EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

University of Tübingen
Working Papers in
Business and Economics

No. 138

An adverse social welfare effect of quadruply gainful trade (2nd ed)

by

Oded Stark, Grzegorz Kosiorowski

Faculty of Economics and Social Sciences
www.wiwi.uni-tuebingen.de

https://publikationen.uni-tuebingen.de/xmlui/handle/10900/95156

An adverse social welfare effect of quadruply gainful trade

by

Oded Stark
Universities of Bonn, Tuebingen, and Warsaw

and

Grzegorz Kosiorowski
Cracow University of Economics

Mailing Address: Oded Stark
ZEF, University of Bonn
Walter-Flex-Strasse 3
D-53113 Bonn
Germany

E-mail Address: ostark@uni-bonn.de

September 2020

Abstract

Acknowledging that individuals dislike having low relative income renders trade less attractive when seen as a technology that integrates two economies by merging separate social spheres into one. We define a "trembling trade" as a situation in which gains from trade are less than losses in relative income, with the result that global social welfare is reduced. We show that a "trembling trade" can arise even when trade is more gainful in four ways: through trade the absolute income of everyone increases, the income gap in both economies is reduced, as is the income gap between the trading economies. However, trade brings populations, economies, or markets that were not previously connected closer together in social space. As a consequence, separate social spheres merge, and people's social space and their comparators are altered. Assuming that people like high (absolute) income and dislike low relative income, the aggregate increase in unhappiness caused by the trade-induced escalation in relative deprivation can result in a negative overall impact of trade on (utilitarian-measured) social welfare, if the *absolute* income gains are not large enough to mitigate the *relative* income losses.

Keywords: Gains from trade; Increase of incomes; Decrease of income gaps; Integration; Change of social space; Low relative income; Quadruply gainful trade; "Trembling trade;" Social welfare

JEL classification: D31; D63; F10; F15; R12

1. Introduction and motivation

At least since Stolper and Samuelson demonstrated in 1941 in a formal general equilibrium model that when an economy leaves a state of autarky to engage in trade with other economies, not everyone's real income rises, a view has been held that if as a result of trade everyone's income is made to rise (redistributive policies can see to that), one of two potential unwarranted consequences of free trade (that the incomes of some people fall) need not be worrisome. A second concern regarding the repercussions of free trade has been a possible rise in the income gap between the trading economies: even when trade between, say, a relatively rich economy P2 and a relatively poor economy P1 confers gains so that everyone's income increases, trade can be viewed with some trepidation if the income gap between P2 and P1 widens. However, when everyone's income increases *and* the income gap between P2 and P1 does not widen, let alone when this gap decreases, then, in general, trade is considered beneficial.

A theoretical foundation of the gains from trade can be traced back to Ricardo's (1817) law of comparative advantage. Rising productivity and increasing national income brought about by international trade, as predicted by Ricardo, are repercussions that were confirmed empirically, for example by MacDougall (1951, 1952), Golub and Hsieh (2000), Melitz (2003), Bernhofen and Brown (2005), Irwin (2005), Amiti and Konings (2007), Goldberg et al. (2010), and Etkes and Zimring (2015). However, since Ricardo's "Principles," economists have also been aware that the distributional consequences of trade are distinct from the rising incomes consequences of trade. Anything that exacerbates inequality has for long been considered problematic, trade included. A concern has been expressed that even when the incomes of all involved increase and the income gap between P2 and P1 narrows, trade can still fail to increase global social welfare when, as a result of engagement in trade, the following happens: the income distribution within P2 or the income distribution within P1 or the income distributions within both widen (the incomes of the richer members of P2 increase by more than the incomes of the poorer members of P2 and / or the incomes of the richer members of P1 increase by more than the incomes of the poorer members of P1). This concern is usually based on the trade models of Ohlin (1933) and Stolper and Samuelson (1941). For example, Berman et al. (1998) view trade as a cause of a simultaneous rise in wage inequality in developed and developing economies. Barro (2000) finds that trade openness increased inequality in developing countries. Lundberg and Squire (2003) identify a positive correlation between trade liberalization and higher within-country income inequality. Goldberg and Pavcnik (2007) observe that across a number of developing countries, significant increases in inequality were a typical consequence of trade liberalization after the 1970s. Feenstra and Hanson (1997), Grossman and Rossi-Hansberg (2008), Burstein and Vogel (2011), and Sampson (2014) list a variety of reasons why greater economic integration between countries may cause wage inequality to rise within countries. Antràs et al. (2017), who measure the gains from trade by adding to the classical real income gain a term that measures the inequality cost, conclude that trade-induced increases in inequality "eat up" about 20% of the US gain from trade. Studying European Union integration, Beckfield (2009) observes both a decrease in inequality between countries, and an increase in inequality within countries. On the other hand, Calderon and Chong (2001) find that, at least in developed countries, an increase in the volume of trade is positively correlated with a long-run decline in the in-country income inequality. What the current paper seeks to establish is that even when on all four counts trade is beneficial (namely when it increases the income of every individual in P1 and in P2, narrows the income gap between P2 and P1, narrows the income gap within P1, and narrows the income gap within P2), trade can still fail to raise global social welfare.

In a recent paper, Stark et al. (2018) present a constructive example why trade that increases the absolute income of every member of the trading economies and narrows the income gap between the economies can at the same time lower global social welfare. The core idea of the paper by Stark et al. is that trade affects social relations which, in turn, are likely to bring about changes in social welfare. Trade may modify social relations in a variety of ways, which include revising social ties, broadening social horizons, and forging new social interactions: trade does not occur only in geographical space; it leaves footprints in the social sphere. The participation of economies in trade can enlarge the social space of the members of the economies. This expansion impacts on how these members value and assess the benefits that trade confers. Stark et al. build on the notion that individuals' preferences are social in nature. This perspective incorporates the concepts of social space, relative income, and relative income deprivation (defined in Section 2 below). Stark et al. use the term “trembling trade” to describe a situation in which the negative effect on the welfare of the trading economies that arises from the trade-induced shake-up of their social spheres is greater than the gains brought about by trade in the form of income increases and reduced income gap between the trading economies.

In this paper we present a general theory of a “trembling trade” - a situation in which gains from trade are less than losses in relative income - and we use the theory to show that even a quadruply gainful trade, meaning trade that increases the absolute income of everyone, reduces the income gap in both of the two trading economies, and narrows the income gap between the trading economies, can lower global social welfare.

Although it is not typical for trade to be quadruply gainful, our strategy is to show that *even* when the economic data appear as favorable as possible, trade can still be detrimental to global social welfare. In fact, the “trembling trade” outcome does not depend on the assumption that trade is *quadruply* gainful (consult Comment 1 in Section 4). Suppose that trade fails to satisfy one of the assumptions that render it being quadruply gainful. For example, suppose that trade raises all incomes, and brings about convergence between countries and declining inequality in the poorer country, but no reduction in inequality in the rich country. In such a case where trade is triply gainful, trade is even more prone to be trembling, and the fact that it is trembling is just less surprising. Logically, then, it suffices to consider the case of quadruply gainful trade and “export” the results of such an analysis to other trade configurations.

We view trade as a process which, as a result of market transactions involving an exchange of goods or services, brings populations, economies, or markets that previously were not connected closer together in social space. As a consequence of this integration, separate social spheres merge, and people’s social space and their comparators are altered. Then, the perceived relative incomes of some individuals, calculated in the context of the broader social space, may decrease even if through trade the absolute income of everyone increases, the income gap within each economy is reduced, as is the income gap between the trading economies. Consequently, in and by itself, the integration of social spaces may exacerbate the discontent that some individuals experience from having low relative income. We represent people's concern about low relative income by means of a measure of relative income deprivation: we say that an individual experiences relative deprivation when his income falls behind the incomes of other individuals who constitute his comparison group. Assuming that people like high (absolute) income and dislike low relative income, we show that the aggregate increase in dismay caused by the trade-induced escalation in relative deprivation can result in a negative overall impact of trade on (utilitarian-measured) social welfare, if the *absolute* income gains are not large enough to mitigate the *relative* income losses.

The trade-generated expansion of a social space can arise via several channels. First, purchased goods convey information about trading partners, in particular about the productivity of labor and about incomes in the partner economy. This channel operates even in the sterile world of perfect Walrasian markets. A second channel results from the interactions of people in non-Walrasian environments where trade requires social interactions aimed at matching trading partners and at mediation which, to different extents, removes the information gap between the trading economies. Third, in order to facilitate and intensify trade, trading partners introduce mechanisms and procedures that also reduce the social divide between them. In the context of international trade, monetary unification (an event that has occurred in Europe seven times since 1999) is one such example.[1] Fourth, trade invites, and is often based on, exchanges of traders, trade representatives, trade delegations, and experts of various types. Presence and exposure foster comparisons. Fifth, trade is built on a study of the needs, preferences, consumption habits, and demands of the partners in trade; the expansion of commercial space brings in its wake an expansion of social space. Sixth, trade is often the precursor of migration, and migrants facilitate and intensify cross-cultural awareness and inter-economy social ties.

[1] Stark and Wlodarczyk (2015, p. 185) reason as follows. "The introduction of a common currency is an instrument of fundamental change in economic and social relations in general, and in interpersonal comparisons of earnings, pay, and incomes in particular. Although, prior to the introduction of the euro as a common currency, individuals in specific European countries were able to compare their incomes with the incomes of individuals in other European countries, the comparison was not immediate; it required effort to convert incomes denominated in different currencies, and it was presumably not done very often. … When a single currency is introduced, the comparison environment changes, enabling, indeed inviting, simpler comparisons with others. For example, with currency unification, workers who perform the same task and who are employed by a manufacturer with plants located in different EMU countries can compare their earnings with each other directly, effortlessly, and routinely."

The remainder of this paper is divided into three main sections. In Section 2 we present an example. In Section 3 we develop a general framework that in subsequent Section 4 enables us to state and prove our main proposition and corollaries, the essence of which are conditions under which, in spite of being quadruply gainful, trade can lower global social welfare. A brief Section 5 concludes.

2. An example

A numerical illustration helps to demonstrate that a "trembling trade" can arise even when trade between two populations, P1 and P2, is quadruply gainful in the sense that:

1. it increases the income of *every* individual in P1 and in P2;
2. it narrows the income gap *within* P1;
3. it narrows the income gap *within* P2;
4. it narrows the income gap *between* P2 and P1.

Let populations P1 and P2 consist, each, of two individuals. We let the initial incomes of the individuals in population P1 be 2 and 4, so the average initial income in P1 is 3. When trade occurs between P1 and P2, incomes 2 and 4 rise to 6 and 7 respectively. Consequently, the average post-trade income in P1 is 6.5, and 6.5 > 3. When trade occurs, the income gap within P1, when measured as the difference between the incomes of the individuals in P1, narrows from 2 to 1. The decreased inequality within population P1 is also indicated by a lower Gini index, GI. For a population of n individuals where x_i is the income of individual *i*, $i \in \{1, 2, \ldots, n\}$, GI is defined as

$$GI \equiv \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} | x_j - x_i |}{2n\sum_{i=1}^{n} x_i}.$$

For $n = 2,$ and $x_2 > x_1$ GI takes the form

$$GI = \frac{x_2 - x_1}{2\left(x_1 + x_2\right)}.$$

Thus, the pre-trade GI of population P1 is $GI_1^{PRE} = \frac{4-2}{2(4+2)} = \frac{1}{6}$, and the post-trade GI of population P1 is $GI_1^{POST} = \frac{7-6}{2(7+6)} = \frac{1}{26} < \frac{1}{6}$.

We let the initial incomes of the individuals in population P2 be 8 and 12, so the average initial income in P2 is 10. When trade occurs with P1, incomes 8 and 12 rise to 10 and 13 respectively. The average post-trade income in P2 is, then, 11.5, and 11.5 > 10. When trade occurs, the income gap within P2 narrows from 4 to 3. The GI of population P2 is lowered: the pre-trade GI of population P2 is $GI_2^{PRE} = \frac{12-8}{2(12+8)} = \frac{1}{10}$, and the post-trade GI of population P2 is $GI_2^{POST} = \frac{13-10}{2(13+10)} = \frac{3}{46} < \frac{1}{10}$.

The initial gap between the average incomes of P2 and P1 is $10-3=7.$ The post-trade gap between the two populations, when measured as the difference between the average incomes of P2 and P1, is $11.5-6.5=5,$ and 5 < 7. The corresponding GI shrinks: the *pre-trade* GI of population $P1 \cup P2,$ that is, the GI when the set of incomes of the individuals in $P1 \cup P2$ is $\{2,4,8,12\}$, is

$$GI_3^{PRE} = \frac{(12-8)+(12-4)+(12-2)+(8-4)+(8-2)+(4-2)}{4(12+8+4+2)} = \frac{17}{52},$$

whereas the *post-trade* GI of population $P1 \cup P2$, that is, when the set of incomes of the individuals is $\{6,7,10,13\}$, is

$$GI_3^{POST} = \frac{(13-10)+(13-7)+(13-6)+(10-7)+(10-6)+(7-6)}{4(13+10+7+6)} = \frac{1}{6} < \frac{17}{52}.$$

We assume that the individuals derive utility from (absolute) income, and disutility from low relative income, and that they assign the weight $\alpha \in (0,1)$ to the disutility from low relative income, and the weight $1-\alpha$ to the utility from (absolute) income. We use a standard measure of relative income deprivation: the aggregate of the excesses of the incomes of others in the individual's comparison group, divided by the size of this group.[2] Namely we assume that the utility function of the individual is given by

$$U(m,\tilde{m}) = (1-\alpha)m - \alpha RD(m,\tilde{m}),$$

where m stands for the individual's income; $\tilde{m}$ is the income vector of the individual's comparison group including the individual (prior to trade, this group is population P1 or population P2, whereas post trade this group is population $P1 \cup P2$); and RD is the disutility experienced by the individual from relative income deprivation, which in turn is defined as

$$RD(m,\tilde{m}) \equiv \frac{1}{|\tilde{m}|}\sum_{x\in\tilde{m}} \max\{x-m,0\},$$

where $|\tilde{m}|$ denotes the size of the vector $\tilde{m}$.

Given this utility characterization, we obtain the following results regarding the changes in a utilitarian measure of social welfare brought about when trade occurs between the two populations. When, for example, we write next $U\left(2,(2,4)\right)$, 2 is m and (2,4) stands for $\tilde{m}$. The level of social welfare of the pre-trade, autarkic P1 is $SW_1^{PRE} = U\left(2,(2,4)\right) + U\left(4,(2,4)\right) = \left(1-\alpha\right)2 + \left(1-\alpha\right)4 - \frac{\alpha}{2}\left(4-2\right) = 6-7\alpha$.

The level of social welfare of the pre-trade, autarkic P2 is

$$SW_2^{PRE} = U\left(8,(8,12)\right) + U\left(12,(8,12)\right) = \left(1-\alpha\right)8 + \left(1-\alpha\right)12 - \frac{\alpha}{2}\left(12-8\right) = 20-22\alpha.$$

The level of social welfare of post-trade P1 when the P1 individuals are in the comparison group with income vector (6,7,10,13), namely of P1 as part of an integrated $P1 \cup P2$, is

$$\begin{aligned}SW_1^{POST} &= U\left(6,(6,7,10,13)\right) + U\left(7,(6,7,10,13)\right) = \left(1-\alpha\right)6 + \left(1-\alpha\right)7 - \frac{\alpha}{4}\left(1+4+7+3+6\right)\\ &= 13-18.25\alpha.\end{aligned}$$

[2] This measure of relative deprivation is equivalent to a measure of relative deprivation of individual *i* defined as the fraction of the individuals in *i*'s comparison group whose incomes are higher than the income of individual *i* times the difference between the average income of the higher income individuals and *i*'s income. Stark (2013) provides a brief foray into the concept of relative deprivation and a presentation of its measures.

The level of social welfare of post-trade P2 when the P2 individuals are in the comparison group with income vector (6,7,10,13), namely of P2 as part of an integrated $P1 \cup P2$, is

$$SW_2^{POST} = U\left(10,(6,7,10,13)\right) + U\left(13,(6,7,10,13)\right) = (1-\alpha)10 + (1-\alpha)13 - \frac{\alpha}{4}(13-10)$$
$$= 23 - 23.75\alpha.$$

The change of global social welfare brought about by trade is

$$(SW_1^{POST} + SW_2^{POST}) - (SW_1^{PRE} + SW_2^{PRE})$$
$$= (23 - 23.75\alpha + 13 - 18.25\alpha) - (20 - 22\alpha + 6 - 7\alpha)$$
$$= 36 - 42\alpha - (26 - 29\alpha) = 10 - 13\alpha.$$

It follows, then, that global social welfare declines if $10 - 13\alpha < 0,$ namely if $\alpha > \dfrac{10}{13}$.

The preceding calculations illustrate a possible occurrence of a "trembling trade" even if trade is quadruply gainful. The example in this section rests on several simplifying assumptions, especially the strong assumption of linearity of the individuals' utility functions. In the next two sections we present a general modeling framework, and we specify conditions under which a "trembling trade" can occur.

3. A theory of a "trembling trade"

The setting that we consider is the following. We let P1 and P2 be two populations of size $n \geq 2$ each, such that individuals $\{1, 2, \ldots, n\}$ belong to population P1, and individuals $\{n+1, n+2, \ldots, 2n\}$ belong to population P2.[3] Prior to the populations trading with each other, members of one population have no ties (and do not compare themselves) with members of the other population. The utility function of individual $i \in \{1, \ldots, 2n\}$ whose income is m_i takes the form

$$U_i(m_i, \tilde{m}_i) = g(m_i) - h(RD(m_i, \tilde{m}_i)), \tag{1}$$

[3] The findings reported in this section and in Section 4 do not depend on the assumption that populations P1 and P2 are of equal size. Making this assumption simplifies the notation, eases the proofs, and allows us to focus on the qualitative aspects of our model.

where the functions g and h are continuous and increasing;[4] $\tilde{m}_i$ is the vector of the incomes of all the individuals in individual i's comparison group (including individual i himself); and $RD(\cdot)$ is a measure of low relative income. As in the example of Section 2, the $RD(\cdot)$ of individual i is defined as

$$RD(m_i, \tilde{m}_i) \equiv \frac{1}{|\tilde{m}_i|} \sum_{m \in \tilde{m}_i} \max\{m - m_i, 0\},$$

where $|\tilde{m}_i|$ denotes the cardinality of the set $\tilde{m}_i$.

We let the individuals in population P1 have pre-trade incomes denoted by $x_1 < x_2 < \ldots < x_n$ (x_i is the income of individual i, $i \in \{1, \ldots, n\}$), and we let the individuals in population P2 have pre-trade incomes denoted by $y_1 < y_2 < \ldots < y_n$ (y_i is the income of individual $n+i$, $i \in \{1, \ldots, n\}$). We define vectors $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$, and we assume that $x \neq y$, namely that with respect to income distributions, populations P1 and P2 are not identical copies of each other. We let

$$G^{PRE}(x, y) = \sum_{i=1}^{n} \left[g(x_i) + g(y_i) \right] \tag{2}$$

be the sum of the pre-trade levels of utility from income of all the individuals in the two populations, and we let

$$H^{PRE}(x, y) = \sum_{i=1}^{n} \left[h(RD(x_i, x)) + h(RD(y_i, y)) \right] \tag{3}$$

be the sum of the pre-trade levels of disutility from relative income deprivation of all the individuals in the two populations.

We assume that when trade occurs between populations P1 and P2, the incomes of the members of population P1 represented by the vector $x = (x_1, x_2, \ldots, x_n)$ increase by the corresponding amounts from the vector $a = (a_1, a_2, \ldots, a_n)$, namely the income of individual i increases by a_i, and that the incomes of the members of population P2 represented by the vector $y = (y_1, y_2, \ldots, y_n)$ increase by the corresponding amounts from

[4] Most of the results that follow can be generalized to the case of perfectly heterogeneous populations: the assumption that the functions g and h are identical for individuals $i \in \{0, 1, \ldots, 2n\}$ can be substituted with individual-specific functions g_i and h_i, respectively. However, such a generalization will make the notations and proofs unnecessarily tedious without adding novel insights.

the vector $b=(b_1,b_2,\ldots,b_n)$, namely the income of individual $n+i$ increases by b_i.[5] We assume that there exists $M\in\mathbb{R}_+$ so that for $i\in\{1,2,\ldots,n\}$, $x_i,y_i,a_i,b_i,x_i+a_i,y_i+b_i\in[0,M]$.[6] We let

$$G^{POST}(x,y,a,b)=\sum_{i=1}^{n}[g(x_i+a_i)+g(y_i+b_i)], \tag{4}$$

and we let

$$\begin{aligned} H^{POST}(x,y,a,b)=\sum_{i=1}^{n}[&h(RD(x_i+a_i,(x+a,y+b))\\ &+h(RD(y_i+b_i,(x+a,y+b))] \end{aligned} \tag{5}$$

be, respectively, the sum of the post-trade levels of utility from income and the sum of the post-trade levels of disutility from relative deprivation. From the continuity of the functions g, h, and RD, it follows that G^{POST} and H^{POST} are continuous functions of x, y, a, and b; that is, these functions are continuous in the initial incomes, and in the income increases brought about by trade. Obviously, $G^{POST}(x,y,0,0)=G^{PRE}(x,y)$.

We let SW_1^{PRE} and SW_2^{PRE} be the pre-trade levels of social welfare of populations P1 and P2, respectively, and we let SW_1^{POST} and SW_2^{POST} be the corresponding levels of social welfare of the two populations when trade occurs. Prior to trade, members of each population compare themselves only with members of their own population; the population that they belong to constitutes their exclusive comparison group. When trade occurs, however, members of each population compare themselves also with members of the other population, so that the members of both populations constitute the comparison group of each individual. This revision of the comparison groups arises from the merging of social spheres brought about by trade, as explained in Section 1.

We measure the social welfare of a population in a utilitarian manner, namely as the sum of the utility levels of the members of the population. We study the possible

[5] The vectors a and b stand for the net gains from trade, namely the gains that accrue after deducting all related costs such as the cost of entering foreign markets, transport, advertising, and the like, referred to, for example, by Melitz (2003), and Helpman et al. (2004).

[6] Although this assumption is made purely for technical reasons (so that the supremum norm on the space C^0 of continuous functions, the domains of which are equal to a given closed interval, will be well defined), the assumption is sensible if we reason that there is an upper limit on income, and that people do not earn less than 0.

prevalence of a "trembling trade" defined, as already mentioned, as a situation in which the gains from trade are overtaken by the losses in terms of relative deprivation, with the result that global social welfare is reduced. That is, a "trembling trade" arises when

$$SW_1^{POST} + SW_2^{POST} < SW_1^{PRE} + SW_2^{PRE}.$$

Based on the definitions provided in (2) through (5), we can rewrite this inequality as

$$[G^{POST}(x,y,a,b) - G^{PRE}(x,y)] - [H^{POST}(x,y,a,b) - H^{PRE}(x,y)] < 0, \tag{6}$$

which constitutes the necessary and sufficient condition for trade to be trembling. The first bracketed term in (6) represents the aggregate gain from trade that arises from increases in incomes, and the second bracketed term in (6) represents the aggregate loss from trade that arises from the increase in the levels of relative deprivation. The difference between the two bracketed terms is the change in global social welfare.

We next define an auxiliary function φ that represents the change in the sum of the levels of social welfare of the two populations when trade occurs. Formally, the function $\varphi : \Omega^2 \times [0, M]^{4n} \to \mathbb{R}$, where $\Omega = \{ f \in C^0([0, M], \mathbb{R}) : f \text{ is increasing}\}$ with the standard C^0-topology, is given by:[7]

$$\varphi(g,h,x,y,a,b) = G^{POST}(x,y,a,b) - G^{PRE}(x,y) - [H^{POST}(x,y,a,b) - H^{PRE}(x,y)]. \tag{7}$$

Because a fixed 6-tuple $(g,h,x,y,a,b) \in \Omega^2 \times [0, M]^{4n}$ fully characterizes the model of trade and of changes of the levels of social welfare, we refer to the arguments of the φ function as the parameters of the model. These parameters are the initial incomes in the two populations, the gains from trade of the members of the two populations, and the utility functions of these members. We refer to $\Omega^2 \times [0, M]^{4n}$ as the space of parameters, and we denote by $T \subset \Omega^2 \times [0, M]^{4n}$ the set of the parameters generating a "trembling trade." From (6) and (7), we get that

$$T = \varphi^{-1}((-\infty, 0)).$$

We now state and prove four lemmas that in combination provide a basis for subsequently stating and proving our main proposition and corollaries.

---

[7] This is the topology given by the supremum norm: for $f \in C^0([0, M], \mathbb{R})$, $\| f \| = \sup_{x \in [0, M]} | f(x) |$.

Lemma 1. If there are no gains from trade (meaning if $(a_1,\dots,a_n)=(b_1,\dots,b_n)=(0,\dots,0)$), then the sum of the post-merger levels of relative deprivation of individuals $\{1,2,\dots,2n\}$ is greater than the sum of the pre-merger levels of relative deprivations of the same individuals, namely

$$\sum_{i=1}^{n}[RD(x_i,(x,y))+RD(y_i,(x,y))]>\sum_{i=1}^{n}[RD(x_i,x)+RD(y_i,y)]. \tag{8}$$

Proof. In the Appendix.

Lemma 2. The function φ is continuous.

Proof. In the Appendix.

Lemma 3. Let $\Omega_h \subset \Omega$ be the set of functions h satisfying the condition $H^{POST}(x,y,0,0)>H^{PRE}(x,y)$ for any $(x,y)\in[0,M]^{2n}$ such that $x\neq y$. Then, for any $g\in\Omega$, $h\in\Omega_h$, and $(x,y)\in[0,M]^{2n}$, assuming once more that $x\neq y$, there exists a U - open neighborhood of $(0,0)$ in $[0,M]^{2n}$ such that for any $(a,b)\in U$, $(g,h,x,y,a,b)\in T$.

Proof. In the Appendix.

The meaning of Lemma 3 is that if only $H^{POST}(x,y,0,0)>H^{PRE}(x,y)$, then for any initial income vectors of the two populations and for any function g transforming income into utility, a "trembling trade" occurs when the income gains from trade are sufficiently small. The assumption $H^{POST}(x,y,0,0)>H^{PRE}(x,y)$ has a natural interpretation: if a merger were to occur of populations P1 and P2 without any gains from trade, then it would increase the aggregate disutility (social dismay) from relative deprivation. This assumption is satisfied for a wide class of functions. For example, as is formally shown in the following lemma, every linear function h belongs to Ω_h.

Lemma 4. Let $L\subset\Omega$ be the set of increasing linear functions, namely $L=\{f:[0,M]\ni t\to\beta t\in\mathbb{R};\ \beta\in(0,\infty)\}$. Then, $L\subset\Omega_h$.

Proof. In the Appendix.

We now have in place the infrastructure required to present the main result of this section: the occurrence of a "trembling trade" is not singular in the space of the parameters of the model (namely the admissible initial incomes, the gains from trade, and the utility functions of the individuals): a "trembling trade" occurs for an open set in this space. In particular, trade is trembling if the initial income vectors of populations P1 and P2 are not identical, the function h (namely the relative deprivation component of the utility function) is sufficiently close to being linear, and the gains from trade are relatively modest. We then have the following proposition.

Proposition 1. T is nonempty and open in $\Omega^2 \times [0,M]^4$. Moreover, $\Omega \times L \times ([0,M]^2$, $\{(x,y) \in [0,M]^2 : x = y\}) \times \{(0,0)\} \subset T$.

Proof. $T = \varphi^{-1}((-\infty,0))$. The set $(-\infty,0)$ is open in $\mathbb{R}$, φ is continuous on account of Lemma 2, and the counter-image of an open set by a continuous function is open. Therefore, T is open. If $(g,h,x,y,a,b) \in \Omega \times L \times ([0,M]^2$, $\{(x,y) \in [0,M]^2 : x = y\}) \times \{(0,0)\}$ then Lemmas 3 and 4 guarantee that $(g,h,x,y,a,b) \in T$. Therefore, $\Omega \times L \times ([0,M]^2$, $\{(x,y) \in [0,M]^2 : x = y\}) \times \{(0,0)\} \subset T$. At the same time, the sets Ω, L, and $[0,M]^2$, $\{(x,y) \in [0,M]^2 : x = y\}$ are nonempty. Thus, $\Omega \times L \times ([0,M]^2$, $\{(x,y) \in [0,M]^2 : x = y\}) \times \{(0,0)\}$ is nonempty and, consequently, so is T. Q.E.D.

4. The case of quadruply gainful trade

To the “trade” model of the preceding section we now add two assumptions: the individuals in population P1 are poorer than the individuals in population P2, and trade does not affect the ordering of the incomes in the merged population. While the first assumption is innocuous, making the second assumption requires commentary. Having the ordering reserved guarantees that population P1 stays poorer than population P2 also when trade occurs. In addition, the changes in relative deprivation (as defined in Section 2) are then smooth, which simplifies the notation without loss of generality. In addition, making the second assumption is appropriate when considering a short period of time after introducing trade, so that incomes did not change rapidly enough to interfere with the ordering of incomes. And finally, both assumptions are in line with the motivating example of Section 2. Formally, we present these two assumptions, respectively, by

$$x_1 < x_2 < \ldots < x_n < y_1 < y_2 < \ldots < y_n, \tag{9}$$

and

$$x_1 + a_1 < x_2 + a_2 < \ldots < x_n + a_n < y_1 + b_1 < y_2 + b_2 < \ldots < y_n + b_n. \tag{10}$$

We say that trade between population P1 and population P2 is quadruply gainful if the following conditions hold:

$$a_1 > a_2 > \ldots > a_n > b_1 > b_2 > \ldots > b_n > 0. \tag{11}$$

A trade of the type described by (11) raises the incomes of all the individuals in the two populations, narrows the income gaps between the individuals in population P1, narrows the income gaps between the individuals in population P2, and narrows the income gap between population P2 and population P1. Thus, when trade occurs that satisfies (11), the income inequalities in populations $P1$, $P2$, and $P1 \cup P2$ (as measured, for example, by the Gini index) decrease.

Comment 1. What we have in (11) is not critical for obtaining the results reported later on in this section: the inequalities in (11) set the most favorable conditions that can be imagined regarding the effect of trade, namely that everyone's income increases while the income inequality between any two individuals decreases. In drawing on (11), we seek to show that even under generously advantageous assumptions, trade can still be trembling. Therefore, it is even more plausible for trade to be trembling when the conditions are less stringent than in (11). Formally, we can substitute condition (11) with another set of strict inequalities, for example $GI_1^{PRE} > GI_1^{POST}$, $GI_2^{PRE} > GI_2^{POST}$, and $GI_{TOTAL}^{PRE} > GI_{TOTAL}^{POST}$, where GI_i denotes the Gini index of population P_i for $i = 1, 2$; GI_{TOTAL} denotes the Gini index of population $P1 \cup P2$. The three corollaries presented below will hold true, and their proof would need to undergo merely cosmetic changes. Naturally, if the vector (x, y, a, b) satisfies (9), (10), and (11), then it also satisfies the latter set of inequalities, so the new set of conditions is merely weaker. If we wanted to weaken condition (11) further, say when we considered a "triply gainful trade" (such that trade raises all incomes, brings about convergence between countries, and entails declining inequality in the poorer country), we would discard one of the inequalities either in condition (11) or from the set of inequalities about the Gini index defined earlier in this comment, such that the results of this section would still carry through.

Under which conditions is a quadruply gainful trade trembling? We let

$X = \{(x, y, a, b) \in [0, M]^{4n}$: assumptions (9), (10), and condition (11) are satisfied$\}$.

That is, X is the set of the initial income vectors and of the gains from trade such that trade is quadruply gainful. X is an open subset of $[0, M]^{4n}$ because it is defined by a set of strict inequalities. Trade is both trembling and quadruply gainful for the set of parameters (g, h, x, y, a, b) if and only if $(g, h, x, y, a, b) \in T_X = T \cap (\Omega^2 \times X)$. From Proposition 1, it follows that the following holds true.

Corollary 1. T_X is nonempty and open in both $\Omega^2 \times X$ and $\Omega^2 \times [0, M]^{4n}$.

Proof. X is open in $[0, M]^{4n}$, thus $\Omega^2 \times X$ is open in $\Omega^2 \times [0, M]^{4n}$. From Proposition 1, T is open in $\Omega^2 \times [0, M]^{4n}$, thus $T_X = T \cap (\Omega^2 \times X)$ is open in $(\Omega^2 \times X)$. Moreover, by definition, $T_X = T \cap (\Omega^2 \times X)$ is an intersection of two open sets in $\Omega^2 \times [0, M]^{4n}$, so T_x is open in $\Omega^2 \times [0, M]^{4n}$. We let $g_0 \in \Omega$, $h_0 \in L$ and $(x_0, y_0) \in [0, M]^{2n}$, and we let x_0 and y_0 satisfy assumption (9), in particular $x_0 \neq y_0$. We know that $(g_0, h_0, x_0, y_0, 0, 0)$ satisfies (9), (10), and a weak-inequality variant of (11), namely

$$a_1 \geq a_2 \geq \ldots \geq a_n \geq b_1 \geq b_2 \geq \ldots \geq b_n \geq 0.$$

Thus, $(g_0, h_0, x_0, y_0, 0, 0) \in \Omega^2 \times \partial X$, so every neighborhood of $(g_0, h_0, x_0, y_0, 0, 0)$ in $\Omega^2 \times [0, M]^{4n}$ has a nonempty intersection with $\Omega^2 \times X$. However, by Proposition 1, $(g_0, h_0, x_0, y_0, 0, 0) \in T$, and T is open. Then, there exists U -nonempty neighborhood of $(g_0, h_0, x_0, y_0, 0, 0)$ in $\Omega^2 \times [0, M]^{4n}$ such that $U \subset T$. Therefore, $U \cap (\Omega^2 \times X)$ is nonempty, and because $U \cap (\Omega^2 \times X) \subset T \cap (\Omega^2 \times X) = T_X$, it follows that T_X is nonempty. Q.E.D.

Drawing on Proposition 1 and Corollary 1, it can be shown that for any set of utility functions of type (1), where $(g, h) \in \Omega \times \Omega_h$, and for any pair of initial income vectors (x, y) satisfying (9), if the gains from trade are sufficiently small, then trade between population P1 and population P2 is trembling. To this end, we have the following corollary.

Corollary 2. We let $(g_0, h_0) \in \Omega \times \Omega_h$, and we let $(x_0, y_0) \in [0, M]^{2n}$ satisfy (9). We let $p : \Omega^2 \times [0, M]^{4n} \to [0, M]^{2n}$ be the projection of $\Omega^2 \times [0, M]^{4n}$ onto its last $2n$ coordinates, namely $p(g, h, x, y, a, b) = (a, b)$. Then, there exists an open subset $V \subset p(\Omega^2 \times X)$ such that for any $(a, b) \in V$ satisfying (10) (with fixed $(x, y) = (x_0, y_0)$) and (11), trade that increases the incomes of members of population P1 by a and of members of population P2 by b is trembling. Moreover, $V \cup \{(0,0)\}$ is a neighborhood of $(0,0) \in [0, M]^{2n}$.

Proof. It suffices to define $V = p(\Omega^2 \times U)$, for U defined in the same way as in the proof of Corollary 1. By definition, V has the properties listed in Corollary 2. Q.E.D.

Let $(x, y, a, b) \in X$ be a fixed set of conditions generating a quadruply gainful trade. The following question remains: is it possible to construct a pair of functions $(g, h) \in \Omega^2$ such that $(g, h, x, y, a, b) \in T_X$, meaning: is there a pair of utility functions of type (1) such that trade is trembling? If we restrict the set of functions by adding the condition that $h \in L$ (that is, if we take h to be linear and increasing), then the necessary and sufficient condition for trade to be trembling is that the level of aggregate relative deprivation of the trading populations P1 and P2 when these populations are brought into the same social space via trade is higher than the sum of the levels of the aggregate relative deprivation of population P1 and of population P2 when apart (that is, in the absence of trade). Formally, this condition is represented by

$$\begin{aligned}&\sum_{i=1}^{n}\left[RD(x_i + a_i, (x+a, y+b)) + RD(y_i + b_i, (x+a, y+b))\right]\\&> \sum_{i=1}^{n}\left[RD(x_i, x) + RD(y_i, y)\right].\end{aligned} \tag{12}$$

We formalize this statement in the next corollary.

Corollary 3. We let $(x, y, a, b) \in X$, and we let $h \in L$. Then a function $g \in \Omega$ such that $(g, h, x, y, a, b) \in T_X$ exists if and only if condition (12) is satisfied.

Proof. The proof comes in two parts. First, we assume that condition (12) is not satisfied, and we fix the parameters of the model $(g, h, x, y, a, b) \in \Omega \times L \times X$. Then, the opposite of (12) holds, namely

$$\sum_{i=1}^{n}[RD(x_i,x)+RD(y_i,y)] \geq \sum_{i=1}^{n}[RD(x_i+a_i,(x+a,y+b))+RD(y_i+b_i,(x+a,y+b))],$$

and by the linearity of h, we get that

$$\sum_{i=1}^{n}[h(RD(x_i,x))+h(RD(y_i,y))] \geq \sum_{i=1}^{n}[h(RD(x_i+a_i,(x+a,y+b))) \\ + h(RD(y_i+b_i,(x+a,y+b)))],$$

which is equivalent to saying that

$$H^{PRE}(x,y) \geq H^{POST}(x,y,a,b).$$

However, g is increasing, so for any vectors a and b that satisfy (11), we get $G^{POST}(x,y,a,b)-G^{PRE}(x,y)>0.$ But then,

$$G^{POST}(x,y,a,b)-G^{PRE}(x,y)-[H^{POST}(x,y,a,b)-H^{PRE}(x,y)]>0,$$

which contradicts (6). Therefore, such a trade is not trembling, and $(g,h,x,y,a,b) \notin T_X$.

Second, we assume that condition (12) is satisfied. We then define $\zeta>0$ as follows:

$$\zeta \equiv \sum_{i=1}^{n}\left[RD(x_i+a_i,(x+a,y+b))+RD(y_i+b_i,(x+a,y+b))\right] \\ -\sum_{i=1}^{n}\left[RD(x_i,x)+RD(y_i,y)\right].$$

From the definition of L we know that there exists $\beta \in (0,\infty)$ such that $h(t)=\beta t$ for every $t \in [0,M]$. We denote $A=\sum_{i=1}^{n}a_i$, $B=\sum_{i=1}^{n}b_i$, and $\gamma^*=\dfrac{\beta\zeta}{A+B}>0.$ We now choose $g \in \Omega$ that satisfies the condition $g(t)<\gamma^* t$ for $t \in (0,M]$. For example, for any $0<\gamma<\gamma^*$, we can define g as

$$g(t)=\gamma t$$

for $t \in [0,M]$. Then,

$$G^{POST}(x,y,a,b)-G^{PRE}(x,y)-[H^{POST}(x,y,a,b)-H^{PRE}(x,y)]=\gamma(A+B)-\beta\zeta<\beta\zeta-\beta\zeta=0.$$

Thus, (6) is satisfied, and trade is trembling for the parameters (g,h,x,y,a,b), namely for $(g,h,x,y,a,b)\in T_X$.[8] Q.E.D.

In the simple case in which g and h are both linear, the utility function of individual i takes a form that is equivalent to

$$U_i(m_i)=(1-\alpha)m_i-\alpha(RD(m_i,\tilde{m}_i)), \tag{13}$$

for some $\alpha\in[0,1]$. The next corollary presents a straightforward generalization of the example constructed in Section 2.

Corollary 4. We let $(x,y,a,b)\in X$, and we let the utility function of individual i be defined by (13). Then, condition (12) is satisfied if and only if there exists $\alpha^*<1$ such that trade is trembling for any $\alpha\in(\alpha^*,1]$.

Proof. If (12) is not satisfied, then trade is not trembling, as per the first part of the proof of Corollary 3.

If (12) is satisfied, then we only need to replicate the second part of the proof of Corollary 3 for $\beta=\alpha$ and $\gamma^*=1-\alpha^*$. Thus, $\alpha^*=1-\dfrac{\zeta}{A+B+\zeta}$, and if $\alpha\in(\alpha^*,1]$, then trade is trembling. Q.E.D.

In the example presented in Section 2, namely for initial incomes $x_1=2, x_2=4, y_1=8, y_2=12$ and gains from trade, $a_1=4, a_2=3, b_1=2, b_2=1$, (12) is satisfied:

$$\frac{1}{4}[(13-10)+(13-7)+(13-6)+(10-7)+(10-6)+(7-6)]=6>3=\frac{1}{2}(4-2)+\frac{1}{2}(12-8),$$

and applying the notation from the proofs of Corollaries 3 and 4, we get that $\zeta=6-3=3$, $A=4+3=7$, $B=2+1=3$, $\alpha^*=1-\dfrac{3}{3+7+3}=\dfrac{10}{13}$. By Corollary 4, for $\alpha\in(\frac{10}{13},1]$ trade is trembling.

Comment 2. An interesting issue to address is what is the relative size of the set T_X of the parameters generating a “trembling trade” in the set of all the parameters for

[8] T_X is open, so $(\hat{g},\hat{h},x,y,a,b)\in T_X$ also holds true for each $(\hat{g},\hat{h})$ that belongs to a sufficiently close neighborhood of (g,h) in Ω^2.

which trade is quadruply gainful, namely $\Omega^2 \times X$? The results presented thus far are existential in nature: while we showed that such a set is open and, therefore, the phenomenon of a quadruply gainful trembling trade is not singular, it is possible that this open set is so small that its real-life applications are limited. In particular, it seems possible that a trembling trade may occur only from small gains from trade, and only for utility functions in which a major part of value arises from the disutility of low relative income. In general, such a problem has no clear solution: we do not have a natural, well-defined measure on the space of parameters so that even the formal definition of the problem is unclear. Fixing utility functions and considering the size of the set of vectors $(x, y, a, b) \in [0, M]^{4n}$ generating a "trembling trade" will provide us with different results, depending on the choice of utility functions.

Having said that, it is helpful to refer again to the example presented in Section 2 in order to somewhat dispel the doubts. Trade in that example is quadruply gainful and is trembling, and the gains from trade are far from tiny: the income of every individual increases by at least 8.3% (in the case of the richest individual), the income of the poorest individual increases by 200%. At the same time, according to our calculations, more than 20% of the space of utility functions satisfying (13) (measured by the parameter α) guarantees that trade is trembling. Thus, we conclude that the set T_X does not have to be very small.

5. Conclusion

Trade that increases the incomes of the members of the trading populations and reduces the income gaps between and within populations can intuitively be considered to be gainful. However, we showed that even under very favorable assumptions as given by a quadruply gainful trade, it is possible that when trade occurs global social welfare may not improve. Acknowledging that individuals take into consideration not only the comfort of absolute income but also the discomfort of low relative income, we noted that the dismay brought about from expansion of the social space of the members of the trading populations can override the gains from trade. This finding could help explain anti-trade sentiments among populations that engage in a seemingly advantageous trade. For example, Nguyen (2015) argues that concern for inequality is an important factor of the decreasing support for liberal trade policies in the American public, and that, in general, individuals who are more concerned about income inequality are more likely to support protectionist measures. Burgoon (2013) finds that income inequalities encourage support for political parties with protectionist and anti-globalist agendas in OECD countries. Consequently, as Marktanner and Sayour (2009) show, countries characterized by higher levels of income inequality are less likely to liberalize trade as they perceive trade to exacerbate inequality. Distaste for low relative income may also be a reason why Scheve and Slaughter (2006) find that anti-trade sentiments are stronger in countries with higher unemployment rates. On the other hand, in developing countries (for example, in the Philippines, as pointed out by Pasadilla and Liao, 2005), even the wealthiest may be concerned about income comparisons with potential trade partners abroad and, therefore, they oppose unrestricted trade.

Our analysis bears significantly on policy design because it implies that often-prescribed policy recommendations aimed at redressing a downside of trade can well fall short. To see this vividly, we refer to a May 2018 interview of the *Economist* magazine with John Van Reenen of MIT.[9] When asked “What are the downsides of free trade?” he replied as follows: “There are well-known downsides. The way I like to think about it is that free trade increases the size of the pie. The overall amount of material wellbeing expands. But just because the size of the pie expands, it doesn’t mean that everyone is better off. There are going to be some losers whose slice of the pie is so much smaller that they would have been better off with less trade. However, because the overall size of the pie has got bigger, the government can compensate the losers which can still make everyone better off.” This response is telling because it implies that when trade occurs, everyone’s income will increase - if not directly because of trade then indirectly because of a governmental redistribution of the higher aggregate income conferred by trade - social welfare will rise. The approach taken in this paper does not share this perception: higher incomes of all members of the trading economies do not necessarily translate into a global welfare gain. Interestingly, in another part of the interview John Van Reenen intimates that “With free trade, you come into more contact with . . . new people.” We

[9] https://www.economist.com/open-future/2018/05/04/a-healthy-re-examination-of-free-trades-benefits-and-shocks

showed that when this consequence is weighed in the calculus of the gains from trade, trade can be trembling.

In closing, it is tempting to ponder what the social welfare fallout would be from a situation in which a relatively poor economy that geographically neighbors a richer economy but trade-wise is detached from the richer economy, embarks on full blown trade with the richer economy. The perspective elaborated in this paper can serve as a warning sign of a possible adverse outcome.

Appendix: Proofs of Lemmas 1-4

Proof of Lemma 1.

From Stark (2013, Claim 1), we know that if two populations merge while the incomes of their members remain unchanged, then the sum of the post-merger levels of relative deprivation of the individuals who belong to these populations is not lower than the sum of the pre-merger levels of relative deprivation of the same individuals. Thus, for inequality (8) to hold strictly, we only need to prove that equality does not occur if $x \neq y$. To this end we assume that $z=(z_1, z_2, \ldots, z_{2n})$ is an ordered vector of incomes of population $P1 \cup P2$ (namely the sequence z consists of elements of the sequences x and y in an ascending order). For $k \in \{1, \ldots, 2n\}$, we define $h_x(k)$ as the number of individuals in population P1 whose incomes are less than or equal to z_k, and we define $h_y(k)$ as the number of individuals in population P2 whose incomes are less than or equal to z_k.

We use the following result from Stark (2013, Proof of Claim 1, p. 7) for populations P1 and P2 of equal size n:

$$\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] = \sum_{j=1}^{2n-1} \left(z_{j+1} - z_j \right) \frac{[h_y(j) - h_x(j)]^2}{2n}. \qquad \text{(A1)}$$

Each of the summands on the right-hand side of (A1) is non-negative and, therefore, $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] \geq 0$. In order for $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)]$ to be equal to zero, all the summands on the right-hand side of (A1) need to be equal to zero. Hence, for each j, either $z_{j+1} = z_j$, or $h_y(j) = h_x(j)$.

Let us assume that $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] = 0$ and based on our earlier assumptions, we have that $x \neq y$. Then, there are an income level w and natural numbers $k \neq l$ so that the incomes of exactly k individuals of population P1 and of exactly l individuals of population P2 are equal to w. We denote by m the number of individuals in population $P1 \cup P2$ whose incomes are less than w. Then, there are three cases to consider:

I. $m = 0$ (namely w is the smallest income in population $P1 \cup P2$). Then, for $j = k + l$, $z_{k+l+1} > w = z_{k+l}$, and $h_y(k+l) = l \neq k = h_x(k+1)$, hence

$$\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] > 0.$$

II. $m = 2n - k - l$ (namely w is the highest income in population $P1 \cup P2$). Then, $z_{2n-k-l+1} = w > z_{2n-k-l}$, and $h_y(2n-k-l) = n - l \neq n - k = h_x(2n-k-l)$, hence $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] > 0.$

III. $0 < m < 2n - k - l$.

Then, $z_{m+1} > z_m$, thus for $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] = 0$ it is necessary that $h_y(m) = h_x(m)$. Then,

$h_x(m+k+l) = h_1(m) + k \neq h_x(m) + l = h_y(m) + l = h_y(m+k+l)$, and

$z_{2n-k-l+1} > w = z_{2n-k-l}$, hence $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] > 0.$

Each of the three cases I, II, and III thus leads to a contradiction. Therefore, when populations P1 and P2 are not identical with respect to their vectors of incomes then, indeed, $\sum_{i=1}^{2n} RD(z_i, z) - \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)] > 0,$ which is equivalent to stating that

$\sum_{i=1}^{n} [RD(x_i, (x, y)) + RD(y_i, (x, y))] > \sum_{i=1}^{n} [RD(x_i, x) + RD(y_i, y)].$ Q.E.D.

Proof of Lemma 2.

Using the definitions of G^{PRE}, G^{POST}, H^{PRE}, and H^{POST} as given in (2) through (5), we can rewrite the formula of function φ presented in (7) as follows:

$$\varphi(g,h,x,y,a,b) = \sum_{i=1}^{n}[g(x_i+a_i)+g(y_i+b_i)-g(x_i)-g(y_i)]$$
$$-\sum_{i=1}^{n}[h(RD(x_i+a_i,(x+a,y+b))+h(RD(y_i+b_i,(x+a,y+b))$$
$$-h(RD(x_i,x)-h(RD(y_i,y))].$$

Because the functions g, h, and RD are continuous, φ is also continuous. Q.E.D.

Proof of Lemma 3.

We fix $g\in\Omega$, $h\in\Omega_h$, $(x,y)\in[0,M]^{2n}$, assuming that $x\neq y$. We let $H^{POST}(x,y,0,0)-H^{PRE}(x,y)=\varepsilon>0$. As already noted following (5), because h is continuous, then H^{POST} is continuous too. And because of the continuity of H^{POST}, there exists U_1 - an open neighborhood of $(0,0)$ such that for each $(a,b)\in U_1$:

$$|H^{POST}(x,y,0,0)-H^{POST}(x,y,a,b)|<\frac{\varepsilon}{2}.$$

In particular, $H^{POST}(x,y,a,b)-H^{PRE}(x,y)>\frac{\varepsilon}{2}>0.$

At the same time, because of the continuity of G^{POST} (recalling (4)), there exists U_2 - an open neighborhood of $(0,0)$ such that for each $(a,b)\in U_2$:

$$|G^{PRE}(x,y)-G^{POST}(x,y,a,b)|=|G^{POST}(x,y,0,0)-G^{POST}(x,y,a,b)|<\frac{\varepsilon}{2}.$$

Therefore, there exists $U=U_1\cap U_2$ - an open neighborhood of $(0,0)$ such that for any $(a,b)\in U$:

$$G^{POST}(x,y,a,b)-G^{PRE}(x,y)\leq|G^{PRE}(x,y)-G^{POST}(x,y,a,b)|<\frac{\varepsilon}{2}$$
$$=\varepsilon-\frac{\varepsilon}{2}<|H^{POST}(x,y,0,0)-H^{PRE}(x,y)|-|H^{POST}(x,y,0,0)-H^{POST}(x,y,a,b)|$$
$$\leq|H^{POST}(x,y,a,b)-H^{PRE}(x,y)|=H^{POST}(x,y,a,b)-H^{PRE}(x,y).$$

Thus, (6) is satisfied, trade is trembling, and $(g,h,x,y,a,b)\in T$. Q.E.D.

Proof of Lemma 4.

We let $\beta\in(0,\infty)$, and we let $h\in L$ be given by $h(t)=\beta t$ for $t\in[0,M]$. By Lemma 1, when $x\neq y$,

$$\sum_{i=1}^{n}[RD(x_i,x)+RD(y_i,y)]<\sum_{i=1}^{n}[RD(x_i,(x,y))+RD(y_i,(x,y))]$$

and, consequently,

$$\sum_{i=1}^{n}[\beta RD(x_i,x)+\beta RD(y_i,y)]<\sum_{i=1}^{n}[\beta RD(x_i,(x,y))+\beta RD(y_i,(x,y))],$$

which is equivalent to

$$\sum_{i=1}^{n}[h(RD(x_i,x))+h(RD(y_i,y))]<\sum_{i=1}^{n}[h(RD(x_i,(x,y)))+h(RD(y_i,(x,y)))],$$

and, thus,

$$H^{PRE}(x,y)<H^{POST}(x,y,0,0).$$

Therefore, $h\in\Omega_h$. Q.E.D.

References

Amiti, Mary and Konings, Jozef (2007). "Trade liberalization, intermediate inputs, and productivity: Evidence from Indonesia." *American Economic Review* 97(5): 1611-1638.

Antràs, Pol, de Gortari, Alonso, and Itskhoki, Oleg (2017). "Globalization, inequality and welfare." *Journal of International Economics* 108: 387-412.

Barro, Robert J. (2000). "Inequality and growth in a panel of countries." *Journal of Economic Growth* 5: 5-32.

Berman, Eli, Bound, John, and Machin, Stephen (1998). "Implications of skill-biased technological change: International evidence." *Quarterly Journal of Economics* 113(4): 1245-1279.

Bernhofen, Daniel and Brown, John (2005). "An empirical assessment of the comparative advantage gains from trade: Evidence from Japan." *American Economic Review* 95(1): 208-225.

Beckfield, Jason (2009). "Remapping inequality in Europe." *International Journal of Comparative Sociology* 50(5-6): 486-509.

Burgoon, Brian (2013). "Inequality and anti-globalization backlash by political parties." *European Union Politics* 14(3): 408-435.

Burstein, Ariel and Vogel, Jonathan (2011). "Factor prices and international trade: A unifying perspective." Cambridge, MA: NBER Working Paper No. 16904.

Calderon, Cesar and Chong, Alberto (2001). "External sector and income inequality in interdependent economies using a dynamic panel data approach." *Economics Letters* 71: 225-231.

Etkes, Haggay and Zimring, Assaf (2015). "When trade stops: Lessons from the Gaza blockade 2007-2010." *Journal of International Economics* 95: 16-27.

Feenstra, Robert C. and Hanson, Gordon H. (1997). "Foreign direct investment and relative wages: Evidence from Mexico's Maquiladoras." *Journal of International Economics* 42: 371-393.

Goldberg, Pinelopi K., Khandelwal, Amit, Pavcnik, Nina, and Topalova, Petia (2010). "Imported intermediate inputs and domestic product growth: Evidence from India." *Quarterly Journal of Economics* 125(4): 1727-1767.

Goldberg, Pinelopi K. and Pavcnik, Nina (2007). “Distributional effects of globalization in developing countries.” *Journal of Economic Literature* 45(1): 39-82.

Golub, Stephen S. and Hsieh, Chang-Tai (2000). “Classical Ricardian theory of comparative advantage revisited.” *Review of International Economics* 8(2): 221-234.

Grossman, Gene M. and Rossi-Hansberg, Esteban (2008). “Trading tasks: A simple theory of offshoring.” *American Economic Review* 98(5): 1978-1997.

Helpman, Elhanan, Melitz, Marc J., and Yeaple, Stephen Ross (2004). “Export versus FDI with heterogeneous firms.” *American Economic Review* 94(1): 300-316.

Irwin, Douglas A. (2005). “The welfare cost of autarky: Evidence from the Jeffersonian trade embargo, 1807-1809.” *Review of International Economics* 13(4): 631-645.

Lundberg, Mattias and Squire, Lyn (2003). “The simultaneous evolution of growth and inequality.” *Economic Journal* 113: 326-344.

MacDougall, George Donald Alastair (1951). “British and American exports: A study suggested by the theory of comparative costs. Part I.” *Economic Journal* 61(244): 697-724.

MacDougall, George Donald Alastair (1952). “British and American exports: A study suggested by the theory of comparative costs. Part II.” *Economic Journal* 62(247): 487-521.

Marktanner, Marcus and Sayour, Nagham (2009). “Does initial inequality prevent trade development? A political-economy approach.” *Trade and Development Review* 2(2): 93-105.

Melitz, Marc J. (2003). “The impact of trade on intra-industry reallocations and aggregate industry productivity.” *Econometrica* 71(6): 1695-1725.

Nguyen, Quynh (2015). “Mind the gap: Inequality aversion and mass support for protectionism.” NCCR Trade Regulation Working Paper No 2015/19.

Ohlin, Bertil (1933). *Interregional and International Trade*. Cambridge, MA: Harvard University Press.

Pasadilla, Gloria M. and Liao, Christine Marie M. (2005). “Who are opposed to free trade in the Philippines?” *Philippine Journal of Development* 59, First Semester 2005(XXXII-1): 1-17.

Ricardo, David (1817). *On the Principles of Political Economy and Taxation.* London: John Murray, Albemarle-Street.

Sampson, Thomas (2014). "Selection into trade and wage inequality." *American Economic Journal: Microeconomics* 6(3): 157-202.

Scheve, Kenneth F. and Slaughter, Matthew J. (2006). "Public opinion, international economic integration, and the welfare state." In: Pranab Bardhan, Samuel Bowles, and Michael Wallerstein (eds.). *Globalization and Egalitarian Redistribution*, Princeton University Press: Princeton, pp. 499-521.

Stark, Oded (2013). "Stressful integration." *European Economic Review* 63:1-9.

Stark, Oded and Włodarczyk, Julia (2015). "European monetary integration and aggregate relative deprivation: The dull side of the shiny Euro." *Economics and Politics* 27(2): 185-203.

Stark, Oded, Zawojska, Ewa, Kohler, Wilhelm, and Szczygielski, Krzysztof (2018). "An adverse social welfare effect of a doubly gainful trade." *Journal of Development Economics* 135: 77-84.

Stolper, Wolfgang and Samuelson, Paul A. (1941). "Protection and real wages." *Review of Economic Studies* 9(1): 58-73.