CAA
2017

Anna Kubicka and Maciej Kasiński
http://dx.doi.org/10.15496/publikation-43229

The Metrological Research of Machu Picchu Settlement: Application of a Cosine Quantogram Method for 3D Laser Data

Anna Kubicka
Faculty of Architecture
Wroclaw University of
Science and Technology
kubicka.ania@gmail.com

Maciej Kasiński
maciejkasinski@gmail.com

Abstract

The purpose of this research is to look for a basic unit or units of measure (quantum), the multiplication of which would help delineate the outline of Inca settlements such as Machu Picchu. By making use of the statistical method developed by D. G. Kendall, the cosine quantogram, and dealing with data acquired through 3D laser scanning, we can answer the question about Inca imperial measurement system. Based on length measurements from the construction level of niches, we can conclude that an imperial system of measure existed. Three basic units of design were used in different ranks and functions of the building, as follows: 0.20 m; 0.41 m; 0.54 m.

Keywords: Inca architecture, length measurement, statistics, metrology, 3D laser scanning

Introduction

The Inca Empire formed a conglomerate of many ethnic groups and languages. The Inca elite formed a small community of around 300,000 - 500,000 people out of the 6 to 14 million people in the empire (Szemiński and Ziółkowski 2014:34-35). In terms of territorial extent in the middle of 15th century, the Inca state covered an area encompassing parts of the of modern nations of Columbia, Ecuador, Peru, Bolivia, Argentina, and Chile. The life of the empire's inhabitants was controlled on many levels by the Inca administrative authorities who managed the labour service for the state (*mit'a*). The Inca urban planning and architecture was characterized by the standardization of forms, structures, and function; however, there was a great deal of variability due to local traditions.

The aim of this research is to verify the hypothesis about the presense of a basic unit (quantum) of length measurement used by the Inca to delineate the architectural complex of Machu Picchu. This paper is focused on the methodological part of the research based on the cosine quantogram method, Monte Carlo verification, and hypothesis testing of equal quanta (basic units of measure) between buildings. We have made use of already known algorithms and statistical techniques. Our own implementation has been developed using the free, open-source software environment R (Kasiński 2019)[1].

The subject of research, Machu Picchu, was an imperial investment of Inca Pachacuti (Rowe 1990:141-145). The general layout of Machu Picchu seems not to have been achieved by chance but rather deliberately planned. It is visible in the spatial organization of the architectural complexes where each sector has a specific border carefully arranged on *andenes* (terraces). Engineering skills visible in wall construction and stonework suggest that a building layout plan must have existed. Inca architects/engineers and stonemasons supervised imperial constructions, but the scale of their influence in the construction process of the settlement plans is unknown because there is no information in Colonial documents. The process of building construction mainly relied on the great workforce of temporally working people (*mi-*

1 The creation of R package would not be possible without R software environment authors (R Core Team 2016; Wand 2015; Wickham et al. 2018; Wickham and Henry 2018), as well as authors of useful R graphical helpers (Hocking 2017; Neuwirth 2014; Wickham 2016).

Measure	María Rostworowski de Diez Canseco [cm]	Wendell C. Bennett [cm]	John Rowland Rowe [cm]	Santiago Agurto Calvo[cm]
sikya	81.0	81.0	84.0	80.0-84.0
cuchuch	_	45.0	45.0	40.0-45.0
chaqui	21.0	20.0	25.0	25.0-30.0
yuku	10.0-12.0	10.0-12.0	10.0-12.0	10.0-15.0

Table 1. Small measurements comparison based on modern scholars' studies of Quechua dictionaries.

tayoq), who were gathered from different parts of the Inca realm to provide a labour tax for the Inca state (Rowe 1946:69). The Inca Empire was composed of dozens if not hundreds of different ethnic and tribal groups (Rowe 1946:185-192), many with distinct languages and maybe with a different tradition of length measurement. There is no information from written sources about work duration or the number of people working and living in Machu Picchu. However, bioarchaeological and skeletal studies of the individuals buried at Machu Picchu (shows the ethnic diversity of the site's population where the skeletons of commoners include natives of both the coast and highlands (Turner and Hewitt 2018; Turner et al. 2009; Verano 2003).

Our knowledge about Inca measurement is based on information from post-conquest chronicles (Betanzos 2004 [1576]:88,98,116) and modern studies (Table 1) of 16th century Quechua dictionaries of Padre Gonzales Holguin or Fray Domingo de Santo Tomas. Based on that we can presume that Inca used an anthropometric system of measurement where the basic unit was adopted from body parts, like fingers (Quechua: *yuku* – distance from the tip of outstretched thumb and forefinger), foot (Spanish: *pie*, Quechua: *chaqui*), or cubit (Spanish: *codo*, Quechua: *khococ*). Because of the lack of written sources or archaeological finds of measuring devices, a statistical approach to these studies is needed to test whether there was a standard system of Inca measurements reflected in the architecture of Inca imperial constructions like Machu Picchu.

Methodology

Machu Picchu in the 3D Point Cloud

For the purpose of digital documentation and these studies, the agricultural and urban zone of Machu Picchu were scanned with a Leica P40 terrestrial 3D laser scanner. Beginning in 2010, research and documentation[2] have covered the whole urban zone of Machu Picchu with more than 600 scanner positions. The 3D point cloud obtained from scans is the most accurate form of measurement for this type of site. The varied landscape and complicated geometry of trapezoidal Inca edifices require a detailed and high-resolution dataset with visible type and course of masonry. The dataset collected from the 3D point cloud was initially transformed into vectorised plans and sections. Each plan contains desired measurements that are collected in the textual database for quantum analysis.

Data Selection

Building plans have three levels of layout: a foundation, a level of niches or windows, and a top level of masonry walls to support a roof construction. These studies are concentrated on non-invasive techniques of documentation, so it is not possible to reach the foundation level or even to establish how deep an individual footing of the building is because of the irregular bedrock of the mountain. However, in the masonry courses of Inca walls, we can observe a horizontal level used for placing windows and niches. Whole building interiors are occupied with niches or windows arranged in a row in specific intervals. Trapezoidal in shape with the shortest side on the top, they are noticeable in all kinds of Inca buildings across the empire, not only in sacred places. Niches are usually situated approximately 120-150 cm above a floor level and their function can be different depending on their location and size (Protzen 1993:221). Based on the observation of the course of

2 Team from 3D Scanning and Modelling Laboratory (Wroclaw University of Science and Technology) and The National Archaeological Park of Machu Picchu.

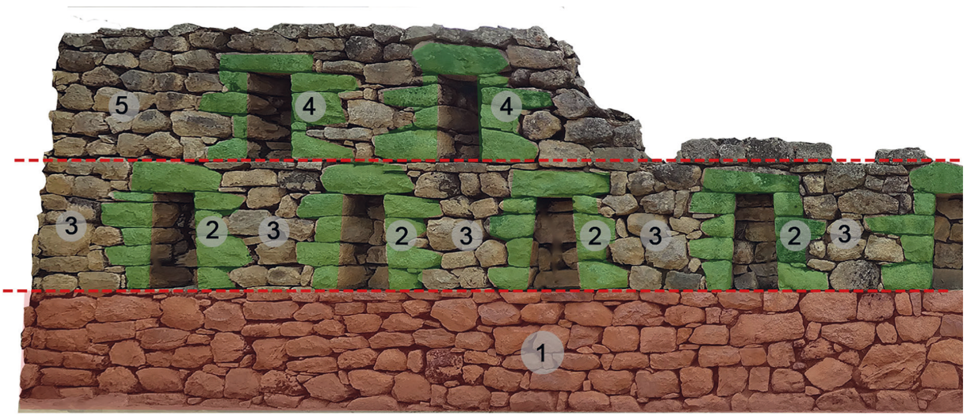

Figure 1. The wall of Kallanka (Sector 5D) with construction phases, orthoimage.

stone blocks in the wall and construction of window's stone frame, Niles (1987) presented a theory that the walls were built up to the desired height of the sills for the niches, after which frames for the niches were constructed at the specific intervals. When all the frames were placed, the gap between them was filled and additional courses of masonry (Figure 1). Length dimensions of niches and width and intervals between them were included in the dataset due to their uniformity in building plans. These measurements are significantly small, in terms of value, as well as large in terms of sample size to conduct the cosine quantogram statistic test. Their role in building layout and construction features potentially suggest the existence of standardized measurements in Inca building plans.

Data Characterisation

We divided the urbanized site of Machu Picchu into separate architectural sectors based on the modern subdivision of the site and common architectural and masonry features (Figure 2). Length measurements [cm] on the level of niches were assigned to each wall of a building. Groups of buildings having the same function, layout plan, and masonry were usually placed on the same terrace or two. This type of grouping was categorised as a sector within the established dataset. First, quantum searched the measurements of each building of the sector. By employing this step, we could exclude a possibility of more than two quanta per dataset, which would cause a distortion in the quanta score and eventually lead to an incorrect interpretation (Mustonen 2012). In most results for each building, there was only one candidate for quantum and for these cases the quantum estimation was calculated for the whole group, but when the results were divided, subgroups for sectors were established.

Data selection is probably one of the main questions behind ancient metrology. Which building elements can be used in the study of metrological units and how did the past builders of Machu Pichu follow the process of measurement execution? In regard to the niches, the question about the horizontal construction process emerged. We examined whether the corner measurements influenced the quantum estimation, but there also arose the question about the starting point of delineation. To find a solution for this question, we constructed five models. The first model excludes extreme left measurements, the second one excludes extreme right measurements. The third excludes both extremes, while in the fourth model all measurements are counted. The fifth, and final, model includes axes measurements (Figure 3). The differences of quantum values do not promote any of the five models but do exclude delineation between axes. Estimation of quantum does not significantly change after exclusion of corner measurements, probably because of the small ratio of corner measurements to interior measurements, which proves that the quantum in the series of measurements from widths and distance between niches exists.

Method Description

The cosine quantogram method used in this study was developed by David George Kendall (1974) for detecting a quantum of an unknown size from a set of data. A statistical model of cosine quantogram has been successfully implemented during the analysis of Mediterranean architectural sites as well as of

Figure 2. The plan of Machu Picchu, the urban zone with sectors marked, based on: Plano sub sectores llaqta Machupicchu (archives of Parque Arqueológico Nacional de Machupicchu).

European medieval urbanism (Cox 2006; Pakkanen 2001; Pakkanen 2004; Pakkanen 2005). In a case of Inca architecture, each building dimension of Machu Picchu can be described as an integer M multiplied by the basic unit q plus an error ε, i.e.

$$X_i = M_i q + \varepsilon_i$$

(Equation 1)

The error might be a result of ancient building execution as well as the modern method of measurement. In the equation ε, which is significantly smaller than q, is analysed and then the formula calculates an amount which clusters around q. Assuming a range of possible quantum values, cosine quantogram is defined as:

$$f(q) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \cos\left(\frac{2\pi X_i}{q}\right)$$

(Equation 2)

Where N is the sample size and $\{X_i\}$ is the set of building dimensions. Applying (Equation 1) to (Equation 2) reduces the integer M and makes the formula depend directly on ε - the remainder of dividing a measurement by q. The value of q that maximizes the formula within a given range is the one with the highest probability of being a quantum. The term $\sqrt{(2/N)}$ adds dependence of the cosine quantogram result on sample size. Unrounding the measurements, also proposed by Kendall (1974), makes the data a bit less sensitive to measurement

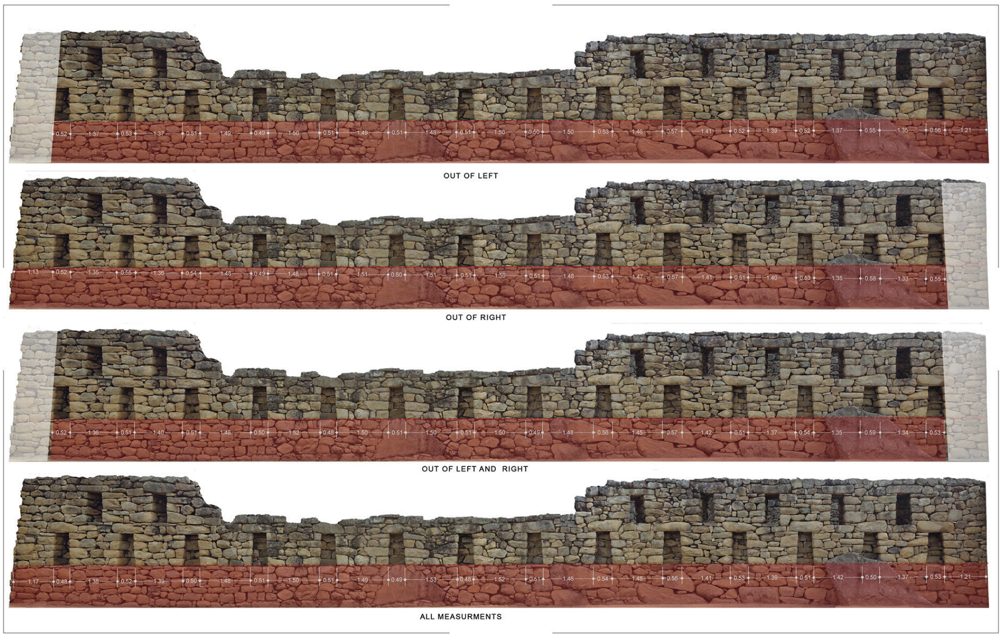

Figure 3. The four models of niches layout execution.

errors of any type. Unrounding is defined as adding a little random noise to each measurement. If measurements are rounded up to the nearest centimeter, a random number from a uniform distribution on the [-0.5cm, 0.5cm] interval is added. We apply unrounding to both original and sampled data before inputting it to the cosine quantogram.

Monte Carlo Validation of Quantum — Bootstrap Confidence Intervals

After applying the cosine quantogram to the niches data, we wanted to make sure the estimated quantum is neither evaluated by chance nor dominated by some outlier measurements. The Monte Carlo bootstrap method is used to construct confidence intervals – its limits determine the range of most possible quantum values. From the original data, a sufficiently large number of equal size samples are drawn with replacement. By "sufficiently" we mean the number of draws such that increasing it does not enhance the confidence of estimation. In our case the empirically determined number of draws is 1000. We tested that rate of convergence to confidence interval limits which is very low after that number. For each sample, after unrounding, the same cosine quantogram procedure is applied. This way we get a bootstrap distribution of the quantum parameter. Assuming 5% significance level, if quantum estimations for the original data fall within the confidence interval, it can be interpreted that the chance of the quantum happening by chance is very low. Specifically, in cosine quantogram problems, parameter distributions might be concentrated around few quanta, which requires a cautious interpretation of confidence intervals (Figure 4).

Quanta Equality Validation with Significance Tests

Evaluation and further verification of cosine quantogram results give rise to the question of whether little differences between estimated quanta lead to a single quantum. At this stage, two similar tasks are defined: first is to statistically verify that quanta in selected sectors are equal; the second is to verify that quanta in buildings within a given sector are equal. For this reason, we construct tests based on the maximum likelihood estimation.

Considering buildings within a given sector, the null hypothesis states that there is no significant difference between quantum for the entire sector and quantum for each building. The alternative hypothesis states that in at least one building, the quantum is different than the general one. In an analogous way, quanta equality is tested between sectors. Assuming data follows a von Mises distribution, we define the likelihood ratio function, which expresses how many times the original data under the null hypothesis model is more likely than under the alternative hypothesis. Without loss of generality, a logarithm of the LR ratio is taken as the test statistic. We construct a distribution of test statistics by employing Monte Carlo methods based on the KDE (kernel density

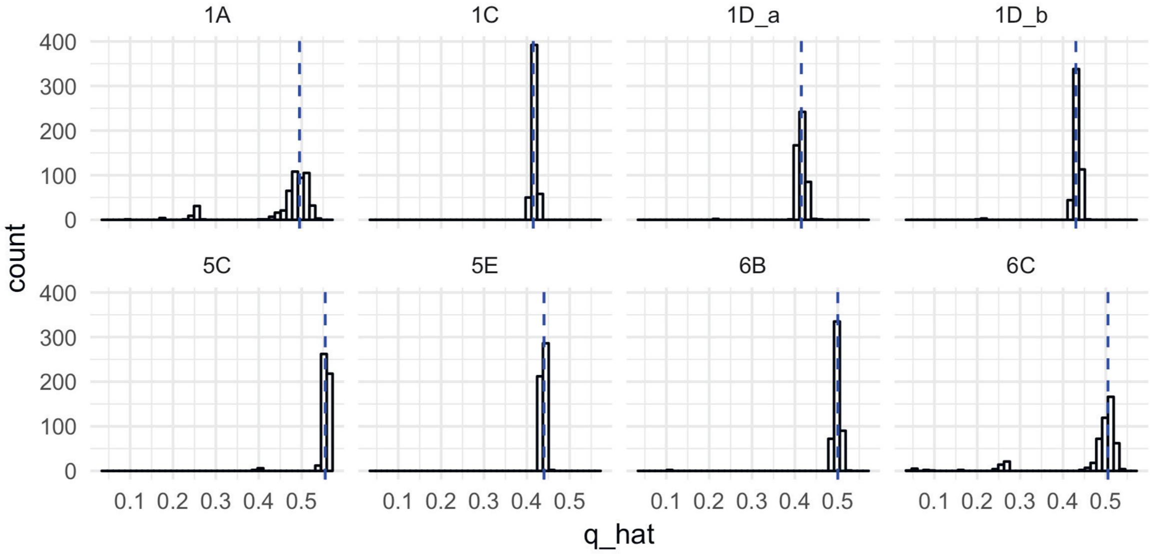

Figure 4. Bootstrap distribution of quantum parameter.

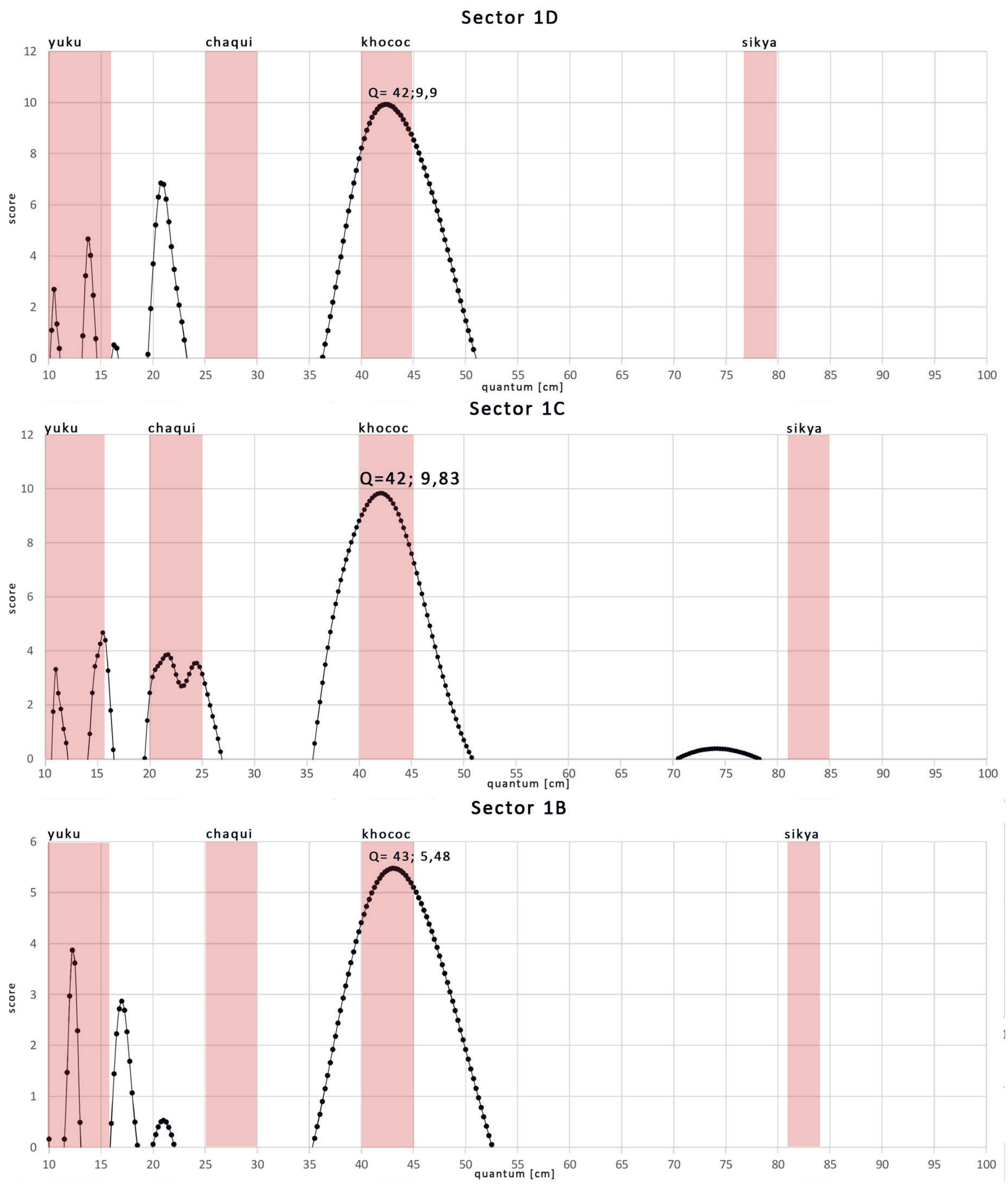

Figure 5. Quantum results for sectors: 1D, 1C, 1B.

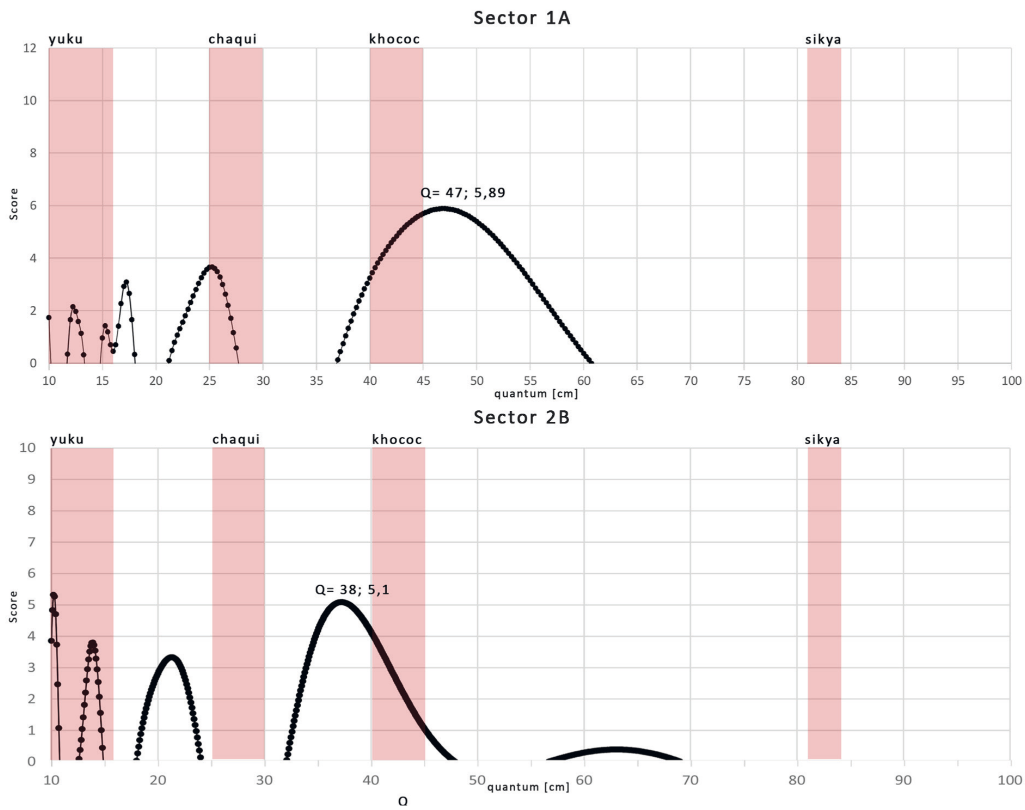

Figure 6. Quantum results for sectors: 1A, 2B.

estimator) model of data. In this approach, different than bootstrapping, samples are drawn from the theoretical distribution of measurements given by density estimation.

If the test statistic from the original data falls outside of the defined confidence interval, then the null hypothesis is rejected in favour of an alternative, which would support the idea that quanta are significantly different. Otherwise, accepting the null hypothesis means that there is a strong ground for quanta equality, but that does not provide us with the ability to ultimately prove it.

Looking for Quantum—Results

The range of dimensions collected from the level of niches is between 25 cm and 150 cm. In order to exclude meaningless quanta, lower and upper bounds of 10 cm and 100 cm were assumed. If smaller units were employed in building design and execution, it is doubtful that they could be discovered in a metrological analysis of rather large building dimensions included in this study.

Quantum results were obtained from a part of the urban area of Machu Picchu. Further research was conducted to test each building and sector at the site. The basic units of design were investigated on the west side of the main plaza (Sector: 1Da, 1Db, 1C, 1B, 1A, 2B) and the east side of the main plaza (Sector: 6B, 6C, 5C, 5E) (Figure 2). The results cover the region from the entrance of the site to the urban zone, which is dominated by buildings with a residential function. This part of the site consists of five modern sectors. Two of them: 1A ("*Casa real*") and 1B ("*Templo del Sol*") have masonry distinguished from the rest of the sectors which consist of the housing zone. Sector 1B is also different in terms of function, with two buildings of cut and fitted masonry, designed as the sacred Temple of the Sun. However, obtained quantum values in the most part (except Sector 1A) of the west side of the main plaza are estimated between 38 - 43 cm (Figures 5 and 6). Considering each sector separately, small differences could occur as a result of ancient measuring execution but planned quantum could be the same for whole architectural groups. Evaluation of this problem was statistically tested for equality of quanta in Sectors 1Da, 1Db, 1C, 1B. Results reveal that the hypothesis about quanta equality is accepted (Figure 7). The possible value range of quantum corresponds with a measure of a cubit (Quechua: *khococ*), which,

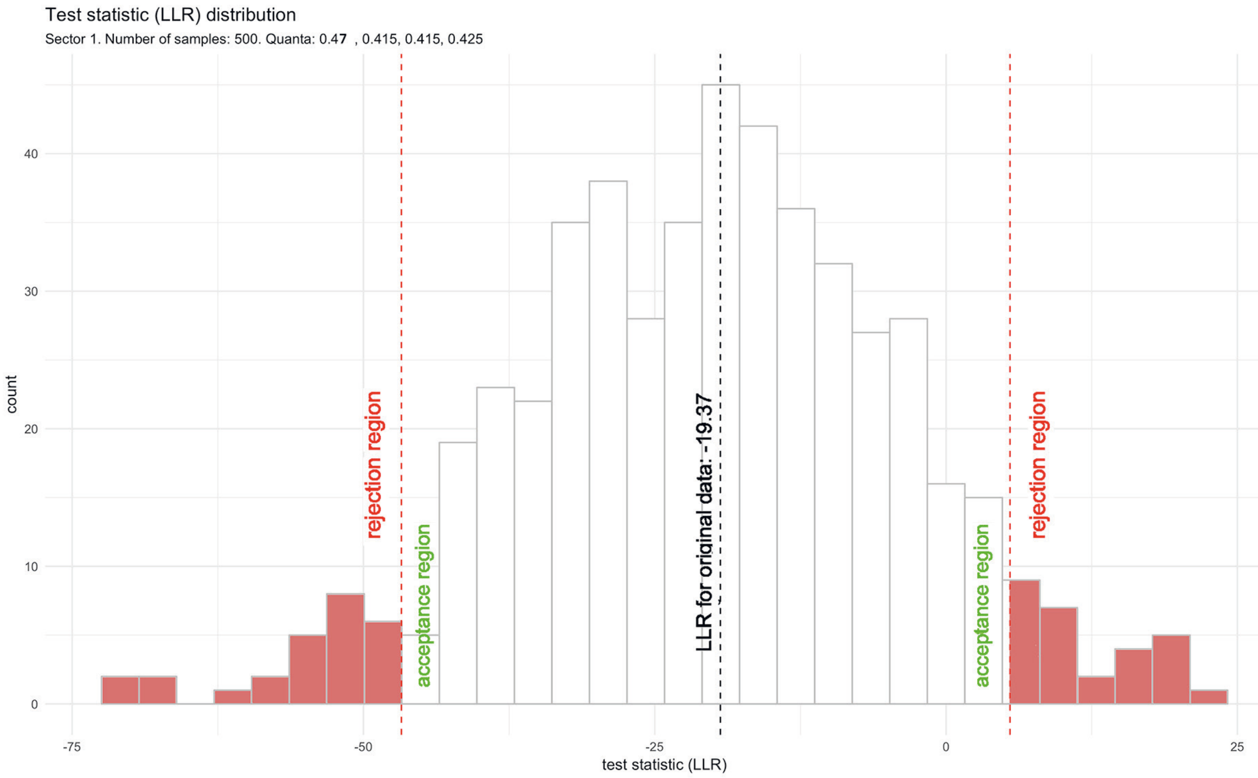

Figure 7. Test statistic (LLR) for equal quantum in sectors on the west side of the main plaza.

based on previous measurement studies, is between 40-45 cm (Rostworowski 1960:10-11). Multiplication of this measurement as a basic unit creates a whole system of length measurements: *sikya* = 81 cm (2*40.5 cm) and *rikra* = 162 cm (4*40.5 cm), which are known from the urban division of Ollantaytambo (Farrington 2013:70-76). While the Inca system of measurements as well as studies of Inca engineering (Wright et al. 2011:14-21; Wright et al. 2016:35-37) and building layout (Lee 1996) have been explored by scholars, they have never been based on metrological studies in the building scale.

The east side of the main plaza consists of high-status households organized in the form of *kancha* – the classic Inca household that contains rectangular units surrounded by an enclosure (Sectors 5C, 6B, 6C, 1A-from the west part). But *kancha* are not the only structures to occur on this side of the plaza. Behind each *kancha* is a type of small, domestic building that is grouped in a row without a courtyard. In Inca times they could have been occupied by servants (Quechua: *yanakuna*) dedicated to each elite compound (Salazar 2004:29-30). Differences between these domestic sectors are also visible in quanta results. For Sectors 5C, 6B, 6C, and 1A, the quantum is in the range of 47-55 cm (3*54 cm = *rikra*) (Figure 8), but in the eight small domestic buildings in Sector 5B and 5D, the value of the quantum is 20 cm (8*20 = *rikra*), which correspond to the size of a hand (k'apa).

WEST SIDE OF PLAZA	QUANTUM ESTIMATION [cm]	EAST SIDE OF PLAZA	QUANTUM ESTIMATION [cm]
1A	47	5B	58
1B	43	5C	55
1C	42	5E	45
1D	42	6B	50
2B	38	6D	44
		6C	50

Table 2. Quantum estimation for two sides of the main plaza.

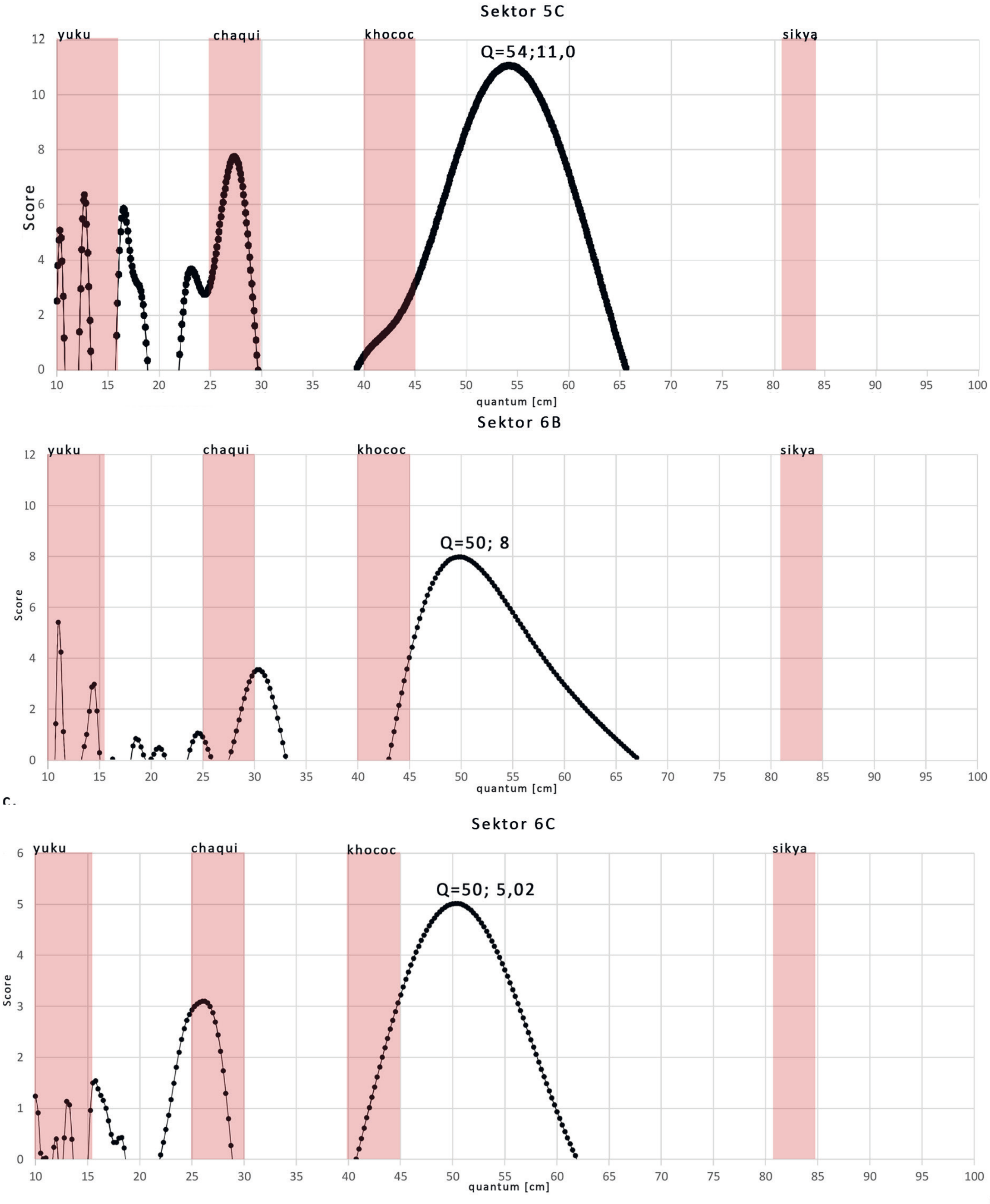

Figure 8. Quantum results for sectors: 5C, 6B, 6C.

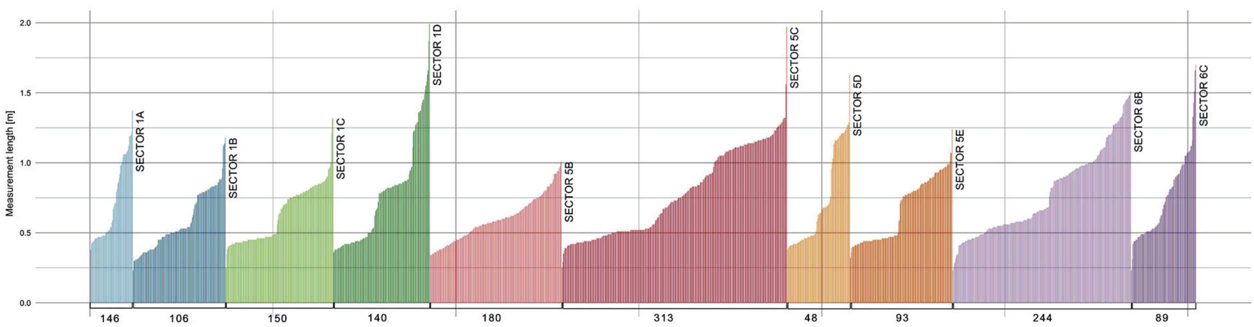

Figure 9. Distribution of length measurements on the level of niches for each sector.

Quantum results from sectors on the west side of the plaza were statistically tested for being equal because of the small differences between quanta within architectural groups. The Inca buildings on Machu Picchu were constructed of semi-cut or unworked fieldstones laid up in an argillaceous mortar. Based on the results we conclude that in this type of masonry it is not possible to distinguish one single unit of measurement. This means that quantum of 40 cm and the quantum of 44 cm are statistically indistinguishable. Apparently, inequality could emerge from ancient measuring execution, but not from the different local traditions of measure. A distinctive characteristic of housing design at Machu Picchu with simple, small houses on the west and elite housing compounds on the east was also evidenced in two different quanta, but in both cases, they are an integer multiple of a fathom (Quechua: *rikra*) 4*41, 3*54, 8*20.

The statistical approach in this metrological analysis revealed that based on length measurements from the construction level of niches we can conclude that an imperial system of measure for the Inca state could exist. Three basic units of design were used in buildings that had different functions and were associated with people of different ranks. These are as follows: 20 cm; 41 cm; 54 cm (Table 2). Further multiplication of these values coincides with other known length measurements from Hispanic sources: 81 cm (*sikya*), 162 cm (*rikra*). The non-invasive technique of documentation does not allow us to measure the layout of the building on foundation level; however, further studies are focused on the theoretical reconstruction of the base level in order to find measurements that correspond with a multiplication of the basic unit of design found in upper parts of Inca buildings.

Acknowledgements

This work was supported by the National Science Centre of Poland under doctoral scholarship ETIUDA 3.

References

Betanzos, J de 2004[1576] *Suma y Narración de los Incas*. Carmen Martín Rubio, M (ed.) Madrid: Polifemo.

Cox, M 2006 Determining Greek Architectural Design Units in the Sanctuary of the Great Gods, Samothrace: Application of and Extensions to the Cosine Quantogram Method Unpublished thesis (PhD), Emory University.

Farrington, I 2013 *Cusco. Urbanism and Archaeology in Inca World*. Gainesville: University Press of Florida.

Hocking, T D 2017 *directlabels: Direct Labels for Multicolor Plots*. R package version 2017.03.31. https://CRAN.R-project.org/package=directlabels

Kasiński, M 2019 *Quantatools: Analysis of quantum in archaeological data with cosine quantogram and related statistical methods*. R package version 0.0.1, 06 December 2017. Available at https://github.com/maciejkasinski/quantatools. [Last accessed 15 February 2019] DOI: 10.5281/zenodo.2566606.

Kendall, D G 1974 Haunting quanta. In: Hodson, F.R (ed.) *The Place of Astronomy in the Ancient World*. London: Royal Society A 276, pp. 231–266. DOI: 10.1098/rsta.1974.0020.

Lee, V 1996 *Design by Numbers. Architectural order among Incas*. Wilson, WY: Sixpac Manco Publications.

Mustonen, S 2012 *Hunting multiple quanta by selective least squares*. [online access at: http://www.survo.fi/papers/HuntingQuanta2012.pdf last accessed 20 July 2017].

Neuwirth, E 2014 *RColorBrewer: ColorBrewer Palettes*. R package version 1.1-2. https://CRAN.R-project.org/package=RColorBrewer).

Niles, S 1987 *Callachaca: Style and status in Inca community*. Iowa City: University of Iowa Press. DOI: 10.2307/j.ctt20h6v13.

Pakkanen, J 2001 Deriving ancient foot unit from building dimensions: a statistical approach employing cosine quantogram analysis. In: Burenhult, G and Arvidsson, J (eds) *Archaeological Informatics: Pushing the Envelope. CAA2001, Proceedings of the 29th Conference, Gotland, April 2001*, Oxford: Archaeopress, pp. 501-506.

Pakkanen, J 2004 The Temple of Zeus at Stratos: New Observations on the Building Design. *Arctos Acta Philologica Fennica*, 38:95-121. DOI: 10.2307/41542136.

Pakkanen, J 2005 The Temple of Athena Alea at Tegea: Revisiting Design-Unit Derivation from Building

Measurements. In: Østby, E (ed.) *Third international seminar on Ancient Arcadia, 7-10 May 2002*, Athens: Norwegian Institute at Athens, pp. 167-183.

Protzen, J P 1993 *Inca Architecture and Construction of Ollantaytambo*, Oxford: Oxford University Press. DOI: 10.1525/aa.1994.96.3.02a00570.

R Core Team 2016 *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, https://www.R-project.org/).

Rostworowski de Diez Conseco, M 1960 *Pesos y medidas en el Peru pre-hispanico*. Lima, Perú: Minerva.

Rowe, J 1946 Inca culture at the time of the Spanish conquest. In: Steward, J H (ed.) *Handbook of South American Indians*, vol. 2, Washington: Smithsonian Institution, pp. 183-330.

Rowe, J H 1990 Machu Picchu a la Luz de Documentos del Siglo XVI. In: *Histórica*, vol.14(1), Lima: Pontificia Universidad Catolica del Peru, Departamento de Humanidades, pp. 139-154.

Salazar, L C 2004 Machu Picchu: Mysterious Royal Estate in the Cloud Forest. In: Burger, R L and Salazar, L C (eds.) *Machu Picchu. Unveiling the Mystery of the Incas*. London: Yale University Press, pp. 21-48.

Szemiński, J and Ziółkowski, M 2014 *Mity, rytuały i polityka Inków*. Warszawa: Państwowy Instytut Wydawniczy.

Turner, B L and Hewitt, B R 2018 The Aqlla and Mitmaqkuna: Diet, Ethnicity, and Status. In: Alconini, S and Covey, A R (eds.) *The Oxford Handbook of the Incas*. Oxford, UK: Oxford University Press, Chapter 3.4. DOI: 10.1093/oxfordhb/9780190219352.013.47

Turner, B L, Kamenov, G D, Kingston, J D, and Armelagos, G J 2009 Insights into immigration and social class at Machu Picchu, Peru based on oxygen, strontium, and lead isotopic analysis. *Journal of Archaeological Science* 36: 317-332. DOI: 10.1016/j.jas.2008.09.018.

Verano, J W 2003 Human Skeletal Remains from Machu Picchu. A Reexamination of the Yale Peabody Museum's Collections. In Burger, R L and Salazar, L C (Eds.) T*he 1912 Yale Peruvian Scientific Expedition Collections from Machu Picchu. Human and Animal Remains*, New Haven, Connecticut: Publications Office, Peabody Museum of Natural History, Yale University.

Wand, M 2015 *KernSmooth: Functions for Kernel Smoothing Supporting* Wand & Jones (1995). R package version 2.23-15. https://CRAN.R-project.org/package=KernSmooth)

Wickham, H 2016 *ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag, New York.

Wickham H, François, R, Henry, L, and Müller, K 2018 *dplyr: A Grammar of Data Manipulation*. R package version 0.7.6. https://CRAN.R-project.org/package=dplyr;

Wickham, H and Henry, L 2018 *tidyr: Easily Tidy Data with 'spread()' and 'gather()' Functions*. R package version 0.8.1. https://CRAN.R-project.org/package=-tidyr;

Wright, K R, Wright, R M, Valencia Zegarra, A, and McEwen, G F 2011 *Moray: Inca Engineering Mystery*. Reston, VA: American Society of Civil Engineers. DOI: 10.1061/9780784410790.

Wright, K R, Gibaja Oviedo, A M, McEwen, G F, Miksad, R W, and Wright, R M 2016 *Incamisana: engineering an Inca water temple*. Reston, VA: American Society of Civil Engineers.
DOI: 10.1061/9780784480595.031