The Spread of COVID-19 in Germany
An Application of the SIRDH Model

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The Spread of COVID-19 in Germany
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Abstract

The paper studies the current COVID-19 pandemic by applying an adapted epidemiologic model, where each individual is in one of the five states “susceptible”, “infected”, “removed”, “immune healthy” or “dead”. We extend the basic model with time-invariant transition rates between these states by allowing for time-dependent infection rates as a consequence of lockdowns and social distancing policies as well as for time-dependent mortality rates as a result of changing infection patterns. Our model proves to be appropriate to calibrate and simulate the dynamics of COVID-19 pandemic in Germany between January and October 2020. We provide deeper insights about some key indicators such as the reproduction number, the effectiveness of non-pharmaceutical interventions, and the development of the infection and mortality rates.

Keywords: COVID-19, SIRDH model, health behavior
JEL Classification: I12, I18

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1. Introduction

Infections with the coronavirus SARS-CoV-2, short for “severe acute respiratory syndrome coronavirus 2” first appeared in Wuhan/China in early December 2019 (Wu et al., 2020), causing the disease COVID-19 (WHO, 2020b). Since then the virus is spreading continuously over the world population, leading not only to millions of infections and hundreds of thousands of deaths (WHO, 2020f) but also having the potential to cause a global economic downturn such as the world has not experienced since 75 years (World Bank, 2020). According to the data provided by the Johns Hopkins University Center for Systems Science and Engineering (JHU CSSE)¹, there are more than a total of 38 million reported cases worldwide that occurred since the beginning in January until mid-October 2020. Despite a variety of public life restrictions such as mask wearing or social distancing, the virus still persists in most parts of the world (JHU CSSE, 2020).

To model the dynamics of infectious diseases, scientists usually apply the SIR model, which is short for “susceptible” S, “infected” I, and “removed” R. Indeed, this model has a long history of successfully simulating the spread of several types of diseases over the last hundred years, ranging from smallpox to measles to seasonal influenza viruses (Hethcote, 2008). At least two aspects of the current corona crisis make an extension of the original SIR model necessary: First, COVID-19 is a disease that is fatal for about 1% of infected individuals (compare Davies et al., 2020; Pastor-Barriuso et al., 2020; Russell et al., 2020; Salje et al., 2020; Verity et al., 2020; Ward et al., 2020). Second, governments imposed several interventions aimed at reducing the transmission of the virus and as a result, infection levels as well as deaths rates were significantly reduced (Flaxman et al., 2020). Therefore, we extend the basic SIR model (i) by taking into account the transition processes from “removed” R to “dead” D or to “immune healthy” H, and (ii) by allowing for time-dependent infection and mortality rates due to governmental interventions as well as behavioral adaptations of people.

The aim of our study is to explain and to simulate the dynamics of the SARS-CoV-2 pandemic in Germany between January and October 2020. Thereby, we provide insights about some key indicators such as the reproduction number, the effectiveness of non-pharmaceutical interventions and the infection fatality rate.

¹ The John Hopkins University provides a constantly updated COVID-19 data set as developed by Dong et al. (2020) in February 2020 in order to demonstrate the development of the global case numbers.
The rest of the paper is organized as follows: Section 2 presents an overview of the development of the coronavirus and provides information about the most important characteristics of the current COVID-19 disease which is not only crucial for being up to date with the state-of-the-art research but even more for designing an appropriate pandemic model. In Section 3, we describe the spread of the virus in Germany between January and October 2020. Section 4 presents our version of the SIRDH model and derives some important indicators such as the reproduction number and the infection level needed for herd immunity. In Section 5, we apply this model to the pandemic development in Germany, including different scenarios such as an uncontrollable spread of COVID-19 compared to the actual development. Finally, Section 6 concludes the paper.

2. The Background of the Pandemic

In this section, we provide an overview of the pandemic background of the corona crisis. First, we report on the development and the status quo of the pandemic and afterwards we summarize the key characteristics of COVID-19.

2.1. Development and Status Quo

SARS-CoV-2 is related to previous coronaviruses like SARS-CoV or MERS-CoV that occurred in 2003 and 2012, respectively (Petersen et al., 2020). While there are still research efforts to clarify the origin of the new coronavirus, currently it is believed that the virus can be traced back to animals, likely bats, with the exact route of transmission for the first human infection being still unclear (WHO, 2020c).

Starting point of the spread of the novel virus is believed to be late December 2019 with many cases tracing back to the seafood market in the city of Wuhan (WHO, 2020c). From there on, the coronavirus spread within the Hubei region and also reached China’s neighboring countries such as Thailand, Japan and South Korea that all reported first cases until the 20th of January, 2020 (WHO, 2020a). Soon, Europe was affected by the virus with the first officially confirmed cases reported in France at the 24th of January, 2020 (Spiteri et al., 2020; Stoecklin et al., 2020). However, compared to China, total infections in Europe and the rest of the world remained relatively low during the following weeks. On the 11th of March, when the WHO decided to declare the crisis as a pandemic, almost 70% of the 118,000 global infections were still attributed to China (WHO, 2020d). However, this picture changed. While China successfully

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2 See also Petersen et al. (2020) for similarities and differences between previous coronavirus outbreaks, influenza viruses and COVID-19.
reduced the spreading of the virus through governmental interferences (Tian et al., 2020), the pandemic unfolded its full impact in Europe where total infections already exceeded documented cases in Asia in late March 2020 (ECDC, 2020).

An important parameter in epidemiology is the basic and effective reproduction number, denoted here as $R_0$ and $R_e$ respectively. The basic reproduction number can be defined as “the average number of secondary cases produced by one infected individual introduced into a population of susceptible individuals” (van den Driessche, 2017, p. 289). In the case of the novel corona virus, $R_0$ is estimated to lie between 2 and 3 (Y. Liu et al., 2020; Tian et al., 2020; WHO, 2020c). In other words, this indicates that an infected individual infects on average between two and three persons during his or her illness. $R_0$ denotes the reproduction number at the very beginning of an outbreak, whereas $R_e$ represents the actual value of the reproduction number and thus varies over time (Chowell et al., 2004). The importance of these epidemiological parameters rely on the fact, that a reproduction number above 1 leads to the potential of a pandemic with many infections, whereas a reproduction number below 1 makes an infectious disease disappear in the long run (van den Driessche, 2017).

In fact, there are three possibilities to reduce the reproduction number and to reach a controllable level of COVID-19 infections: First, governments can rely on so-called non-pharmaceutical interventions (NPIs) (Ferguson et al., 2020). NPIs can best be described with the term “social distancing” (R. M. Anderson et al., 2020). This generic term includes a variety of measures, all of which aim to either bring infected and uninfected persons together at a lower rate or to keep a certain physical distance to make a transmission less likely. Social distancing policies used in the COVID-19 pandemic are for example school and business closures, ban of mass gatherings, closing of borders or curfews (see You Li et al. (2020) for the use and effectiveness of these NPIs). During the first wave\(^3\) of the pandemic, many European countries followed a lockdown\(^4\) strategy that led to a decline of the reproduction number of more than 80% (Flaxman et al., 2020). In New Zealand, a suppression strategy not only reduced the reproduction number but even led to zero new cases in early May 2020 (Cousins, 2020). In addition, there is also evidence that interventions such as wearing a mask or keeping minimum distances between persons, can also contribute to hinder the virus from spreading. In their meta-analysis study, Chu et al. (2020) estimate that the risk of transmission is five times less likely if individuals keep a distance of more than one meter with even greater effects if distances are larger. While the effectiveness to stop the virus from spreading depends on the mask type (Chu

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3 Understood as the time between March and April 2020 characterized by relatively high infection levels.
4 Lockdown is understood as a broad term for various social distancing policies implemented at the same time.
et al., 2020), experiments by Fischer et al. (2020) show that already surgical face masks or simple cotton masks can have an impact on reducing droplet transmission. Moreover, despite this experimental evidence there are also studies that analyze the development in various regions such as the city of Jena (Mitze et al., 2020) or New York, Wuhan and Italy (Zhang et al., 2020) with the conclusion that mask wearing in public is associated with a decline in case numbers.

As a note, studies also point out the risks of scenarios in which governments would have not decided to fight the spread of COVID-19. For example, Ferguson et al. (2020) estimate that in the UK and the US, about 80% of the population would have been infected within four months. The situation presented in such a scenario would not only lead to an overload of the health care system but cause several hundreds of thousands of deaths (Ferguson et al., 2020). This is supported by another study of Flaxman et al. (2020) which suggests that without social distancing, total deaths in the five biggest European countries (France, Germany, Italy, UK, and Spain) could have been in total above 2.9 million already in early May 2020.

Another approach to fight the virus is the so-called mitigation strategy with the long-term goal to reach herd immunity (Drozd & Tavares, 2020). Herd immunity is reached when the majority of the population got exposed to the virus – implying immunity against a re-infection – and thus with less and less individuals being susceptible to the virus, the spread of the virus eventually declines (Hethcote, 2008). However, there are certain risks and drawbacks associated with a mitigation strategy. On the one hand, as Ferguson et al. (2020) and Bock et al. (2020) point out, mitigation strategies aiming at acquiring herd immunity within a short time period are hardly applicable for COVID-19, as the number of intensive care patients and deadly outcomes would be too high for health systems to handle. On the other hand, studies show that generated antibodies might disappear several weeks after an infection (Ibarraondo et al., 2020; Seow et al., 2020). Indeed, the study by To et al. (2020) confirmed that it is possible to be infected again after being exposed to the virus.

Moreover, many so-called seroprevalence studies that analyze the existence of antibodies point in the same direction: While the extent to which the population of European countries got exposed to the virus in Spring 2020 is significantly larger than documented by national health authorities or institutions such as the Johns Hopkins University, total infection levels are still in the single digit percentage area and therefore too low to constitute an effective barrier to hinder the virus from spreading (Flaxman et al., 2020; Pollán et al., 2020; RKI, 2020f; Salje et al., 2020).
However, herd immunity might be reached with the help of vaccination. In general, vaccine development is characterized by two different stages, namely the pre-clinical and clinical stage, followed by a market entry after the vaccine is approved by the concerning regulatory institution (Pronker et al., 2013). On average, such a process is estimated to take about ten years with an average success rate of 6% (Pronker et al., 2013). However, in the case of COVID-19, these gradual development steps are speeded up and clinical trials did already start as soon as March 2020 (Lurie et al., 2020). According to an overview provided by the WHO (2020f), in October 2020 almost 200 vaccine manufactures and research institutes were working on the development of vaccines for COVID-19 and there are in total ten companies currently testing their product in the last out of three clinical trial stages (WHO, 2020e). However, it remains speculative whether approved vaccines will be available already in late 2020 as expressed by Agrawal et al. (2020) or only in 2021 (R. M. Anderson et al., 2020).

In sum, herd immunity via infection or vaccination remains rather unlikely in autumn and winter 2020. When effective social distancing measures are cancelled at an insufficiently high level of immunity in the population, then a resurrection of COVID-19 cases is a likely scenario (R. M. Anderson et al., 2020; Ferguson et al., 2020; Salje et al., 2020). In fact, according to the numbers provided by JHU CSSE (2020), after most European countries overcame the first wave of coronavirus infections, COVID-19 resurged in the late Summer 2020 on the continent with case numbers starting to increase again, thereby reaching or exceeding similar daily detected case numbers as in March and April 2020.

2.2. Key Characteristics of COVID-19

The novel coronavirus is either transmitted directly from one individual to another (Chan et al., 2020; Ghinai, McPherson, et al., 2020) or through contaminated objects – so called fomites (WHO, 2020c). When it comes to the transmission from one person to another, the virus passes from an infected to a susceptible individual via so-called droplets or aerosols (Prather et al., 2020). Infection via droplets mainly occurs through coughing or sneezing whereby the infected person is close to other uninfected individuals (Asadi et al., 2020). In contrast to those rather large droplets, aerosols are smaller and remain in the air much longer with the possibility to travel several meters (Prather et al., 2020). At the beginning of the pandemic, aerosol transmission was believed to be only a minor factor (WHO, 2020c), however, recently published studies suggest otherwise (see e.g. E. L. Anderson et al., 2020; Lednicky et al., 2020; Yuguo Li et al., 2020; Morawska & Cao, 2020; Zhang et al., 2020). In general, indoor environments are associated with a higher probability of a virus transmission (Morawska &
Cao, 2020) with estimations suggesting that an infection with SARS-CoV-2 may be more than ten times as likely as compared to an outdoor environment (Nishiura et al., 2020). Especially overcrowded rooms and insufficient ventilation are a breeding ground for virus infections (Yuguo Li et al., 2020; Morawska & Cao, 2020; Nishiura et al., 2020). There exist many reports tracing back infections to single events or clusters such as nightclubs (Kang et al., 2020), parties or even funerals (Ghinai, Woods, et al., 2020), indoor restaurant visits (Lu et al., 2020), sport groups (Jang et al., 2020), choirs (Hamner et al., 2020), schools (Stein-Zamir et al., 2020), business meeting rooms or religious facilities like churches (Pung et al., 2020), or cruise ships like the Diamond Princess (Tabata et al., 2020).

The symptoms of corona patients vary in degree of severity ranging from mild to severe symptoms even leading to death and there are no unique symptoms indicating an infection by COVID-19 (WHO, 2020c). However, some symptoms seem to occur more frequently than others. As the investigation of several symptomatic corona patients by Lechien et al. (2020) and Zhu et al., (2020) for Europe and China suggest, the most common symptoms are among others headache, cough, fever, loss of smell or taste or muscle soreness. Patients also report long term damages such as lasting fatigue (Arnold et al., 2020; Carfì et al., 2020). In contrast, about 33% of infected may overcome an infection without experiencing symptoms at all (Pollán et al., 2020; Ward et al., 2020).

In general, existing pre-conditions (such as diabetes or chronic lung diseases), an unhealthy way of life and being of higher age are associated with an increased risk for COVID-19, meaning those individuals have a higher probability of needing additional treatment in hospitals or dying in the course of the disease (CDC, 2020; Hamer et al., 2020; Verity et al., 2020). On a global and regional level, the study by Clark et al. (2020) estimates which fraction of a population may have a higher probability to experience a severe disease progression due to pre-existing conditions or age. These results suggest that about one in five individuals worldwide could experience severe symptoms in the course of a SARS-CoV-2 infection and about four out of 100 infected would need to be transferred to an hospital (Clark et al., 2020). Especially for Germany, due to a higher proportion of elderly in the population, the estimations by Clark et al. (2020) are slightly higher compared to world average. However, simply being young and healthy does not imply that those individuals remain unaffected by the virus. There are also reported cases in which healthy children needed treatment in hospitals (Götzinger et al., 2020). Moreover, it is crucial to know how many people die from or with COVID-19. One way to measure the mortality rate of the corona virus is to use the so-called case fatality rate (CFR), which is the ratio between the total number of deaths and official confirmed cases (Vanella et
The CFR of early October 2020 for Germany ranks according to JHU CSSE, (2020) at 2.8%, which is in the range of other European countries such as France (4.0%) or Spain (3.6%), but significantly lower than for example the relative high rate of Italy of about 9.5%. However, there are many drawbacks associated with using the CFR in the case of COVID-19. First, differences in the amount of testing or age distributions may explain discrepancies and thus make the CFR unsuitable to compare different countries (Vanella et al., 2020). Second, the CFR does not account for undocumented cases (Streeck et al., 2020). Consequently, this rate may overestimate the lethality of COVID-19 (Russell et al., 2020). Therefore, one can use the so-called infection fatality rate (IFR) which can be defined as “the average number of deaths per infection” (Perez-Saez et al., 2020, p. 1). Estimations suggest an overall IFR of approximately between 0.5% and 1.3% for various countries such as the UK, France, China and Spain (Davies et al., 2020; Pastor-Barriuso et al., 2020; Russell et al., 2020; Salje et al., 2020; Verity et al., 2020; Ward et al., 2020). Up to this date, there is no peer-reviewed study proposing an IFR especially for Germany as a whole. A pre-print study by Streeck et al. (2020) estimates a range between 0.36% and 0.41% for the North Rhine-Westphalian municipality of Gangelt and is therefore below the reported numbers above. It is important to note that the IFR strongly differs with respect to age. In one study the IFR for those younger than 60 is below 0.2%, for those aged older than 60 is approximately 3.3% and those being aged 80 or older are at an especially high risk with an IFR of 7.8% (Verity et al., 2020)5.

The duration of the disease depends on the degree of severity with those experiencing only mild symptoms needing approximately between one and two weeks to recover, whereas those individuals with very serious disease progression may need up to six weeks to be symptom free (WHO, 2020c). However, experiencing symptoms for several weeks does not automatically imply that these patients are also infectious for the same period (Boscolo-Rizzo et al., 2020). Analyses suggest an average infectivity period between seven and ten days after the development of symptoms with the degree of infectivity being significantly lower at the end of the infectious period (He et al., 2020; Kampen et al., 2020; Singanayagam et al., 2020; Wölfel et al., 2020).

5 While the estimated numbers in similar studies slightly differ (compare Pastor-Barriuso et al., 2020; Russell et al., 2020; Salje et al., 2020; Verity et al., 2020), those studies clearly indicate that the mortality risk for the younger population is relatively low whereas the older generation has a relatively high risk of deceasing in the course of the disease.
3. The Phases of COVID-19 in Germany

In order to calibrate the model, we have to distinguish at least three different phases of the pandemic: Phase I captures the period before the lockdown when the pandemic spread out without important policy measures to reduce the infection rate. This case will be captured by the basic SIRDH model with time-invariant transition rates. Phase II corresponds to the lockdown period and phase III refers to the period in which the policy measures have been partly relaxed leading to a slow, but steady revival of active case numbers. These two phases will be captured by an extended SIRDH model with time-dependent transition rates. This division into three different phases is illustrated in Figure 1.

![Figure 1: Development of Active COVID-19 Cases Divided into Three Different Phases](image)

### 3.1. Phase I: The Pre-Lockdown Development

According to the data provided by JHU CSSE (2020), the first officially reported COVID-19 case already occurred at 27th of January 2020, but recorded case numbers only slightly increased such that, in late February, only 20 cases in total were officially reported. However, this picture totally changed in the following weeks. Notably, early March was associated with two superspreading events. First, while total infections were still below 300, over 40% of the reported cases were attributed to a carnival party in the Heinsberg region of Nord-Rhine-Westphalia (RKI, 2020a). Second, many infections occurred in Ischgl/Austria which is a popular skiing resort. This place is also associated with contributing to the spread of the virus after the German tourists returned from their holidays (Felbermayr et al., 2020).
Eventually, the number of cases rapidly increased and after total infections exceeded the one-thousand-mark on the 8th of March, it only took ten days to surpass a total of ten thousand confirmed infections (JHU CSSE, 2020). According to Hartl et al. (2020), in the first three weeks of March 2020, confirmed cases duplicated within three days and the number of new infections increased by approximately 27% per day. There are several scenario analyses pointing out that an overwhelming of the health care system associated with a huge number of deaths could have occurred if the German governments would have not decided to interfere via various social distancing policies. For example the estimations by Flaxman et al. (2020) suggest, that without any restrictions at all, Germany could have experienced more than 500,000 deaths in early May. In addition, the Ministry of the Interior discussed several scenarios for the COVID-19 development in Germany and estimated that more than 300,000 beds in intensive care units could have been needed (BMI, 2020).

3.2. Phase II: The Lockdown

Around mid-March, the German government imposed several policies with the most prominent ones listed in Table 1.

<table>
<thead>
<tr>
<th>Social Distancing Policies</th>
<th>Date</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cancellation of Mass Gatherings</td>
<td>11.03.2020</td>
<td>(RKI, 2020b)</td>
</tr>
<tr>
<td>School and Border Closures</td>
<td>16.03.2020</td>
<td>(RKI, 2020c)</td>
</tr>
<tr>
<td>Closure of Non-Essential Stores</td>
<td>22.03.2020</td>
<td>(BReg, 2020a, 2020b)</td>
</tr>
<tr>
<td>Contact Ban</td>
<td>22.03.2020</td>
<td></td>
</tr>
<tr>
<td>Physical Distance</td>
<td>22.03.2020</td>
<td></td>
</tr>
<tr>
<td>Face Masks</td>
<td>29.04.2020</td>
<td>(RKI, 2020d)</td>
</tr>
<tr>
<td>Corona App</td>
<td>16.06.2020</td>
<td>(RKI, 2020g)</td>
</tr>
<tr>
<td>Intensified Regulations for local “Hot Spots”</td>
<td>14.10.2020</td>
<td>(BReg, 2020d)</td>
</tr>
</tbody>
</table>

As the estimations by Hartl et al. (2020) show, the pace of virus transmission already decreased around the 20th of March. Dehning et al. (2020) estimate that the approximated reproduction number $R=1.15$ in mid-march was less than half compared to the beginning of the pandemic. However, a reproduction number slightly above 1 means that several thousands of infected individuals again infect several thousands of new individuals. Therefore Dehning et al. (2020)
conclude, that besides interventions such as closure of schools or mass gathering bans, further restrictions were needed to finally suppress the virus transmission in Germany.

In fact, on the 22nd of March 2020, the German government imposed three significant social distancing policies leading to nationwide lockdown: First, restaurants, museums and similar institutions and leisure activities were closed, with only the retail sector such as drugstores, supermarkets or pharmacies remaining open to supply the population (BReg, 2020a). Second, nationwide contact restrictions or contact bans were introduced, meaning that, with exceptions for families and households, only two persons were allowed to meet at the same time and it was required in public life to keep a physical distance of 1.5 meters between individuals (BReg, 2020b). According to Dehning et al. (2020), it was due to these final measures that the reproduction number could be reduced below 1. Moreover, the effects of the social distancing measures introduced in mid-March were reflected in a change of individual behavior. As shown by Schlosser et al. (2020), individual mobility declined down to 40% in the respective period, meaning that individuals were less often on the move than before the lockdown. Thus, daily case numbers did not continue to grow, but new infections peaked in late March and early April 2020 with a maximum of almost 7,000 new confirmed cases a day (JHU CSSE, 2020).

Apart from that, in early April, there was still uncertainty whether the spread of the corona virus would lead to an overwhelming of the health care system and if it would come to so-called triage situations, in which critical case numbers are so high that physicians cannot guarantee sufficient care for all patients (Stang et al., 2020). However, according to data provided by the DIVI (Deutsche Interdisziplinäre Vereinigung für Intensiv- und Notfallmedizin), with a maximum of slightly more than 2,900 occupied beds during the first wave, the overall share of corona patients at the intensive-care-unit did not exceed the overall capacity, as more than 10,000 beds were still available for medical treatment (compare DIVI, 2020). Notably, after some cities such as Jena or federal states like Saxony already started some days earlier, the wearing of face masks for public transport and shopping became mandatory for all federal states on the 29th April (RKI, 2020d).

To summarize, in the lockdown period, case numbers dropped continuously. In May 2020, for the first time since mid-March, less than 1,000 daily infections were reported repeatedly and estimated active infections decreased by more than 70% from approximately 70,000 in April to less than 20,000 (compare JHU CSSE, 2020).
3.3. Phase III: The Post-Lockdown Period

Eventually, the NPIs introduced in March and April 2020, leading to a nationwide lockdown, were relaxed gradually. While some restrictions such as the ban of mass gatherings or masks remained in place in most German regions, non-essential stores, educational and public institutions as well as many leisure activities reopened under certain restrictions (BReg, 2020c). As shown in Figure 1, despite those liftings, Germany did not immediately experience a significant rise in case numbers leading to a similar situation as compared to March or April. Case numbers increased only sporadically for a very short period. For example, infections in a meat production facility in North Rhine-Westphalia led to over 1,000 new cases, pushing the effective reproduction number above 1 and causing a temporal, local lockdown in the respective region (RKI, 2020e). In general, the course of infection in the post lockdown period might be partially driven by cluster infections, as the daily situational reports provided by the Robert-Koch-Institute often register infections related to family parties or production facilities such as the above mentioned slaughterhouses (see for example RKI, 2020e, 2020i). As a note, compared to the peak of the pandemic in April, the CFR decreased during the post lockdown period from 4.7% to currently 2.8% in October 2020 (compare JHU CSSE, 2020). In other words, less persons died per documented infection in the months after the lockdown. The reason is that in earlier periods of the infection occurrence, the elderly accounted for a larger fraction of total infections, compared to later periods in which overall infections occurred more frequently in individuals younger than age 60 (RKI, 2020i). As discussed in previous chapters, age is an important variable when it comes to the likelihood of infected experiencing a more critical outcome.

In summer 2020, the situation changed compared to the previous months of June and May, characterized by low infection numbers, as daily confirmed cases exceeded the one-thousand-mark for several days in August 2020 (JHU CSSE, 2020), leading the RKI to state that the development is “concerning” (RKI, 2020h, p. 1). The COVID-19 situation continued to deteriorate in Autumn 2020 and on the 15th of October, with 7,600 daily confirmed cases, the numbers reached an all-time high and thus exceeded the infection occurrence during the first wave followed by a nation-wide lockdown (JHU CSSE, 2020). Due to the resurgence of the virus, the German legislative agreed to intensify their COVID-19 strategy on the 14th of October 2020 to especially target regional outbreaks exceeding a certain threshold of infections by implementing stricter regulations such as additional masking rules or contact restrictions (BReg, 2020d). At the beginning of October, a slightly positive trend in the number of daily deaths could be observed in the JHU CSSE (2020) data. However, it remains to be seen to what
extent this is a lasting trend and whether the death numbers will return to the level observed during the first wave.

4. The Theoretical Model

In order to trace the described development of the pandemic, we apply a highly stylized theoretical model, the so-called SIRDH model. It consists of the dynamic transitions between the states S (susceptible), I (infected), R (removed), D (death), and H (immune healthy). With respect to the phase I, we use the basic model with time-invariant transition rates. The phases II and III, however, are characterized by governmental interventions, ranging from the introduction to the relaxation of requirements. Therefore, we will use an extended version of the model with time-dependent transition rates.

4.1. The Basic SIRDH Model with Time-Invariant Transition Rates

The classical SIR models are based on the work by Kermack and McKendrick (1927) and are since then widely used to analyze the spread of diseases of the last two centuries (Hethcote, 2008; Z. Ma, 2009). Among many others, the standard SIR model or variations of it are used to study notable infectious diseases such as the well-known influenza virus in 1918 and more recently for research in Ebola or the SARS outbreak in 2003 (see e.g. Chowell et al., 2004; Ng et al., 2003; Weitz & Dushoff, 2015; Yu et al., 2017). Since the beginning of the COVID-19 pandemic in China in late December 2019, there exists a wide range of studies based on the SIR type models to calibrate or predict the spread of the novel corona virus. The specification of these models has to be adapted to basic characteristics of the COVID-19 pandemic as reported above: At first this means that an appropriate model must be able to capture the dynamic process of the spread of the virus under changing circumstances such as governmental interventions and relaxations of these interventions (e.g. Atkeson, 2020; James et al., 2020). Second, it should be able to provide information about the critical share of the population that must be infected in order for the whole population to acquire herd-immunity (Britton et al., 2020) or information about some key epidemiological indicators such as the (basic) reproduction number (R. Li et al., 2020). Dehning et al. (2020) use the SIR model to assess for Germany how NPIs reduced the infection rate and how the pandemic might have developed if the government had reacted in a different way to the spread of the virus. Toda (2020) provides similar estimations for the infection rate for the period of late March 2020 for about 30 countries. Similar, adding a death compartment to the equations, Fernández-Villaverde & Jones (2020) provide an analysis about the mortality of COVID-19 including various countries as
well as certain regions such as the City of New York, Madrid or Stockholm. As a note, some studies such as Toda (2020) combine epidemiological and economic features to assess which governmental intervention strategy may be the best. Moreover, various studies show how a variable infection rate can be used to account for the use of NPIs. For example, Atkeson (2020) and Baker (2020) describe from a theoretical perspective how time-dependent transmission rates change the dynamics of the SIR model and Rossi and Ianni (2020) show that it is possible to use a decreasing exponential function to simulate the COVID-19 development in Italy. In sum, these studies demonstrate that adequately adapted SIR models are very appropriate to simulate the current COVID-19 pandemic.

We use a variant of the model of Fernández-Villaverde and Jones (2020) to simulate the COVID-19 pandemic in Germany. The transition dynamics of the model, here referred to as the SIRDH model, is represented in Figure 2

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**Figure 2: Transition Dynamics of the SIRDH Model**

At the beginning of the process, nearly the whole population $N$ consists of susceptible individuals $S$. This means that, in the absence of vaccination, anyone can be potentially infected with the virus (WHO, 2020c). Susceptible may get the COVID-19 disease via contact with infected individuals for example through droplets or aerosol transmission. These contacts are captured by the infection rate $\beta$. The infected compartment $I$ refers to those who are exposed to SARS-CoV-2. At the rate $\gamma$, these individuals lose their infectivity and remove from state $I$ to state $R$. It is assumed that those who are removed from the disease acquire immunity rendering a re-infection with the same virus impossible. Moreover, after a period of $\frac{1}{\delta}$ days on average, implying the exit rate $\theta$, the individuals move either to the dead compartment ($D$) with probability $\delta$ or to the healthy compartment ($H$) with the counter probability $1 - \delta$. The five components sum up to $S(t) + I(t) + R(t) + D(t) + H(t) = N$, where the population size $N$ is assumed to be time-invariant. This pandemical dynamics is described by the following set of differential equations:
\[ \frac{dS}{dt} = -\beta \frac{IS}{N}, \quad \beta > 0 \]  \hfill (1)

\[ \frac{dl}{dt} = \beta \frac{IS}{N} - \gamma I, \quad 0 < \gamma < \beta \]  \hfill (2)

\[ \frac{dR}{dt} = \gamma I - \theta R, \quad \theta > 0 \]  \hfill (3)

\[ \frac{dD}{dt} = \delta \theta R, \quad 0 < \delta < 1 \]  \hfill (4)

\[ \frac{dH}{dt} = (1 - \delta) \theta R \]  \hfill (5)

At the beginning of the pandemic, it holds that \( S(0) \approx N \) so that (2) can be rewritten as
\[ \frac{dl}{dt} = (\beta - \gamma)I. \]

This differential equation implies exponential growth at the constant rate \((\beta - \gamma)\). Later on, the time path for the number of infected individuals is given by the differential equation
\[ \frac{dl}{dt} = \gamma I(R_e - 1), \]  \hfill (6)
where \( R_e = \frac{\beta S}{\gamma N} \) is the effective reproduction number and \( R_0 = \frac{\beta}{\gamma} \) is the basic reproduction number at the beginning of the pandemic when \( \frac{S(0)}{N} \approx 1 \). As can be seen from (6), in the case of \( R_e > 1 \) the virus spreads through the whole population.

Given the short-run capacity constraints of the hospitals’ intense care units, it is very important to calculate the maximum number \( I_{\text{max}} \) of infected persons. Due to the separability of the differential equations (1) and (2), we are able to solve the SIRDH model with respect to this number. Division of (2) by (1) gives
\[ \frac{dl}{dS} = \frac{dl/dt}{dS/dt} = \frac{R_0}{S} - 1. \]

Integration with respect to S leads to
\[ I(S) = R_0 \ln S - S + A. \]

Due to the initial condition \( I(S = N) = R_0 \ln N - N + A = 0 \), we obtain for the constant of integration \( A = N - R_0 \ln N \) so that
\[ I(S) = R_0 \ln(S/N) + N - S. \]

The first-order condition for the unique maximum of \( I(S) \) requires \( S = R_0 \) and hence
\[ I_{\text{max}}(S = R_0) = 1 - (1 + \ln R_0)/R_0. \]  \hfill (7)
By using the basic reproduction number, it is also possible to derive another key epidemiological parameter, namely the herd immunity threshold \( h = (N - S_h)/N \). This threshold value indicates the share of the immunologically healthy population that is necessary to avoid the spread of the pandemic. Since this critical value is given by \( R_e = \frac{R_0 S_h}{N} = 1 \), one obtains \( S_h = N/R_0 \) and thus

\[
h = 1 - \frac{1}{R_0}.
\]

(8)

Herd immunity may not only be reached through individuals becoming infected and recovered but also with the help of vaccination. Therefore, (8) equivalently corresponds to the share of the population needed to be vaccinated such that the number of infected individuals \( I(t) \) declines over time (van den Driessche, 2017).

Figure 3: Numerical Simulation of a Pandemic Development

A numerical example of the SIRDH dynamics is illustrated in Figure 3. Here, the total population is standardized to \( N=1 \). The initial conditions are given by \( S(0) = 0.99 \) and \( I(0) = 0.01 \). The transition rates are given by \( \beta = 0.3 \) and \( \gamma = \theta = 0.1 \) leading to the basic reproduction number of \( R_0 = 3 \). The maximum share of infected people is calculated as \( I_{\text{max}} \approx 0.3 \) and the threshold value for herd immunity corresponds to the share \( h \approx 0.67 \) of immunologically healthy persons. The probability of death is given by \( \delta = 0.01 \). This implies that, in the long run, the population converges to the share \( H = 0.99 \) of immunologically healthy people and the share \( D= 0.01 \) of deceased people.
4.2. The SIRDH Model with Time-Dependent Infection and Mortality Rates

The model described so far is characterized by fixed parameter values for all transition rates. However, for several reasons this is not true for the second and third phases of the COVID-19 pandemic. Of course, all governments reacted more or less to the development and imposed various restrictions to condemn the spread of SARS-CoV-2. Moreover, even without governmental policy, the individuals as a whole adapt their behavior during pandemics in order to lower the transmission of the virus (for earlier infection disease outbreaks see e.g. Funk et al. (2009) and R. Liu et al. (2007)). For this reason, we relax the assumption of fixed transition rates and introduce a time-dependent infection rate $\beta(t)$ as well as a time-dependent mortality $\delta(t)$.

To specify a variable infection rate $\beta(t)$, we follow the approach of Chowell et al. (2004) and assume a fixed transmission rate $\beta_0$ in the first phase and a decreasing logistic function for the second. Additionally, we use an increasing logistic function for the third phase. This allows us to capture the three main stages of the German pandemic, namely the pre-lockdown, lockdown, and the post-lockdown period. We specify the development of the infection rate as

$$
\beta(t) = \begin{cases} 
\beta_0 & \text{for } t \leq l \\
\beta_1 + (\beta_0 - \beta_1)e^{-k_1(t-l)} & \text{for } l < t \leq m \\
\beta_2 - (\beta_2 - \beta_1)e^{-k_2(t-m)} & \text{for } m < t 
\end{cases}
$$

where $l$ and $m$ correspond to the lockdown and lifting dates and the parameters $\beta_1 < \beta_2 < \beta_0$ and $k_1, k_2 > 0$ determine the time path of the infection rate. With respect to the mortality rate, we assume two periods and specify the development as

$$
\delta(t) = \begin{cases} 
\delta_0 & \text{for } t \leq m \\
\delta_1 + (\delta_0 - \delta_1)e^{-k_3(t-m)} & \text{for } t > m 
\end{cases}
$$

where the parameters $\delta_1 < \delta_0$ and $k_3 > 0$ determine the time path of the mortality rate. As the mortality rate was relatively high in the phases I and II but declined during phase III, $\delta(t)$ starts to decrease at the same date $m$ as the lifting date in (11) does.

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6 Chowell et al. (2004) show how governmental interferences during a past Ebola outbreak in Africa are associated with a reduction of the reproduction number.
5. Application of the SIRDH Model on the Development in Germany

The following empirical analysis is based on discrete time, whereby one period corresponds to one day.

5.1. Time-Invariant Infection and Mortality Rates

As a first step, we simulate a hypothetical scenario, in which the transition of SARS-CoV-2 corresponds to the pre-lockdown period during phase I of the pandemic in Germany. It is assumed that the spread of the virus is not influenced through behavioral adaptions or governmental interferences as a response to COVID-19. We set the parameters as follows (see the Appendix for the calibration of all variables and parameters): Regarding the infectivity period we choose a value of 10 days leading to $\gamma = 0.1$ (compare He et al., 2020; Kampen et al., 2020; Singanayagam et al., 2020; Wölfel et al., 2020). For the less critical COVID-19 patients it takes about two weeks to fully recover from the illness (WHO, 2020c) which led us to set $\theta \approx 0.25$. For the infection rate we considered the time between January and mid-March 2020 and fitted it to the data provided by JHU CSSE (2020). This led us to set a constant value of $\beta = 0.3$ resulting in a basic reproduction number of $R_0 = 3$. Finally, we compared three different values for the mortality rate $\delta$ given by 0.005, 0.01 and 0.015 which is within the range of several reports estimating the infection fatality rate (IFR) of COVID-19 (compare Davies et al., 2020; Pastor-Barriuso et al., 2020; Russell et al., 2020; Salje et al., 2020; Verity et al., 2020; Ward et al., 2020). The graphical solutions of the simulations are shown in the figures below.

![Figure 4: Simulated Daily Infections vs. Real Development in Phase I](image-url)
As can be seen in Figure 4, a reproduction number of 3 yields a good approximation of daily infections until the 21\textsuperscript{st} of March 2020. However, from there on, the simulation starts to take off, leading to tens of thousands of infections by the end of March. This trend continues resulting in more than 3,000,000 infections per day in May 2020.

![Figure 5: Hypothetical Active Infections](image)

During the peak of the hypothetical scenario, a maximum share of about 30\% of the German population is infected corresponding to around 26,000,000 of exposed individuals at the same time (see Figure 5). Taking Clark's calculations as an approximation that about 6\% of the population needs to be transferred to the hospital after an infection indicates that health care systems in Germany would be at high risk to be overwhelmed. In this scenario, with millions of infections, the German population eventually reaches herd immunity, and the number of infections declines. Thus, the virus disappears by itself in the long run.
By Autumn 2020, with almost 80,000,000 total infections, almost the entire German population got exposed to SARS-CoV-2 (see Figure 6). In contrast, assuming that a successful vaccine distribution for about 67% ($\approx 55,000,000$) of the population would have been available at the beginning of the outbreak, the spreading of the virus would have been immediately stopped after the first infections.

Finally, figure 7 shows the development of total deaths for the three infection-fatality-rates of 0.5%, 1% and 1.5%. Depending on the rate, this results in approximately 400,000 to 1,200,000 total deaths by the end of the pandemic.
5.2. Time-Dependent Infection and Mortality Rates

Let us now relax the assumption of time-invariant infection and mortality rates and instead calibrate $\beta(t)$ and $\delta(t)$ to the data provided by JHU CSSE (2020). All remaining parameters are set as shown above.

To simulate the overall development of COVID-19 cases in Germany, we make two simplifications. First, we assume that the effective reproduction number is constant during the pre-lockdown period. Second, we interpret the post-lockdown period as a slow, but gradual increase of the reproduction number and exclude any ups and downs of daily case numbers. As above, we set $\beta_0$ to be 0.3 in phase I which is kept constant until late March 2020.

![Figure 8: Simulated Reproduction Number](image)

By the end of March, we reduced $\beta_0$ gradually to $\beta_1 = 0.059$. Here, the reproduction number is pushed to a minimum value of $R_e \approx 0.59$ which is reached in early May 2020. This development covers the lockdown-period. Afterwards, we increased $\beta_1$ again starting in mid-May which marks the beginning of the post-lockdown period. Eventually, the infection rate gradually converges towards $\beta_2 = 0.14$. This increase leads to the fact that in mid-October $R_e$ corresponds to around 1.3 (see Figure 8).

Finally, we simulated the total deaths of the pandemic in Germany. Regarding the pre- and lockdown period we assume a constant mortality rate of $\delta_0 = 0.0485$.\footnote{In this version, the mortality rate corresponds to the case-fatality rate as only documented cases are considered.} Afterwards, the rate

![Reproduction Number Treshold](image)
declines in the post-lockdown period and eventually reaches $\delta_1 = 0.007$. The graphical solutions are presented in the figures below.

![Figure 9: Simulated Daily Infections vs Real Development](image1)

As Figure 9 shows, the extended SIRDH model provides a good fit between the simulation and the data. Similar to the previous analysis, the first wave equals an inverted U-shape. However, due to the variable infection rate, the daily case numbers are significantly lower and do not exceed 7,000 daily infections in the first wave. As $R_e$ is pushed below 1 in the lockdown period case numbers eventually decline. However, in the long run it is given that $R_e > 1$ and thus the growth rate of new infections is positive, leading to a second wave of COVID-19 infections.

![Figure 10: Simulated Total Deaths vs. Real Development](image2)
As a result of the time-dependent infection and mortality rates, it is also possible to reconstruct the development of total deaths and total confirmed infections quite accurate (see Figure 10 and 11).

![Figure 11: Simulated Total Cases vs. Real Development](image)

6. Summary and Conclusion

This paper dealt with the emergence of COVID-19 and investigated how the development of the disease in Germany can be ex-post simulated by applying a combination of the basic and an extended SIRDH model. We summarized the development of the COVID-19 crisis and presented key characteristics of SARS-CoV-2. In a very stylized but helpful setting, we distinguished three different phases. The first phase refers to the pre-lockdown stage characterized by exponential growth. The second phase corresponds to the time between March and May 2020 in which the German government controlled the infection process by implementing various non-pharmaceutical interventions leading to a nationwide lockdown. Finally, the third post-lockdown stage describes the gradual relaxation of restrictions which led to a revival of the virus in late Summer/Autumn 2020. This time structure laid the foundation for our design of the epidemiological model.

Based on Fernández-Villaverde & Jones (2020), we first simulated the SIRDH model using time-invariant transition rates. The results serve as a “what if” analysis and show that without any behavioral or governmental interventions, a spread of COVID-19 would potentially lead to
several millions of infections, at the same time accompanied by hundreds of thousands of deaths.

The findings are in line with the results of Flaxman et al. (2020) who estimate that over 500,000 Germans could have died by May 2020, if the spread of the virus would not have been suppressed by social distancing policies. These results are backed up by Ferguson et al. (2020) who calculate similar death numbers for the UK and the US and state that intensive care unit capacities would not have been enough, if the virus would have spread rapidly through the population. Such estimations are only hypothetical, and our results need to be interpreted with caution. However, they serve as a benchmark to show the potential unfolding dynamics of COVID-19.

To include a decreasing case fatality rate, observed during the post-lockdown period, and to account for the fact that the German government imposed several social distancing policies to fight the spread of the virus, the SIRDH model was modified with respect to time-dependent infection and mortality rates. The use of a first constant, then decreasing, and finally increasing infection rate allowed us to model the pandemic in Germany regarding daily new and total infections and deaths quite accurate.

An interesting extension of our approach would be to include a cosine function in order to allow for oscillating infection rates. Such an approach is for example used to account for seasonality effects of other infectious diseases such as measles (Chen & Epureanu, 2017) or influenza (Dushoff et al., 2004) and is also applied to model the current COVID-19 pandemic (Neher et al., 2020). Furthermore, we assumed that individuals acquire permanent immunity after an infection. While we showed that this rationale can unproblematically be used to reconstruct the development of the first and second corona waves in 2020, it remains to be seen whether this assumption is also valid in the long run. The study by To et al. (2020) has confirmed that it is possible to be infected again after being exposed to the virus. If this turns out to be a regular pattern, it will be necessary to further extend our SIRDH version to a SIRDHS version, in which infected individuals lose their immunity after a certain period of time and move back to the susceptible state $S$ (see, e.g. Hethcote (2008)).

These alternatives may become increasingly important when no effective vaccine will be developed and distributed in the near future. However, during the final editing of this paper, it has become known that effective vaccination is very likely to be available in the first half of 2021. Let us hope the best.
Appendix

Calibrated Values for the Variables and Parameters

<table>
<thead>
<tr>
<th>Total Population</th>
<th>N</th>
<th>83,783,942</th>
<th>Worldometer (2020)</th>
</tr>
</thead>
</table>

**Initial Conditions for the Variables**

<table>
<thead>
<tr>
<th>Susceptible</th>
<th>S(0)</th>
<th>83,783,941</th>
<th>N - I(0) - R(0) - H(0) - D(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infected</td>
<td>I(0)</td>
<td>1</td>
<td>JHU CSSE (2020)</td>
</tr>
<tr>
<td>Removed</td>
<td>R(0)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Healthy</td>
<td>H(0)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Dead</td>
<td>D(0)</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Fixed Parameter Values**

<table>
<thead>
<tr>
<th>Duration of Infectivity</th>
<th>10 days</th>
<th>He et al. (2020, p. 672), Kampen et al. (2020, p. 2), Singanayagam et al. (2020, p. 1), Wölfel et al. (2020, p. 466)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Removing Rate</td>
<td>γ</td>
<td>$\frac{1}{10} = 0.1$</td>
</tr>
<tr>
<td>Duration until</td>
<td>4 days</td>
<td>WHO (2020c, p. 14)</td>
</tr>
<tr>
<td>Recovery or Death</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exit Rate from the</td>
<td>$\theta$</td>
<td>$\frac{1}{4} = 0.25$</td>
</tr>
<tr>
<td>State R</td>
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<td></td>
</tr>
<tr>
<td>Mortality Rate</td>
<td>$\delta$</td>
<td>0.005-0.015</td>
</tr>
<tr>
<td>(Infection Fatality</td>
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<td></td>
</tr>
<tr>
<td>Rate)</td>
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</tr>
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</table>

**Fitted Parameter Values**

<table>
<thead>
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<th>Pre-Lockdown Infection Rate</th>
<th>$\beta_0$</th>
<th>0.3</th>
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</thead>
<tbody>
<tr>
<td>Lockdown Infection Rate</td>
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</tr>
<tr>
<td>Lockdown Parameter</td>
<td>$k_1$</td>
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<td>Lockdown Date</td>
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<td>54</td>
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<td></td>
<td></td>
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<tr>
<td>Post-Lockdown Infection Rate</td>
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<td>Lifting Parameter</td>
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<td>Lifting Date</td>
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<tr>
<td></td>
<td></td>
<td>11.05.2020</td>
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<tr>
<td>Mortality Rate in Phases I</td>
<td>$\delta_0$</td>
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</tr>
<tr>
<td>and II (Case Fatality Rate)</td>
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<tr>
<td>Mortality Rate in Phase III</td>
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<td>0.007</td>
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<tr>
<td>(Case Fatality Rate)</td>
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<tr>
<td>Mortality Parameter</td>
<td>$k_3$</td>
<td>0.03</td>
</tr>
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</table>

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References


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