbracket$$

In the ‘than’-clause, the corresponding ‘little’ phrase undergoes comparative deletion. When reconstructed, it is QRed as well. Crucially, Heim postulates two scope sites for the movement of the embedded ‘little’ phrase. In (228)a it moves locally below the modal and in (228)b it moves across the modal.

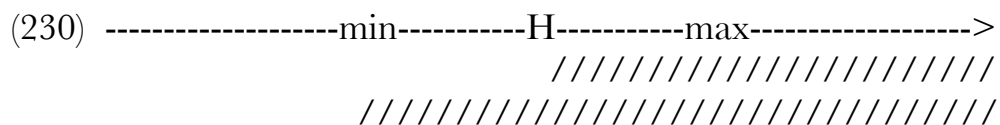
- (228) a. $\lambda 3$ can $\llbracket [t3 \text{ little}] \lambda 4$ [a plane fly t4 high]
- b. $\lambda 3$ $\llbracket [t3 \text{ little}] \lambda 4$ [can [a plane fly t4 high]]

By plugging in the meanings of the embedded clause we arrive at the following truth conditions:

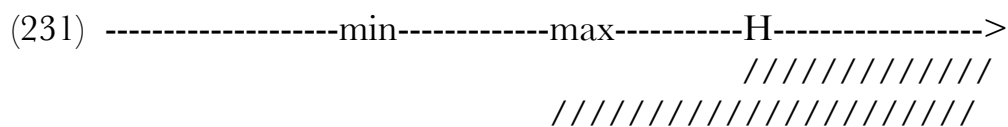
- (229) a. $[\lambda d \text{ALTITUDE}_w(\text{the helicopter}) < d] \subset$
 $[\lambda d \exists w' \in \text{Acc}_w: \text{ALTITUDE}_{w'}(\text{a plane}) < d]$
- b. $[\lambda d \text{ALTITUDE}_w(\text{the helicopter}) < d] \subset$
 $[\lambda d \forall w' \in \text{Acc}_w: \text{ALTITUDE}_{w'}(\text{a plane}) < d]$

(229)a corresponds to a comparison with the minimal altitude of a plane. Two final segments of the scale are compared here. The segment whose

lower bound is the actual altitude of the helicopter is claimed to be included in the segment whose lower bound is the minimal altitude of a plane, as illustrated in (230).



(229)b corresponds to a comparison with the maximal altitude of a plane. The comparative complement provides a set of degrees whose lower bound is the maximal altitude that a plane can reach, see (231).



Rullmann admits that his decomposition hypothesis relies on rather dubious syntactic processes. What concerns him most is the availability of ambiguity with negative pole antonyms in examples like (221) repeated below.

(221) The helicopter was flying lower than a plane can fly.

If one is committed to the ‘syntactic copying’ account with ‘less high’, one has to treat ‘lower’ in parallel and assume a kind of decomposition that would make available the relevant building blocks, namely, ‘little’, ‘high’, and ‘-er.’ This is a central hypothesis in (Heim, 2007), (Büring, 2007a), (Büring, 2007b) and (Heim, 2008), all of which assume the negation theory of antonymy according to which a negative pole adjective in an antonym pair expresses internal negation of its positive pole counterpart, see (232). The approaches differ in their assumptions concerning the level of derivation at which the decomposition into negation and the positive pole adjective takes place.

$$(232) \llbracket \text{low} \rrbracket = \lambda_w \lambda_d \lambda_x \neg \text{ALTITUDE}_w(x) \geq d$$

(Heim, 2008) treats the ambiguity of (221) by analogy to the ambiguity in

'less' comparatives. She argues, contra (Büring, 2007b), that the negative meaning component of a negative antonym cannot be split from the rest of the adjective in the syntax, i.e. there is no separate abstract lexical item 'little' that participates in spelling out negative pole adjectives. She posits an additional lexical item 'little' with a function similar to the scopally active 'little' in (Heim, 2007). It is stipulated that the process of comparative deletion sanctions the reconstruction of the elided embedded predicate in the form of 'little' + 'high', as long as it is semantically identical to 'low'. The analysis of comparative complements then proceeds as in the case of 'less high' elaborated above.

2.4.3 POLARITY?

Judging from the analyses discussed in the previous section, the claim that Rullmann ambiguity is restricted to comparatives with negative pole adjectives and 'less' has significant consequences for the semantics of antonyms. However, it is not unanimously accepted. For example, (Meier, 2002) argues that comparative sentences with positive pole adjectives and the embedded 'allowed' can be ambiguous as well. She considers the following example:

(233) Chuck is driving faster than he is allowed to drive.

Out of the blue, this sentence conveys that Chuck is exceeding the maximal speed limit, i.e. it receives a more-than-maximum interpretation. Meier discusses a scenario in which a different interpretation of (233) could be prominent, namely, a more-than-minimum reading.

(234) Chuck is driving a truck full of eggs on a New Jersey highway. It is true that the slower Chuck drives the less eggs break. Moreover, Chuck wants to bring as many eggs as possible to their final destination and he does not want to risk a speeding ticket. The road traffic regulations limit the minimum speed on New Jersey highways to 45mph and the maximum speed to 65mph. All in all, it seems desirable that Chuck is driving as slow as possible within the regulations.

Meier argues that in the scenario sketched above (233) can be true if Chuck exceeds 45 mph but not the maximum speed. This judgement is rather shaky but Meier's point that the role of the context is crucial in making this or that reading prominent is worthwhile. Example (235)b (Vera Hohaus, p.c.) lends itself to a more-than-minimum interpretation much more readily than Meier's example if it is uttered in context (235)a, which makes a more-than-maximum interpretation unlikely.

- (235) a. Diese Autorennstrecke ist für Anfänger gut geeignet, weil es hier auch erlaubt ist nur 60km/h schnell zu fahren.
 'This race track is especially good for beginners, because it is allowed to drive 60 km/h on it.'
- b. Schon nach wenigen Runden hat Michael sich getraut schneller zu fahren, als erlaubt ist.
 'Already after a few laps Michael had the courage to drive faster than was allowed.'

Consider also the following variant of Rullmann's example:

- (236) Because the plane was flying higher than a helicopter can fly, it could be detected by the radar.

In the given context the focus is on the minimal height an aircraft can sustain. It is well-known that the lower the altitude of an aircraft the harder it is to detect its presence or speed. It is therefore natural to understand (236) as expressing comparison with the minimal altitude of a helicopter.

The same point is made in (Boguslavski, 2001), who argues that the polarity of the predicate is not responsible for the availability of different interpretations. He gives the following pair of examples in Russian which have two distinct interpretations.

- (237) a. Самолет летел выше, чем было можно.
 plane fly-PAST higher than was permitted
- b. Самолет летел выше, чем мог бы.
 plane fly-PAST higher than can-PAST SUBJ

Sentence (237)a conveys that the pilot exceeded the maximal altitude. In

contrast, the prominent interpretation of (237)b is a more-than-minimum reading. (237)b would be natural in a context in which it is desirable to fly at the lowest possible altitude, say at an aircraft show. Note that (237)a and (237)b feature different possibility modals. Example (237)a involves a deontic possibility, i.e. a possibility in view of a law. In (237)b, subjunctive morphology signals a counterfactual interpretation. Counterfactual modality is assumed to be related to some facts or circumstances of the world of evaluation. This interpretation is expected given the aircraft show scenario. The kind of facts that are relevant for the evaluation of (237)b may pertain to the technical parameters of the aircraft or the pilot's skills and experience.

Importantly, there are naturally occurring examples with positive pole adjectives that have a more-than-minimum interpretation. We have already come across some examples of this kind in section 2.3.4.2. Most frequently such examples involve counterfactual possibility but other kinds of modal interpretation may also be compatible with a more-than-minimum reading. Below I repeat Russian example (194) with the deontic **разрешено**/'be allowed'.

- (194) У большинства народов браки заключаются в подавляющем числе случаев позже, чем разрешено по закону, причем для мужчин это характернее, чем для женщин.
 'In most cultures people predominantly get married later than allowed by the law; and it is more typical of men than women.'

These data strongly suggest that it is not the polarity of the predicate but rather some other material in the embedded clause that triggers Rullmann ambiguity. The following two examples by Boguslavski demonstrate that the ambiguity is not restricted to comparative complements. Like all previous examples in this subsection, (238) and (239) feature a degree term in the scope of a possibility modal.

- (238) Этот самолет может лететь на высоте 100 метров.
 this plane can fly at height 100 metre
 ‘This plane can fly 100 m high.’

Normally, one would use (238) to specify the maximal altitude of the plane. However, anyone who is aware of altitudes that planes can reach infers that 100 m refers to the minimal altitude. The measure phrase receives an ‘at least’ interpretation. In (239), an ‘at least’ interpretation of the measure phrase is not only prominent, there is hardly any context in which this sentence can have an ‘at most’ reading.

- (239) Микроскоп дает возможность различать структуры с
 microscope give possibility resolve structures with
 расстоянием между элементами до 0,20мкм.
 distance between elements to 0.20μm
 ‘A microscope can resolve images of structures with distances of
 0.2 μm between elements.’

As a rule, an ‘at least’ reading of a numeral appears less natural than an ‘at most’ reading. However, as the previous two examples show it may be forced under certain circumstances. Consider the following example in Meier’s context (234), in which the minimum speed limit is as relevant as the maximum one.

- (240) Chuck is allowed to drive 45mph fast on this highway.

‘45mph’ could be understood to refer to the maximum as well as to the minimum in this example leading to an ‘at most’ or to an ‘at least’ interpretation, respectively. If we embed (240) in a ‘than’-clause, as in Meier’s example repeated below, we may expect a comparison with the maximum as well as with the minimum. These are the readings that arguably obtain for (233).

- (233) Chuck is driving faster than he is allowed to drive.

More often than not, we deal with an ‘at most’ interpretation of a measure expression under a possibility modal. Interestingly, two out of

three examples in (Geurts, 2006) that demonstrate the availability of an ‘at most’ interpretation with bare numerals happen to feature possibility modals, cf. (241)a and (241)c. If there is indeed a correlation between the interpretation of numerals and the available readings under the comparative this might explain the preference for a more-than-maximum reading over a more-than-minimum reading.

- (241) a. She can have 2000 calories without putting on weight.
- b. The council houses are big enough for families with three kids.
- c. You may attend six courses.

To sum up, a measure phrase embedded under a possibility modal can have an ‘at least’ as well as an ‘at most’ interpretation. Proponents of Meier’s position that Rullmann ambiguity is not polarity sensitive may pursue the following line of reasoning. If a possibility statement is embedded under the comparative this leads to comparison with one of the two possible cut-off points: the maximum or the minimum. If the context favours an ‘at least’ reading of the numeral, the resulting comparative gets a more-than-minimum reading. Otherwise, a more-than-maximum interpretation obtains.

To sum up, I considered the distribution of Rullmann ambiguity in comparative sentences with embedded possibility modals. Under the view that this kind of ambiguity is restricted to negative pole predicates, it has to fall out from the interpretation of antonyms. I brought some empirical arguments in favour a different view that relates Rullmann ambiguity to the availability of an ‘at least’ and an ‘at most’ interpretation of a degree term in the scope of a possibility modal rather than to the polarity of the gradable predicate involved.

2.5 CONCLUSION

The upshot of this section is tripartite. First, we have seen that treating comparatives with ‘be required’ and ‘be allowed’ in matrix clauses as cases of structural ambiguity is problematic. On the one hand, the readings that are derived by assigning the comparative morpheme the widest scope possible reveal a pattern recognisable in non-comparative contexts. Ambiguous comparatives with ‘be required’ express a comparison with the required minimum, which is prominent in other constructions with this modal. Similarly, ‘be allowed’ triggers interpretations that involve a reference to a maximally permitted degree. For non-comparative constructions displaying the same kind of ambiguity as comparatives a scope solution is implausible, e.g. for sentences with prepositional temporal clauses. The goal is now to provide a unified analysis of intensional comparatives and sentences with ‘before’-phrases and to relate the apparent wide scope readings in comparatives to the similar interpretations of the relevant modals in other scalar contexts.

Second, I argued that ‘than’-clauses with polarity sensitive indefinites, possibility modals and disjunction must be treated independently of comparative complements with necessity modals like ‘have to’. The remaining expressions invariably take scope outside of comparative complements. If the first two classes are successfully handled, scope-based analyses lose much of their motivation and the task of explaining the behaviour of the apparently wide scope elements becomes less complex.

Third, I suggested that Rullmann ambiguity is not polarity related. Rather it is a result of an ‘at least’ or an ‘at most’ reading of a degree term in the scope of a possibility operator. It is necessary to spell out pragmatic factors leading to these readings and explain why one reading is preferred to another.

In the remainder of the thesis I focus on two aspects of the preceding discussion. I begin by considering necessity modals that trigger ambiguities

in contexts discussed in (Heim, 2001) and produce more-than-minimum readings in intensional ‘than’-clauses. I argue that both phenomena follow from the interpretation of these modals. I then investigate the behaviour of possibility modals and show that the fact that they may give rise to pragmatic implicatures in scalar contexts explains their apparent narrow scope readings in comparative complements as well as ambiguities in Heim’s and Rullmann’s environments.

Chapter 3. Comparing to What is Necessary

3.1 STRUCTURE

The main goal of this section is to answer the following question:

- (54) What kind of modals give rise to the minimal requirement reading (MinR)? Why does MinR obtain with these modals and not with others?

The chapter is organised in the following way. In section 3.2 I introduce the relevant data and give a preview of the main claims. In section 3.3 I first look into the class of modals that give rise to MinR to pin down the kind of modal flavour they are associated with. A distinct feature of MinR modals is their goal-orientation. I show that a distinction between weak and strong modals within the class of goal-oriented modals corresponds one-to-one to a distinction between MinR and non-MinR modals. I discuss some well-known differences between weak and strong modals with the aim of delineating the class of MinR modals. In section 3.4 I develop an analysis of strong modals and explain the source of MinR. In section 3.5 I turn to weak weak and account for their lack of MinR.

3.2 BACKGROUND

In the forgoing survey of the data on intensional comparatives, necessity modals figure as a non-homogeneous class. First, (Heim, 2001) points out that not all necessity modals lead to an ambiguity in comparatives with ‘exactly’ differentials or ‘less’. In the following pair of examples both sentences normally convey that the paper is 15 pages long in all worlds complying with the requirement. Sentence (43) can, however, additionally mean that 15 pages is the required minimal length of the paper.

(43) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

(242) (The draft is ten pages long.) The paper should be exactly five pages longer than that.

Second, not all necessity modals receive an apparent wide scope interpretation in comparative complements. To see the contrast between those that do and those that do not, consider the following pair of sentences:

- (243) a. Sean got more points than he had to in order to win.
b. Sean got more (penalty) points than he should have in order to win.

Sentence (243)a expresses a comparison with the minimally required score. It is true iff Sean’s actual score exceeds his score in some worlds in which he wins but not in all such worlds, i.e. the minimally sufficient score. In contrast, sentence (243)b expresses a comparison with the maximally permitted score. For this sentence to be true, Sean’s actual score must be greater than his score in any of the accessible worlds in which Sean wins.

The same kind of difference between necessity modals is visible in other scalar contexts. Consider the pair below.

- (244) a. Sean has to get 90 points in order to win.
b. Sean should get 90 points in order to win.

While (244)a claims that getting less than 90 points is not enough for Sean to win, (244)b does not imply anything of the kind. Like in the comparative sentences in (43) and (243)a, the variant with ‘have to’ refers to the minimally required amount. Generally, whether a necessity modal leads to an ambiguity in Heim’s contexts and receives an apparent narrow scope interpretation in comparative complements depends on its capacity to refer to the minimal requirement outside comparative constructions. I therefore suggest that the phenomena illustrated by (43), (242) and (243) should receive a uniform account. The difference in the interpretation of (243)a and (243)b and the availability of an additional reading of (43) should follow from the account of the contrast in (244).

I call the interpretation shared by the ‘have to’ examples above the minimal requirement reading (MinR). MinR is expected only in contexts that make goals, intentions or wishes of some agents salient. The kind of modals that express necessity or possibility in view of someone’s goals or wishes have been called anankastic in the recent literature, e.g. (von Stechow *et al.*, 2006). The main claim of this chapter is that MinR is rooted in anankastic modality, more precisely, MinR is related to a realistic ordering source used by an anankastic modal for ranking its accessible worlds. My second claim is that goal-oriented modals that do not have MinR are weak anankastic modals that constitute an additional layer of deontic or bouletic modality on top of a covert anankastic modal. Such treatment of non-MinR modals is shown to make welcome predictions concerning their behaviour in scalar contexts and some other distinctive properties they have.

3.3 PINNING DOWN THE KIND OF MODALITY

In this section I go over a number of empirical tests with the aim of giving an descriptive characterisation of MinR modals. Subsection 3.3.1 lays out the background on anankastic modals. In subsection 3.3.2 I discuss differences between weak and strong anankastic modals focussing on four phenomena: modal force, subjunctive morphology, interaction with negation and availability of MinR. The conclusion is that the ‘strong/weak modals’ distinction corresponds to the ‘MinR/non-MinR modals’ one.

3.3.1 ANANKASTIC MODALS

Necessity modals that lead to MinR are characterised by being goal-oriented. Descriptively, this means that they are used to express a condition necessary for achieving someone’s goals or satisfying someone’s wishes. I will refer to this kind of modality as anankastic modality, following (von Stechow *et al.*, 2006). Anankastic modals have been recently a focus of attention in the literature on modality due to various phenomena they are associated with.

Building on the work by (Sæbø, 2001), several authors attempted to develop an analysis of sentences like (245), see (von Fintel and Iatridou, 2005), (Huitink, 2005), (von Stechow *et al.*, 2006). Such sentences were termed anankastic conditionals. A challenging aspect in the analysis of anankastic conditionals is the role of the bouletic ‘if’ clause. For the current discussion it is, however, more important how the modal in the matrix clause is treated. I will discuss the accessibility relation on ‘have to’ in the following sections.

(245) If you want to go to Harlem, you have to take the A train.

The second, often discussed environment that features anankastic modals is the scope of the exclusive particle ‘only’. Sentences like (246) are known as the sufficiency modal construction (SMC), see (Huitink, 2005),

(Krasikova and Zhechev, 2006), (von Fintel and Iatridou, 2007). Not all necessity modals that occur in anankastic conditionals may participate in the SMC. In the following, I will use the SMC diagnostic as an efficient test for the availability of MinR.

(246) You only have to go to the North End to get good cheese.

Finally, (von Fintel and Iatridou, 2008) observe that anankastic modals like some other kinds of modals come in two varieties: weak and strong modals. They discuss modal predicates like ‘ought to’, which are usually judged to express a weaker requirement than strong modals like ‘have to’, cf. (247). The distinction between weak and strong modals will be crucial for delineating the class of MinR modals.

(247) To go to Ashfield, you ought to take Route 2, but you don’t have to.

What kind of modal auxiliaries can effect an anankastic reading? It is well known since Kratzer’s work on modality, cf. (Kratzer, 1991), that though some modals are associated with specific kinds of modality, in most cases a modal can have different modal flavours and it is the role of the context to narrow down the choice to one. For example, the following sentence can mean different things depending on what kind of accessibility relation ‘have to’ is associated with.

(248) Müller has to be on the train to Hamburg.

I may utter (248) to express epistemic certainty. For example, the proposition that Müller is on the train could be a necessity in view of what I know about Müller’s travel schedule. The epistemic interpretation is plausible if (248) occurs in a context like (249).

(249) If my watch is correct, Müller has to be on the train to Hamburg.

I may also utter (248) to express a condition necessary for achieving some goal. This is an anankastic interpretation, made prominent in (250).

(250) Müller has to be on the train to Hamburg if he wants to meet Schmidt today.

An anankastic modal statement can be supplemented by an ‘in order to’-complement. It was first noted in (Bech, 1955) that ‘in order to’-clauses have two different interpretations in necessity statements. On the one hand, an ‘in order to’-complement may restrict the modal to yield an anankastic reading. In this case, it expresses a goal for the achievement of which the proposition under the modal is a necessary condition. For example, on its anankastic reading, (251) conveys that going to Hamburg is necessary for meeting Schmidt. On the other hand, an ‘in order to’ clause may not associate with the modal but simply specify the purpose of the event introduced by the complement of the modal. On such a reading, (251) means that it is required of Müller to go to Hamburg for the purpose of meeting Schmidt.

(251) Müller has to go to Hamburg in order to meet Schmidt.

If the ‘in order to’ clause is fronted, only an anankastic interpretation is possible. For example, sentence (252) does not have a purpose reading.

(252) In order to meet Schmidt, Müller has to go to Hamburg.

One can use the ‘fronting’ test to decide whether an anankastic reading is available with this or that modal. The idea is to check whether a suitable ‘in order to’ complement can be coherently attached to the beginning of the relevant modal statement. As an example, take the German modal predicate ‘sollen’. According to (Kratzer, 1981), it usually has a bouletic modal flavour. ‘In order to’ complements receive only a purpose interpretation in sentences with ‘sollen’. Example (253) conveys that in view of the speaker’s wishes, the addressee is required to wash the dishes for the purpose of helping her mother. Forcing an anankastic reading by fronting the infinitival complement, as in (254), leads to unacceptability.

(253) Du sollst das Geschirrabspülen, um deiner Mutter zu helfen.
you soll-2ps the dishes wash for your mother to help
'You shall wash the dishes to help your mother.'

(254) #Um deiner Mutter zu helfen, sollst du das Geschirrabspülen.
for your mother to help soll-2ps you the dishes wash
lit: 'To help your mother you shall wash the dishes.'

However, if 'sollen' carries subjunctive morphology it passes the fronting test.²² In contrast to (254), (255) is not deviant. This suggests that the subjunctive 'sollen' has an anankastic use.

(255) Um deiner Mutter zu helfen, solltest du das Geschirrabspülen.
for your mother to help soll-past you the dishes wash
'To help your mother you should wash the dishes.'

As another example, consider 'have to', which has an anankastic interpretation. If the 'in order to' clause follows the complement of the modal, two interpretations are available. Example (256) either implies that Müller's itinerary has an entry like: «location: train to Hamburg; purpose: meeting with Schmidt»; or it contains a note saying that the only location where Müller can meet Schmidt is Hamburg or the train to Hamburg.

(256) According to Müller's itinerary, he has to be on the train to Hamburg in order to meet Schmidt.

If the infinitival clause is fronted, only the second interpretation is available, cf. (257).

(257) According to Müller's itinerary, in order to meet Schmidt, he has to be on the train to Hamburg.

To conclude, MinR modals are goal-oriented and can be subsumed under the category of anankastic modals. Descriptively, anankastics are those modals that can be restricted by 'in order to' complements. An 'in order to' complement of an anankastic modal should be distinguished from a

22. Present subjunctive morphology corresponds to the preterite in German.

purpose ‘in order to’ clause modifying the proposition embedded under the modal.

3.3.2 WEAK AND STRONG ANANKASTIC MODALS

Anankastic modals come in two varieties known as weak and strong modals. A weak modal is said to have a weaker modal force or lead to a weaker claim than a strong one. The following remark by (Sloman, 1970), cited in (von Fintel and Iatridou, 2008), gives a first impression of what is behind this distinction:

- (258) “For instance ‘If you want to get to London by noon, then you *ought to* go by train’ picks out the best means without excluding the possibility of others, whereas ‘If you want to get to London by noon then you *have to* (*must, will be obliged to* etc.) go by train’ implies that no other means exists.”

The intuition behind the weakening of modal force is captured by the fact that a weak anankastic statement can be true in a context in which a strong one is false. (von Fintel and Iatridou, 2008) demonstrate this by the following context and the pair of examples.

- (259) Imagine that to cross Siberia to go to Vladivostok you can take one of two trains: the Russian train or the Chinese train. The Chinese train is significantly more comfortable.²³
- a. To go to Vladivostok, you have to take the Chinese train.
 - b. To go to Vladivostok, you ought to take the Chinese train.

Whereas (259)a is judged false in the given scenario, (259)b can well be true. It appears that in (259)b, besides the goal expressed by the infinitival clause, some secondary goals or considerations of the speaker are taken into account. It is not that you take the Chinese train in every world in which you travel to Vladivostok, but rather, in every world in which you travel to Vladivostok comfortably.

23. This scenario was first discussed in (von Stechow *et al.*, 2006).

Another feature of weak modals is their subjunctive form. In most languages surveyed by (von Fintel and Iatridou, 2008) a weak modal is formed by augmenting a strong one with the subjunctive morphology. They call such languages transparent. An example is Russian. The strong anankastic **надо** / ‘necessary’ can be ‘weakened’ by the subjunctive particle **бы**.

(260) Тебе надо ехать китайским поездом.
 you necessary go Chinese train-INSTR
 ‘You have to take the Chinese train.’

(261) Тебе надо бы ехать китайским поездом.
 you necessary SUBJ go Chinese train-INSTR
 ‘You should take the Chinese train.’

Even weak modals in the so-called non-transparent languages, like English or German, have a subjunctive feel about them. Von Fintel and Iatridou note that English ‘ought to’ used to express the past subjunctive of ‘owe’ in the meaning ‘possess’. In German, weak necessity can be expressed by the preterite form of ‘sollen’, which corresponds to the subjunctive, cf. (255) repeated below.

(255) Um deiner Mutter zu helfen, solltest du das Geschirr abspülen.
 for your mother to help soll-past you the dishes wash
 ‘To help your mother you should wash the dishes.’

The distinction between weak and strong anankastic modals captures a number of interesting properties that these modals display in negative and scalar environments including their ability to trigger SuffR. In the following subsection I focus on some of them.

3.3.2.1. SCOPE OF NEGATION

Weak anankastic modals are known to always take wide scope with respect to negation, whereas strong modals can be interpreted in the scope of negation. This creates a sharp contrast between strong and weak modals and can be used as an empirical test to tell a weak modal from a strong one. Consider the following minimal pair.

- (262) a. You ought not to/should not leave. (modal > negation)
 b. You do not have to leave. (negation > modal)

According to (262)a, it is required of you not to leave, which suggests that the negation operates on the embedded proposition ‘that you leave’ but not to the entire modal statement. In (262)b, on the other hand, it is not the complement of ‘have to’ that is negated but the requirement itself, suggesting that the modal is in the scope of negation.

The syntactic position of the negative marker might be responsible for the semantic scope. However, this hypothesis does not appear to be empirically supported. In the following pairs from English and German b-examples feature negation below a modal on the surface. They contrast with the corresponding a-variants in the semantic scope of negation.

- (263) a. You should not leave. (modal > negation)
 b. You need not leave (negation > modal)

- (264) a. Du solltest nicht gehen (modal > negation)
 you shall-PAST.2PS NEG go
 ‘You should not leave.’
 b. Du brauchst nicht zu gehen (negation > modal)
 you need NEG to go
 ‘You do not have to leave.’

An opposite pattern is observed in Russian. Examples (265)a and (265)b differ in the presence/absence of the subjunctive particle. The negative marker appears above the modal in both cases. However, only in (265)b does its semantic scope reflect its surface position.

- (265) a. Тебе не надо бы идти. (modal > negation)
 you NEG need SUBJ go
 ‘You should not go.’
 b. Тебе не надо идти. (negation > modal)
 you NEG need go
 ‘You should not go.’

This difference between weak and strong modals is also manifested when they are embedded under negative particles. The following pair of examples features anankastic modals under the exclusive particle ‘only’.

- (266) You only have to bring your pass to get admitted to the embassy.
‘You don’t have to do bring anything else.’
‘It is not required of you to bring anything else.’
- (267) You should only bring your pass to get admitted to the embassy.
‘You should not bring anything else.’
‘It is required of you not to bring anything else.’

As the paraphrases below the examples suggest, ‘only’ is interpreted above ‘have to’ but below ‘should’. As a result, sentence (266) negates the necessity of bringing anything else, whereas (267) expresses a prohibition on bringing anything else.

Other negative particles reveal the same pattern. For example, the counterfactual reading of ‘almost’ is available with ‘have to’ but impossible with ‘should’. Compare the following examples.

- (268) a. She almost had to write this paper from scratch,
but then she was allowed to leave it as it was.
- b. She almost should have written this paper from scratch but
#then she was allowed to leave it as it was.

Unlike (268)a, (268)b cannot imply that the necessity to write the paper from scratch was close to being imposed on her but was not actualised, hence the infelicity of the continuation.

To conclude, weak modals are known to avoid the semantic scope of negation. Negation operates on the propositional complement of a weak modal, regardless of its surface position with respect to the modal. This distinguishes weak modals from strong ones, which can be interpreted in the scope of sentential negation and other negative expressions.

3.3.2.2. MINIMAL REQUIREMENT READING

Strong anankastic modals differ from weak ones in their ability to trigger a minimal requirement reading (MinR). This difference can be tracked in a number of scalar contexts. In this section I focus on two MinR environments, ‘have to’ in the scope of the exclusive particle ‘only’ and a degree-denoting expression in the scope of ‘have to’.

Embedding of necessity statements under ‘only’ has been discussed at length in the literature on anankastic modals (see (Huitink, 2005), (Krasikova and Zhechev, 2006), von (von Fintel and Iatridou, 2007)). Consider examples (266) and (267), repeated below. Interpreting a modal in the scope of ‘only’ triggers an inference that the necessity it expresses is minimally sufficient, cf. (266). Obviously, since weak modals scope above ‘only’ they do not lead to such an inference, cf. (267).

(266) You only have to bring your pass to get admitted to the embassy.
 ‘You don’t have to do bring anything else.’
 ‘It is not required of you to bring anything else.’
 ⇒ Bringing your pass is sufficient.

(267) You should only bring your pass to get admitted to the embassy.
 ‘You should not bring anything else.’
 ‘It is required of you not to bring anything else.’
 ⇏ Bringing your pass is sufficient.

(von Fintel and Iatridou, 2007) call sentences like (266) sufficiency modal constructions (SMC). Intuitively, the sufficiency meaning component results from presupposing the necessity of bringing the pass and denying the necessity of bringing anything *in addition to it*. So bringing the pass is understood to be a minimally sufficient action for the achievement of the relevant goal, i.e. getting admitted to the embassy, as schematised below.

(269) -----min----->
 not sufficient bring your pass not necessary

Embedding a degree denoting expression under the modal provides an

explicit scale, on which the prejacent of ‘only’ is compared to its alternatives. For example, in (270) the age scale constitutes such an ordering. The sentence conveys that 18 years is a minimally sufficient age for getting a driving license. Being younger than 18 is understood to be insufficient and being older than 18 unnecessary.

(270) You only have to be 18 years old to get a driving license in Germany.

Leaving out ‘only’ in example (270) does not remove the sufficiency inference as the continuation in (271) shows. In contrast, a weak modal does not lead to such an inference in this context, cf. (272).

(271) You have to be 18 years old to get a driving license in Germany
#but you could also get it before you turn 18.

(272) You should be 18 years old to get a driving license in Germany,
but you could also get it before you turn 18.

This is the second MinR environment – a degree-denoting expression in the scope of a strong anankastic modal. I use the term ‘degree-denoting expression’ to refer not only to overt numerals or measure phrases like ‘60 km/h’ but also to degree variables. The second environment thus encompasses a large range of degree constructions featuring gradable predicates, which project degree variables.

Let us first look at the case of overt numerals. (Geurts, 2006) calls the minimal requirement reading of (270) an ‘at least’ reading of the numeral. This terminology refers to the numerical scale and appears to be less general than the term MinR. Sentence (271) indeed implies that one has to be *at least* 18 years to get a driving license, which renders the age of 18 years minimally sufficient. There are, however, examples of a sufficiency inference corresponding to an ‘at most’ reading of the numeral. Consider example (273).

(273) You have to get up at 5 o’clock to catch the first train.

This sentence conveys that getting after 5 o'clock is insufficient, i.e. too late, for catching the first train. This means that one has to get up at 5 o'clock at the latest. This is neatly captured by the following sufficiency scale:

(274) -----min----->(time)
 not necessary 5 o'clock not sufficient

We can, of course, construct examples with time adverbials that have an 'at the earliest' reading, cf. (275) – (276).

(275) You have to arrive at 12 o'clock to be able to check in.

(276) -----min----->(time)
 not sufficient 12 o'clock not necessary

What (273) and (275) have in common is that they refer to the minimally sufficient time. Crucially, their counterparts in (277) – (278) featuring 'should' do not make such a reference, as the provided continuations suggest.

(277) You should get up at 5 o'clock to catch the first train, but could also make it if you get up later.

(278) You should arrive at 12 o'clock to be able to check in, but you could also occupy a room if you arrive earlier.

Intensional comparatives present somewhat more complex examples of the second MinR environment. The degree-denoting expression is either the degree argument of the matrix adjective, as in the two types of comparatives discussed in (Heim, 2001), or the degree argument of the adjective embedded in an intensional 'than'-clause. Below I give examples for these three types of constructions.

The first type is a 'less' comparative. For example, (279)a may convey that the addressee wrote more than minimally required, see (279)b, which corresponds to MinR.

(279) a. (Your draft is 10 pages.) You need to write less than that.

b. You are not required to write as much as that.

Replacing ‘have to’ by a weak modal makes MinR unavailable, as can be seen on the contrast in (280).

- (280) a. You need to write less than that. You could have spared a couple of pages.
b. You should have written less than that. #You could have spared a couple of pages.

The second type is a comparative with an ‘exactly’ differential. In the following pair of examples only (281)a can be understood to mean that 90 points is the minimally required score, which corresponds to MinR. It can also mean that Sean can only win if he scores exactly 90 points, the reading it shares with (281)b.

- (281) a. (Jimmy scored 85 points.) Sean has to get exactly 5 points more than that in order to win.
b. (Jimmy scored 85 points.) Sean should get exactly 5 points more than that in order to win.

Finally, the third type is a comparative with an intensional complement. In this case, MinR corresponds to a comparison with the minimally required degree as the standard. An example is given in (282). It conveys that the number of courses attended by the addressee exceeds the minimal requirement. As a result, we can get an inference that the addressee could have attended fewer courses and still met the requirement, cf. (282)a. The sentence therefore does not report on a violation of a rule and we do not get an inference that the addressee failed to meet the requirement, cf. (282)b.

- (282) You attended more courses than you had to.
a. \Rightarrow You could have attended fewer courses.
b. \Rightarrow You should have attended fewer courses.

In contrast, example (283), which contains a weak modal, implies that the number of courses attended by the addressee exceeds the maximally

allowed limit. It does not have MinR.

(283) You attended more courses than you should have.

a. \Rightarrow You could have attended fewer courses.

b. \Rightarrow You should have attended fewer courses.

To sum up, a common thread running through a number of constructions with anankastic modals is a reference to an amount minimally sufficient for achieving a goal salient in the context. I call this inference a minimal requirement reading. MinR surfaces in anankastic sentences if they are embedded under ‘only’ or feature degree-denoting expressions. The latter class is very diverse, including overt measure phrases as well degree-denoting variables. Thus, MinR may arise in simple modal statements with overt measure expressions and more complex intensional comparatives. In each environment the availability of MinR depends on the “force” of the modal. Only strong anankastic modals have MinR. If an anankastic modal is weak, i.e. if it is marked by subjunctive morphology and cannot be interpreted in the scope of negation, MinR does not obtain.

3.4 ACCOUNTING FOR THE MINIMAL REQUIREMENT READING

To track down the source of MinR we need to look into the interpretation of anankastic modals. This is the task of this section. It is structured as follows. In subsection 3.4.1, I consider two alternative explanations of MinR and motivate my approach as well as preview the outcome. In subsections 3.4.2 – 3.4.4, I spell out an analysis of the anankastic ‘have to’ and demonstrate that it predicts MinR in ‘have to’ sentences with measure expressions or numerals. In section 3.4.7, I turn to more complex environments and show how MinR is derived in intensional comparatives.

3.4.1 PLOT

The term sufficiency inference, I have been using to characterise the MinR, may suggest that an anankastic modal makes its complement to a condition sufficient for achieving a goal supplied by the ‘in order to’ clause. Indeed, this is how the proposition embedded under an anankastic modal is often understood. For example, the following sentence conveys that pressing the button is sufficient for restarting the printer.

(284) To restart the printer, you have to press the button.

However, this is not a general property of anankastic modals. The following example is underspecified with respect to whether the proposition expressed by the complement is sufficient for achieving the given goal.

(285) In order to get this job, you need to learn French.

Obviously, this kind of underspecification does not hinge on the semantics of the modal but on the nature of the goal/means relation which holds between the eventualities expressed by the complement and the infinitival clause, as the following variants without modals suggest. In (286)a the action of pressing the button is understood to be sufficient for restarting the printer, whereas (286)b does not carry a sufficiency implication.

- (286) a. You pressed the button to restart the printer.
 b. You learned French to get this job.

What is meant by MinR of anankastic sentences in the context of this work is not a propositional-level sufficiency inference observed in (284) and (286)a but a degree-level one. As discussed in section 3.3.2.2, it is observed in some anankastic sentences with gradable expressions, e.g. comparatives and constructions with numerals and measure phrases. Consider an example with a numeral below.

- (287) To win, Sean has to score 90 points.

Does Kratzer’s analysis of necessity modals introduced in section 1.2.3 predict that 90 points refers to the minimally sufficient score? Assuming Kratzer’s rule for necessity modals, we derive the following truth conditions for this example.

- (288) $\forall w' \in \cap f(w): w' \in [\lambda w \text{ Sean gets 90 points in } w]$

Following common practice, we may assume that $f(w)$ is a set of propositions compatible with Sean’s goal to win in w and ignore the role of the ordering source for now. Then, example (287) is predicted true iff Sean scores 90 points in every world in which his goal to win is achieved. The fact that Sean scores 90 points in all of his goal worlds does not imply that 90 points are minimally sufficient for his victory. Note that this outcome hinges on the exact contribution of “gets 90 points”. If we assume that it has an ‘at least’ reading, the minimal requirement inference can be easily derived as a result of enriching the truth conditions in (289)a by a scalar implicature along the lines in (289)b. (289)b adds the condition that there are some goal-winning worlds in which Sean scores less than 90 points.

- (289) a. $\forall w' \in \cap f(w): w' \in [\lambda w \text{ Sean gets at least 90 points in } w]$
 b. $\forall n > 90: \neg \forall w' \in \cap f(w): w' \in [\lambda w \text{ Sean gets at least } n \text{ points in } w]$

There are several questions that a solution in terms of pragmatic enrichment raises. First, it is *prima facie* unclear how this account can differentiate between ‘have to’ and other modals, like ‘should’, with which the MinR is not possible, compare (287) to (290) where the phrase ‘90 points’ does not refer to the minimal requirement.²⁴

(290) To win, Sean should score 90 points.

Secondly, scalar implicatures are known to disappear in downward entailing contexts, like the scope of negation, whereas the sufficiency inference does not. Consider the negation of (287).

(291) To win, Sean does not have to score 90 points.

This sentence negates that 90 points are minimally required for winning. The required minimum is conveyed to be below 90 points, which suggests that the sufficiency inference is part of the truth-conditional content. If it were an implicature, we would expect it not to contribute to the meaning of the negated sentence. (291) would be expected to make no reference to the minimal requirement and amount to the claim that Sean can win with a score different from 90 points, the negation of the non-MinR reading. However, contrary to this expectation, MinR survives under negation.

Finally, an ‘at least’ meaning assumption does not generalise to cases like (273) repeated below, where one would have to work with an ‘at most’ meaning.

(273) You have to get up at 5 o’clock to catch the first train.

To make correct predictions in every case, the direction of the scale on which the exhaustification mechanism operates should not be fixed, but rather pragmatically determined. This is a standard, independently

24. A strategy that immediately suggests itself here would be argue that weak modals can not be strengthened for the same reason they cannot be interpreted in the scope of negation.

motivated assumption, whereby the context determines the inference pattern and the direction of the scale crucial for the computation of the scalar implicature. However, one is left with the feeling that this direction depends on the contextual parameters determining the modal interpretation. It is the modal that is associated with the sufficiency scale I informally introduced in section 3.3.2.2. This consideration motivates an account of MinR that is not based on a pragmatic enrichment but instead derives this interpretation from the properties of anankastic modals.

To give a preview of my proposal, I suggest that the sufficiency scale corresponds to the possibility scale, in the sense of (Lewis, 1973b), used by an anankastic modal to compare its propositional complement to alternative propositions. In the following I sketch my account of the availability of MinR in example (287) and its absence in example (290). The truth condition derived for (287) are roughly the following:

- (292) That Sean achieves his goal to win and gets 90 points is a better possibility in w than that he achieves his goal to win and gets n points, where n is a score different from 90 points.

Two ingredients of (292) are noteworthy. First, the notion of comparative possibility is based on the relation of similarity to the index world. The criterion for deciding which of two propositions is a better possibility is their proximity to the world of evaluation. Second, the propositions compared to each other vary in the place of the numeral. In the given example, we compare the possibility of a proposition based on Sean's score. The worlds closest to w are those where the addressee wins and nothing unusual from the perspective of w happens. A far-fetched world could be a world in which the addressee wins without a sufficient score as well as a world in which the addressee takes too much effort. If we reason like this, (292) is met iff scoring 100 points is minimally sufficient for winning. This is how the MinR comes about. The main goal of this chapter is to provide independent motivation for the assumptions underlying (292), namely,

similarity to the evaluation world as a type of ordering employed by anankastic modals and the use of alternatives.

The analysis of the ‘should’ version in (290) proceeds differently. Suppose that (290) is uttered by Sean’s coach who wishes Sean to win with a considerable lead. The sentence is predicted to have the following truth conditions.

- (293) For any world w compatible with Sean’s coach’s preferences: that Sean achieves his goal to win and gets 90 points is a better possibility in w than that he achieves his goal to win and gets n points, where n is a score different from 90 points.

In contrast to the ‘have to’ case, deciding what is a better possibility depends on the perspective of Sean’s coach. The factor of the effort made does not play a role whatsoever if Sean’s coach prefers Sean to always win with a big advantage at whatever cost. By using a non-MinR modal we are considering the closeness to someones’s desirable/preferred state of affairs not to what is the case in reality. The truth conditions in (293) are therefore met iff scoring 90 points is the most preferable way of winning in Sean’s coach’s view, not necessarily the minimally sufficient one. As a result, (290) does not have MinR.

As envisioned at the outset of the thesis, I use this outcome to derive the differences in the interpretation of comparatives with MinR and non-MinR modals, e.g. (294) – (295).

- (294) a. Sean scored more points than he had to.
b. Sean got more points than he should have.
- (295) a. Sean has to score less than 90 points.
b. Sean should score less than 90 points.

The common feature of these examples is that they are based on the following template.

- (296) Sean has/should to score d -many points.

Since the ‘have to’ version of (296) triggers the MinR that Sean is minimally required to score d -many points, its comparative counterparts in (294)a and (295)a also have this inference and are interpreted along the following lines:

(297) Sean’s actual score is greater than the minimal score sufficient for Sean’s winning.

(298) The score sufficient for Sean’s winning is smaller than 90 points.

The b-examples in (294) and (295) are not predicted to have such interpretations under the present account.

3.4.2 ANANKASTIC CONDITIONALS AS COUNTERFACTUALS

Consider an example of an anankastic sentence:

(299) In order to get to Harlem, you have to take the A train.

In this section I develop an analysis of (299) in Kratzer’s framework for analysing modals and conditionals, (Kratzer, 1981). While doing this, I am guided by the following two questions:

- (300) a. What is the semantic contribution of an ‘in order to’-clause?
b. What kind of conversational backgrounds is ‘have to’ associated with?

3.4.2.1. ANTECEDENT

A distinctive feature of anankastic modals is their goal-orientation, which is anchored in their compatibility with ‘in order to’-clauses. A modal auxiliary can receive different interpretations; a universal modal, in particular, can express different kinds of necessity, e.g. a necessity in view of some circumstances or in view of the available information sources. There are no modal auxiliaries, to the best of my knowledge, that exclusively express anankastic necessity. However, fronting an ‘in order to’ clause removes ambiguity and forces an anankastic reading. Example (299), with the fronted complement, means that taking the A train is a necessary condition for

achieving the goal of getting to Harlem. Compare example (299) to the following one, which can additionally express an obligation to take the A train with the aim of getting to Harlem.

(299) In order to get to Harlem, you have to take the A train.

(301) You have to take the A train in order to get to Harlem.

What is the role of an ‘in order to’-clause in determining the truth of an anankastic modal statement? Obviously, it has to be composed with the modal for the anankastic reading to be possible. This conclusion goes back to (Bech, 1955), who observes that ‘in order to’-clauses are ambiguous when they occur with a modal: they either restrict the modal and thereby produce an anankastic interpretation, or they simply express a goal. Bech calls the first use of ‘in order to’ *determinative* and the second one *indeterminative*. Under an indeterminative, i.e. non-anankastic, interpretation, an ‘in order to’-clause expresses a *causa finalis*. This interpretation is available without a necessity modal as well.

(302) You take the A train to get to Harlem.

If we adopt the theory of causation in (Lewis, 1973a), we can paraphrase (302) as suggested in (303).

(303) ‘You take the A train, and you want to get to Harlem, and if you did not want to get to Harlem, you would not take the A train.’

A determinative, i.e. anankastic, interpretation hinges on the presence of a necessity modal. In this case, an ‘in order to’-clause, in addition to expressing a goal, restricts the quantification domain of the modal. Intuitively, sentence (299) is true iff in every world in which the addressee’s goal to get to Harlem is actualised she has taken the A train. This intuition can be captured if we follow (von Stechow *et al.*, 2006) and (von Fintel and Iatridou, 2005) in treating (299) as a conditional with the antecedent ‘that you take the A train’ and the consequent ‘that you get to Harlem’. In normal hypothetical conditionals antecedents are expressed by ‘if’-clauses.

Recall that in Kratzer’s analysis the function of an ‘if’-clause is to determine the range of worlds that a modal operator quantifies over. (von Fintel and Iatridou, 2005) propose to treat ‘in order to’-clauses in anankastic sentences as playing the same role. Consider the truth conditions they derive for an anankastic sentence:

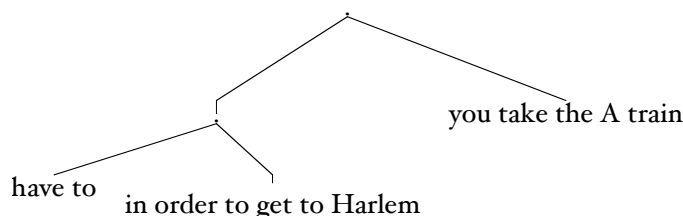
- (304) ‘To p , have to q ’ is true relative to modal base $f(w)$ iff all the worlds in $f(w)$ where p is achieved are q -worlds.

(von Fintel and Iatridou, 2005) call proposition p expressed by the ‘in order to’-clause ‘designated goal’ argument of an anankastic modal. p together with the modal base $f(w)$ restrict the worlds over which ‘have to’ quantifies. In this sense, it serves as an antecedent of what I will call an anankastic conditional. Von Fintel and Iatridou do not specify what it means to be a goal. I take ‘ p is achieved’ to mean ‘ $\text{GOAL}(p, q)$ and p ’, where $\text{GOAL}(p, q)$ could be understood as ‘if it were not for the wish that p , q wouldn’t be true.’ Spelling out the precise meaning of $\text{GOAL}(p, q)$ would take me too far afield. I adopt the proposal to treat an ‘in order to’-clause as an antecedent and leave the ‘goal’ component unspecified:

- (305) ‘To p , have to q ’ is true relative to modal base $f(w)$ iff all the worlds in $f(w)$ where $\text{GOAL}(p, q)$ and p hold are q -worlds.

Under this analysis, the infinitival clause can be treated as an argument of ‘have to.’ This presupposes a structure like (306) where the ‘in order to’-clause is bracketed with the modal.

- (306)



Leaving aside the accessibility relation for now, the meaning of ‘have to’ is roughly the following:

(307) $\llbracket \text{have to} \rrbracket^f(p)(q)$ is true in w iff all the worlds in $f(w)$ where $\text{GOAL}(p, q)$ and p is true are q -worlds.

Alternatively, the infinitival clause can be composed with the rest of the sentence by a special rule in the spirit of Kratzer’s rule for ‘if’-clauses. This alternative is outlined in (308). In what follows, for simplicity, I shall stick to the first option and treat the ‘in order to’-clause as an argument of the modal.

(308) $\llbracket \text{to } p, \text{ have to } q \rrbracket^f = \llbracket \text{have to } q \rrbracket^{f*}$,
 where $\forall w: f^*(w) = f(w) \cup \{\llbracket p \rrbracket^f\} \cup \{\text{GOAL}(p, q)\}$

To sum up so far, anankastic modal statements are necessity conditionals with ‘in order to’ antecedents. An anankastic modal quantifies over worlds in which the proposition expressed by an ‘in order to’-clause is true and is the goal of the event expressed by the consequent.

3.4.2.2. ACCESSIBILITY

In this subsection, I discuss the kind of accessibility relation that anankastic modals are associated with. A number of papers concerned with this kind of modality take for granted that anankastic modals involve a circumstantial modal base, see (Sæbø, 2001), (von Fintel and Iatridou, 2005), (Huitink, 2005). To motivate this assumption, let us look at an example. Von Fintel and Iatridou suggest that ‘have to’ in sentence (299), repeated below, is evaluated relative to f that includes circumstances related to the goal of getting to Harlem, for instance the location of the addressee, relevant train schedules and other facts related to transportation systems. Those facts have to hold in the worlds over which ‘have to’ quantifies.

(299) In order to get to Harlem, you have to take the A train.

Recall that according to von Fintel and Iatridou’s rule, which I adopted in the previous section, an anankastic conditional is true iff the consequent holds in all worlds accessible from the modal base in which the goal

expressed by the antecedent is achieved, cf. (305).

- (305) ‘To p , have to q ’ is true relative to modal base $f(w)$ iff all the worlds in $f(w)$ where $\text{GOAL}(p, q)$ and p hold are q -worlds.

By this rule, sentence (299) is true in world w iff the following holds:

- (309) $\forall w': w' \in \cap (f(w) \cup \{\lambda w \text{ the addressee goes to Harlem in } w\} \cup \{\text{GOAL}(\lambda w \text{ the addressee goes to Harlem in } w, \lambda w \text{ the addressee takes the A train } w)\}) \rightarrow w' \in [\lambda w \text{ the addressee takes the A train } w]$,
‘All the worlds in $f(w)$ where the addressee’s goal to get to Harlem is achieved are the worlds in which the addressee takes the A train.’

Assume that $f(w)$ contains the fact that only the A train goes to Harlem. In this situation, (299) is correctly predicted true under this analysis.

Despite a common-sense conclusion that facts are crucial for evaluating an anankastic conditional, (von Stechow *et al.*, 2006) see a problem with positing a circumstantial modal base for anankastic modals. They note that it is not clear what facts count as relevant and should be included in the modal base. Even ‘circumstances related to the salient goal’ is a far too vague definition to pin down the kind of facts that restrict an anankastic modal. Below I give a pertinent citation from (von Stechow *et al.*, 2006), where they express a concern regarding the role of a circumstantial modal base in an analysis by (Huitink, 2005).

(310) “There are many ways to go to Harlem. You can take the A train, you can take a taxi, you can ask someone to give you a ride, you can walk all the way through Manhattan, you can even pretend to be an emergency and call for the ambulance. In other words, the facts are that you will reach Harlem, by taking the A train, by walking through Manhattan and so on. If all of these were equally relevant for the evaluation of the truth of (21) [If you want to go to Harlem you ought to take the A train], the AC would be predicted false under Huitink’s analysis. So how do we know that of all these facts the only thing that matters is that you will reach Harlem if you take the A train? We think the answer is that taking the A train is the easiest way to get to Harlem. The other ways mentioned are more remote possibilities. So we really need a theory that chooses the least remote possibility among several possibilities.” (von Stechow *et al.*, 2006)

To account for the intuition that remote possibilities leading to the achievement of a goal are ignored, (von Stechow *et al.*, 2006) propose to treat anankastic conditionals as Lewisian variably strict conditionals. Such a treatment was applied to the analysis of counterfactual conditionals in (Lewis, 1973b) and later in (Kratzer, 1981). The main idea is that possible worlds can be ordered relative to how much they depart from the ideal represented by the world of evaluation.

(311) “Given a counterfactual to be evaluated as true or false at a world, such an ordering serves to divide the worlds where the antecedent holds into two classes. There are those that differ minimally from the given world; and there are those that differ more-than-minimally, gratuitously. Then we may ignore the latter, and call the counterfactual true iff the consequent holds throughout the worlds in the former class.” (Lewis, 1981)

Following (von Stechow *et al.*, 2006), I treat anankastic conditionals as counterfactuals. To keep track of the assumptions made so far, let me formulate this proposal in Kratzer’s framework. To capture Lewis’s insight that the truth of a counterfactual depends on the relation of similarity to the actual world, Kratzer defines an ordering source employed by a counterfac-

tual modal as a function that assigns to any world a set of propositions that characterise it uniquely. She calls this kind of ordering totally realistic.

(312) Totally realistic conversational backgrounds: *In view of what is the case...*

A totally realistic conversational background is a function g that such that for any $w \in W$, $\cap g(w) = \{w\}$

The second conversational background, i.e. the modal base, does not contribute any further restriction on the accessible worlds and is therefore assumed to be empty.

(313) Empty conversational background:

The empty conversational background is the function f such that for any $w \in W$, $f(w) = \emptyset$

Integrating these parameters into von Fintel and Iatridou's rule gives us the following semantics for an anankastic conditional:

(314) 'To p , have to q ' is true relative to a totally realistic ordering source $g(w)$ and an empty modal base $f(w)$ iff all the worlds in $f(w)$ that come closest to $g(w)$ and where $\text{GOAL}(p, q)$ and p are true are q -worlds.

We can now define the meaning of 'have to'.

(315) $\llbracket \text{have to} \rrbracket(p)(q)$ is true relative to totally realistic ordering source $g(w)$ and empty modal base $f(w)$ iff
 $\forall w' \in \cap(f(w) \cup \{q\} \cup \{\text{GOAL}(p, q)\}): \text{Max}_{g(w)}(w') \rightarrow w' \in p$

Applying this analysis to the anankastic conditional in (299) results in the truth condition in (316). I abstract here from the 'in order to' clause function of specifying the goal of the taking-the-A-train event.

(299) In order to get to Harlem, you have to take the A train.

(316) $\forall w'$: you get to Harlem in $w' \wedge \text{Max}_{g(w)}(w') \rightarrow$ you take the A train in w' , where $g(w)$ is a totally realistic ordering source.

As intended, this analysis overcomes the problem of 'remote possibilities' addressed in (von Stechow *et al.*, 2006). Consider a scenario in which only the A train stops in Harlem. For any world in which one achieves the goal

of getting to Harlem by such expensive or time-consuming means as taking a taxi or walking all the way through, there is a closer world in which one reaches Harlem on the A train. Even more remote are the worlds in which one gets to Harlem on a different train, given the transportation system.

To summarise this section, following the proposals in (von Stechow *et al.*, 2006) and (von Fintel and Iatridou, 2005), I adopted an analysis of anankastic sentences as counterfactual conditionals couched in Kratzer's semantics for modals. More concretely, the quantification domain of the anankastic 'have to' is restricted by a totally realistic ordering source and the proposition expressed by the 'in order to'-clause, which plays the role of the antecedent.

3.4.3 INTRODUCING ALTERNATIVES

The aim of this subsection is to adjust the definition of 'have to' provided above, in order to capture context sensitivity of anankastic modals. I am going to demonstrate that the interpretation of anankastic conditionals, like some other intensional expressions, is focus sensitive.

By way of motivation for the refinement I want to propose, consider the following context:

- (317) Müller has to negotiate with Schmidt. He can meet Schmidt in Hamburg or Stuttgart. He may only drive or travel by train. He can get to Stuttgart by train, but there are no trains to Hamburg that depart early enough for him to make it to the meeting.

In this context, sentence (318) with the focus on the directional phrase is judged false. Driving to Stuttgart to meet Schmidt is as good a possibility given the facts as driving to Hamburg. The sentence, however, becomes true if we switch the focus from 'Hamburg' to 'drive', as shown in (319).

- (318) To meet Schmidt, Müller has to drive to [Hamburg]_F.
(319) To meet Schmidt, Müller has to [drive]_F to Hamburg.

It turns out that a change in the focus structure has an impact on the truth

of an anankastic conditional. Intuitively, we compare possibilities of different propositions depending on what is at issue. In (318) not driving to Hamburg is compared to driving somewhere else, while in (319) it is compared to getting to Hamburg by some other means. This has to be reflected in the meaning of ‘have to’.

The argument that focused phrases in the scope of some expressions lead to meaning differences goes back to (Dretske, 1972), who is concerned with the focus dependence of some propositional attitude predicates. In the following subsection I shall draw an analogy between anankastic modals and a certain class of attitudes, namely desire predicates.

3.4.3.1. ANALOGY TO DESIRE REPORTS

Anankastic modals are not exceptional in being focus sensitive. There are other intensional predicates and adverbs of quantification that have been observed to be sensitive to focused constituents in their scope, see e.g. (Beaver and Clark, 2003) on focus sensitivity of ‘always.’ (Villalta, 2008) discusses desire predicates and argues that their semantics needs to incorporate a reference to contextual alternatives determined by the focus structure in their scope. Villalta’s proposal builds on the comparative semantics of ‘want’ developed in (Heim, 1992), parallel to the treatment of anankastic modals adopted here, and is aimed at capturing the context sensitivity of desire reports. As such, her analysis is worth while looking at.

Let us first consider Heim’s treatment of desire predicates. Inspired by Stalnaker’s conjecture that every desire report involves a hidden conditional, Heim proposes that a statement with an attitude verb like ‘want’ expresses a counterfactual. The intuition behind this idea is that by saying (320) I express a belief that if I get well I will be in a more desirable state than if I do not.

(320) I want to get well.

Formally, this paraphrase is expressed by the following condition, where

$\text{Dox}_{\text{speaker}}(w)$ is the set of doxastic alternatives or beliefs of the speaker from the perspective of world w :

- (321) For every $w' \in \text{Dox}_{\text{speaker}}(w)$: every world maximally similar to w' in which the speaker gets well is more desirable to the speaker in w than any world maximally similar to w' in which she does not.

The motivation for introducing (321) in the place of the standard Hintikka-style rule, which states that ‘ α want φ ’ is true if and only if φ holds in all bouletic alternatives of α , is similar to the one that lead me to the counterfactual analysis of goal-oriented modals. Heim discusses a number of examples from (Stalnaker, 1984), showing that (321) avoids some fallacious inferences that would have been validated by the Hintikka-style rule. For instance, according to the latter if I get well in all worlds that conform to my desires then I have been sick in all such worlds. (320) certainly does not entail that I want to have been sick. Heim’s rule (321) solves this problem. It allows for ‘I want to have been sick’ to be false in a situation in which ‘I want to get well’ is true. The crucial factor is, of course, the similarity ordering. We may plausibly assume that the worlds in which I have not been sick, though being more desirable, are less close to the actual one. One can now show that (320) can be true without the speaker wanting to have been sick.

Villalta argues in (Villalta, 2006) and (Villalta, 2008) that negation of the desire content, e.g. in (320) the proposition that I don’t get well, should be replaced by the set of contextually salient alternatives to the desired proposition. She discusses example (322) in a scenario in which Victoria’s wish competes with a more likely and less desirable alternative on the one hand and a less likely but more attractive one on the other hand, see (323)

- (322) Victoria wishes Sofia would bring an apple pie.

(323) Sofia has promised to bring a dessert to the picnic. Victoria believes that there are three possibilities for what she may actually do. She could prepare a chocolate cake, even though Victoria considers that extremely unlikely because it represents far too much work. She might bring an apple pie, which Victoria considers very likely since she can just buy it at the bakery nearby. Or Sofia might bring ice-cream, which seems most likely to Victoria, since she usually has some in her freezer. Victoria prefers the chocolate cake over the apple pie and the apple pie over the ice-cream.

Villalta shows that (322) is predicted to be true in this scenario by Heim’s rule in (321), contrary to our intuition. To see why, one needs to consider the worlds compatible with Victoria’s doxastic alternatives in which Sofia does not bring an apple pie and that are only minimally different from what Victoria believes. In the given scenario, these are the worlds in which Sofia brings ice-cream, others being too remote from Victoria’s beliefs. They are less desirable than the apple pie worlds. This is enough to verify the sentence. Villalta concludes that a more adequate formulation of the truth conditions of ‘ α want φ ’ should be based on the comparison of φ to its contextual alternatives and the comparison of φ to $\neg\varphi$ should be viewed as a special case. Villalta’s final proposal for the meaning of ‘want’ is the following.

(324) $\llbracket \text{want}_C \rrbracket^g(p)(a)(w) = 1$ iff $\forall q : q \neq p \ \& \ q \in g(C) : p \succ_{DESa,w} q$,
 where $\forall p, q : p \succ_{DESa,w} q$ iff p is more desirable to a in w than q ²⁵

The major difference to Heim’s definition is the universal quantification over contextual alternatives to the prejacent of ‘want’, i.e. its propositional argument, in place of the negation of the prejacent. This solves the problem with the availability of less likely but more desirable alternatives that Heim’s analysis faces in contexts like (323).

25. To keep the exposition simple I leave out the precise definition of ‘comparative desirability’ and the presupposition of ‘want,’ which requires that the alternatives be doxastically accessible, and refer the reader to (Villalta, 2008).

Another welcome result is that the alternative based semantics allows one to deal with the focus sensitivity of ‘want’. Consider the following pair of examples in the context of the picnic scenario from above. Villalta observes that (325) is true, while (326) is not.

(325) Victoria wants Sofia to bring a [chocolate cake]_F.

(326) Victoria wants [Sofia]_F to bring a chocolate cake.

The difference falls out from the difference in the value of the contextual variable on ‘want’ in the two cases. Under the assumption that the focussed constituents of the prejacent determine the form of alternatives, the values of C correspond to the sets $\{\lambda w \exists x P(x) \ \& \ \text{Sofia brings } x \text{ in } w: P \in D_{et}\}$ and $\{\lambda w x \text{ brings a chocolate cake in } w: x \in D_e\}$, respectively. Comparing the prejacent to alternatives of the first type according to the lexical rule in (324) renders sentence (325) true. In contrast, (326) which is associated with second type of alternatives, is correctly predicted false.

To conclude, Villalta’s revision of Heim’s semantics for desire predicates is successful in capturing their context dependence and introduces a mechanism to deal with their focus sensitivity. In the next subsection, I make the same kind of revision to the semantics of anankastic modals.

3.4.3.2. CONTEXTUAL RESTRICTION

Recall that my proposal for the meaning of ‘have to’ is to treat it as expressing necessity in view of a totally realistic ordering.

(315) $\llbracket \text{have to} \rrbracket(p)(q)$ is true relative to totally realistic ordering source $g(w)$ and empty modal base $f(w)$ iff
 $\forall w' \in \cap(f(w) \cup \{q\} \cup \{\text{GOAL}(p, q)\}): \text{Max}_{g(w)}(w') \rightarrow w' \in p$

To implement Villalta-style revision, i.e. to make ‘have to’ sensitive to the contextual alternatives to p , I propose to reformulate the lexical rule in (315), by using the relation of comparative possibility, an alternative way to express the truth conditions of a counterfactual conditional formulated in (Lewis, 1973c). The definition of comparative possibility relation from

(Kratzer, 1981), which I give below, is equivalent to that given in (Lewis, 1973c). For convenience, I stick to Kratzer's formulation.

- (327) A proposition p is more possible than a proposition q in a world w in view of a modal base f and an ordering source g iff
- a. For all $u \in f(w)$ such that $u \in q$ there is a world $v \in f(w)$ such that $v \leq_{g(w)} u$ and $v \in p$.
 - b. There is a world $u \in f(w)$ such that $u \in p$ and there is no world $v \in f(w)$ such that $v \in q$ and $v \leq_{g(w)} u$.

Lewis shows that this modal relation may be used to formulate the truth conditions of a conditional sentence with an antecedent k and consequent r in the following way.²⁶

- (328) A proposition that k and r is more possible than a proposition that k and not r in a world w in view of a modal base f and an ordering source g .

Since Kratzer's truth conditions of counterfactual conditionals are equivalent to Lewis's, see (Lewis, 1981) for the proof, we may safely replace the truth conditions for anankastic conditionals adopted above by the reformulation in (328). This leads us to the following lexical rule for 'have to'.

- (329) $\llbracket \text{have to} \rrbracket(p)(q)$ is true in w iff
 [that p & q] is more possible than [that p & not q] in w
 in view of a totally realistic $g(w)$ and an empty $f(w)$.

According to this entry, 'have to' compares possibilities of two propositions relative to what is the case in the actual world. More specifically, it relates the possibility of the conjunction of its complement with the antecedent to the possibility of the conjunction of its complement with the negation of the antecedent.

26. More precisely, the reformulation that Lewis proposes is equivalent to the original version of the truth conditions of counterfactuals without the Limit Assumption, not the the simplified meaning of necessity I introduced above.

Following Villalta, I assume that the truth of an anankastic conditional is sensitive to a set of contextually determined alternatives. I build Villalta-style revision into the semantics of ‘have to’ as follows:

- (330) $\llbracket \text{have to}_C \rrbracket^h(w)(p)(q)$ iff for all $k \in h(C)$ such that $k \neq q$ [that $p \ \& \ q$] is more possible than [that $p \ \& \ k$] in w in view of a totally realistic $g(w)$ and an empty $f(w)$.

Let us turn back to examples (318) – (319) with focussed constituents, where this revision is crucial.

(318) To meet Schmidt, Müller has to drive to [Hamburg]_F.

(319) To meet Schmidt, Müller has to [drive]_F to Hamburg.

Abstracting away from the details of association with focus, let us assume that the alternatives on ‘have to’ correspond to the focus semantic value of the prejacent in the sense of (Rooth, 1985). The focus semantic value of a constituent is a set of meanings matching the semantic type of this constituent, formed by abstracting over the value of the focussed element.²⁷

In our examples, the focussed semantic values of the prejacent of ‘have to’ in (318) – (319) are roughly the following sets of propositions:

(331) $\{\lambda w \exists e: \text{drive}(e)(w) \wedge \text{agent}(e)(M)(w) \wedge \text{goal}(e)(x)(w): x \in D_c\}$

(332) $\{\lambda w \exists e: P(e)(w) \wedge \text{agent}(e)(M)(w) \wedge \text{goal}(e)(\text{Hamburg})(w): P \in D_{ct}\}$

Let these sets be the contents of the contextual restriction on ‘have to’ in the two cases at hand. Given the revised rule in (330), we derive the following truth conditions for (318) and (319):

27. Another way to form alternative sets is to pick set-type elements from tuples of structured propositions in a structured meaning approach to focus, see (von Stechow, 1985).

- (333) For all k , such that $k \in \{\text{that Müller drives to } x: x \in D_e\}$ & $k \neq$ that Müller drives to Hamburg
 [that Müller drives to Hamburg and Müller meets Schmidt] is more possible than [that k and Müller meets Schmidt] in w in view of $g(w)$ and $f(w)$
- (334) For all k , such that $k \in \{\text{that Müller Ps to Hamburg: } P \in D_{et}\}$ & $k \neq$ that Müller drives to Hamburg
 [that Müller drives to Hamburg and Müller meets Schmidt] is more possible than [that k and Müller meets Schmidt] in w in view of $g(w)$ and $f(w)$

According to (333), (318) is true iff the fact that Müller drives to Hamburg and meets Schmidt is a better possibility than the fact that he drives to a different location and meets Schmidt. This condition is not met in scenario (317) repeated below. Driving to Stuttgart appears to be as good a possibility as driving to Hamburg.

- (317) Müller has to negotiate with Schmidt. He can meet Schmidt in Hamburg or Stuttgart. He may only drive or travel by train. He can get to Stuttgart by train, but there are no trains to Hamburg that depart early enough for him to make it to the meeting.

On the other hand, the truth condition in (334) is met in the scenario above, given that driving to Hamburg for the purpose of meeting Schmidt is a better possibility than getting to Hamburg by any other relevant transportation means. Sentence (319) is thus correctly predicted to be true.

To conclude, I showed here that the focus sensitivity of ‘have to’ can be accounted for by restricting its comparison class to the focus alternatives. According to the revised rule, ‘have to’ compares the possibility of the prejacent, given the goal expressed by the antecedent, to the possibility of the focus alternatives to the prejacent, given the same goal.

3.4.4 SUFFICIENCY INFERENCE

I now turn to the main goal of this section, namely deriving MinR of anankastic modals in scalar contexts, i.e. in the SMC and in anankastic conditionals featuring a degree-denoting expression in the consequent.

3.4.4.1. MINR WITH A NUMERAL

Sentence (335) below implies that 90 points is the score minimally sufficient for winning. Let me show that the analysis of anankastic conditionals I proposed in the previous sections derives this reading.

(335) In order to win, Sean has to get 90 points.

Recall that ‘have to’ compares propositions according to their relative possibility at the index world and the comparison class is provided by the contextual variable it carries. Under the assumption that ‘have to’ takes the proposition expressed by the ‘in order to’-clause and the embedded proposition as its two arguments, the LF for example (335) looks as follows:

(336) [have to_C [Sean win]] [Sean gets 90 points]²⁸

Let me provide a scenario in which (335) should come out true and check whether the current analysis makes correct predictions.

(337) Sean is taking part in a final race of a championship. Given his score so far, 90 points are sufficient for him to win the championship. John wants to spare his car and win without a big lead.

First, we need to decide on the value of the comparison class *C*. The form of contextual alternatives is determined by the context of utterance and, more concretely, by the relevant question under discussion. The latter may but need not be specified by means of intonational focus. If present, a scalar term, ‘90 points’ in the example at hand, determines the QUD without being prosodically prominent, see (Krifka, 1999). Most naturally, example (335) is taken to answer the following question.

(338) How many points does Sean have to score in order to win?

It is therefore plausible to assume that the comparison class value on the

28. For a more detailed analysis of the prejacent see section 3.4.5.1. For the present discussion, it suffices to know that I assume an ‘exactly’ interpretation of the numeral.

modal looks as follows.

$$(339) \quad g(C) \subseteq \{\lambda w \text{ Sean gets } d \text{ points in } w: d \in D_d\}$$

With this value fixed, we may compute the extension of our example in scenario (337). For convenience I repeat the lexical meaning of ‘have to’.

$$(330) \quad \text{For an empty modal base } f \text{ and a totally realistic ordering source } g: \\ \llbracket \text{have to}_c \rrbracket^h(w)(p)(q) \text{ iff for all } k \in h(C) \text{ such that } k \neq q \\ \text{[that } p \text{ \& } q] \text{ is more possible than [that } p \text{ \& } k] \text{ in } w \text{ in view of } g(w) \\ \text{and } f(w)$$

By applying this entry, we assign the following extension to (335):

$$(340) \quad \text{For any proposition } k \in \{\lambda w \text{ Sean gets } d \text{ points in } w: d \in D_d\} \text{ \&} \\ k \neq \lambda w \text{ Sean gets 90 points in } w: \\ \llbracket \lambda w \text{ Sean wins in } w \text{ and Sean gets 90 points in } w \rrbracket \text{ is more possible} \\ \text{in view of } g(w) \text{ than } \llbracket \lambda w \text{ Sean wins in } w \text{ \& } k(w) \rrbracket$$

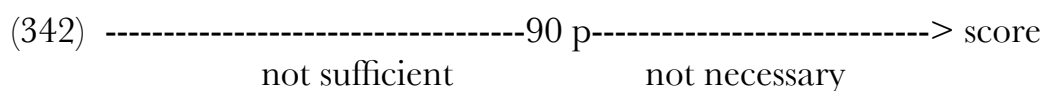
Given scenario (337), the ordering source contains at least the following two facts:

$$(341) \quad g(w) = \{\text{that Sean chooses the easiest means to achieve his goal;} \\ \text{that the minimal score sufficient for winning is 90 points, ...}\}$$

To see whether the truth conditions in (340) are met, we compare Sean’s getting 90 points and winning to Sean’s getting more or less than 90 points and winning. Taking into account the fact that 90 points is the minimal requirement, Sean’s winning by getting less than 90 points is much more remote than his winning by getting 90 points. Likewise, winning with a score greater than 90 points is a more remote possibility than winning with 90 points, given the fact that it requires more effort on Sean’s part. Therefore, (340) is fulfilled and sentence (335) is correctly predicted true.

Let me now explain the source of MinR in more general terms. It comes about in contexts which make salient an effort scale measuring difficulty or cost of actions leading to the achievement of the relevant goal. Quite often this scale is mapped to the numerical scale associated with a degree-denoting expression. In the example at hand, the greater the score is, the

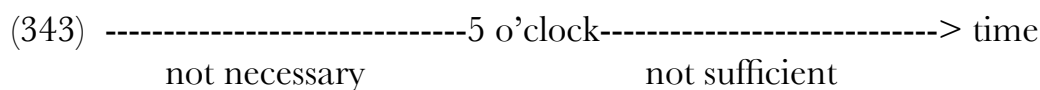
more difficult it is to get it. Normally, we add the general preference for the easiest way to achieve one's goals to the factual background that enters the totally realistic ordering source associated with 'have to'. Therefore the relative possibility of a proposition is partially determined by the measure of effort it involves. This has the effect that the prejacent of 'have to' is understood to be the easiest means leading to the goal. Turning back to example (335), by virtue of being the easiest way to win, scoring 90 points renders all easier means, i.e. scoring less than 90 points insufficient and all more difficult means, i.e. scoring more than 90 points, unnecessary, as schematised below:



Note that we do not simply use a numerical scale to estimate comparative possibility of propositions, but associate it with an effort scale. An effort scale and the numerical scale may in fact point in different directions. This is the case in example (273), repeated below:

(273) You have to get up at 5 o'clock to catch the first train.

Given our world knowledge, the earlier one gets up the more effort is involved. Therefore, getting up at 5 o'clock performs the following partition on its alternatives:



As a result we understand (273) as conveying that getting up after 5 o'clock is insufficient and getting up before that time is unnecessary.

To sum up, MinR results from mapping a lexical scale, that is, a scale associated with a gradable predicate or a numeral, to an effort scale which, if made prominent in a context, is crucial for comparing different ways of achieving a goal on a possibility scale contributed by an anankastic modal. As such, sufficiency inference amounts to understanding the proposition

expressed by the complement of the modal as the easiest way to achieve the goal expressed by the ‘in order to’-clause.

Naturally, we may deal with a context in which no mapping of the lexical scale to the effort scale and, as a result, to the possibility scale occurs. In such context MinR does not obtain. Suppose, (335) is uttered in the following context.

(344) Sean is playing darts. Given his hits so far, 90 points would reduce his score to zero and he would win.

Here, the numerical scale does not correspond to the effort scale as in the racing scenario. Scoring 90 points is not understood to be easier than scoring 100 points. Rather, comparative possibility is determined by precision. This explains the absence of the inference that 90 points is minimally sufficient.

3.4.4.2. MINR UNDER ‘ONLY’

The second MinR environment is the sufficiency modal construction (SMC), exemplified by the following sentence.

(345) In order to win, Sean only has to get 90 points.

This sentence conveys that getting 90 points is a sufficient and easy way to win. If we compare (345) to the prejacent of ‘only’ in (335) repeated below, only the ‘easiness’ meaning component is the new information contributed by ‘only’. The sufficiency inference is already associated with the prejacent. In this subsection I elaborate the analysis of (345) and, in particular, the treatment of ‘only’, in order to show how this result is derived.

(335) In order to win, Sean has to get 90 points.

The standard analysis of ‘only’ goes back to (Horn, 1969) and draws on the idea that the meaning of ‘only’ comprises two components: a negative assertion and a positive presupposition. For instance, sentence (346) asserts that I did not see any person different from Gemma and presupposes

that I saw Gemma. More formally, on the truth conditional level, ‘only’ applies to its prejacent [λw I saw Gemma in w] and excludes all relevant propositional alternatives to it, see (347). The form of the alternatives is determined by the focus structure of the prejacent.

(346) I saw only Gemma.

(347) $\llbracket \text{only}_C \rrbracket^g(w)(\lambda w \text{ I saw Gemma in } w) =$
 $\forall q \in g(C): q \neq \lambda w \text{ I saw Gemma in } w \rightarrow \neg q(w)$

There is another use of ‘only’, which is called ‘scalar’. (Beaver, 2004) gives the following example.

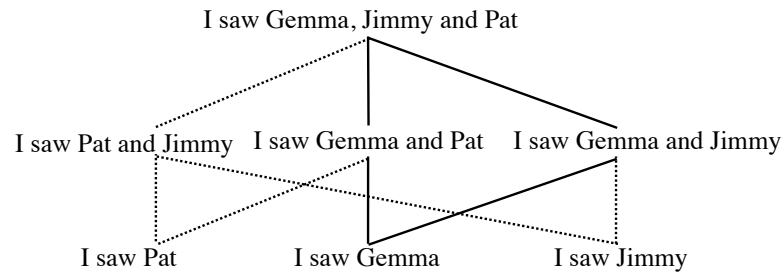
(348) Jane is only a lieutenant.

On its natural interpretation, (348) asserts that Jane has no higher rank than that of lieutenant. The excluded alternatives are therefore propositions of the form [λw Jane is an X in w], where X is a higher rank than lieutenant. To derive this interpretation, we need to assume that ‘only’ induces an ordering on the propositions in C and then uses this ordering to exclude alternatives, as captured by the entry in (349).

(349) $\llbracket \text{only}_{C, R} \rrbracket^g(w)(p) = \forall q \in g(C): q >_{g(R)} p \rightarrow \neg q(w),$
 where $g(R)$ is a contextually given salient linear order on propositions and C is the field of $g(R)$.

It is an open issue whether the scalar use is just a particular instance of the contribution of ‘only’ or both cases can get a unified account. (Beaver, 2004) suggests that we could treat the apparently non-scalar uses of ‘only’ as a special case of the scalar ‘only’ with the logical entailment as the relevant ordering. For example, for (346) one could consider the following partial ordering.

(350)



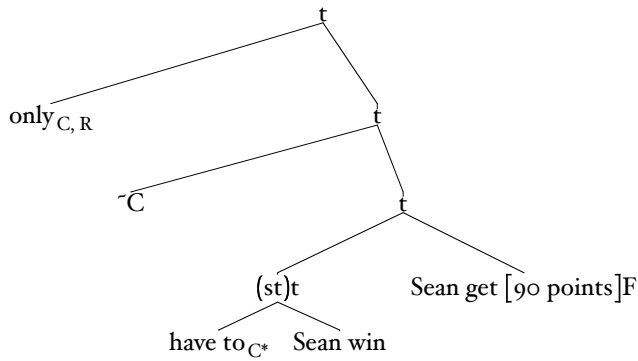
Under the assumption that a property is true of a group if it is true of its parts, $[\lambda w \text{ I saw Gemma in } w]$ is logically entailed by $[\lambda w \text{ I saw Gemma and Jimmy in } w]$, $[\lambda w \text{ I saw Gemma and Pat in } w]$, $[\lambda w \text{ I saw Gemma, Jimmy and Pat in } w]$. Excluding these alternatives, as outlined in (351), results in the extension equivalent to that of (347), namely, that I saw nobody but Gemma in w .

$$(351) \llbracket \text{only}_C \rrbracket^g(w)(\lambda w \text{ I saw Gemma in } w) = \forall q \in g(C): q \subset \lambda w \text{ I saw Gemma in } w \rightarrow \neg q(w)$$

I suggest that ‘only’ in the SMC is best characterised as a scalar operator. Its prejacent is an anankastic conditional for the evaluation of which the context makes salient an effort scale. ‘Only’ uses that scale to rank the alternatives to its prejacent.

I follow (Rooth, 1985) in assuming that the alternatives of ‘only’ are constrained to be a subset of the focus semantic value of the prejacent. To implement this, the \sim operator is attached to the prejacent. The \sim operator introduces a variable whose value is constrained to be a subset of the focus semantic value of the constituent that \sim adjoins to. This variable provides an antecedent for the alternative set variable restricting ‘only’. The LF for (345) is the following:

(352)



By default, the focus is on the degree term and the value of C has the following form:

$$(353) \quad g(C) \subseteq \{ \lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ Sean win in } w) (\lambda w \text{ SCORE}_w(\text{Sean}) = d) : d \in D_d \}$$

Parallel to Beaver's example in (348), a contextually inferred scale is used to induce an ordering on the propositions in $g(C)$. In the former case, the context provides an ordering of military ranks; in the latter case, it provides an effort scale for ranking propositions, compare (354) and (355).

$$(354) \quad [\lambda w \text{ Jane is an } X \text{ in } w] >_{g(R)} [\lambda w \text{ Jane is an } X' \text{ in } w], \\ \text{iff } X \text{ is a higher military rank than } X'.$$

$$(355) \quad [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ Sean win in } w)(q)] >_{g(R)} \\ [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ Sean win in } w)(q')], \\ \text{iff } q \text{ involves more effort than } q'.$$

If we assume that the higher the score is, the more effort is involved, the ordering of the alternative propositions in $g(C)$, in the example at hand, looks as follows:

$$(356) \quad \dots \\ [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ S win in } w) (\lambda w \text{ SCORE}_w(S) = 100 \text{ p})] >_{g(R)} \\ [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ S win in } w) (\lambda w \text{ SCORE}_w(S) = 90 \text{ p})] >_{g(R)} \\ [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w) (\lambda w \text{ S win in } w) (\lambda w \text{ SCORE}_w(S) = 80 \text{ p})] >_{g(R)} \\ \dots$$

With the values of C and R fixed like that, applying ‘only’ to its prejacent, using the lexical entry in (349), leads to the following truth conditions.

$$(357) \lambda w \forall q \in g(C): \\ q >_R [\lambda w \llbracket \text{have to}_{C^*} \rrbracket^g(w)(\lambda w S \text{ win in } w)(\lambda w \text{ SCORE}_w(S) = 90 \text{ p})] \rightarrow \\ \neg q(w)$$

According to these truth conditions, any alternative of the form ‘Sean has to get n points to win’ that involves more effort than the prejacent is false. Given that more effort corresponds to a higher score, which is reflected in the value of R, we may simplify the derived meaning as follows:

$$(358) \lambda w \forall n > 90 \text{ points}: \\ \neg \llbracket \text{have to}_{C^*} \rrbracket^g(w)(\lambda w S \text{ win in } w)(\lambda w \text{ SCORE}_w(S) = n \text{ points})$$

Finally, feeding in the lexical meaning of ‘have to’ into (358) derives the following result:

$$(359) \forall n > 90 \text{ points}: \text{ that Sean wins and gets 90 points is not more possible in view of a totally realistic } g(w) \text{ than that Sean wins and gets } n \text{ points.}$$

I make the standard assumption that ‘only’ presupposes the truth of its prejacent, i.e. it is only defined if condition (360) is met.

$$(360) \llbracket \text{have to}_{C^*} \rrbracket^g(w)(\lambda w S \text{ win in } w)(\lambda w \text{ SCORE}_w(S) = 90 \text{ p}) = \\ \forall n \neq 90: \text{ that Sean wins and gets 90 points is more possible in view of a totally realistic ordering than that Sean wins and gets } n \text{ points.}$$

The prejacent in (360) already implies that getting more than 90 points is not necessary. As a result, the semantic contribution of ‘only’ in (359) comes out to be part of its presupposition. I take this to be a welcome prediction. Compare our example to its prejacent repeated below.

$$(345) \text{ In order to win, Sean only has to get 90 points.}$$

$$(335) \text{ In order to win, Sean has to get 90 points.}$$

Both the SMC sentence in (345) and its prejacent in (335) convey that getting 90 points is sufficient. The difference between them is that the

former additionally implies that getting 90 points counts as an easy way to win. This implication is due to the contribution of ‘only,’ which ranks the prejacent low on the effort scale. The ‘diminishing’ function of ‘only’ in SMC is discussed by (von Stechow and Iatridou, 2007), who propose that the easiness meaning component is an implicature characteristic of exclusive particles. The common property of such expressions is to pick as their prejacent an element which is ranked low on a relevant scale, cf. Beaver’s example in (348). Following (von Stechow and Iatridou, 2007), I assume that in the SMC the sufficient action is marked as easy by an implicature associated with the use of an exclusive particle.

In conclusion, I want to address an issue which caused much debate in connection with the treatment of SMC. Consider von Stechow and Iatridou’s paradigmatic example in (361).

(361) To get good cheese, you only have to go to the North End.

Von Stechow and Iatridou observe that (361) does not presuppose that going to the North End is a necessary condition for getting good cheese. The sentence can be uttered in a situation in which one can buy good cheese in other places as well. They call the fallacy of the inference in (362) the Prejacent Problem and try to solve it by weakening the semantics of the SMC.

(362) You only have to go to the North End.
~ You have to go to the North End.

To see the source of the Prejacent Problem, it is instructive to look at a scalar case. Compare the inference in (362) to the one in (363).

(363) Sean only has to get 90 points.
~ Sean has to get 90 points.

If the prejacent contains a degree-denoting expression, the inference pattern goes through. This finds a straightforward explanation under the present analysis. Let us consider the interpretation of (364), given in (365).

(364) To get good cheese, you have to go to the North End.

(365) $\lambda w \forall k \in g(C) \& k \neq \lambda w$ you go to the North End: [λw you go to the North End in w and you get good cheese in w] is more possible in view of a totally realistic $g(w)$ than [λw you get good cheese in w & $k(w)$]

Most plausibly, $g(C)$ contains propositions of the form [λw you go to x], where x is a prominent location in the given context. The context may also provide an effort scale ranking alternatives as to how much effort they involve, say, depending on how far away they are from the current location of the addressee. For example, consider the following scenario.

(366) The big cheese shop in the North End is the only place in Boston where you can get good cheese. Unfortunately, the supermarkets next to your house do not sell good cheese. Another possibility to get good cheese is to go to a cheese shop in the next town, which is far away and you are in a hurry.

In such a context, getting good cheese in the North End is indeed the best possibility in view of what is the case and (364) is judged true. If the context does not rank alternatives on an effort scale but they are viewed as equally easy ways of getting good cheese, (364) comes out false. Consider a kind of context discussed in (von Stechow and Iatridou, 2007).

(367) There are several supermarkets in the Boston area where you can buy good cheese. You can also go to the North End cheese shop in Boston which sells good cheese as well.

In this case the only factor that determines the possibility of a proposition is the availability of good cheese. Every supermarket in the Boston area is as good a possibility as the North End cheese shop from this perspective. Therefore condition (365) is not met. Getting good cheese in the North End is not the best possibility.

Thus, if an anankastic conditional does not contain a degree-denoting expression, the ranking of alternatives is not necessarily given in the context. However, such a ranking has to be prominent for the scalar ‘only’ in

an SMC to infer a scale for the exclusion of its alternatives. Note that the SMC (361) is not felicitous in the ‘unranked alternatives’ scenario in (367). The Prejacent Problem arises when we use a context like (367) to evaluate the prejacent of ‘only’. To conclude, the inference in (362) is as valid as the one in (363), as long as the SMC and the anankastic conditional expressing its prejacent are evaluated with respect to the same ranked set of alternatives.

3.4.5 MINR IN COMPARATIVES

3.4.5.1. MINR IN COMPARATIVE COMPLEMENTS

In the following, I extend the analysis developed above to comparatives with intensional complements, like (368).

(368) Sean got more points than he had to in order to win.

Let us consider this example in the scenario of the previous section repeated below:

(337) Sean is taking part in a final race of a championship. Given his score so far, 90 points are sufficient for him to win the championship. John wants to spare his car and win without a big lead.

In this scenario, (368) conveys that Sean scored more than minimally required, i.e. Sean's actual score is compared to his score in a world in which he won by having scored what was minimally sufficient. My goal is to show that the analysis of anankastic conditionals I adopted above predicts this reading.

I assume that the embedded clause of (368) undergoes the process of comparative deletion and the elided material is reconstructed at the level of interpretation. Furthermore, the degree argument position of the gradable 'many' is abstracted over as a result of the movement of the wh-operator originating in that position. The skeleton of the structure of the 'than'-clause is given in (369).

(369) λd he had to get d many points in order to win

Let me now elaborate on the analysis of 'many'. Following (Hackl, 2009), I treat 'many' as a gradable predicate of plural individuals, see (370), with the requirement that its degree argument is a cardinality measure.

(370) $\llbracket \text{many} \rrbracket = \lambda w \lambda d \lambda x$ x has d -many atomic parts in w

The comparative complement has the following structure.

(371) λd [have to_C [he win]] [$\exists x$ [x [d many] points]] [he get x]]

The analysis of the main clause is parallel, the only difference being that that degree argument slot of ‘many’ is occupied by the comparative morpheme:

(372) $\exists d \exists x$ [d -er than-clause] [[x [d many] points]] [Sean get x]]

To provide ‘-er’ with the standard-of-comparison degree, the degree property expressed by the ‘than’-clause is coerced into a definite description by applying to it the definite term DEF.

Let us compute the meaning of the entire sentence. We start with the property of degrees expressed by the comparative complement. The preja-cent of the embedded ‘have to’ has the following meaning.

(373) $\lambda w \exists!x$: Sean get x in w & x are points in w and x has d-many atomic parts in w

I use $\exists!x$ instead of a simple existential quantifier to guarantee an ‘exactly’ meaning of the degree term. I remain neutral as to how this interpretation obtains exactly, see (Geurts, 2006) for a range of technical alternatives. For the sake of exposition, I use the following abbreviation instead of (373).

(374) $\lambda w \text{SCORE}_w(\text{Sean}) = 90$

The degree argument of ‘many’ is bound by the lambda abstractor and, as a result, we obtain a property of degrees. I again assume that alternatives in C have the form [$\lambda w' \text{SCORE}_{w'}(\text{Sean}) = n$], i.e. they vary in the degree variable.

(375) $\lambda w \lambda d$ [[have to_C]](w)($\lambda w' S$ win in w')($\lambda w' \text{SCORE}_{w'}(S) = d$) =
 $\lambda w \lambda d \forall k \in \{\lambda w' \text{SCORE}_{w'}(S) = n : n \in D_d\}$ & $k \neq \lambda w' \text{SCORE}_{w'}(S) = d$:
 [$\lambda w' \text{Sean wins in } w'$ and $\text{SCORE}_{w'}(\text{Sean}) = d$] is more possible in
 view of a totally realistic $g(w)$ than [$\lambda w' \text{Sean wins in } w' \ \& \ k(w')$]

In the scenario at hand, the only degree that falls under this description is the one corresponding to 90 points, for this is the score that is most likely to

lead to Sean’s victory, given Sean’s current score and his preference for as little effort as possible. Therefore the extension of (375) in the given scenario is a singleton set containing the score of 90 points. This set can be coerced to its unique element by the application of DEF. Thus, the standard-of-comparison degree, which the comparative receives as its first argument, is a degree description referring to the minimal score sufficient for Sean to achieve his goal. I abbreviate it as ‘90 points’.

Applying the comparative to the ‘than’-clause gives us the property which is true of a degree if it exceeds 90 points. This is the restriction of the comparative indefinite. Combining it with the rest of the matrix clause, results in a comparison of Sean’s actual score the minimally required score.

(376) $\lambda w \exists d \exists !x$: Sean get x in w & x are points in w and x has d -many atomic parts in w & $d > 90$ points

I have shown that given an adequate semantic analysis of ‘have to’, MinR of comparatives with embedded anankastic conditionals, i.e. the so called more-than-minimum reading, does not present a challenge for an analysis of comparative constructions. The analysis I laid out relies on standard assumptions concerning the contribution of the comparative morpheme and the denotation of ‘than’-clauses. In particular, I analyse the comparative morpheme as expressing ‘>’ relation between two degrees. ‘Than’-clauses are treated as properties of degrees coerced into degree descriptions by the application of an implicit definite term.

3.4.5.2. MINR IN MAIN CLAUSES OF COMPARATIVES

Another MinR environment are two kinds of intensional comparatives discussed in (Heim, 2001) – ‘less’-comparatives and comparatives with ‘exactly’ differentials. Consider first an example of a ‘less’ comparative given in (377)a. Paraphrases in (377)b and (377)c correspond to the two readings of this sentence. R1 in (377)b is a MinR. On this reading the sentence conveys that Sean’s minimally sufficient score is under 100 points, that is, it

is smaller than Jimmy's score. R2 in (377)c does not imply anything about the minimally sufficient score.

- (377) a. (Jimmy scored 100 points.)
 To win, Sean has to get fewer points than that.
- b. Sean is not required to get as much as that to win. (R1)
- c. Sean can only win if he gets fewer than 100 points. (R2)

I suggest that these distinct interpretations result from two potential alternative values for the contextual variable on the modal. Let me elaborate this proposal. I assume the following LF for (377)a.

- (378) [have to_C [S. win]] [∃x∃d [d -er 100p] [x [few d] points] [S. get x]]

The lexical entry of 'few' is parallel to that of its positive pole counterpart 'many'. The only difference is in the direction of the scales they employ to order their degree arguments.

- (379) a. [[many]] = $\lambda w \lambda d \in D_{>many} \lambda x$ x has d atomic parts in w
- b. [[few]] = $\lambda w \lambda d \in D_{>few} \lambda x$ x has d atomic parts in w

We first compute the meaning of the prejacent of 'have to'.

- (380) $\lambda w \exists d \exists ! x$ [[-er]](w)(100 points)(d) & [[few]](w)(d)(x) & [[points]](w)(x) & [[Sean get]](w)(x) =

$\lambda w \exists d \exists ! x$: $d >_{few} 100$ points & x has d atomic parts in w & x are points in w & Sean gets x in w =

$\lambda w \exists d \exists ! x$: $d < 100$ points & x has d atomic parts in w & x are points in w & Sean gets x in w =

$\lambda w \text{SCORE}_w(\text{Sean}) < 100$ points

Applying 'have to' to its prejacent results in the following proposition.

- (381) $\lambda w \forall k \in g(C) \& k \neq \lambda w \text{SCORE}_w(\text{Sean}) < 100$ points:
 [[$\lambda w'$ Sean wins in w' and $\text{SCORE}_{w'}(\text{Sean}) < 100$ points]] is more possible in view of a totally realistic $g(w)$ than [[$\lambda w'$ Sean wins in w' & $k(w')$]]

To compute the extension, we need to fix the value of C. One candidate would be a set of propositions that vary in the place of the comparative relation ‘fewer’, see (383). Such set is an appropriate value for C, if the focus falls on ‘fewer’, which, in turn, is the case if the relevant question under discussion is, for example, (382).

(382) How many points does Sean has to get in order to win, compared to Jimmy?

(383) $g(C) = \{\lambda w \exists d \exists x! : x \text{ has } d \text{ atomic parts in } w \ \& \ d < 100 \text{ points} \ \& \ x \text{ are points in } w \ \& \ \text{Sean gets } x \text{ in } w;$
 $\lambda w \exists d \exists x! : x \text{ has } d \text{ atomic parts in } w \ \& \ d \geq 100 \text{ points} \ \& \ x \text{ are points in } w \ \& \ \text{Sean gets } x \text{ in } w\}$

Given this restriction set and the meaning of the prejacent in (381), the sentence is predicted true iff Sean’s winning with a score that is fewer than 100 points is more possible, in view of what is the case, than his winning with 100 or more points, see (384). These truth conditions corresponds to R2 in (377)c, that is, on this reading (377)a conveys that getting fewer than 100 points is a necessary condition for winning.

(384) $[\lambda w' \text{ Sean wins in } w' \ \& \ \text{SCORE}_{w'}(\text{Sean}) < 100]$ is more possible in view of a totally realistic $g(w)$ than $[\lambda w' \text{ Sean wins in } w' \ \& \ \text{SCORE}_{w'}(\text{Sean}) \geq 100]$ (R2)

Another relevant question under discussion could be a question inquiring after the minimally required score, for example:

(385) How many points does Sean has to get in order to win?

To reflect this kind of topic, the alternatives need to vary in the position of the degree term corresponding to Sean’s score. To achieve this, the degree argument of ‘few’ is left free at the level of the prejacent and the existential closure over degrees is introduced above the modal, as shown in (386). The alternative set can now be assigned the value in (387).

(386) $[\exists d \text{ have to}_C [\text{S. win}]] [\exists x [d \text{ -er } 100p]] [x [\text{few } d] \text{ points}] [\text{S. get } x]]$

(387) $g(C) = \{\lambda w \exists x!: x \text{ has } d \text{ atomic parts in } w \ \& \ d < 100 \text{ points} \ \& \ x \text{ are points in } w \ \& \ \text{Sean gets } x \text{ in } w: d \in D_d\}$

Assuming this alternative set, the extension of (377)a is the following: that Sean wins with a score d which is below 100 points is a better possibility given the facts than that he wins with a score d' different from d which is also below 100 points, see (388).

(388) $\exists d \forall d' \in D_d: d' \neq d \rightarrow [\lambda w' S \text{ wins in } w' \text{ and } \text{SCORE}_{w'}(S) = d \wedge d < 100 \text{ points}] \text{ is more possible in view of a totally realistic } g(w) \text{ than } [\lambda w' S \text{ wins in } w' \ \& \ \text{SCORE}_{w'}(S) = d' \wedge d' < 100 \text{ points}]$ (R1)

Given Sean's intention to win with the minimum effort. It is additionally asserted to be smaller than 100 points, the necessary score is understood to be minimally sufficient. Why is this so? (388) is analogous to the truth conditions in (340) of the sentence (287), which I repeat below.

(287) To win, Sean has to score 90 points.

(340) For any proposition $k \in \{\lambda w \text{ Sean gets } d \text{ points in } w: d \in D_d\}$ & $k \neq \lambda w \text{ Sean gets 90 points in } w:$
 $[\lambda w \text{ Sean wins in } w \text{ and Sean gets 90 points in } w]$ is more possible in view of $g(w)$ than $[\lambda w \text{ Sean wins in } w \ \& \ k(w)]$

The reasoning in (340) that scoring 90 points is the easiest and therefore the minimally sufficient way for Sean to win may equally well apply in (388), where d is understood to be minimally sufficient. The following informal paraphrases for the derived truth conditions of these two sentences highlight the parallel.

(389) a. Sean's minimally sufficient score is 90 points.
 b. There is a degree d , s.t. Sean's minimally sufficient score is d & d is less than 100 points. (R1)

The paraphrase in (389)b may suggest that the derived truth conditions are too weak, allowing for the sentence to be true in a situation in which it is equally possible that Sean wins with a score greater than 100.

Note, that according to (388), we compare scores below 100 and one of them results in the best possibility. Nothing prevents a proposition of the form ‘that Sean wins with a score greater than 100 points’ to be an even better possibility.²⁹ However, since the possibility scale corresponds to the reversed numerical scale in the given context, the range of possibilities corresponding to the scores over 100 points are lower than those below 100 points. They are excluded from being better possibilities due to the monotonicity built into the semantics of the modal. Put informally, if we scan the lower part of the scale and pin down the minimally sufficient degree, we do not have to check the upper span to make sure that it is really minimally sufficient.

Let us turn to ambiguous comparatives with ‘exactly’ differentials. An example is given in (390)a. R1 and R2 are two possible interpretations of this sentence. R1 corresponds to SuffR.

- (390) a. (Pat scored 80 points.) In order to win, Sean has to get exactly 10 points more than that.
- b. The minimal score Sean is required to get is exactly 90 points.
(R1)
- c. Sean can only win if he gets exactly 90 points. (R2)

Before we look at the derivation of these readings, let me comment on the contribution of ‘exactly’. I adopt a pragmatic approach to the interpretation of ‘exactly’ in the spirit of (Lasersohn, 1999), where it is assumed to be a so called pragmatic slack regulator. ‘Exactly’, similarly to some other expressions of its kind, e.g. ‘all,’ ‘perfectly’ etc., signals how much pragmatically licensed deviation from truth is allowed for practical purposes. To analyse slack regulators, Lasersohn develops a notion of pragmatic halos. Pragmatic halos are sets approximating the denotation of an expression computed on the basis of a recursive set of rules. They are fixed in part

29. This objection has been raised by Sigrid Beck, p.c.

pragmatically and in part structurally through the use of slack regulators. From a semantic point of view, ‘exactly’ is an identity function on degrees. In (390)a it contracts the pragmatic halo associated with a degree corresponding to 10 points. Depending on a context, it could be a set containing 10 points and, say, another two close values, viz. {9p, 10p, 11p}. After the application of ‘exactly’ the semantic value of ‘10 points’ does not change, while the pragmatic halo set contracts to include only the center of the original halo, i.e. {10p}. For an implementation, I refer the reader to (Laser-
sohn, 1999).

Note, that the use of ‘exactly’ may influence what counts as a better possibility in a context and therefore have an effect on the truth conditions. Consider example (390)a. There are two possibilities for how the possibility scale may look like. On the one hand, it may be determined by the effort scale. In this case, if the measure of effort increases with each additional point, then the possibility scale is the inverse of the score scale. On the other hand, the possibility may be determined by precision, which is plausible in the presence of ‘exactly’. Then the best score is the most precise score. Bearing these two alternatives in mind, let us look at the analysis of (390)a. Abstracting away from the details of the treatment of differential measure phrases, the following truth conditions are derived in the current analysis.

$$(391) \quad \lambda w \exists d \forall k \in g(C) \ \& \ k \neq [\lambda w \text{ SCORE}_{w'}(S) = d \wedge d > 80 \text{ p} \wedge \text{DIFF}(d, 80\text{p}) = 10\text{p}]: [\lambda w' S \text{ wins in } w' \text{ and } \text{SCORE}_{w'}(S) = d \wedge d > 80 \text{ p} \wedge \text{DIFF}(d, 80\text{p}) = 10\text{p}] \text{ is more possible in view of a totally realistic } g(w) \text{ than } [\lambda w' S \text{ wins in } w' \ \& \ k(w')]^{30}$$

It is plausible that the numeral in the differential phrase is the focus of the utterance. Consequently, the set of alternatives to the prejacent takes the

30. The degree variable in the prejacent of ‘have to’ can be bound by an existential closure either within the scope of the modal or higher up, nothing hinges on the this choice here.

following shape:

$$(392) \quad g(C) = \{\lambda_w \text{ SCORE}_w(S) = d \wedge \text{DIFF}(d, 80p) = d' : d' \in D_d\}$$

The truth conditions in (391) are met iff Sean's winning with a score which is exactly 10 points greater than Pat's score is a better possibility in view of the facts than his winning with a score which is not exactly 10 points greater than Pat's. To verify this, we need to know what counts as most possible. If it is precision that counts most, the sentence conveys that getting exactly 90 points is the best possibility given the goal without implying that it is a minimal requirement. Getting more than 90 points is considered a remote possibility for the same reason that getting less than 90 points is. The effort involved is not taken into consideration, rather precision. This interpretation corresponds to reading (390)c. Alternatively, if getting a high score counts as difficult, the sentence conveys that getting exactly 90 points is the best possibility, given the goal, implying that it is minimally sufficient. In that case, getting more than 90 points is understood to be a remote possibility due to the unnecessary effort it involves. This is a reading paraphrased in (390)b.³¹

In this section, I have shown that ambiguities discussed in (Heim, 2001) can be accounted for without resorting to a scoping strategy. *MinR*, which is a hallmark of anankastic modals in scalar contexts, constitutes the first set of readings. The other set of readings surfaces if some contextual conditions for *MinR* are not met. In particular, *MinR* is blocked if the possibility scale cannot be mapped to the degree scale associated with the gradable predicate. In 'less' comparatives this happens if the content of the implicit alternative set contains just the prejacent and its negation. In comparatives with 'exactly' differentials the use of 'exactly' usually hinders

31. The same result is derived if the alternatives vary in the position of the degree variable corresponding to Sean's score. In that case, the existential closure operation has to apply above the modal.

the mapping of an effort scale to a relevant degree scale by introducing a different criterion for measuring possibility, namely precision.

3.4.6 ALTERNATIVES

Much of the discussion in this section revolved around the notion of contextual alternatives. The present account of MinR hinges on restricting the relevant modal by an appropriate set of alternative propositions. I proposed that that set is determined by the information structure in the complement of the modal. The alternatives are compared by the modal on a possibility scale. In this section I assess some consequences of this assumption.

I argued that a focussed element in the prejacent that influences the form of alternatives on the modal is crucial for determining the extent of similarity to the ideal. The focussed element may turn out to be a scalar item, i.e. an element that lexically encodes its membership in a scale, viz. measure phrases and items forming Horn scales. For example, if a numeral is focussed, the numerical scale is employed for measuring comparative possibility of propositions that are sufficient for the achievement of the goal. For concreteness, consider example (287), repeated below. The possibility of a proposition of the form [that Sean gets n points and achieves his goal to win] in w depends on the value of n . If 90 points are minimally sufficient the possibility decreases with any value under 90.

(287) To win, Sean has to score 90 points.

The lexically provided scale by virtue of corresponding the effort scale may contribute an additional criterion for deciding what is more possible: possibility decreases with an increase in effort. In (287), the effort scale is mapped to a numerical scale. Consider an example of a non-numerical scale.

(393) Pat has to correct [SOME]_F of the homework.

The focus on ‘some’, forming a Horn scale with ‘all’, generates an effort scale which renders the proposition that Pat fulfils his duties and corrects some of the homework a better possibility than the proposition that Pat fulfils his duties and corrects all of the homework. This derives the MinR that correcting some of the homework is sufficient.

The prediction of my proposal is that position of the focus and more generally the topic/focus structure of the prejacent is crucial for the availability of MinR. If the degree term in the scope of an anankastic modal is prominent, the effort scale is likely to be prominent in the context, too, and is mapped to the scale that degree term is associated with. Otherwise, the effort scale may not be regarded at all for comparing propositions on a possibility scale. Consider the variant of (393) with the focus on the subject.

(394) [PAT]_F has to correct some of the homework.

Shifting the focus to ‘Pat’ prevents, or at least hinders, the mapping of the possibility scale to the effort scale. In this case, we expect ‘have to’ to compare the alternatives of the form ‘X corrects some of the homework’ on the possibility scale. It is difficult to come up with a context in which one such alternative could be judged to be an easier means for achieving the goal than another.

A scale may also be inferred from the focussed element pragmatically. For example, in von Stechow and Iatridou’s example in (361), repeated below, the North End is compared to other more or less distant places of getting good cheese. We may infer a scale in (395).

(361) To get good cheese, you only have to go to the North End.

(395) the nearest shop $<_R$ the North End $<_R$ Italy,
 where $<_R$ is closer to the location of the utterance.

If a scale has to be inferred from the context and this process not facilitated by the presence of ‘only’, MinR is not as readily available. Recall that this is the main trigger for the putative Prejacent Problem discussed in section

3.4.4.2. If the prejacent of ‘only’ in (361) is evaluated in a context that does not make salient an effort scale, there appears to be a conflict between the assertion and the presupposition of (361). From the assertion we infer sufficiency of going to the North End due to the contribution ‘only’ that introduces an effort scale. However, we need not infer this from the presupposed prejacent of ‘only’. On MinR, the prejacent entails that going to the North End is minimally sufficient for getting good cheese, it does not exclude the possibility of getting good cheese in other places. On non-MinR, it entails that one can only get good cheese if one goes to the North End. Only on the MinR do the inferences of the assertion and the presupposition match.

In the presence of a scalar item, MinR is more prominent than non-MinR, unless the focus does not fall on this item. As a result, the Prejacent problem does not arise in scalar contexts. There is no question that (345), repeated below, presupposes (287).

(345) In order to win, Sean only has to get 90 points.

Though the focus on a scalar expression results in the kind of alternatives that lead to MinR, I suggested that there may be pragmatic factors at work, blocking MinR in scalar environments. In particular, this happens when the use of ‘exactly’ prevents mapping the degree scale to the effort scale. For example, (396) may be uttered in a context in which it is precision that counts most, which is made salient by the use of ‘exactly’. In such a context, the effort scale cannot correspond to a numerical scale and the sentence does not receive MinR.

(396) Sean has to get exactly 90 points in order to win.

‘Exactly’ is more compatible with a non-MinR context. Put informally, it tends to switch off the dependence of the measure of effort on the degree scale. However, this can be done without ‘exactly’, too. Recall, the darts scenario in (344), repeated below. In section 3.6, I will come back the

pragmatic factors that impede MinR and discuss them from a broader perspective.

- (344) Sean is playing darts. Given his hits so far, 90 points would reduce his score to zero and he would win.

To conclude, MinR of an anankastic modal hinges on the kind of alternatives the modal picks to compare its complement to. Crucially, the alternatives have to be ranked on an effort scale. MinR is therefore facilitated by the use of scalar items in the complement and it is less prominent in non-scalar environments.

3.4.7 CONCLUSION

I presented an analysis of anankastic conditionals that has the following ingredients. First, the antecedent of an anankastic conditional is the restricting ‘in order to’-clause. Accessible worlds quantified over by an anankastic modal are restricted to those in which the proposition expressed by the ‘in order to’ clause is asserted to be true and a goal of the action expressed by the consequent.

Second, anankastic modals express comparative possibility in the sense of (Lewis, 1973c) and (Kratzer, 1981). They compare propositions according to how remote they are from what is the case in the world of evaluation. As such, they are associated with a totally realistic ordering source.

Third, the truth conditions of an anankastic conditional depend on the information structure of the consequent. An anankastic modals is restricted by an implicit contextual variable containing alternatives to its prejacent whose value is determined pragmatically and can be influenced by the intonational focus in the prejacent. This accounts for the focus sensitivity of anankastic modals.

I have shown that an analysis based on these assumptions predicts the availability of MinR in scalar contexts. The special property of scalar contexts responsible for MinR is the prominence of an effort scale ranking al-

ternative propositions that constitute the comparison class for an anankastic modal according to how difficult they are. Provided that the comparative possibility ordering is totally realistic, being less difficult amounts to being a better possibility. As a result, the prejacent of an anankastic modal is understood to be the easiest way or the minimal requirement for the achievement of the relevant goal.

3.5 WEAK ANANKASTIC MODALS

The aim of this section is to explain why *MinR* is not possible with all kinds of anankastic modals. In section 3.3.2, I pointed out that there is a variety of anankastic modals, the so-called weak modals, that have a number of distinctive properties setting them apart from other goal-oriented necessity modals. Among those properties are their apparently weaker modal force, the inability to scope below negation and the lack of *MinR*. Weak anankastic (henceforth *WA*) modals are marked by subjunctive morphology.

To account for the specific features of *WA* conditionals I suggest to treat them as involving two layers of modality. Here is the gist of my proposal:

1. *WA* conditionals involve two universal modal layers: a higher bouletic modal and a lower anankastic one.
2. The modal auxiliary in a *WA* conditional is a spell out of an anankastic modal. It carries an uninterpretable feature *SUBJUNCTIVE* which is licensed by a corresponding interpretable feature on a higher abstract attitude predicate.

The section is structured as follows. In section 3.5.1 I work out my proposal for German *WA* conditionals with ‘*sollten*’. Section 3.5.2 is concerned with the role of subjunctive. In sections 3.5.3 – 3.5.4 I show that my proposal is successful in predicting the unavailability of *MinR* in weak anankastic conditionals.

3.5.1 TWO MODAL LAYERS

(von Stechow *et al.*, 2006) suggest that *WA* conditionals are associated with two kinds of conversational backgrounds, one of them relating to wishes, preferences or recommendations of the speaker or some other person. Building on their proposal, I treat *WA* conditionals as involving two modal layers. A *WA* conditional is analysed as an anankastic conditional embedded under a bouletic attitude.

To elaborate the proposal, I suggest to consider the case of German WA conditional with ‘sollten’ discussed above. I assume that the modal predicate ‘sollten’ spells out two modal operators. The lower anankastic modal is represented as ANANKE and the higher bouletic modal as BOUL. In line (397), I give the skeleton of the LF for example (255), repeated below.

(255) Umdeiner Mutter zu helfen,solltest du das Geschirr abspülen.
 for your mother to help soll-2psyou the dishes wash
 ‘To help your mother you should wash the dishes.’

(397) BOUL [[ANANKE_C [you help your mother]] you wash the dishes]

While ANANKE is responsible for the anankastic modality and is analysed as the anankastic ‘have to’, the contribution of BOUL should be explicated in more detail. The first issue I want to address in this connection is the kind of conversational background BOUL is associated with. For this purpose, it is worth while considering the interpretation of the non-subjunctive forms of modal auxiliaries involved in WA conditionals. (Kratzer, 1981) observes that ‘sollen’ is generally associated with a bouletic ordering source which is based on someone’s wishes or preferences.

(398) “‘Sollen’ expresses necessity. It requires an ordering source corresponding to what is good, planned or recommended, or by what someone wants, plans or recommends.”, (Kratzer, 1981)

She gives a number of examples with a bouletic modal flavour which I cite below.

- (399) a. Ein Richard Wagner Festspielhaus sollte nach den Entwürfen des Architekten Semper gebaut werden.
(In view of the plans of King Ludwig II of Bavaria, a Richard Wagner festival hall was to be built after the designs of the architect Semper.)
- b. Ich bitt’ euch gar schön, der hochwürdige Herr Pfarrer soll kommen.
(Gauzner Michl is dying. In view of what he wants, a priest must come.)
- c. Sechs Tage soltu erbeiten und alle deine Werck thun.
(In view of what God wants, it is necessary that you work six days a week.)

- d. Ich soll ein Bäcker werden.
(In view of what my parents want, it is necessary that I become a baker.)

Turning to the subjunctive forms in WA conditionals, they seem to inherit the bouletic modal flavour of their non-subjunctive pro-forms. Consider the following example with ‘sollten’.

- (400) Er sollte schöne Zähne, muss aber keinen durchtrainierten Körper haben, um Erfolg zu haben.
(In view of somebody’s wishes, he should have beautiful teeth, but he doesn’t need a well-trained physique in order to have a success.)³²

This parallel between WA conditionals and their non-subjunctive non-anankastic counterparts can be captured in the present nested modality approach if BOUL is treated in the spirit of Kratzer’s analysis of ‘sollen’. In effect, BOUL is a necessity modal requiring a circumstantial modal base and a bouletic ordering source. The modal base marks as accessible those worlds which are compatible with the relevant circumstances. The ordering source then selects among them those worlds that make as many wishes from some ideal true as possible. The meaning rule for BOUL in (401) is based on the notion of human necessity from (Kratzer, 1981), see (402).

- (401) $\llbracket \text{BOUL} \rrbracket(w)(p) = 1$ iff p is a human necessity relative to $g(w)$ and $f(w)$, where $g(w)$ is a bouletic ordering source and $f(w)$ is a circumstantial modal base.

- (402) Human Necessity
 A proposition p is a human necessity in a world w with respect to a modal base f and an ordering source g iff
 for all $u \in \cap f(w)$, there is a $v \in \cap f(w)$ such that
- a. $v <_{g(w)} u$
 - b. For all $z \in \cap f(w)$: If $z <_{g(w)} v$, then $z \in p$.

The definition of human necessity does not presuppose the existence of the closest worlds. As Kratzer notes, assuming that they exist would simplify it

32. <http://www.news.at/articles/0625/610/143563/sommerflirt-er-zachne-koerper>

considerably. It could then be paraphrased as follows: a proposition is a human necessity iff it is true in all worlds from the modal base that come closest to the ideal represented by the ordering source. What kind of propositions could be part of that ideal? Turning to example (255), we can assume that $g(w)$ on BOUL contains the addressee's mother wishes, i.e. $g(w) = \{p: p \text{ is compatible with what the addressee's mother wishes in } w\}$.

Let us now consider the impact of embedding ANANKE under BOUL. According to the analysis of anankastic conditionals I adopted above, the complement of BOUL expresses the following proposition:

- (403) $\lambda w \forall k \in g(C) \ \& \ k \neq [\lambda w \text{ you wash the dishes in } w]:$
 $[\lambda w \text{ you help your mother in } w \text{ and you wash the dishes in } w]$ is
 more possible in view of a totally realistic $g(w)$ than $[\lambda w \text{ you help}$
 your mother in $w \ \& \ k(w)]$

This anankastic conditional is embedded under BOUL and as a result evaluated in worlds that are closest to the ideal represented by the addressee's mother's wishes. The meaning of the entire weak conditional is therefore the following.

- (404) λw For all circumstantially accessible worlds w' that come closest
 to what your mother wishes in w :
 $\forall k \in g(C) \ \& \ k \neq \lambda w \text{ you wash the dishes in } w:$
 $[\lambda w \text{ you help your mother in } w \text{ and you wash the dishes in } w]$ is
 more possible in view of a totally realistic $g(w')$ than $[\lambda w \text{ you help}$
 your mother $\ \& \ k(w)]$

This analysis predicts the apparent weakening of the modal force in weak anankastic conditionals. Consider the pair of sentences in (259), repeated below. Recall that while (259)a is false in the scenario at hand, (259)b is undoubtedly true.

- (259) Imagine that to cross Siberia to go to Vladivostok you can take one
 of two trains: the Russian train or the Chinese train. The Chinese
 train is significantly more comfortable.
- a. To go to Vladivostok, you have to take the Chinese train.

- b. To go to Vladivostok, you ought to take the Chinese train.

Under the present account, ANANKE quantifies over a set of worlds that come closest to the ideal determined by the higher bouletic modal BOUL. This set is different from the quantification domain of the anankastic ‘have to’ in example (259)a, where the actual world serves as an ideal. Hence, the following discourse is not contradictory:

- (405) To go to Vladivostok, you ought to take the Chinese train but you don’t have to.

For concreteness sake, consider the truth conditions for this example.

- (406) In view of what is the case in the worlds compatible with the speaker’s recommendations, that you take the Chinese train and get to Vladivostok is more possible than that you don’t take the Chinese train and get there. But in view of what is the case in the actual world, it is not more possible.

This is a consistent statement which is met in the scenario in (259).

To sum up, a WA conditional is analysed as an anankastic conditional embedded under an additional modal layer, which shifts the interpretation of the underlying anankastic modal to a different index. This accounts for the notorious weakening effect associated with WA conditionals.

In this subsection, we looked at the specific case of German WA conditionals with ‘sollten’. I suggested that the overt modal predicate is a spell out of an anankastic modal embedded under a bouletic attitude. Presumably, the higher modal in a WA conditional does not have to be bouletic, but may have some other kind of modal flavour. An option to have different kinds of modal interpretation on top of the anankastic ANANKE might explain the difference in the interpretation of weak conditionals with ‘should’ and ‘ought to’ in English or ‘sollten’ and ‘müssten’ in German. The latter pair of modal auxiliaries is discussed in (Kratzer, 1981), where it is pointed out that they involve different kinds of ordering sources.

3.5.2 THE ROLE OF SUBJUNCTIVE

In this section, I briefly consider the contribution of subjunctive morphology to the meaning of WA conditionals. As before, I stick to the case of German and refrain from making more general claims.

(von Stechow and Grønn, 2009) develop a morphological feature system in which subjunctive morphology on a verb stem is treated as an uninterpretable feature [uSub] which needs to be licensed by a corresponding interpretable feature, [iSub], carried by some element higher up in the structure. Drawing on the fact that German subjunctive forms do not occur in a matrix context, he proposes that the licenser of subjunctive should be an attitude predicate. For example, in example (407) below *glauben*/'believe' carries [iSub] and thus licenses the present and past subjunctive forms 'sei' and 'wäre' in its complement.

- (407) Hans glaubt, dass Marie krank sei/wäre.
Hans believe that Marie ill be-SUBJ.PRES/be-SUBJ.PAST
'Hans believes that Marie is ill.'

In view of this theory, it is plausible to assume that the subjunctive morphology on a WA modal also corresponds to an uninterpretable feature which must be licensed by an attitude. In the present set up, this boils down to treating ANANKE as the holder of [uSub] and BOUL as its licenser carrying [iSub], as demonstrated below for the German example in (255).

- (408) BOUL_[iSub][[ANANKE_{C[uSub]}✓[you help your mother]] you wash the dishes]

The abstract representation provided here is admittedly a simplification. However, I take it to be sufficiently explicit for the present purposes. A more thorough investigation into the form of WA modals is called for. It is however beyond the scope of this thesis. What we need to take from this discussion is that subjunctive morphology in WA conditionals reflects the

presence of an abstract attitude predicate above the anankastic modal. In German, the requirement of a higher attitude predicate falls out from the fact that propositional attitudes are capable of licensing uninterpretable subjunctive morphology.

3.5.3 NEGATION

The nested modality approach endorsed here makes a welcome prediction regarding the behaviour of WA conditionals under negation. To see this, let us consider introducing a negation above BOUL.

(409) NEG[BOUL[[ANANKE_C [you help your mother]] you wash the dishes]]

This LF corresponds to the following interpretation.

(410) λw Not for all circumstantially accessible worlds w' that come closest to what your mother wishes in w :
 $\forall k \in g(C) \ \& \ k \neq [\lambda w$ you wash the dishes in w]:
 $[\lambda w$ you help your mother in w and you wash the dishes in w] is more possible in view of a totally realistic $g(w')$ than
 $[\lambda w$ you help your mother in w & $k(w)$]

The truth conditions in (410) are met iff in at least some ideal world the embedded anankastic conditional is false, i.e. in some desirable worlds the fact that the addressee helps her mother by washing dishes is not a better possibility than the fact that she helps her mother by some other salient means. It turns out that this condition cannot be met, given the basic assumptions of the theory of modals I rely on. Let us see why this is so.

(410) could only be met, if the worlds coming closest to the bouletic ideal were assigned different degrees on the similarity scale, i.e. if a proposition could be a better possibility relative to one of them than to another. However, by virtue of representing the ideal with respect to which the embedded ANANKE is evaluated, they have to be assigned the same degree on the similarity scale. For each such ideal world w there is no world that co-

mes closer to the bouletic ideal than w itself.³³ In other words, the ideal has to be centered and no world in it can be different from the others with respect to how possible some proposition is in that world.

A negated WA conditional is incompatible with this centering constraint, which, I suggest, is the reason for the absence of the wide scope reading of negation. This explains away the unavailable reading but does not yet derive the available narrow scope reading of these sentences. Intuitively, (411) conveys that, in view of what the addressee's mother wishes, she would help her mother if she didn't wash the dishes.

- (411) Um deiner Mutter zu helfen, solltest du das Geschirr nicht abspülen.
 'To help your mother you should not wash the dishes.'

The truth conditions corresponding to the position of negation below ANANKE capture this intuition.

- (412) λw For all circumstantially accessible worlds w' that come closest to what your mother wishes in w :
 $\forall k \in g(C) \ \& \ k \neq [\lambda w$ you do not wash the dishes in $w]$:
 $[\lambda w$ you help your mother in w and you do not wash the dishes in $w]$ is more possible in view of a totally realistic $g(w')$ than
 $[\lambda w$ you help your mother in $w \ \& \ k(w)]$

To see how the low-scope-of-negation reading arises, it is instructive to look at the semantic/pragmatic theories of Neg-Raising. The common core of these theories is that Neg-Raising predicates, like 'should', are associated with the so-called 'excluded middle' (EM) assumption, see (Gajewski, 2007) for a recent literature overview. For example, (Bartsch, 1973) proposes that some propositional attitudes trigger the EM presupposition, which has a dramatic effect under negation. Let me briefly explain how this account

33. Lewis calls this requirement 'centering', cf. (Lewis, 1981), p. 221:

(i) w does, and all other worlds do not, belong to every proposition in $f(w)$.

goes. I represent the semantic meaning of a WA conditional in (413)a below. The EM presupposition is given in (413)b.

- (413) a. Assertion: For all desirable worlds w : p is true in w .
 b. Presupposition: For all desirable worlds w : p is true in w or for all desirable worlds w : p is false in w .

The negation of the assertion and the excluded middle presupposition imply the low-scope-of-negation reading:

- (414) For all desirable words w : p is false in w .

In the present set up, the EM presupposition can be seen as a filter excluding all those contexts in which the centering constraint cannot be met. There are two ways to implement this idea. One can assume, in the spirit of pragmatic theories of Neg-Raising, that the low scope of negation in WA conditionals is a pragmatic effect. It is a result of discarding the situations in which the wide scope reading of negation is true, in order to comply with the centering constraint. This would, however, predict that (411) has what we may call the middle-scope reading of negation, which is intuitively unavailable.

- (415) λw For all circumstantially accessible worlds w' that come closest to what your mother wishes in w :
 $\forall k \in g(C) \ \& \ k \neq [\lambda w$ you do not wash the dishes in $w]$:
 $[\lambda w$ you help your mother in w and you wash the dishes in $w]$ is no more possible in view of a totally realistic $g(w')$ than
 $[\lambda w$ you help your mother in $w \ \& \ k(w)]$

Another way to go is to assume that the centering constraint blocks the wide-scope reading and forces negation to be interpreted on the complement of ANANKE, as sketched below. It would have to be stipulated that negation cannot land between the two modal layers for some syntactic reason. I leave this issue for future research.

- (416) BOUL[[ANANKE_C [you help your mother]] NEG[you wash the dishes]]

The upshot of this section is that embedding an anankastic conditional under an attitude blocks the wide scope of negation relative to the modal. The wide-scope-of-negation reading cannot arise because it is incompatible with the centering constraint requiring that the worlds representing the ideal relative to which the embedded anankastic conditional is evaluated be assigned the same degree of similarity. As a result, negation is interpreted on the prejacent of the modal.

3.5.4 ABSENCE OF MINR

Last but not the least, the present analysis offers an explanation for the lack of MinR in WA conditionals. Recall that MinR hinges on an anankastic interpretation, it however does not surface in any anankastic conditional. The presence of subjunctive morphology blocks MinR. Let us see on an example why this happens. Consider the minimal pair below.

(335) In order to win, Sean has to get 90 points.

(417) In order to win, Sean ought to get 90 points.

I argued that the crucial factor for the availability of MinR in (335) is a totally realistic ordering which ‘have to’ is associated with. The ideal that the modal uses for ordering its propositional alternatives contains the fact that one picks the easiest, cheapest, fastest means to achieve one’s goal. By default, this consideration is employed for ranking propositions according to how remote they are from the actual world. For example, in the context below, the effort scale corresponding to the score scale determines proximity to the actual world. As a result, the necessary condition expressed by an anankastic conditional is always understood as a minimally sufficient one in a scalar context.

(337) Sean is taking part in a final race of a championship. Given his score so far, 90 points are sufficient for him to win the championship. John wants to spare his car and win without a big lead.

In a WA conditional, facts of the actual world are not taken into account

for the evaluation of the embedded ANANKE. The ideal is represented by what is good, planned, recommended, wished for or, possibly, dictated by some rules, laws and moral norms. This is a consequence of the presence of the licensing attitude predicate BOUL. As a result, being low on an effort scale does not necessarily rank a proposition as a good possibility relative to an ideal. In example (417), winning by 90 points is not conveyed to be a better possibility in view of what is considered easy in the actual world so as to be appropriate in the scenario in (337), rather it is understood to be a better possibility in view of some other, bouletic ideal. The context, however, does not make any such ordering salient for the use of ‘ought to’ to be felicitous. This explains why (417) sounds odd in the given scenario.

Suppose now that (417) is uttered by Sean’s coach who for some reason prefers Sean to have a considerable lead over his rivals. BOUL may be assumed to order accessible worlds relative to how close they come to Sean’s coach’s preferences. The best worlds do not have to be compatible with the fact that Sean chooses the easiest means to achieve his goals. Therefore, the embedded ANANKE does not use the effort scale for measuring closeness to the ideal. What counts as close to the ideal in that case is winning with a big advantage. Winning by getting 90 points is understood as the best possibility, by virtue of securing Sean a big advantage over his rivals rather than by virtue of being the easiest way to win. By this reasoning, the necessary condition conveyed by a WA conditional is not understood to be a minimally sufficient one.

To conclude, the nested modality approach accounts for three distinctive properties of WA conditionals mentioned at the outset of this chapter. Under this analysis, anankastic modals in such conditionals are evaluated in the scope of an abstract propositional attitude predicate, i.e relative to a bouletic or deontic ideal, not the ideal represented by the facts of the actual world. This shift from the actual world to a different index blocks MinR

and forces negation to be interpreted low in the structure. It also accounts for the apparent weaker modal force of WA conditionals.

3.6 DISTRIBUTION OF MINR

I suggested that MinR is pragmatically rather than semantically anchored. Essentially, it arises when the context of utterance provides an effort scale which can be used for ranking propositions on the possibility scale and, importantly, can be mapped to the lexical scale associated with the gradable predicate. This section is concerned with how the context can influence the availability of MinR. I discuss some interfering pragmatic factors which have already been touched upon in the previous sections. The effort scale may not play a role in estimating the relative possibility of a proposition. This happens if the mapping of a lexical scale to an effort scale is hindered by what counts as difficult in the context or it is not specified what counts as difficult.

The effort scale need not be mapped to the degree scale. There are different factors that may affect this mapping. Most notably, it hinges on the kind of propositional alternatives the modal chooses to compare and therefore on the focus structure of its prejacent. Specifically, the alternatives need to vary in the degree argument of the gradable predicate for MinR to arise. If they do not, we usually deal with a context in which the effort scale is not prominent.

For example, if in Heim's example in (73), repeated below, the focus is on 'less', it conveys that meeting the requirement by writing a paper shorter than 10 pages is a better possibility than by writing a paper as long as 10 pages. This interpretation can only arise in a context in which 'less than 10 pages' is for some reason better than 'no less than 10 pages' in view of the relevant goal. Apparently, such contexts do not make prominent an effort scale as the measure of comparative possibility.

- (73) (The draft is ten pages.) The paper is required to be less long than that.

Another example of an effort scale playing no role are sentences with ‘exactly’. I repeat the relevant example from (Heim, 2001) below.

- (43) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

In this case, is not the form of alternatives that switches off the effort scale. On its prominent reading, the sentence conveys that if the paper is exactly 15 pages long, it fares better, given the requirements, than if it has a length different from 15 pages. The use of ‘exactly’ signals that the difficulty is not in writing more. Therefore, we do not use the effort scale to compare meeting the requirement by writing a 15 pages long paper to meeting the requirement by writing a paper of a different length. There is no need to employ the effort scale to decide which of the lengths is more possible. ‘More possible’ means ‘more precise’ in this context, so anything different from 15 pages is not as good a possibility.

The special focus pattern on the preajacent and the use of lexical items like ‘exactly’ is not necessary for blocking MinR. The context may fail to supply the ranking of alternatives according to the effort they involve altogether. This often happens if there is no lexically provided scale and it is not obvious what constitutes the scale. In the following example, passing a driving test is not immediately understood to be minimally sufficient for getting a driving license. The reason is that we need to come up a context that makes prominent other more and less difficult ways of getting a driving license, which is not given by default.

- (418) To get a driving license, you have to pass a driving test.

This kind of blocking may also occur in a scalar context if there is some criterion different from the effort involved that is used for comparing alternatives on the possibility scale. Mostly, the effort scale is irrelevant if it is precision that counts, as in the darts scenario discussed in section 3.4.4.1.

Finally, the mapping of an effort scale to a possibility scale is hindered in WA conditionals. In such cases, the possibility of a proposition given the goal is determined not relative to the actual world but relative to some bouletic or other kind of ideal, that is, relative to someone's wishes, preferences, expectations, recommendations and so on. It is only natural to expect that considerations of cost and effort need not be taken into account in such worlds.³⁴ As a result, being low on an effort scale does not necessarily rank a proposition as a good possibility. Consider the minimal pair below.

- (419) a. To fulfil your course duties, you have to correct some of the exams.
- b. To fulfil your course duties, you ought to correct some of the exams.

On its natural interpretation, (419)a compares the possibility that the addressee fulfils his/her duty by correcting some of the exams, given the facts, to the possibility that she/he does so by correcting all of the exams or correcting none. In (419)b, a similar comparison may be made but the possibilities are evaluated relative to the preferences/wishes/recommendations of the speaker or some other salient individual, not relative to the facts. Whereas the former kind of comparison may create the inference that correcting some of the exams is the easiest way for the addressee to fulfil his/her course duties, this is unlikely with the second kind of comparison, for which the considerations of time, effort and the like are irrelevant. The addressee rather infers that correcting some of the exams must in some way be an optimal way to satisfy the relevant goal and somebody's preferences. Hence the absence of MinR.

34. They need not be important, but they may play a role, given the appropriate context. Suppose, if someone genuinely wishes you won with the least effort and utters "To win, you should get 90 points." they imply that 90 points is minimally sufficient in their view.

3.7 CONCLUSION

The task of this chapter was to answer the following questions.

- (54) What kind of modals give rise to the minimal requirement reading (MinR)? Why does MinR obtain with these modals and not with others?

The main claims of the section are the following:

1. MinR obtains with anankastic modals. Anankastic modals are necessity modals expressing goal-oriented modality. They are restricted by ‘in order to’-clauses and require a totally realistic ordering source. Since the role of an ‘in order to’-clause is similar to that of an antecedent of a conditional, I call sentences with anankastic modals anankastic conditionals. Anankastic modals are sensitive to the focus structure of their complements. I treat an anankastic modal as comparing possibilities of propositions in the world of evaluation, in the spirit of the analysis for counterfactuals in (Lewis, 1973c) and (Kratzer, 1981).
2. MinR is triggered in scalar contexts which provide an anankastic modal with an effort scale used for ranking the propositions in its comparison class. Given such an ordering the complement of an anankastic modal is understood to be a requirement involving the least effort. This is the essence of MinR.
3. An anankastic modal that carries subjunctive morphology spells out two modal operators: a low anankastic modal and a high propositional attitude predicate. I call such structures WA conditionals. Such conditionals do not have MinR because anankastic modals in them are evaluated with respect to an ordering source different from that used in unembedded anankastic conditionals. The facts of the actual world, in particular, the ranking of propositions on an effort scale, are not taken into account for that kind of ordering.

The most important general outcome of my treatment of anankastic conditionals is that it adequately characterises their properties in scalar contexts. In particular, the analysis I proposed accounts for the sufficiency inference associated with the sufficiency modal construction in which an anankastic modals acts as a prejacent of the scalar ‘only’. My account dismisses the so called Prejacent Problem, which forced some researchers to adopt non-compositional analyses of SMC, cf. (Huitink, 2005), (von Stechow and Iatridou, 2007). Another welcome result of my analysis is that it recognises and successfully deals with focus sensitivity of anankastic modals. Finally, I proposed a novel account of weak anankastic conditionals as nested modal structures. The strength of this proposal is that it spells out the role of the subjunctive morphology and captures a half bouletic, half goal-oriented modal flavour of WA conditionals and their apparently weaker modal force.

The consequences of my proposal for the theory of comparatives are twofold. First, I showed that MinR of anankastic conditionals in intensional comparative complements corresponds to the so-called more-than-minimum interpretation of such comparatives. The present analysis correctly predicts that comparatives with embedded weak anankastic conditionals do not have such a reading. This removes the necessity to apply comparative specific mechanisms, such as introducing an abstract negation in the comparative complement or a type shifter from degrees to degree sets, in order to explain distinct interpretations of comparatives with ‘have to’ and ‘should’ in (243) repeated below:

- (243) a. Sean got more points than he had to in order to win.
b. Sean got more (penalty) points than he should have in order to win.

Most importantly, the treatment I proposed does not face the overgeneration problem of theories treating (243)a and (243)b as cases of structural ambiguity, for example, those developed in (Heim, 2006) and

(Schwarzschild, 2008). It should, however, be noted that (243)b, like most other comparatives with embedded quantifiers, expresses a comparison with the maximum in ‘interval’ scenarios. An account of this fact is beyond the scope of this work. I pointed to some promising proposals in section 2.3.4.4.

Second, MinR of comparatives with anankastic modals in main clauses corresponds to the reading that is derived by scoping the comparative operator above the modal in (Heim, 2001) and is the result of the wide scope interpretation of ‘exactly’ differentials in (Oda, 2008). The relevant constructions and their interpretations are repeated below.

- (43) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.
‘The minimally sufficient length of the paper is 15 pages.’
- (73) (The draft is ten pages.) The paper is required to be less long than that.
‘The minimally sufficient length of the paper is less than ten pages long.’

As in the case of intensional ‘than’-clauses, my analysis has the advantage of explaining why the relevant reading, i.e. MinR, is not possible with a weak anankastic conditional like (242), repeated below.

- (242) (The draft is ten pages long.) The paper should be exactly five pages longer than that.
‘The minimally sufficient length of the paper is 15 pages.’

Thus, the second consequence of my proposal for anankastic modals is that it refutes a weighty argument in favour of a quantificational treatment of the comparative morpheme and also dismisses the need to scope out ‘exactly’ measure phrases in differential comparatives.

Chapter 4. Comparing to What is Possible

This chapter is concerned with ambiguous comparatives containing possibility modals. There are three types of such comparatives whose interpretation calls for an explanation and is the main focus of the following discussion.

The first type is a ‘less’ comparative with a possibility predicate in the main clause, whose ambiguity was first pointed out in (Heim, 2001). Heim’s example is repeated below.

- (74) (The draft is ten pages.) The paper is allowed to be less long than that.

One reading of this sentence says that the paper is not allowed to be as long as the draft. The other reading says that it is possible for the paper to be shorter than the draft, leaving it open whether it is allowed to be as long as the draft.

The second kind of ambiguity was also first addressed in (Heim, 2001). It is observed in a differential comparative like (79) below.

- (79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

The preferable reading of this sentence is a rather weak interpretation conveying that it is allowed for the paper to be 15 pages long. Under some circumstances, (79) may additionally imply that the paper is not allowed to be longer than 15 pages. In the following I refer to the ambiguities in (74) and (79) as Heim ambiguities.

The third construction I am going to consider is known as Rullmann ambiguity.

- (218) The helicopter was flying less high than a plane can fly.

This sentence conveys that the altitude of the helicopter was below the

maximum or the minimum altitude of a plane. Rullmann ambiguity arguably surfaces only in negative pole comparatives with embedded possibility modals.

Throughout the chapter I am guided by the idea, envisioned at the beginning of the thesis, that the weaker readings of the first two constructions are the plain assertions and the stronger interpretations result from a pragmatic enrichment. The same kind of enrichment is at work in Rullmann ambiguity examples. I call the pragmatically enriched meanings ‘extreme’ readings (ExR). In (74) and (79) the extreme is the maximally allowed length of the paper. In (218) the extreme is either set to the maximal or the minimal altitude of a plane. The main questions to be addressed in this section are the following:

- (60) What is the source of ‘extreme’ readings (ExR) in comparatives with possibility modals? Under which conditions is the extreme set to the minimum and the maximum?

I claim that the pragmatic enrichment amounting to ExR is due to Gricean implicatures. An important outcome concerning Rullmann ambiguity is that it is not polarity sensitive. Pragmatic enrichment can target comparatives with negative as well as positive pole predicates. Thus, the two readings of (218) will be argued to have the same source as a more-than-maximum reading of examples like (129).

- (129) Pat ate more than he was allowed to.

The chapter is structured as follows. In section 4.1 I summarise the arguments against treating the three kinds of ambiguities introduced above as structural ones and motivate a non-structural approach. The bulk of this summary is based on the results from chapters 2.2 and 2.4. In section 4.2 I implement a pragmatic approach to the ambiguities. In the remaining part I address two potential problems that such a pragmatic approach faces. The first problem is the dependence of Rullmann ambiguity on negative polarity in some contexts. The second concern is the difference in the

strength of ExR in ‘less’ comparatives as predicted by a scope approach versus by a pragmatic approach.³⁵

35. I am very grateful to Maribel Romero for pointing out this problem to me and the audience at SuB 14 in Vienna for an inspiring discussion following her question.

4.1 AGAINST STRUCTURAL AMBIGUITY

The mainstream approach to the ambiguities under discussion is structural. Its central claim is that each of the three kinds of comparatives discussed above can be assigned two logical forms differing in the position of the comparative morpheme relative some other element in the structure. In section 2.2, I sketched two proposals to account for Heim ambiguities, namely (Heim, 2001) and (Oda, 2008), and the proposal in (Heim, 2007), which builds on the analysis in (Heim, 2001) and extends it to Rullmann ambiguity. In 4.1.1, I sketch the analysis in (Heim, 2007) which, unlike other analyses, is general enough to account for all cases of ambiguity. In the second part of this subsection, I point out the main challenges to a structural ambiguity approach.

4.1.1 HEIM (2007)

In Heim's approach, the ambiguity in sentences with 'less' results from the mobility of 'less' and the availability of two landing sites for it in the presence of intensional predicates. 'Less' spells out the comparative morpheme and the negative element 'little'. Being a degree negation, cf. the lexical entry in (223), 'little' scopally interacts with possibility modals.

$$(223) \llbracket \text{little} \rrbracket = \lambda d \lambda P \neg P(d)$$

Let me demonstrate how this derives the interpretations of (74) and (218) given below.

(74) (The draft is ten pages.) The paper is allowed to be less long than that.

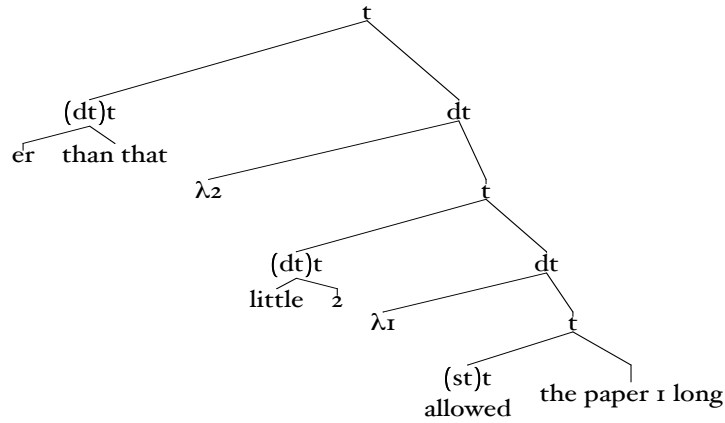
- (76) a. A length of the paper under ten pages is permitted.
b. The paper is only allowed to be shorter than ten pages.

(218) The helicopter was flying less high than a plane can fly.

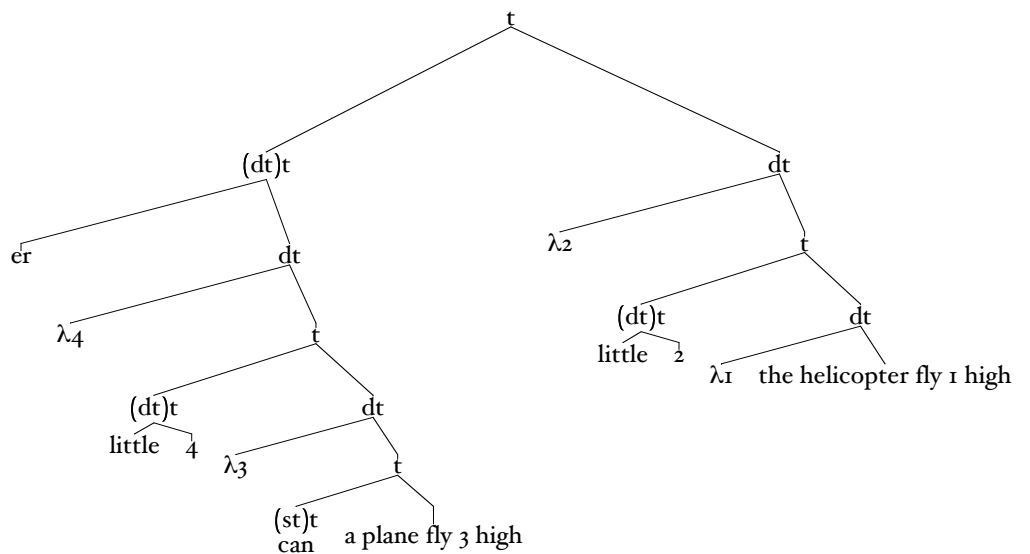
- (420) a. The helicopter was flying below the minimal altitude of a plane.
 b. The helicopter was flying below the maximal altitude of a plane.

If 'little' lands above the modal the readings (76)b and (420)b are derived. The logical forms of (74) and (218) corresponding to this option are given in (421)a and (421)b, respectively. In both cases, a 'little' phrase crosses a modal: in (421)a 'little' ends up above 'allowed' within the matrix clause, while in (421)b the embedded 'little' moves over 'can'.

(421) a.



b.

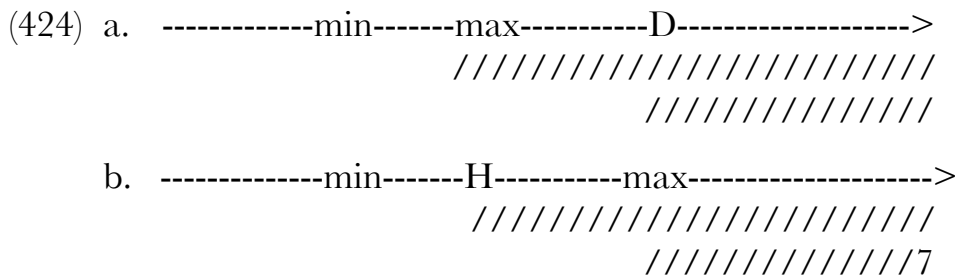


The resulting negated degree sets, which constitute the scope and the restriction of the comparative operator, respectively, correspond to the complements of the original ones.

- (422) a. $\lambda d \notin [\lambda d' \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) \geq d'] =$
 $\lambda d \neg \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) \geq d =$
 $\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) < d$
- b. $\lambda d \notin [\lambda d' \exists w' \in \text{Acc}_w: \text{ALTITUDE}_w(\text{a plane}) \geq d'] =$
 $\lambda d \neg \exists w' \in \text{Acc}_w: \text{ALTITUDE}_w(\text{a plane}) \geq d =$
 $\lambda d \forall w' \in \text{Acc}_w: \text{ALTITUDE}_w(\text{a plane}) < d$

The set in (422)a contains degrees of length that lie above the maximal length across the worlds compliant with the rules and, by analogy, (422)b defines a set of height degrees that exceed the maximal possible altitude of a plane. The comparative operator, defined as a set inclusion, relates those sets to the degree sets stretching upwards from the length of the draft and the altitude of the helicopter, correspondingly. This derives comparison with the maximum readings, cf. (423)a and (423)b, and their schematic representations in (424)a and (424)b, respectively.

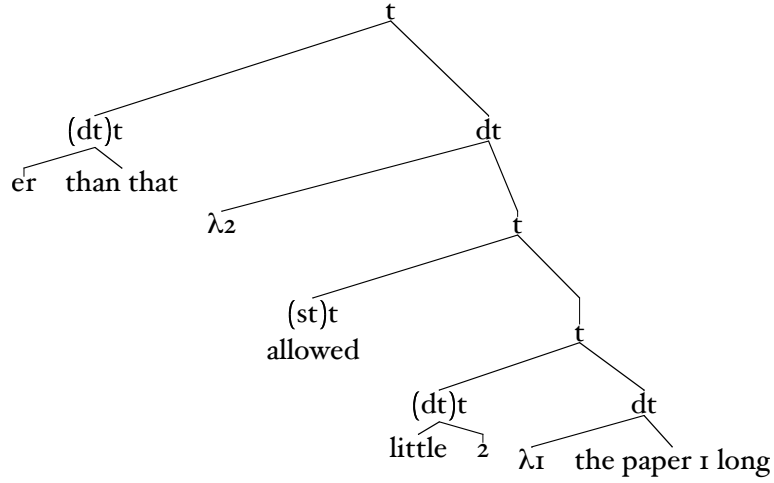
- (423) a. $[\lambda d \text{LENGTH}_w(\text{dr}) < d] \subset [\lambda d \forall w' \in \text{Acc}_w: \text{LENGTH}_w(p) < d]$
 b. $[\lambda d \forall w' \in \text{Acc}_w: \text{ALTITUDE}_w(p) < d] \subset [\lambda d \text{ALTITUDE}_w(h) < d]$



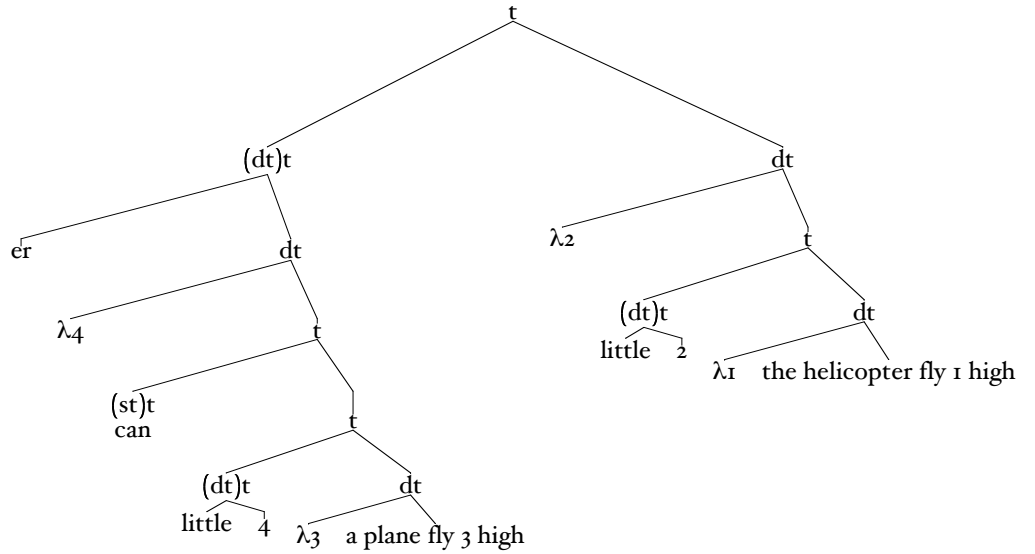
The reading in (424)a says that the maximally allowed length of the paper is below the length of the draft, cf. (76)b. The reading in (424)b says that the altitude of the helicopter is smaller than the maximal possible altitude of a plane, cf. (420)b.

The local movement of ‘little’ phrases produces the other set of readings. The LFs, their interpretations and the corresponding schematic representations are given below:

(425) a.



b.



(426) a. $[\lambda d \text{ LENGTH}_w(d) < d] \subset [\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) < d]$

b. $[\lambda d \exists w' \in \text{Acc}_w: \text{ALTITUDE}_{w'}(p) < d] \subset [\lambda d \text{ ALTITUDE}_w(h) < d]$

- (427) a. -----min-----D-----max----->
 ///////////////////////////////////
 //////////////////////////////////
- b. -----H-----min-----max----->
 ///////////////////////////////////
 //////////////////////////////////

According to (426)a, the minimally allowed length of the paper is below the length of the draft, which corresponds to the weak reading in (76)a. (426)b says that the minimal altitude of a plane is below the actual altitude of the helicopter, which corresponds to Rullmann’s less-than-minimum reading, cf. (420)a.

If there is no degree negation, structural ambiguity can still be derived by assigning the comparative operator wide or narrow scope relative to a possibility modal in the matrix clause. For example, this leads to two distinct interpretation in comparatives with ‘exactly’ differentials. This is a prediction that (Heim, 2007) inherits from (Heim, 2001), where the comparative is treated as a degree operator undergoing QR at the LF.

For example, the two readings of (79), repeated below, are represented as a wide scope of the comparative in (428)a and a narrow scope of the comparative in (428)b, for more details on the analysis of this example see section 2.2.2.

(79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

- (428) a. $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) \geq d) = 15 \text{ pages}$
 ‘The maximally allowed length of the paper is 15 pages.’
- b. $\exists w' \in \text{Acc}_w: \text{MAX}(\lambda d \text{LENGTH}_w(p) \geq d) = 15 \text{ pages}$
 ‘It is possible for the paper to be 15 pages long.’

Heim’s approach is thus capable of accounting for all three cases I listed at the beginning of this section. Heim points out another two advantages of her analysis. First, it uncovers the link between negative polarity and the availability of two readings in Rullmann’s examples. Indeed, the

variant of (218) with a positive pole ‘higher’ in place of ‘less high’, repeated below, is judged unambiguous. It can only compare the altitude of the helicopter with the maximal altitude of a plane and unlike its negative counterpart does not express a comparison with the minimum.

(222) The helicopter was flying higher than a plane can fly.

Second, it can be extended to sentences featuring anankastic necessity modals instead of possibility modals, the kind of examples I handled in section 3.4.5.2. For example, (73) and (78) are reported to have a reading under which the length of the draft is compared to the minimal required length of the paper. They may also compare the length of the draft to the unique required length of the paper.

(73) (The draft is ten pages.) The paper is required to be less long than that.

(78) (The draft is ten pages long.) The paper is required to be exactly five pages longer than that.

To complete the picture, (Heim, 2007) gives an example of Rullmann ambiguity with a necessity modal. Her sentence in (429) may convey that Paul ate less than “the strictures of the diet would have allowed him to eat”, a less-than-maximum reading made prominent by the continuation in (429)a. It may also say that “Paul didn’t eat enough to maintain his weight”, a less-than-minimum interpretation prominent in (429)b. The wide scope of the embedded ‘little’ over ‘necessary’ results in comparison with the minimum, i.e. reading (429)b. The narrow scope of ‘little’ gives rise to the reading in (429)a.

(429) Paul ate less than was necessary.

a. ... to comply with the diet.

b. ... to maintain his weight.

On a closer look, it is not the scope of ‘little’ but much rather the interpretation of ‘necessary’ that is crucial for the ambiguity in (429). Note that

both readings are MinRs insofar as they talk about a degree minimally sufficient for the achievement of a goal, i.e. Paul is said to have done more or less than sufficient for the purpose at hand. ‘In order to’ clauses determine what kind of goal restricts the accessible worlds quantified over by the modal. If the goal is to comply with the diet, the sentence conveys that Paul did more than minimally sufficient by having eaten even less than would have done. If the goal is to maintain one’s weight, the sentence means that Paul did less than sufficient, by having eaten less than would have done.

4.1.2 CHALLENGES TO A SCOPE APPROACH

A scope analysis faces a number of challenges some of which have been already addressed in (Heim, 2007), others have not been discussed yet. In this subsection I list five problems of a scope approach.

First, it appears that Heim ambiguity obtains outside of comparatives, as already pointed out in section 2.2.4. Consider the following examples:

(430) You are allowed to arrive at 10 p.m.

(431) You are allowed to arrive earlier than 10 p.m.

(432) You are allowed to arrive before 10 p.m.

Depending on what kind of information is relevant to the addressee each of these sentences has two different interpretations. They may be answers to the question in (433).

(433) When are we allowed to arrive?

In that case, they report on the latest permissible time of arrival. Thus, sentence (430) conveys that the addressee is allowed to arrive at 10 p.m. at the latest. Sentences (431) and (432) both convey that the latest time of arrival compliant with the rules is before 10 p.m. However, each of these sentences, can make a weaker claim when it is an answer to a relevant alternative question in (434).

(434) a. Are we allowed to arrive at 10 p.m.?

- b. Are we allowed to arrive earlier than 10 p.m.?
- c. Are we allowed to arrive before 10 p.m.?

In this case sentence (430) means that 10 p.m. is a permissible time of arrival leaving it open whether arriving before or after 10 p.m. would violate the regulations. By analogy, sentences (431) and (432) simply communicate that arriving before 10 p.m. is compliant with the regulations. The two readings of example (431) correspond to Heim ambiguity and the ambiguity of this example receives a straightforward account in (Heim, 2007). However, neither (430) nor (432) can be handled by (Heim, 2007), despite the obvious relatedness and probably the same nature of the available interpretations. Neither of them contains a comparative morpheme or any other degree operator whose scope can be manipulated, see section 2.2.4.2 for a discussion of ‘before’ clauses.

Turning to Rullmann ambiguity, it is not restricted to comparatives either. For example, the comparative clause of (435), when unembedded as in (436), has two strong readings. 60 km/h may refer to the maximal speed limit as well as to the minimal one. These interpretations pattern with the less-than-maximum and the less-than-minimum reading of (435).

(435) Michael is driving less fast than allowed on this race track.

(436) It is allowed to drive 60 km/h on this race track.

Similarly, if (433) happens to be a request for the earliest permitted time of arrival, sentence (430) can be used as an answer, implying that the addressee is not allowed to arrive until 10 p.m. Sentences (431) and (432) convey that the earliest permitted time of arrival is before 10 p.m.

The common feature of examples (430) through (432) is the presence of a degree-denoting expression under a certain type of possibility modal. Notably, not any possibility modal gives rise to an ambiguity. This raises a second problem for a scope analysis, already noted in (Heim, 2001) and

(Heim, 2007). Consider the variants of (430) – (432) with the epistemic ‘might’ in place of the deontic ‘be allowed’.

(437) You might arrive at 10 p.m.

(438) You might arrive earlier than 10 p.m.

(439) You might arrive before 10 p.m.

Neither of the examples in (437) – (439) may convey the latest time at which the addressee will possibly arrive in view of the information available to the speaker. They have weaker meanings. Sentence (437) says that the addressee will possibly arrive at 10 p.m., leaving it open whether a later or an earlier arrival is a possibility too. Likewise, (438) and (439) say that the addressee arriving before 10 p.m. is compatible with what the speaker knows. Accounting for the discrepancy between (430) – (432) and (437) – (439) within a scope approach would most likely amount to formulating a restriction on the narrow scope of ‘might’ with respect to ‘-er’, as Heim suggests. ‘Might’ is notorious for its pervasive high scope behaviour. This would leave us with a set of readings which correspond to the wide scope of ‘might’ relative to the comparative, i.e. the weaker readings. However, where exactly such a restriction stems from remains an open issue so far.

The third challenge for a scope analysis is the fact that ExRs in Heim ambiguity are usually judged less natural than weaker ones. Uttered in a context appropriate for this kind of reading, e.g. preceded by wh-question (433), the comparative sentence in (431) is judged degraded by most speakers. It is, however, reported to improve if the modal is preceded by ‘only’ as in example (440).

(440) You are only allowed to arrive earlier than 10 p.m.

The role of ‘only’ remains a mystery under a scope approach, and so does the strong preference for the weak interpretation when (430) is uttered out of the blue.

The fourth challenge faced by a scope approach is the ambiguity of comparatives with negative adjectives, like (221), repeated below. Similar to (218), sentence (221) has a less-than-minimum and a less-than-maximum reading. Since in a scope approach the locus of ambiguity is the position of ‘little’ in the embedded clause, its proponents are forced to extract a component with the meaning of ‘little’ from ‘lower’, see (Heim, 2007), (Heim, 2008), (Büring, 2007a). The discussion of a decompositional treatment of negative antonyms in (Heim, 2008) makes it clear that this creates as many puzzles as it solves problems, see 2.4.2 for more details.

(218) The helicopter was flying less high than a plane can fly.

(221) The helicopter was flying lower than a plane can fly.

The fifth challenge, which is related to the treatment of Rullmann ambiguity, is the availability of empirical evidence showing that this kind of ambiguity does not depend on negative polarity, see section 2.4.3 for a discussion. The assumption that Rullmann ambiguity is restricted to comparatives with negative pole predicates is questioned in (Meier, 2002). I repeat Meier’s example below. Along with a prominent more-than-maximum reading, it is reported to have a more-than-minimum reading, e.g in a situation in which Chuck is a driver of a truck transporting a fragile load.

(233) Chuck is driving faster than he is allowed to drive.

Comparatives with counterfactual possibility modals present somewhat more convincing evidence for the availability of ambiguity with positive pole adjectives. For example, sentence (441) is clearly ambiguous when uttered out of the blue. The context in (441)a highlights a more-than-minimum interpretation, while (441)b suggests a more-than-maximum reading.

(441) Mary made more mistakes than Bill could have.

a. if he had done his best.

- b. if he had made no effort at all.

If Rullmann ambiguity is not rooted in negative polarity, we cannot rely on the non-local movement of ‘little’ in the comparative complement as proposed in (Heim, 2007). In (233) and (441) the source of ambiguity obviously lies somewhere else. Note that the embedded clause of (441) is again ambiguous per se, parallel to the other examples of Rullmann ambiguity discussed here. Thus, example (442) can be understood conveying either the minimal or the maximal number of mistakes Bill could have made.

(442) Bill could have made 5 mistakes.

To sum up this section, a scope approach to the ambiguity in comparatives with possibility modals fails to explain why this kind of ambiguity is possible outside of comparatives and only with a certain kind of modals. It has nothing to say on the preference for the weak reading and the role of ‘only’ in promoting the strong reading. It commits its proponents to a decompositional analysis of negative pole adjectives. Finally, its prediction that the ambiguity is dependent on the polarity is questionable. This gives us enough reason to consider a scope approach untenable and search for a different account of ambiguity. One alternative is to follow in the footsteps of (Fox and Hackl, 2006) who take strong readings of ambiguous sentences with ‘before’ to be a result of a pragmatic enrichment of weak readings.

4.2 PRAGMATIC ENRICHMENT

In section 2.2.4.2, I pointed to a possibility of a pragmatic account of the strong reading of example (116), paraphrased in (443) below.

(116) You are allowed to arrive before 10 p.m.

(443) The latest possible time of your arrival precedes 10 p.m.

(Fox and Hackl, 2006) propose that this sentence can convey the latest allowed time of arrival as a result of a scalar implicature. The scalar implicature, they discuss, amounts to negating that the addressee is allowed to arrive before any time following 10 o'clock. The crucial assumption that underlies this kind of pragmatic strengthening is the possibility of the so called free choice interpretation of (116). If this sentence is understood to mean that the addressee is allowed to arrive *any time* before 10 o'clock in a certain context, this can generate the scalar implicature corresponding to the problematic strong reading.

The goal of this section is to spell out the mechanism behind the proposal sketched in (Fox and Hackl, 2006). The structure of this section is as follows. Subsection 4.2.1 is concerned with Heim ambiguity in 'less' comparatives. I make precise the kind of reasoning that motivates the free choice interpretation and how this interpretation feeds a further pragmatic enrichment step, namely, strengthening by a scalar implicature. In the following subsections, I attempt to extend this analysis to other cases of Heim ambiguity and to Rullmann ambiguity.

4.2.1 HEIM AMBIGUITY IN 'LESS' COMPARATIVES

Let us consider example (74), repeated below for convenience. The task is to explain why it can implicate that the paper is not allowed to be ten pages or longer.

(74) (The draft is ten pages.) The paper is allowed to be less long than that.

Under the assumption that ‘be allowed’ existentially quantifies over the worlds compliant with whatever laws are in force in the actual world, we derive the following truth condition for (74).

$$(444) \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_{w'}(p) = d \wedge d < 10 \text{ pages}$$

These truth conditions correspond to the weak reading of the sentence: it is predicted to be true iff there is a permitted state of affairs under which the length of the paper is less than ten pages. Thus, the addressee gets informed that the paper is allowed to have some length in the interval spanning from zero to 10 pages. In some contexts, it would be an informative enough statement; in others, it would be natural for the addressee to want to know if this span can be narrowed down. Put differently, these truth conditions generate the ignorance inference that the speaker does not know whether there exists the lower and/or the upper bound on the admissible page length. However, if the speaker is known to be an authority on this issue, this may trigger a chain of pragmatic steps with the aim to get rid of the ignorance inference.

This situation is reminiscent of the cases discussed under the rubric ‘free choice effect’. For example, (Kratzer and Shimoyama, 2002) address the question on the part of the addressee of why the speaker chooses the widest domain of quantification by using the free choice existential ‘irgendein’ in sentences like “Sie darf irgendeinen Arzt heiraten”/“She is allowed to marry some doctor or other”. The choice of the widest domain leads the addressee to consider why smaller domains have not been preferred. This consideration results in the free choice implicature that any doctor is such that she is allowed to marry him.

I assume that reporting on the allowed span by the use of ‘less than 10 pages’ may trigger the same kind of reaction on the part of the addressee, that is, an inquiry into why a smaller span has not been chosen, and even-

tually lead to the free choice implicature that any length is allowed, see (445).

$$(445) \lambda w \forall d < 10 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) = d$$

In the case at hand, we also deal with the manipulation of the quantifier domain. Here, the domain of the degree quantifier is constrained by the restriction ‘less long than that’ to the degrees that lie below 10 pages. Other domain restrictions could have been chosen by the speaker. The following set represents possible alternatives.

$$(446) \{p: \exists D \subseteq \{0, \dots, 10\text{pp}\}: p = \lambda w \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_{w'}(p) = d \wedge d \in D\}$$

The speaker has chosen (444) among other alternatives and this calls for an explanation. Kratzer and Shimoyama show this question brings about what they call a distribution requirement, in the case at hand the strengthened meaning in (445).

A pragmatic principle at the core of their reasoning is avoiding false exhaustivity inferences. According to this principle, if one utters S and the context makes salient an alternative utterance S' the reason for not uttering the disjunction of S and S' instead of S is that the alternative S' is taken to be false. Now, suppose the speaker uttered a sentence of the form “it is possible that A or B”. Making a plausible assumption that the speaker believes in the truth of “it is possible that A”, the addressee may conclude that the speaker couldn’t have avoided the utterance of the latter because of the falsity of “it is possible that B”. Symmetrically, the addressee reaches the conclusion that the speaker couldn’t have avoided the utterance of “it is possible that B” because of the falsity of “it is possible that A”. We derive that the two alternatives are equivalent. This, together with the truth of the utte-

red disjunction derives the distribution requirement, namely “it is possible that A and it is possible that B”.³⁶

Turning to our example, relying on the principle of avoiding false exhaustivity inferences, we derive that all alternatives from the set (446) are equivalent to each other.

$$(447) \quad \forall q,r \in \{p: \exists D \subseteq \{0, \dots, 10pp\}: p = \lambda_w \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_w(p) = d \wedge d \in D\}: q \leftrightarrow r$$

In conjunction with the plain meaning of disjunction, this results in the following distribution requirement, which implies the free choice interpretation in (445) that we are after.

$$(448) \quad \forall D \subseteq \{0, \dots, 10pp\}: \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_w(p) = d \wedge d \in D$$

It is easy to show that the free choice interpretation can give rise to the scalar implicature that for any length d greater than 10 pages the paper is not allowed to be less long than d . Scalar implicatures arise in contexts that make prominent stronger alternative assertions result. In the spirit of (Fox, 2007), the effect of strengthening by a scalar implicature can be best replicated by embedding the relevant sentence under ‘only’, which negates stronger alternatives to its prejacent. Consider the variant of (74) with the overt

36. Here is a proof (P(X) stands for “it is possible that X”).

$$(1) P(A \vee B)$$

$$(2) P(A) \leftrightarrow P(B)$$

$$\therefore (3) P(A) \& P(B)$$

Assume that $\neg P(A)$.

$$(4) \neg P(B) \text{ (follows from (2) and the assumption)}$$

$$(5) \neg P(A) \& \neg P(B) \text{ (by (4) and the assumption)}$$

$$(6) \neg P(A \vee B) \text{ (by (5))}$$

$$(6) \text{ contradicts (1)} \Rightarrow P(A)$$

Likewise, we show that $P(B)$ and therefore $P(A) \& P(B)$. Q.E.D.

‘only’, given below. This sentence conveys that the paper is not allowed to be as long as the draft or longer. This is precisely the ExR of this sentence.

(449) The draft is ten pages long. The paper is only allowed to be less long than that.

Uttering (74) makes prominent the alternative assertions with the length of the draft, ‘ten pages’, replaced by other degrees on the length scale.

(450) $\lambda_w \forall d < n: \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) = d$, for $n \neq 10$ pp

As Kratzer and Shimoyama note, alternatives are made prominent for a reason, they cannot be “contextualised away”. The plausible reason in the case at hand is to strengthen the claim made by negating the logically stronger alternative claims. Following the neo-Gricean program in (Sauerland, 2004), we strengthen (74) by excluding its alternatives with n ranked higher on the numerical scale than 10. As a result we get the conjunction of the free choice interpretation and the Gricean quantity implicature.

(451) $\lambda_w \forall d < 10 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) = d \wedge$
 $\forall d' \geq 10 \text{ pp}: \neg \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(p) = d'$

Applied to a world, (451) entails that the maximally allowed length of the paper is under ten pages, which corresponds to the ExR we are after. The fact that the presence of the overt ‘only’, whose semantics is based on the exclusion of alternatives, makes ExR prominent in Heim ambiguity examples with ‘less’ supports the idea that this reading is a result of pragmatic enrichment. It also explains why many speakers can only get ExR if ‘only’ is pronounced.

The main argument that the free choice effect should be derived pragmatically, by the system that computes scalar implicatures comes from the fact that it is not traceable in downward-entailing contexts. If a sentence with a FC interpretation is embedded under negation only the part of mea-

ning to the exclusion of the FC implicature is negated. This effect is revealed by our example as well. Consider its negation.

(452) (The draft is ten pages.) The paper is not allowed to be less long than that.

This sentence unambiguously excludes the situation in which the paper is shorter than 10 pages and complies with the rules, which corresponds to the negation of the plain meaning given in (444). If we were to negate the strong meaning in (451), the result would be a weaker assertion. Abstracting from the truth conditions we have derived, the negation of Heim's paraphrase of the strong meaning is given below. It does not capture the intuitive meaning of (452).

(453) The paper is allowed to be as long as that.

To sum up, I have shown that Heim ambiguity in 'less' comparatives is a result of a pragmatic enrichment of the basic meanings of comparative sentences by a quantity implicature. Crucially, the quantity implicature hinges on the free choice interpretation of 'less' comparatives. The latter arises in contexts where the speaker is considered to be an authority or have an opinion or evidence on what is allowed. Treating ExR as a result of a free choice effect has a welcome prediction for sentences with epistemic possibility modals like (438), repeated below.

(438) You might arrive earlier than 10 p.m.

The use of the epistemic 'might' indicates the speaker's uncertainty about the truth of the embedded proposition. The speaker obviously lacks the relevant evidence. In such a situation, an inference about the ignorance of the speaker is plausible. It is not revised and the literal meaning does not get strengthened. Hence, sentence (438) does not reveal Heim ambiguity.

4.2.2 DEGREE TERMS UNDER POSSIBILITY

The goal of this section is to apply the pragmatic approach we used to account for Heim ambiguities to other cases of ambiguities in possibility statements with scalar items. I want to propose that, like in ‘less’ comparatives, different interpretations of comparatives with ‘exactly’-differentials and Rullmann ambiguities arise as a pragmatic implicature. We start by looking at the simplest case, featuring a numeral in the scope of a possibility modal and convince ourselves that the pragmatic strengthening outlined in the previous section can apply to it, too. To show that that both kinds of examples allow for a unified treatment, I then make precise some assumptions on the treatment of numerals and degree adjectives. In the rest of the section, we consider ‘exactly’-comparatives and Rullmann examples.

If a numeral occurs in the scope of a possibility modal it can refer to the minimum as well as the maximum of what is possible, allowed or could be/have been the case. In the following sentence, the phrase €100 can refer to the minimum as well as to the maximum Mary could have spent.

(454) Mary could have spent €100.

Suppose this sentence is uttered in a situation in which Mary was to organise a party and she’d been saving up money to buy a new laptop. Mary’s primary goal was to avoid spending too much for the party. Mary had €100 at her disposal. In this scenario, example (454) may convey that €100 is the maximum sum she could have spent, as the following continuation suggests.

(455) Mary could have spent €100. She did away with a rather modest sum.

On the other hand, (454) could be uttered in a context in which Mary’s goal was to prepare a fabulous meal for her family, whatever it might cost her. With the continuation in (456), €100 refers to the minimal sum Mary could have spent.

(456) Mary could have spent €100. She didn't have to be that extravagant.

How do these readings come about? I propose that they have the same source as the strong reading of 'less' comparatives. Both of them are instances of ExR, which means that they correspond to the free choice reading of the sentence strengthened by a scalar implicature. To be precise, the Max ExR made prominent in (455), is derived in two steps, as outlined in (457), while the Min ExR in (456), is the result of the strengthening steps shown in (458).

- (457) a. There is a possible state of affairs in which Mary spends at most €100.
- b. For every sum $d \leq €100$, there is a possible state of affairs in which Mary spends d .
- c. For every sum $d \leq €100$, there is a possible state of affairs in which Mary spends d ; and for every sum $d' > €100$, there is no possible state of affairs in which Mary spends d' .
- (458) a. There is a possible state of affairs in which Mary spends at least €100.
- b. For every sum $d \geq €100$, there is a possible state of affairs in which Mary spends d .
- c. For every sum $d \geq €100$, there is a possible state of affairs in which Mary spends d ; and for every sum $d' < €100$, there is no possible state of affairs in which Mary spends d' .

The legitimacy of steps outlined in b. and c. lines has been argued for in the previous section. The question that needs to be addressed here is how do we exactly arrive at the truth conditions in each a. line.

Given our assumptions so far, the plain meaning of (454) can be represented as follows. I gloss over the contribution of tense and assume that 'spend' is associated with the measure function EXPENSE to make a parallel to gradable adjectives.

$$(459) \quad \lambda w \exists w' \in \text{Acc}_w \text{EXPENSE}_{w'}(\text{Mary}) = €100$$

This requirement is met iff there is an accessible world in which Mary’s expense equals €100, an interpretation not corresponding to either of the paraphrases in the a. lines of (457) and (458). Both of the latter involve a monotone interpretation of the degree term. There are two ways monotonicity can enter the meaning of examples like this one: through the meaning of the measure phrase or the entry of the gradable predicate. The second alternative being more general, I propose to revise the lexical entry for degree predicates introduced in section 1.2.2. by making the assumption that such predicates are monotone in their degree argument. The lexical entry of the monotone ‘spend’ is given in (460).

$$(460) \llbracket \text{spend} \rrbracket = \lambda w \lambda d \lambda x \text{EXPENSE}_w(x) \geq d$$

With this definition of the ‘spend’, we derive the truth conditions paraphrased in (458)a, see (461). These may generate an ignorance inference about the limits of what is possible: if the speaker has all the relevant information why did she choose to say that a possible sum is within the unbound range starting from €100 and did not specify a smaller interval. This ignorance inference leads to the implicatures in (458)b and (458)c.

$$(461) \lambda w \exists w' \in \text{Acc}_w \text{EXPENSE}_{w'}(\text{Mary}) \geq \text{€}100$$

In order to derive the ‘at most’ interpretation in (457)a, I assume that the direction of the scale provided by the gradable predicate is not fixed once and for all but determined by the context, see, for example, (Krifka, 2007) on a similar assumption in the analysis of ‘at most’ readings. (Rullmann, 1995) contains an illuminating discussion on how pragmatics determines the direction of an inference in possibility statements. He compares the pair of sentences in (462). Our world knowledge tells us to interpret the numeral in (462)a as meaning ‘at least \$3000’, while that in (462)b as meaning ‘at most \$3000’.

- (462) a. A professor can live on \$3000.
 b. A professor can spend \$3000.

In the case at hand, the scale on which alternative assertions are ranked is determined by the open proposition ‘Mary could have spent d much’. The ‘party’ scenario licences the inference from “Mary could have spent d much” to “Mary could have spent $d-n$ much”. If one saves up money by spending €100, it is certainly true that one saves up money by spending less than €100, cf. (463). In that case, the resulting plain meaning of (456) is an ‘at most’ reading, paraphrased in (457)a and given in (464) below.

- (463) Mary could have spent €100 (and saved up money). \Rightarrow
 Mary could have spent €90 (and saved up money).
 Mary could have spent €100 (and saved up money). \nRightarrow
 Mary could have spent €110 (and saved up money).

- (464) $\lambda w \exists w' \in \text{Acc}_w \text{EXPENSE}_{w'}(\text{Mary}) \leq \text{€}100$

In the ‘family meal’ scenario, the context licenses an inverse inference pattern, namely from ‘Mary can spend d much’ to ‘Mary can spend $d+n$ much’. If Mary’s primary wish is to cook a fabulous meal then spending €100 is higher on the pragmatic scale than spending more than €100, cf. (465). As a result, the plain assertion corresponds to (458)a and (462).

- (465) Mary could have spent €100 (and cooked a fabulous meal). \Rightarrow
 Mary could have spent €200 (and cooked a fabulous meal).
 Mary could have spent €100 (and cooked a fabulous meal). \nRightarrow
 Mary could have spent €90 (and cooked a fabulous meal).

Given (462) and (464), we can straightforwardly derive the Max and Min ExR as a result of a free choice and a scalar implicature.

The pragmatic strategy just described can be applied to the comparative complement of (466) as well.

- (466) Mary spent more than she could have.

If the context licences an ‘at least’ interpretation of the bound degree in the

scope of ‘could’, after the relevant strengthening steps, the embedded clause denotes a set of degrees d that satisfy two conditions: any sum greater or equal to d could have been spent by Mary and any sum less than d could not have been spent by Mary. This corresponds to the singleton set containing the minimum that Mary could have spent. DEF picks out the unique element from this set and passes it to the comparative morpheme. A more-than-minimum reading results.

- (467) $\lambda w \text{ EXPENSE}_w(\mathbf{M}) >_{\text{DEF}} (\lambda d \forall d' \geq d : \exists w' \in \text{Acc}_w : \text{EXPENSE}_{w'}(\mathbf{M}) = d' \wedge \neg \exists d'' < d : \exists w' \in \text{Acc}_w : \text{EXPENSE}_{w'}(\text{Mary}) = d'')$
 ‘Mary’s expense in the actual world exceeds the minimum she could have spent.’

If the context licenses an ‘at most’ reading of the embedded degree variable, the resulting reading of the comparative corresponds to a comparison with the maximum, given below.

- (468) $\lambda w \text{ EXPENSE}_w(\mathbf{M}) >_{\text{DEF}} (\lambda d \forall d' \leq d : \exists w' \in \text{Acc}_w : \text{EXPENSE}_{w'}(\mathbf{M}) = d' \wedge \neg \exists d'' > d : \exists w' \in \text{Acc}_w : \text{EXPENSE}_{w'}(\text{Mary}) = d'')$
 ‘Mary’s expense in the actual world exceeds the maximum she could have spent.’

Strengthening is obviously obligatory in such cases. If we chose the plain meaning of the embedded proposition, the resulting set of degrees would not be a singleton and DEF would end up undefined. Thus, Rullmann ambiguity follows from the availability of two ways to apply the pragmatic reasoning that allows to derive a free choice interpretation of sentences with degree-denoting terms in the scope of a possibility modal and strengthen this interpretation by a scalar implicature.

The last ambiguity case to be considered is a comparative with an ‘exactly’-differential. I repeat the relevant example from (Heim, 2001).

- (79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

Assuming that the direction of the scale used by the gradable predicate is determined pragmatically, the plain meaning of (79) either conveys that

there is a permitted state of affairs in which the paper is exactly 15 pages or longer, see (469), or that there is a permitted state of affairs in which it is exactly 15 pages or shorter, see (470).

$$(469) \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_{w'}(\text{paper}) \geq d \wedge d > \text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_{w'}(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$$

$$(470) \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_{w'}(\text{paper}) \leq d \wedge d > \text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_{w'}(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$$

These interpretations being rather uninformative, the addressee may choose an stronger exhaustified meaning of the embedded scalar proposition, resulting in the truth conditions below, which are met iff there is a world complying with the rules in which the paper is exactly 15 pages.

$$(471) \exists w' \in \text{Acc}_w: \exists d: \text{LENGTH}_{w'}(\text{paper}) = d \wedge d > \text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_{w'}(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$$

However, even this strengthened interpretation generates an inference that the speaker does not know whether the paper is allowed to be longer or shorter than 15 pages, which is might seem implausible to the addressee. She may go back to the interpretation (469) and (470) and inquire about why the speaker did not choose to specify the permitted length interval instead of providing spans stretching from 15 pages, which eventually would result in free choice interpretations given in (472) and (473), respectively.

$$(472) \forall d \geq 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(\text{paper}) = d \wedge d > \text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_{w'}(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$$

$$(473) \forall d \leq 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_{w'}(\text{paper}) = d \wedge d > \text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_{w'}(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$$

These can be strengthened by scalar implicatures and lead to the Min and Max ExR.

- (474) $\forall d \geq 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) = d \wedge d > \text{LENGTH}_w(\text{draft})$
 $\wedge \text{DISTANCE}(\text{LENGTH}_w(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp} \ \&$
 $\neg \exists d < 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) = d \wedge d >$
 $\text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_w(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$
- (475) $\forall d \leq 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) = d \wedge d > \text{LENGTH}_w(\text{draft})$
 $\wedge \text{DISTANCE}(\text{LENGTH}_w(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp} \ \&$
 $\neg \exists d > 15 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) = d \wedge d >$
 $\text{LENGTH}_w(\text{draft}) \wedge \text{DISTANCE}(\text{LENGTH}_w(\text{paper}), \text{LENGTH}_w(\text{draft})) = 5 \text{ pp}$

The second reading, the Max ExR given in (475), corresponds to Heim's wide scope of the comparative relative to 'be allowed'. Under this reading, the paper is not allowed to be longer than 15 pages. The Min ExR, derived in (474), conveys that the paper is not allowed to be shorter than 15 pages. It might arise in a context in which people write too little and there is a rule specifying the minimal page count. Admittedly, the Min ExR is less natural than the Max ExR. In the next section, I will come back to this issue and provide an explanation for the observed asymmetry.

To sum up, I argued that all three examples of ambiguity, viz. 'less' comparatives, comparatives with 'exactly' differentials and comparatives with a possibility modal in 'than' clauses may and sometimes have to be pragmatically strengthened. Thus, Heim and Rullmann ambiguities are not structural but result from the possibility to choose between plain assertions and pragmatically enriched meanings. In Heim ambiguity examples, strengthening is optional and pragmatically enriched ExRs coexist with plain meanings. In Rullmann ambiguity examples strengthening is obligatory. Without it, the comparative clause would fail to refer to a unique degree. In all three cases the crucial requirement for the derivation of the free choice implicature is that the truth conditions may be satisfied by more than one value of the degree-denoting expression in the scope of the modal. In 'less' comparatives this condition is immediately met: the degree term is said to stand in '<' relation to the subject of comparison. In the other two

cases, monotonicity is introduced through the meaning of gradable adjectives.

4.3 CONSEQUENCES OF A PRAGMATIC APPROACH

In this subsection I want to address some consequences of a pragmatic approach to Rullmann and Heim ambiguities. First, in a pragmatic approach the availability of Rullmann ambiguity is not predicted to depend on the polarity of the gradable predicate involved. However, it is widely accepted that sentences with positive pole adjectives are not ambiguous, cf. (Rullmann, 1995), (Heim, 2007), (Büring, 2007a). The second consequence is that the ExR derived in a pragmatic approach is logically stronger than the ExR derived in a structural ambiguity approach. There happen to be contexts in which the weaker meaning derived in Heim's approach is more appropriate.

4.3.1 CONSTRAINTS ON RULLMANN AMBIGUITY

I suggested that Rullmann ambiguity might not be restricted to comparatives with negative pole adjectives by providing some counterexamples to this claim. It is worth while looking at the cases which are reported to lack one of the relevant readings. It turns out that the lack of ambiguity is not directly related to the polarity of gradable predicates, as the common wisdom has it, but resides in certain pragmatic restrictions on the use of this or that kind of comparative sentence. There are at least three factors the interplay of which is crucial for the availability of two readings. They are the modal flavour of the embedded possibility modal, the polarity of the gradable predicate and the preference for the maximum or the minimum ExR outside of the comparative environment.

Let us first consider the case of deontic possibility modals. A deontic modal in a comparative complement is generally not compatible with a more-than-minimum reading. This might be the reason Meier's example in (233), repeated below, is mostly judged to lack that interpretation. One tends to compare Chuck's actual speed with the upper speed limit even if the context at hand does not favour this type of comparison.

(233) Chuck is driving faster than he is allowed to drive.

The lack of the more-than-minimum interpretation of (233) suggests that one can only use this sentence to convey that Chuck does not comply with the regulations. One cannot use it to convey that Chuck is conform to the rules. Comparatives with negative pole adjectives reveal the same pattern. For example, the prominent reading of (476) says that Chuck is driving below the minimum speed, violating traffic regulations. The less-than-maximum reading, though reported to be available, is less natural.

(476) Chuck is driving slower than he is allowed to drive.

Generally speaking, if we assume that the shaded area in the middle of the speed scale in (477) corresponds to how fast one is allowed to drive, a comparative sentence with a deontic possibility modal would set Chuck's actual speed to X1 or X3 but hardly to X2.

(477) -----X1-----////////////////X2////////////////-----X3----->

On reflection, it is not surprising that one avoids to use a comparative construction to claim that Chuck's speed falls within the compliance interval. A comparative sentence can only relate Chuck's speed to one of the bounds of the admissible interval. It cannot refer to both bounds simultaneously. If Chuck is said to drive faster than the minimum or slower than the maximum this meaning appears too weak to ever surface. In both cases nothing can prevent the hearer from inferring that the speaker does not know whether Chuck complies with the rules or not. I propose that this is the reason why the corresponding readings are blocked with deontic modals. A comparative sentence can be used to report on the excess or shortage of some gradable property resulting in a rule violation but it cannot be used to express compliance with a rule, for it cannot refer to the minimum and the maximum set by the rule simultaneously, which is necessary for a 'rule compliance' report.

Thus, a pragmatic approach assigns a pair of readings to a Rullmann ambiguity sentence, regardless of the polarity of the gradable predicate involved. However, the ambiguity is always resolved in favour of the strongest reading. This reminds of the strongest meaning hypothesis (henceforth SMH), a general principle for resolving ambiguity first suggested in (Dalrymple *et al.*, 1998) in connection with the interpretation of reciprocals. If a principle of this sort is at work in Rullmann comparatives, a more-than-maximum reading is a strongly preferred interpretation of a positive pole comparative with a deontic modal, since it logically entails the corresponding more-than-minimum reading, e.g. whenever Chuck's speed is reported to exceed the maximum, it lies above the minimum too. Likewise, in a negative pole comparative a less-than-minimum reading is preferred to a less-than-maximum one: the latter is entailed by the former.

A slight asymmetry between negative and positive pole cases calls for an explanation, though. To be more precise, the occasional availability of a less-than-maximum reading is at odds with the SMH. I suggest that the third factor at stake, along with the polarity and the modal force, is the preferred interpretation of a degree term outside of the comparative complement. Consider example (240). This sentence normally conveys that 45mph is the maximum Chuck is allowed to drive.

(240) Chuck is allowed to drive 45mph fast on this highway.

Likewise, Heim ambiguity in 'exactly' comparatives is normally resolved in favour of a maximal ExR. Under its strengthened interpretation, the following example reports on the allowed maximum rather than minimum.

(79) (The draft is ten pages long.) The paper is allowed to be exactly five pages longer than that.

I suggest that the reason maximum readings are strongly preferred to minimum readings is the availability of two competing ways to express a minimum reading resulting in a semantic blocking effect. Concretely, to re-

port on the minimal compliant speed, we use a variant with a necessity modal instead of one with a possibility modal, that is, (240) is in competition with (478).

(478) Chuck has to drive 45mph fast on this driveway.

The reason (240) loses and (478) wins the competition is that the latter expresses the relevant reading straightforwardly, while for the former it is a result of pragmatic enrichment. The MinR of (478) is rooted in the meaning of the anankastic ‘have to’, which compares alternative propositions according to their possibility given the relevant goal. This interpretation is readily available if the possibility scale can be mapped to the effort scale. Min ExR of (240), on the other hand, arises via a Gricean route, as a result of a free choice and a scalar implicature. Given the choice between MinR and Min ExR, we opt for the former. As a result, the preferred reading of strengthened possibility statements is a Max ExR.

Turning to Rullmann sentences, we now have an explanation for the distribution of Min and Max ExR across positive and negative pole comparatives. I repeat here the relevant examples.

(233) Chuck is driving faster than he is allowed to drive.

(476) Chuck is driving slower than he is allowed to drive.

In positive comparatives, like (233), Max ExR is a preferred reading according to the SMH and it does not enter the kind of semantic competition described above, whereas Min ExR fares worse according to the both parameters. As a result, positive comparatives are reported to lack Min ExR, see (Beck, 2010a). In negative comparatives, the situation is more complicated. Both readings are in conflict with one of the constraints: Min ExR is subject to semantic competition with necessity statements and Max ExR is a weaker one and should be blocked by the SMH. As a result, there is no reading which is strongly preferred over the other and both are reported to be available.

Comparatives with non-deontic modals are not subject to the pragmatic constraints discussed above. However, they must conform to a different kind of usage rules. Consider an example with a counterfactual modal from the previous section.

(466) Mary spent more than she could have.

Unless the embedded clause talks of an event which is different from that of the matrix clause event, this sentence has a more-than-minimum reading only. A more-than-maximum interpretation would make (466) inconsistent. The span covering Mary's expenses in counterfactual worlds cannot be exceeded by Mary in the actual world without producing a contradiction. Why should that be so? Counterfactual modals belong to a class of root modals. Root modality is a handy term covering those kinds of modal interpretations that are sensitive to the facts in the world of evaluation, i.e. requiring realistic conversational backgrounds. Thus, what Mary could have spent should be compatible with the actual circumstances. Mary's actual expense therefore cannot exceed what is possible in view of the facts. This explains the lack of a more-than-maximum interpretation in (466). The same reasoning can be used to explain why (479) appears to lack a less-than-minimum reading.

(479) Mary spent less than she could have.

Note that if the events of the embedded and the matrix clause are explicitly made distinct, the inconsistency effect can be lifted.³⁷ For example, this happens if the subjects of the main and the embedded clause do not co-refer. Consider the following sentence:

(480) Mary spent more than Peter could have spent.

This example has a more-than-minimum reading. It is especially prominent

37. I thank Daniel Büring for drawing my attention to this issue and Vera Hohaus for an insightful discussion of it.

in a context of Mary and Peter spending money for the same purpose, to be more precise, if Mary and Peter are considered the agents of the exactly same event associated with the same circumstances. If Mary and Peter are understood to be the agents of different events, say, Mary organising a party and Peter buying a laptop, a more-than-maximum interpretation becomes possible. The same holds for example (441) repeated below. The circumstance of the two events with Bill or Mary being the agents are most likely quite different. Suppose that Bill is the best student in his class and is normally expected to make less mistakes than anyone else.

(441) Mary made more mistakes than Bill could have.

The inconsistency effect can also be lifted if we explicitly distinguish the two events as in (481) below. Sentence (481) can be understood saying that Mary exceeded the maximum she could have spent under different circumstances.

(481) Mary spent more than she could have if she had had to organize a wedding party.

To sum up, the lack of Rullmann ambiguity in some examples does not hinge on the absence of degree negation. I argued that the apparent polarity sensitivity of Rullmann ambiguity is not semantic but pragmatic – it varies with the kind of modal flavour and is sensitive to which interpretation of the degree term embedded under the modal is preferred.

4.3.2 CANCELLATION

Let me repeat the truth conditions corresponding to the ExR of (74) derived by (Heim, 2007) and those derived in a pragmatic approach proposed here.

(74) (The draft is ten pages.) The paper is allowed to be less long than that.

(Heim, 2007) predicts that (74) is true on its ExR iff the maximum length the paper is allowed to have is below the length of the draft, cf. (482).

(482) $\text{MAX}(\lambda d \exists w' \in \text{Acc}_w: \text{LENGTH}_w(\text{paper}) \geq d) < 10 \text{ pages}$

It turns out that the ExR derived in a pragmatic approach, cf. (451), corresponds to a stronger claim.

(451) $\lambda w \forall d < 10 \text{ pp}: \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) = d \wedge$
 $\forall d' \geq 10 \text{ pp}: \neg \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) = d'$

According to (451), any length under 10 pages is compliant with the rules. Therefore the maximum lies just below 10 pages. This may seem too strict a requirement. Consider the following discourse.

(483) (The draft is ten pages long.) The paper is (only) allowed to be less long than that. But I don't know if there is any length under 10 pages it's not allowed to have.

The last remark of the speaker of (483) contradicts the ExR in (451). However, this discourse is not contradictory. Heim's truth condition in (482) is compatible with the continuation in (483) and *prima facie* appears more adequate than the meaning in (451).

On a closer examination though, the fact that (483) is consistent is not a knock-down argument for a pragmatic theory, as one would think, but is, in fact, predicted by it. The last sentence of (483) contradicts the free choice interpretation of (74), i.e. the claim that the paper is allowed to have any length under 10 pages. Since the free choice interpretation has a status of an implicature in this approach, it is expected that it can be cancelled under certain circumstances, e.g. if the context makes clear that the speaker is not opinionated enough for the implicature to emerge. I would guess that this is what is happening in (483). The first conjunct of (451) is replaced by the weak interpretation of (74) and the resulting truth conditions are the following:

(484) $\lambda w \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) < 10 \text{ pp} \wedge$
 $\forall d' \geq 10 \text{ pp}: \neg \exists w' \in \text{Acc}_w: \text{LENGTH}_w(p) = d'$

So far so good but this explanation raises a further question. How is it possible to cancel the free choice implicature without cancelling the scalar implicature that lives on it? To give a satisfactory answer to this question one would have to develop a theory of implicature cancellation, which has not been done so far. I therefore have to confine myself to a couple of informal remarks.

In a localist approach to pragmatic implicatures, cancellation amounts to backtracking and replacing a parse with an exhaustification operator by a corresponding parse without, cf. (Fox *et al.*, t.a.). This produces an incorrect result for the case at hand, since the computation of the scalar implicature, i.e. the second conjunct in (451), depends on the presence of the two EXH operators responsible for the free choice interpretation. To solve the problem, one needs a dynamic framework for computing and cancelling scalar implicatures that enables one to store the original strengthened meaning and intersect it with the meaning computed in a backtracking step. This would allow to cancel what is contradictory and preserve what is consistent with the following discourse.

In a global neo-Gricean approach to implicatures no grammatical mechanism is introduced to account for strengthening effects, see (Sauerland, 2004) Scalar implicatures are added at the top level after the semantic meaning has been computed. Therefore cancellation targets only the components of the strengthened meaning that produce contradiction. This approach seems better suited to account for the cancellation effect discussed in this section. I leave it open whether a global approach could generate the strengthening effects discussed here, including strengthening at the level of a comparative complement.

4.4 CONCLUSION

The main claim of this section is that ambiguity in comparatives with possibility modals is not structural. It arises as a result of pragmatic enrichment of plain meanings of the modal statements involved. Heim and Rullmann ambiguities can be treated in parallel: the ExR of comparatives with ‘allowed’ in the main clause have the same source as a comparison with the maximum or the minimum arising when ‘allowed’ occurs in a ‘than’-clause.

This approach meets the challenges of the structural ambiguity analysis proposed by (Heim, 2007). It offers a unified account of ambiguity in comparatives, possibility statements with measure phrases and numerals and sentences with temporal prepositions. The lack of ambiguity in comparatives with epistemic modals falls out from the status of ExR as a pragmatically strengthened meaning: epistemic possibility modals are associated with ignorance inferences and therefore do not give rise to pragmatic implicatures. Since a pragmatic approach does not predict a dependence of ExR on the polarity of the gradable predicate, it makes more adequate empirical predictions on the distribution of ambiguities. Finally, it uncovers the role of ‘only’ in facilitating ExR. ExR is a result of a scalar implicature that is realised as part of the basic meaning when ‘only’ is pronounced.

Pursuing a pragmatic approach instead of a scope one has important consequences for the semantics of comparatives. First, it refutes crucial semantic arguments for the degree operator analysis of the comparative morpheme: in this approach, Heim and Rullmann ambiguities do not provide an evidence for the LF mobility of ‘-er’. Another important claim is that ambiguity in comparatives with embedded modals is not anchored in the semantics of antonyms, in particular, it does not motivate decomposition of negative pole adjectives into a negation and a positive pole counterpart.

Chapter 5. Conclusion

Section 2 specified two main research questions of this work. The first one is the treatment of comparatives with anankastic necessity modals. Such modals can lead to Heim ambiguities and a comparison with the required minimum in comparative complements. The second task, which I proposed to tackle independently of the first one, is the treatment of comparatives with possibility modals. On the one hand, they also exhibit Heim ambiguities. On the other hand, when occurring in ‘than’-clauses, possibility modals can lead to Rullmann ambiguity, which has been traditionally linked to the polarity of the gradable predicate involved.

I suggested that, under certain circumstances, comparatives with anankastic necessity modals may have the so called minimal requirement reading (MinR). The availability of this reading causes Heim ambiguity and corresponds to a comparison with the required minimum when a modal occurs in a ‘than’-clause. The first series of questions to be addressed in this work is repeated below.

- (54) What kind of modals give rise to the minimal requirement reading (MinR)? Why does MinR obtain with these modals and not with others?

Regarding comparatives with possibility modals, I proposed to treat the stronger reading of examples of Heim ambiguity and both readings of examples of Rullmann ambiguity as having the same source and, concretely, stemming from a specific pragmatic treatment of degree terms in the scope of possibility modals. I called that interpretation an ‘extreme’ reading (ExR) because it involves a reference to the minimum or the maximum permitted/possible degree. My goal was to track down the source of ExR and explain its distribution, as reflected in the second series of questions.

- (60) What is the source of ‘extreme’ readings (ExR) in comparatives with possibility modals? Under which conditions is the extreme set to the minimum and the maximum?

To answer the questions in (54) I revised the assumptions concerning the interpretation of necessity modals made in the literature on comparatives. The crucial ingredients of my treatment of MinR modals is their requirement of a totally realistic ordering source and their context dependence reflected in the focus sensitivity. More specifically, an anankastic ‘have to’ conveys that the possibility of the proposition expressed by its complement given the goal expressed by the ‘in order to’ clause exceeds the possibility of any propositional alternative to its complement given the same goal, in view of what is the case in the world of evaluation. MinR is a hallmark of anankastic modals in contexts that make prominent an effort scale. In such contexts a necessary condition expressed by an anankastic modal is understood to be the easiest way to achieve a relevant goal. If an anankastic modal is embedded under a modal that is not associated with a realistic ordering, this effect does not arise and no MinR is possible. I argued that this is what happens in the so called weak anankastic conditionals, in which the modal is usually marked by subjunctive morphology reflecting the presence of two modal layers.

Turning to questions in (60), I proposed that ExR is a result of pragmatic enrichment of the basic meaning of possibility modals with embedded degree terms. It is possible in contexts in which such possibility statements can receive a free choice interpretation that has been treated as a pragmatic implicature. ExR can be derived by strengthening the free choice interpretation by a scalar implicature. Whether ExR corresponds to a comparison with the minimum or the maximum is determined by purely pragmatic factors. Any possibility is available in an appropriate pragmatic setting, though the general requirement to avoid inconsistencies and uninformative claims as well as a preference for expressing the same meaning by

semantic rather than pragmatic means may restrict the distribution of this or that ExR.

My proposals have two consequences for the semantics of degree constructions. First, they present an improvement on structural ambiguity approaches to Heim and Rullmann ambiguities and the treatment of MinR of anankastic modals. The analyses I proposed provide a unified account of ExR and MinR in comparative and non-comparative contexts. They do not lead to the overgeneration problem faced by structural ambiguity approaches, neither of which has an efficient spelled out mechanism for banning the MinR with WA modals or ExR with epistemic possibility modals. An outcome for the analysis of comparatives adopted at the beginning of this thesis is that ambiguity in intensional comparatives does not motivate a shift from degrees to intervals and a degree operator treatment of the comparative morpheme. The second consequence concerns the treatment of antonyms. Throughout I assumed a Cresswell-style definition of antonyms instead of the negation theory of antonymy widely accepted in most contemporary analysis of degree constructions. The main asset of the negation theory of antonymy relevant in the light of the present discussion is that it provides a basis for the decompositional analyses of negative pole antonyms which in their turn are crucial for a structural account of Heim and Rullmann ambiguities. Having developed a successful alternative to structural analyses, I have shown that decomposition of negative pole adjectives and, more generally, the negation theory of antonymy are not crucial for explaining the relevant ambiguities.

There are at least two remaining issues from the list of desiderata in (von Stechow, 1984) closely related to the main concern of this thesis which I addressed only indirectly. The first one is the derivation of the so called more-than-maximum interpretation of conjunction and universal quantifiers in comparative complements. I argued that universal modals systematically trigger this interpretation under the comparative. MinR cannot be

viewed as an exception. MinR is not a comparison with a minimum but a comparison with a minimally required amount. In this sense it is a special case of a more-than-maximum interpretation. The second issue is the extension of my derivation of ExR in sentences with Rullmann ambiguity to comparatives with embedded existential quantifiers and disjunction.

I conclude with the general contribution of my work outside the realm of comparatives. I investigated the behaviour of anankastic conditionals in scalar contexts and explained why they trigger a sufficiency inference. Along the way, I proposed a semantic account of differences between strong and weak anankastic conditionals. Another important outcome is the inclusion of examples of Heim and Rullmann ambiguities in the range of constructions that trigger a free choice effect.

Bibliography

- Bartsch, Renate. 1973. "Negative Transportation" gibt es nicht. *Linguistische Berichte* 27.
- Bartsch, Renate and Theo Vennemann. 1972. The Grammar of Relative Adjectives and Comparison. *Linguistische Berichte* 21: 19–32.
- Beaver, David. 2004. Five 'Only' Pieces. *Theoretical Linguistics* 30: 45–64.
- Beaver, David and Brady Clark. 2003. 'Always' and 'Only': Why Not All Focus Sensitive Operators Are Alike. *Natural Language Semantics* 11(4): 323–62.
- Beaver, David and Cleo Condoravdi. 2004. A Uniform Analysis of 'Before' and 'After'. In Young, R.B. and Zhou, Y. (eds.) *Proceedings of Semantics and Linguistic Theory 13*. 37–54. Cornell: CLC Publications.
- Bech, Gunnar. 1955. *Studien Über Das Deutsche Verbum Infinitum*. København: Det Kongelige Danske Akademie av Videnskaberne.
- Beck, Sigrid. 2009. *DegP Scope Reanalysed*. University of Tübingen. Unpublished manuscript. URL http://www.uni-tuebingen.de/eli/sigrid.beck/papers/DegPScope11_09.pdf.
- Beck, Sigrid. 2010a. *Lucinda Driving Too Fast Again – the Scalar Properties of Ambiguous Than-Clauses*. University of Tübingen. Unpublished manuscript. URL http://www.uni-tuebingen.de/eli/sigrid.beck/papers/DegPScope11_09.pdf.
- Beck, Sigrid. 2010b. Quantifiers in Than-Clauses. *Semantics and Pragmatics* 3: 1–72.
- Boguslavski, Igor M. 2001. Modal'Nost', Sravnitel'Nost' I Otricanie / Modality, Comparison and Negation. *Russkij jazyk v nauchnom osveshchenii* 1: 27–52.
- Bresnan, Joan. 1973. Syntax of the Comparative Clause Construction in English. *Linguistic Inquiry* 4: 275–343.
- Büiring, Daniel. 2007a. 'More' Or 'Less'. *Proceedings from the Annual Meeting of the Chicago Linguistic Society* 43(2): 3–17.
- Büiring, Daniel. 2007b. Cross-Polar Nomalies. In Friedman, Tova and Gibson, Masayuki (eds.) Ithaca, NY: CLC Publications, Cornell University.
- Condoravdi, Cleo. 2001. Temporal Interpretation of Modals: Modals for the Present and for the Past, In Beaver, David I. et al. (eds.) *The Construction of Meaning*. 59–88. Stanford: CSLI Publications.

- Cresswell, Max. 1976. The Semantics of Degree. In Partee, Barbara H. (ed.) *Montague Grammar*. 261–92. New York: Academic Press.
- Dalrymple, Mary, Makoto Kanazawa, Yookyung Kim, Sam Mchombo, Stanly Peters. 1998. Reciprocal Expressions and the Concept of Reciprocity. *Linguistics and Philosophy* 21: 159–210.
- Dretske, Fred I. 1972. Contrastive Statements. *Philosophical Review* 81(4): 411–37.
- von Fintel, Kai and Sabine Iatridou. 2005. *What to Do if You Want to Go to Harlem: Anankastic Conditionals and Related Matters*. Handout of a talk delivered at Linguistics Colloquium at the University of Texas at Austin on May 25th. URL <http://web.mit.edu/fintel/www/harlem-austin.pdf>.
- von Fintel, Kai and Sabine Iatridou. 2007. Anatomy of a Modal Construction. *Linguistic Inquiry* 38(3): 445–83.
- von Fintel, Kai and Sabine Iatridou. 2008. How to Say ‘Ought’ in Foreign: The Composition of Weak Necessity Modals. In Guéron, Jacqueline and Lecarme, Jacqueline (eds.) *Time and Modality*. 115–41. Dordrecht: Springer.
- Fox, Danny. 2007. Free Choice and the Theory of Scalar Implicatures. In Sauerland, Uli and Stateva, Penka (eds.) *Presupposition and Implicature in Compositional Semantics*. 71–120. Palgrave.
- Fox, Danny, Gennaro Chierchia and Benjamin Spector. t.a. The Grammatical View of Scalar Implicatures and the Relationship Between Semantics and Pragmatics. In Portner, Paul, Maienborn, Claudia and von Heusinger, Klaus (eds.) *Semantics: An International Handbook of Natural Language Meaning*. Berlin: Mouton de Gruyter.
- Fox, Danny and Martin Hackl. 2006. On the Universal Density of Measurement. *Linguistics and Philosophy* 29(5): 537–86.
- Gajewski, Jon. 2007. Neg-Raising and Polarity. *Linguistics and Philosophy* 30(3): 289–328.
- Gajewski, Jon. 2008. More on Quantifiers in Comparative Clauses. In Gajewski, J, Friedman, T and Ito, S (eds.) *Proceedings of SALT 18*. 341–57. Ithaca, NY: Cornell University.
- Geurts, Bart. 2006. Take “Five”: The Meaning and Use of a Number Word. In Vogeleer, Svetlana and Tasmowski, Liliane (eds.) *Non-Definiteness and Plurality*. 311–29. Amsterdam/Philadelphia: Benjamins.
- Hackl, Martin. 2009. On the Grammar and Processing of Proportional Quantifiers: ‘Most’ Versus ‘More Than Half’. *Natural Language Semantics* 17: 63–98.
- Heim, Irene. 1988. *The Semantics of Definite and Indefinite Noun Phrases*.

- Outstanding Dissertations in Linguistics. New York: Garland.
- Heim, Irene. 1992. Presupposition Projection and the Semantics of Attitude Verbs. *Journal of Semantics* 9: 183–221.
- Heim, Irene. 2006. *Remarks on Comparative Clauses as Generalized Quantifiers*. MIT. Unpublished manuscript. URL <http://semanticsarchive.net/Archive/mJiMDBIN>.
- Heim, Irene. 2007. Little. In Gibson, Masayuki and Howell, Jonathan (eds.) 35–58. Ithaca: CLC Publications.
- Heim, Irene. 2008. Decomposing Antonyms? In Grønn, Atle (ed.) 212–25. Oslo: Department of Literature, Area Studies and European Languages, University of Oslo.
- Heim, Irene. 2001. Degree Operators and Scope. In Féry, Caroline and Sternefeld, Wolfgang (eds.) *Audiatur Vox Sapientiae. A Festschrift for Arnim von Stechow*. 214–39. Berlin: Akademie Verlag.
- Heim, Irene and Angelika Kratzer. 1998. *Semantics in Generative Grammar*. Oxford: Blackwell.
- Horn, Laurence. 1969. A Presuppositional Approach to ‘Only’ and ‘Even’. In Binnick, Robert I. et al. (eds.) *Papers From the Fifth Regional Meeting of the Chicago Linguistic Society*. 98–107. Chicago: University of Chicago.
- Huitink, Janneke. 2005. Analyzing Anankastic Conditionals and Sufficiency Modals. In Blaho, Sylvia, Vicente, Luis and Schoorlemmer, Erik (eds.) *Proceedings of Console 13*. 135–56. University of Leiden.
- Kennedy, Christopher. 1997. Projecting the Adjective: The Syntax and Semantics of Gradability and Comparison. Ph.D. Thesis. UC Santa Cruz.
- Kennedy, Christopher. 2001. Polar Opposition and the Ontology of Degrees. *Linguistics and Philosophy* 24(1): 33–70.
- Klein, Ewan. 1980. A Semantics for Positive and Comparative Adjectives. *Linguistics and Philosophy* 4: 1–45.
- Klein, Ewan. 1991. Comparatives. In Wunderlich, Dieter and von Stechow, Arnim (eds.) *Semantik. Ein Internationales Handbuch Zeitgenössischer Forschung*. 673–91. Berlin, New York: Walter de Gruyter.
- Krantz, D.H., R.D Luce, P. Suppes, A. Tversky. 1971. *Foundations of Measurement*. New York: Academic Press.
- Krasikova, Sveta. 2008. Quantifiers in Comparatives. In Grønn, Atle (ed.) *Proceedings of Sinn und Bedeutung 12*. 337/352. Oslo: Department of Literature, Area Studies and European Languages, University of Oslo.
- Krasikova, Sveta and Ventsislav Zhechev. 2006. You Only Need a Scalar ‘Only’. In Ebert, Christian and Endriss, Cornelia (eds.) *Proceedings of*

- Sinn und Bedeutung* 10. Berlin: ZAS.
- Kratzer, Angelika. 1977. What ‘Must’ and ‘Can’ Must and Can Mean. *Linguistics and Philosophy* 1: 337–55.
- Kratzer, Angelika. 1981. The Notional Category of Modality. In Eikmeyer, H. J. and Rieser, H. (eds.) *Words, Worlds, and Contexts*. 38–74. Berlin: de Gruyter.
- Kratzer, Angelika. 1991. Modality. In Wunderlich, Dieter and von Stechow, Arnim (eds.) *Semantik. Ein Internationales Handbuch Zeitgenössischer Forschung*. Berlin, New York: Walter de Gruyter.
- Kratzer, Angelika and Junko Shimoyama. 2002. Indeterminate Pronouns: The View From Japanese. Paper presented at *the 3rd Tokyo Conference on Psycholinguistics*.
- Krifka, Manfred. 1999. At Least Some Determiners Aren’t Determiners. In Turner, Ken (ed.) *The Semantics/Pragmatics Interface From Different Points of View*. 257–91. Elsevier Science B.V.
- Krifka, Manfred. 2007. *More on the Difference Between ‘More Than Two’ and ‘At Least Three’*. Handout of a talk delivered in University of California at Santa Cruz URL <http://amor.rz.hu-berlin.de/~h2816i3x/Talks/SantaCruz2007.pdf>.
- Larson, Richard K. 1988. Scope and Comparatives. *Linguistics and Philosophy* 11(1): 1–26.
- Lasnik, Peter. 1999. Pragmatic Halos. *Language* 75(3): 522–51.
- Lewis, David. 1970. General Semantics. *Synthese* 22: 18–67.
- Lewis, David. 1973a. Causation. *Journal of Philosophy* 70: 556–67.
- Lewis, David. 1973b. *Counterfactuals*. Oxford: Basil Blackwell.
- Lewis, David. 1973c. Counterfactuals and Comparative Possibility. *Journal of Philosophical Logic* 2(4): 418–46.
- Lewis, David. 1981. Ordering Semantics and Premise Semantics for Counterfactuals. *Journal of Philosophical Logic* 10.
- Marques, Rui. 2004. Towards a Semantics of Comparative Constructions in Portuguese. Ph.D. Dissertation. Universidade de Lisboa.
- Meier, Cecile. 2002. Maximality and Minimality in Comparatives. In Katz, Graham, Reinhard, Sabine and Reuter, Philip (eds.) *Proceedings of Sinn und Bedeutung* 6. 275–87. Osnabrück: University of Osnabrück.
- Oda, Toshiko. 2008. Degree Constructions in Japanese. Ph.D. Dissertation. University of Connecticut.
- van Rooij, Robert. 2008. Comparatives and Quantifiers. In Bonami, Oliver and Hofherr, P. Cabredo (eds.) *Empirical Issues in Syntax and Semantics*.

393–414.

- Rooth, Mats. 1985. Association With Focus. Ph.D. Dissertation. University of Massachusetts at Amherst.
- Rullmann, Hotze. 1995. Maximality in the Semantics of Wh-Constructions. Ph.D. dissertation. University of Massachusetts/Amherst.
- Russell, Bertrand. 1905. On Denoting. *Mind* 14: 479–93.
- Sæbø, Kjell Johan. 2001. Necessary Conditions in a Natural Language. In Féry, Caroline and Sternefeld, Wolfgang (eds.) *Audiatur Vox Sapientiae. A Festschrift for Arnim von Stechow*. Berlin: Akademie Verlag.
- Sassoon, Galit. 2007. Vagueness, Gradability and Typicality. A Comprehensive Semantic Analysis. Ph.D. Thesis. Tel Aviv University.
- Sauerland, Uli. 2004. Scalar Implicatures in Complex Sentences. *Linguistics and Philosophy* 27(3): 367–91.
- Schwarzschild, Roger. 2004. *Scope Splitting in the Comparative*. Handout of a talk delivered at MIT Colloquium on October 15th.
- Schwarzschild, Roger. 2008. The Semantics of Comparatives and Other Degree Constructions. *Language and Linguistics Compass* 2.2: 308–31.
- Schwarzschild, Roger and Karina Wilkinson. 2002. Quantifiers in Comparatives: A Semantics of Degree Based on Intervals. *Natural Language Semantics* 10(1): 1–41.
- Seuren, Peter A. 1973. The Comparative. In Kiefer, Ferenc and Ruwet, N. (eds.) *Generative Grammar in Europe*. 528–64. Dordrecht: Riedel.
- Sloman, Aaron. 1970. Ought and Better. *Mind* 79(315): 385–94.
- Stalnaker, Robert. 1984. *Inquiry*. Cambridge, MA: MIT Press.
- von Stechow, Arnim. 1984. Comparing Semantic Theories of Comparison. *Journal of Semantics* 3: 1–77.
- von Stechow, Arnim. 1985. Focusing and Backgrounding Operators. *Technical report. Universität Konstanz*.
- von Stechow, Arnim. 2002. Temporal Prepositional Phrases With Quantifiers: Some Additions to Pratt and Francez (2001). *Linguistics and Philosophy* 25(5-6): 755–800.
- von Stechow, Arnim. 2005. Semantishes und Morphologisches Tempus: Zur Temporalen Orientierung von Einstellungen und Modalen. *Neue Beiträge zur Germanistik* 4(2): 9–54.
- von Stechow, Arnim. 2008. *Topics in Degree Semantics: 4 Lectures. Handout 1: Degrees*. Paris. Unpublished manuscript. URL <http://www.sfs.uni-tuebingen.de/~astechow/Lehre/Paris08/Stechow/Degrees1.pdf>.

- von Stechow, Arnim and Atle Grønn. 2009. *The (Non-)Interpretation of Subordinate Tense*. Handout of a talk delivered in Göttingen on November 9th. URL http://www2.sfs.uni-tuebingen.de/~arnim10/Handouts/SOT_Goettingen.pdf.
- von Stechow, Arnim, Sveta Krasikova and Doris Penka. 2006. Anankastic Conditionals Again. In Solstad, Torgrim, Grønn, Atle and Haug, Dag (eds.) *A Festschrift for Kjell Johan Sæbo*. Oslo: University of Oslo, Department of Literature, Area Studies and European Languages.
- Villalta, Elisabeth. 2006. Context Dependence in the Interpretation of Questions and Subjunctives. Ph.D. Dissertation. Universität Tübingen.
- Villalta, Elisabeth. 2008. Mood and Gradability: An Investigation of the Subjunctive Mood in Spanish. *Linguistics and Philosophy* 31: 467–522.