Online Searching Behavior and Digital Time

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Danksagung

Die Zeit dieser Promotion war eine schöne, wenn auch herausfordernde Zeit. An dieser Stelle möchte ich mich zuerst bei Herrn Prof. Dr. M. Stadler und Herrn Prod. Dr. W. Neus für die gute und freundliche Beratung, für den fachlichen Input und die bereichernden Gespräche bedanken. Über die Promotion hinaus bedanke ich mich bei Herrn Stadler für die vielen Gestaltungsmöglichkeiten, die er mir im Bereich der Lehre angeboten hat. Es hat Spaß gemacht!

Rückblickend ist diese Arbeit in einer goldenen Zeit entstanden. Die ersten Ideen sind kurz nach der Geburt unseres ersten Kindes gekommen - und so schön die Babyzeit war, so war es doch ein wohltuender Ausgleich, kognitive Arbeit zu leisten. Leider fällt der Abschuss der Arbeit mit dem Tod meines Mannes zusammen. Wer hätte denken können, dass die Exit-Wahrscheinlichkeit, die dem ersten Modell Salz verleiht, uns privat so unerbittlich treffen würde. Nun schließe ich diese Kapitel mit größter Wehmut, mit einem weiterhin brennenden Schmerz aber tatsächlich auch mit Dankbarkeit für diese schöne Zeit der Promotion.

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Chapter 1

Introduction

E-commerce has initially raised the expectations that it would at last be the realization of the perfect competition commonly assumed in economic theory - efficient, frictionless markets under perfect information (Brynjolfsson and Smith, 2000). It was soon clear that the expectations about a lower level of prices, homogeneity in price, and higher price elasticity are at most only partially fulfilled (Smith et al., 2001). And while the online availability of information on products leads to very low search costs in the online channel, it is now a well-known and widely discussed paradoxon that consumers' search behavior remains lower than optimal: the search pattern of consumers shows a strong preference for few, prominent retailers (see e.g. Johnson et al. (2004) or the literature discussion in De los Santos et al. (2012), confirmed more recently in De los Santos (2018)). On the other hand, online publicity has made the worldwide expansion of a calendar discount sales tradition such as Black Friday possible: firms are confronted to the competition by new pricing strategies and have to align their offer. The following work provides explanations for these phenomena, focusing at the impact of time and of consumers searching behavior, adopting three different perspectives on the topic: a classical consumers search model; an alternative *firms* search model which accounts for the new big-data world where firms track their consumers individually; and a model of intertemporal decisions which accounts for the fact that the worldwide online market sets incentives for firms to align with other firms' pricing strategies, which makes their offers more transparent and also more predictable for consumers.

Consumers acceptance of the online channel is often modeled in a self-evident way as an actual *reluctance against* the online channel. Distrust against a new, unfamiliar, not yet mature technology which is, on top, difficult to understand, is not surprising. But while such form of distrust is natural in pioneer literature,[1] it is quite surprising to observe

[1] To give an example from a still frequently cited paper, consider Liang and Huang (1998): the formulation of the questionnaire in it feels nowadays biased against online commerce

it also in later papers, as for example in the qualitative study in Kacen et al. (2013), where disadvantages as uncertainty about handling, exchange, authenticity or lacking the shopping experience and client counseling are said to overweight the benefits in term of variety of choice or lower price. The rise of e-commerce contradicts this posture: the overall increase in the volume of online sales, amounting to more than 4.25 trillion US dollars in 2020,[2] is speaking for itself: consumers adopt e-commerce. Estimations of considerable consumers' surplus from the Internet, such as those listed in Goldfarb and Tucker (2019) (section 8.4), suggests that consumers maximizing their utility when choosing the online channel.

A major concern linked to the rise of e-commerce is that local stores would get "endangered by extinction", as put in Güsken et al. (2020). The authors formulated concerns that owner-managed retail stores would miss the skills and the knowledge necessary for the implementation of multichannel strategies, and would not be able to compete against the rising e-commerce. Expanding to the online channel is, however, no panacea, as this new channel might offset the old one: online success might cannibalize local sales (Bar-Gill and Reichman (2021)).

This concern of cannibalism is at the core of our analysis, but we must first delimit our perspective from neighboring topics. The eventuality of a cannibalistic behavior of firms has already been testified for the case when a producer introduces direct marketing additionally to its local retailer supply-chain (see e.g. Chiang et al. (2003), where a preference parameter for direct sales is introduced, which might be re-interpreted as preference for online sales; or Tsay and Agrawal (2004), where multiple channel strategies are considered): direct marketing might or might not be beneficial to the interested parties. However, we do not consider a chain of producer and retailer, but only retailers. The danger of cannibalistic behavior has also been mentioned with respect to information externalities: consumers might inspect a product in a local store, and browse subsequently the Internet for the best price, a behavior called *showrooming*. Inversely, consumers might get information about a product online (from experts or fellow consumers assessments) and buy in a local store, a counterpart called *webrooming* behavior, and that is none less widespread (Arora and Sahney, 2017). Both showrooming and webrooming are, basically, free-riding behaviors. Here again, depending on the level of search costs, information externalities might or might not be beneficial to the retailers. Webrooming has been put forward as an instrument to attract consumers in local stores (Kim et al., 2022), which leads us back to the topic of local stores as endangered species. Even though the topic of information externalities is related to the question of the searching behavior of consumers, it is not

[2] See `https://www.statista.com/statistics/379046/worldwide-retail-e-commerce-sales/`, visited 2021-12-12.

part of the following chapters. Our analysis focuses instead on the role of time and of the timing of consumers decisions.

Our first model, in chapter 2, analyzes the role of time in a classical frame for the trade-off between traditional and online channel for consumers and firms. Classical search models in the tradition of Weitzman (1979) enhance the role of time in the searching behavior: the fundamental question for the consumer is whether the expected costs of searching further on can be rewarded by a higher satisfaction with the result of the search. Time can explain the self-restriction of consumers in their searching patterns. If they do not expect to find a significantly better offer than the first one, they might save their time and terminate the search very soon: because of their high time preference, the marginal costs of searching soon offset the discounted marginal benefit of searching. Consumers' impatience is indeed a central explanatory factor for suboptimal searching behavior (Stigler, 1961). This approach inspires a strand of literature examining the nature,[3] geographical development,[4] and branch specificity[5] of searching costs. In the first model of this work, following Hendershott and Zhang (2006), we focus at the role of time. We further look at the specific trade-off in the case when consumers can choose between buying in a traditional, brick-and-mortar shop or from an online seller, and the impact of consumers' behavior on the decision of traditional shops to expand online and to offer, additionally to or instead of their brick-and-mortar shop, an online shop. This search model is able to account for the heterogeneity of prices both in traditional and in online retail: this heterogeneity results from the assumption that consumers' valuations of the product are heterogeneous. In this search model, online retail leads to a self-selection of consumers in the two retail channels, depending on their valuation for the good. This model implies a shift of welfare from the firms towards consumers and a specialization of the traditional channel in high-valuation-consumers, and no extinction of the traditional channel. The major contributions of this chapter to the existing literature are the following: first, it embeds the trade-off between traditional and online channel for consumers and firms in the well-established and very attractive framework of Weitzman (1979) and Hendershott and Zhang (2006), which allows for complex assumptions and for a focus at the role of time preferences. Second, it explains the above mentioned subsistence of price heterogeneity in the online channel. Third, it offers an new explanation for the empirical finding that firms follow different pricing strategies in their traditional and online channel, and that consumers are willing to accept these differences in prices (Homburg et al., 2014).

[3] An interesting example is Dutta and Das (2017), which differentiates impatience by assessing the respective role of cognitive factors (education and internet experience) and monetary factors (income and internet costs).

[4] E.g.Taiwan in Liang and Huang (1998) or Beijing in Clemes et al. (2014).

[5] E.g. Park et al. (2009) for health information; Klein and Ford (2003) for the automobile branch; or Dutta and Das (2017) for laptops and mobile phones.

Common experience with online marketing suggests however that this classical framework does not completely fit today's reality. In the age of big data, where individual consumers data is a new, tradable input to firms, where individual online behavior gets tracked, profiled and analyzed, a very simple fact can explain why the searching behavior of consumers is so limited: firms are struggling for consumers attention with the ultimate aim to present them with the products they want to buy before they even start to search, as formulated in the method for "anticipatory shipping" patented by Amazon.[6]. E. Calvano and M. Polo outline that getting consumers attention is no less than a way to relax competition: *"[m]any websites and apps are in the business of harvesting and reselling human attention. (...) New technologies that use data to profile users and follow them as they traverse the internet (e.g. cookies) scale down competition for attention at the individual level"* (Calvano and Polo, 2021).[7] In our second model, in chapter 3, we therefore turn the tables and assume that the search effort is occurred by the firms, not by the consumers. We develop a brand-new model relying on an established working-horse: the Salop circle (Salop, 1979) appropriate to capture competition among local stores. We add a second layer: above the Salop circle, there is a "cloud", where online retailers are gathered. The competitive feature most relevant to local stores is the distance to them; the competitive feature most relevant to online retailers is their visibility for consumers. We are not aware of any other model explicitly integrating virtual space in a location model, and modeling the relationship between real and virtual stores with the classical instruments of microeconomics. This novelty is the main contribution of the model. The main findings are the following. First, from a given degree of acceptance of the online channel on, all firms have an incentive to participate in the online market; this results fits the situation since the COVID-19-lockdowns, which durably increased the acceptance of the online channel. Second, the online channel enhances competition and leads to a shift of welfare in the benefit of consumers; this is consistent with our first model. Third, multichannel firms have an incentive to subsidize their online shop to the detriment of their traditional shop. These results are consistent with empirical observations.

Both our first and our second model are quite distant from the existing literature on the topic, with respect to their perspective as well as to their analysis methods. The decision between the online and the local channel is at the core of a number of behavioral studies, resulting in less (Kollmann et al. (2012), Schröder and Zaharia (2008)) or more

[6]Amazon Technologies Inc. Reno NV (2013), available under `https://patents.google.com/patent/US8615473B2/en` (visited 2021-12-12).

[7]As a complement, let us mention that this individual tracking even makes the eventuality of first-degree price discrimination less theoretical, as firms can even gather information on the individual willingness to pay.

(Chang et al., 2005) complex typologies of consumers, accessorily of products and websites. Beyond these behavioral approaches, the existing literature on e-commerce is mainly quantitative - and indeed, digital technologies and the access to never ending sources of data about individual buying behavior are attractive not only to firms, but also to researchers. We complement these approaches with a purely theoretical one.

The third model, in chapter 4, addresses a different dimension of consumers searching behavior. The online presence of firms widens the scope of offers accessible to the individual consumers. They get information about (or: are made attentive to) more offers and promotions, and can access new discount sales traditions like Black Friday which was, initially, restricted to a given geographic area. As the sales tradition gets known and accessible via Internet, consumers consider this information in their intertemporal planning of consumption expenditure. Here, we do not distinguish between online and traditional channel, because it is not relevant; relevant is the publicity around the calendar sales tradition. Indeed, most firms opt for omnichannel strategies and the phenomenon of Black Friday has expanded as well to online as also to traditional shops. The searching behavior is determined by the large expansion of the sales tradition via online publicity, but consumers decision is in first place about when they decide to buy. Our model shows that there is a self-explaining dynamic: as the tradition gets known, further firms have an incentive to participate in it, in order to remain competitive. The pervasive success of Black Friday promotion hints at it being a dominant pricing strategy. We analyse it here as a coherent pricing strategy over a (yearly) period and prove that it does not merely result from a price war for the increased demand linked to Christmas Shopping (at first glance, price war models like Rotemberg and Saloner (1986) and Stadler (2015) could also explain the Black Friday-phenomenon, assuming an increased demand in the pre-Christmas time): Black Friday in itself generates a demand cycle via its publicity. The model analyses Black Friday as a strategy of intertemporal price discrimination among a continuously renewed flow of strategic consumers and proposes a new solution to the Coase conjecture (Coase, 1972). Black Friday induces a market segmentation beneficial to overall welfare as the strong enhancement in firm's surplus, and the small welfare gain by consumers with low valuations for the good who get able to buy at all at the discounted price, overcompensate the degradation affecting most consumers' surpluses. Again, the main contribution to literature is the novelty of the model itself. It is a contribution to the literature on intertemporal pricing decisions and a theoretical modelization explaining the rapid expansion of the Black Friday tradition.

Chapter 5 concludes.

Chapter 2

Time Preference and Searching behavior

We consider the decision by traditional, brick-and-mortar shops to offer additionally an online shop. This decision has been, in the context of the COVID-19 pandemic and the consequent restrictions, of an existential nature: selling online has represented an emergency solution to lockdown measures. The trend to expand online was however observable long prior to the pandemic, as the competitive pressure from online alternatives had become sensible for traditional shops. However justified the fears of extinction for the local retail industry, the development of online retail has been considered as a promise for more transparency and an increased price competition to the benefit, lastly, of consumers. However, empirical evidence suggests that even though price levels are lower in online retail than in traditional retail, prices are still heterogeneous (Smith et al., 2001).

We develop here a simple, classical search model able to account for the heterogeneity of prices both in traditional and in online retail, and to analyse the ultimate consequences of the dichotomy between online and traditional shopping. In this search model, online retail leads to a self-selection of consumers in the two retail channels, depending on their valuation for the good, and changes the market structure in traditional retail. We consider the welfare implications of this transformation.

The following sections are structured as follow. First, we give an overview on the flourishing literature on the topic of online retail. Then, we summarize the assumptions of the model and observe the benchmark case without any online retail. After that, we compute the full game, analyzing consumers' search behavior, firms' price channel choice and setting strategies in each channel, and the resulting distribution of prices. Last, we conclude with a comparison of sellers and consumers surpluses in each case.

2.1 Neighboring Literature

This paper has to be located in the direct lineage of the classic search models by Weitzman (1979), Spulber (1996) and Hendershott and Zhang (2006). The seminal search mechanism developed in Weitzman (1979) is indeed common to these models. Spulber (1996) extends it with the idea that firms and consumers might have different ask and bid prices and that the search activity - with time discounting as search costs - can be considered a decentralized form of market making. Finally, the model by Hendershott and Zhang (2006) applies this search model to a game with direct and intermediated sales by an upstream firm. We apply a comparable search process to the case when decentralized firms and consumers can operate via two channels, a traditional one and an online one.

This very classic and analytic setting focuses at the role of time. The discounting of consumers surpluses is the only source of search costs. Time might here be regarded as effort or as a form of opportunity costs - the time spent searching cannot be spent otherwise.

However classic the search setting, this model displays a slightly unusual light on the topic of e-commerce. It has nothing do to with the idea of a perfect market, nor with the concern of an extinction of the brick-and-mortar commerce. Indeed, the structure of the traditional channel in the full game, compared with the benchmark without online channel, is stable: the maximal price does not shrink, the number of firms is unchanged, only a part of the demand migrates towards the online channel. This specificity of the model must be commented upon for different reasons.

First, this stability might result from the fact that the considered market is far from perfect information: search is necessary. The model might therefore be more adequate for markets that are less transparent, for example because they are not standardized (e.g. workmen services or bank services) or because firms follow a strategy of obfuscation. Search models are indeed the first choice in the literature on obfuscation, as it is the logical next step that firms might strategically set the level of search costs in order to discriminate among their consumers (see e.g. Perloff and Salop (1985), Ellison and Wolitzky (2012) and Petrikaitė (2018)).

Second, this hints at the fact that the behavior of consumers with a high willingness to pay is not influenced by the online expansion, which is line with the empirical result in Kluge and Fassnacht (2015) or Chatterjee and Kumar (2017) that the behavior of consumers in the high-price, traditional channel is not much affected by the online channel.

Third, the restriction on unit costs in the online channel questions the empirical observation that the unit costs are lower for online shops (Garicano et al. (2001); Zhu (2004)). This observation mostly relies on the idea that the online channel might be more cost-efficient. In our model, however, unit costs are equally distributed in the two channels; the reason why the active firms have lower unit costs is that consumers reservation prices are lower.

The reason why this model does not hint at an extinction of the traditional commerce might be simply that it assumes a multitude of decentralized, independent firms. Some of the consequences - the heterogeneity in prices, the difference in prices in the two channels - are well fitting empirical observation. The negligible market power of each and any single firm, however, is the dark side of this modeling decision. The results can therefore only apply to markets where there is no predominance of one seller.

2.2 Model Assumptions

The theoretical frame is similar to the search model as used in Hendershott and Zhang (2006). Because the model framework is known, mathematical proofs are kept short in this chapter. n potential sellers offer a homogeneous product. Each potential seller who finds it profitable runs a physical, "traditional" shop; each potential seller who finds it profitable runs an online shop. Whether a given seller offers none, one or both forms of shop depends on the respective unit costs. Sellers are heterogeneous with respect to these costs: their transaction unit costs for traditional retail, k_t, as well as for online retail, k_o, are each uniformly distributed on $[0, 1]$.

Each seller i sets price p_t^i in the traditional shop and, if he runs an online shop, p_o^i in the online shop, to maximize the expected discounted profits. We assume that stock-keeping is not possible. The firms' discount rate is assumed to be ρ for all firms.

We consider a flow of heterogeneous consumers with a constant, normalized mass of one. The valuations of consumers for the product, v, are uniformly, identically and independently distributed in the unit interval $[0; 1]$. Each consumers buy maximally one unit of the good. Information about the good's features is perfect: there is no eventuality of deception. A share λ of the pool of consumers is steadily renewed: in any infinitesimal time period of length δt, a particular consumer may exit the market with probability $\lambda \delta t$, whereby λ is exogenously given. In continuous time, the survival probability i.e. the probability of not exiting the market in a time laps of length t is $e^{-\lambda t}$.[1] Exiting consumers are replaced by new ones drawn randomly from the same distribution.

[1] $\lim_{x \to \infty} \left(1 - \frac{\lambda t}{x}\right)^x = e^{-\lambda t}$

Consumers browse the offer: in each infinitesimal period, they pick a shop randomly, and observe the price. If they decide to buy, they do so at the end of the period; otherwise, they might either exit the market with a probability depending on λ, or go with the search in the next period and pick a further shop. Consumers know the distribution of prices in the shops, but they do not know *ex ante* which is the particular firm associated to each price. The prices of the different shops are independent and identically distributed, which has the implication that knowing the price in one specific shop does not enable the consumer to learn anything about the other offers: there is no information externality. The only search costs are time costs linked to the continuous time discounting of consumers' surplus with the time preference rate ρ; for simplicity, we assume that the discount rate of firms and consumers are equal.

2.3 Benchmark: Inexistence of Online Shopping

As a benchmark, we consider the case when the product is sold uniquely via traditional retail. The search procedure described here is similar to Weitzman (1979).

2.3.1 Consumers' Decision

Consumers sample among the $n^b \leq n$ firms actually running a traditional shop in the benchmark case. We denote by $\underline{p_b}$ the lowest, and by $\overline{p_b}$ the highest price that might be offered in the benchmark case. Consumers can fall back on an initial reward $y_0 = 0$: if they don't engage in search, or if they engage in search but don't buy, they have neither costs nor benefit. If they engage in search, we denote as y^k the maximum sampled reward from the $k \leq n^b$ sampled offers, i.e. the difference between the consumer's valuation v and the lowest price among the sampled offers.

Value Function

Be $V^b(y^k)$ the value function for a consumer, after having sampled a set of k offers. This function has consider following eventualities. At the beginning of each search period, the consumer might terminate the search and either don't buy, collecting the fall-back reward ($y_0 = 0$); or buy the best of the offers sampled, collecting the reward y^k. Alternatively, he can decide to go on with the search. He will be able to do so only if doesn't exit the market during the search period, i.e. with probability $e^{-\lambda t}$. If he does not exit, he discovers the price p_b^{k+1} and at the end of the period, two outcomes are possible: on the one hand, he might find out that $v - p_b^{k+1}$ is no better than y^k i.e. that the newly found price is higher than at least one of the formerly observed prices, an eventuality with expected discounted reward $e^{-(\rho+\lambda)t} \cdot V^b(y^{k+1}) \cdot \int_{p_b^k}^{\overline{p_b}} f(p)\ dp$, where the best reward is unchanged: $y^{k+1} = y^k$. Second, $v - p_b^{k+1}$ might be higher than y^t, an eventuality with expected discounted reward $e^{-(\rho+\lambda)t} \cdot V^b(y^{k+1}) \cdot \int_{\underline{p_b}}^{p_b^k} f(p)\ dp$, where the best reward is updated: $y^{k+1} = v - p_b^{k+1}$.

Overall, in the benchmark, consumers' maximization satisfies the recursive relation:

$$V^b(y^{k+1}) = \max\left\{0, y^k, e^{-(\rho+\lambda)t}\left[V^b(y^k) \cdot \int_{p_b^k}^{\overline{p_b}} f(p)\ dp + V^b(v - p_b^{k+1}) \cdot \int_{\underline{p_b}}^{p_b^k} f(p)\ dp\right]\right\}$$

Stopping Rule

A consumer who did not exit the market until then decides to terminate the search and buy when he finds a shop offering a price lower than or equal to his reservation price r_b. This reservation price is defined by the following recursive equation as the value for which the consumer is indifferent between buying or going on with the search[2]:

$$v - r_b = e^{-(\rho+\lambda)t}\left(\int_{\underline{p_b}}^{r_b} \begin{bmatrix}\text{expected utility}\\ \text{if buying}\\ \text{from next seller}\end{bmatrix} f(p)\ dp + \int_{r_b}^{\overline{p_b}} \begin{bmatrix}\text{expected utility}\\ \text{if next seller's}\\ \text{price is too high}\end{bmatrix} f(p)\ dp\right)$$

$$v - r_b = e^{-(\rho+\lambda)t}\left[\int_{\underline{p_b}}^{r_b} (v-p) f(p)\ dp + (v - r_b)\int_{r_b}^{\overline{p_b}} f(p)\ dp\right] \tag{2.1}$$

Integration leads to a relationship between valuation and reservation price:[3]

$$v_b(r_b) = r_b + \frac{e^{-(\rho+\lambda)t}}{1 - e^{-(\rho+\lambda)t}} \int_{\underline{p_b}}^{r_b} F(p)dp \tag{2.2}$$

We summarize the quotient related to the time discounting and the eventuality of exit as function $\Phi(t)$:

$$\Phi(t) = \frac{e^{-(\rho+\lambda)t}}{1 - e^{-(\rho+\lambda)t}} \tag{2.3}$$

Consumers go on searching if their valuation is lower than the critical one:

$$v(r_b) \leqslant v_b(r_b) = r_b + \Phi(t) \int_{\underline{p_t}}^{r_b} F(p)dp \tag{2.4}$$

The critical valuation $v_b(r_b)$ is continuous and strictly increasing in the reservation r_b: consumers with higher valuations have higher reservation values, and consequently they should find prices less than or equal to their reservation price faster: in expectation, they engage in less search. When consider ascending reservation prices, the first consumer

[2]Similar to Weitzman (1979); Spulber (1996) and Hendershott and Zhang (2006)

[3]Using, for the first integral, the integration rule $\int u\ dv = uv| - \int v\ du$ with the help functions: $u = v - p, \quad du = -dp; \quad dv = f(p)dp \quad v = F(p)$ and noticing that $f(\underline{p_b}) = 0, \quad f(\overline{p_b}) = 1$.

susceptible of buying has a reservation price $\overline{r_b}$ equal to the lowest price, and by (2.2), this reservation price is equal to his valuation:

$$\underline{p_b} = \underline{r_b} = \underline{v_b} \tag{2.5}$$

Remark that this value is determined by firms' price setting: the lowest price defines the minimum valuation and reservation price for which buying is possible.

In each search round, all consumers with a reservation price above $\underline{p_b}$ will start a search process, while consumers with a lower reservation price choose the fall-back option and abstain from search.

As $\int_{\underline{p_b}}^{r_b} F(p)dp$ increases with r_b, the difference between valuation and reservation price increases for higher valuations and reservation prices: consumers with higher valuations expect higher utility gains to make up for the time costs of search. Be $\overline{r_b}$ the reservation price of the consumer with highest valuation $v = 1$. $\overline{r_b}$ is the unique and unambiguous solution of the equation:

$$\overline{r_b} + \Phi(t) \int_{\underline{p_t}}^{\overline{r_b}} F(p)dp = 1$$

which implies that the highest reservation value is strictly lower than the highest valuation, 1. As $\overline{r_b}$ is the highest price susceptible of being accepted by consumers, no firm can offer a price above it - it would sell nothing. The highest price is maximally equal to $\overline{r_b}$. We assume that firms participate in the market as soon as they have the perspective to cover their costs, i.e. to reach zero or strictly positive profits. There will be one firm among the continuum of firms that can just achieve zero profits while offering the maximum reservation price as a price: $\overline{p_b} = \overline{r_b}$. We can therefore transform the expression for the highest reservation price:

$$\overline{r_b} = 1 - \Phi(t) \int_{\underline{p_b}}^{\overline{p_b}} F(p)dp \tag{2.6}$$

Remark that, on the contrary of the lower boundary $\underline{r_d}$, the upper boundary is defined by consumers and not by firms: the maximal reservation price defines the upper affordable price.

The impact of the eventuality of exit, of consumers' time preference rate and of the period length is reflected in the factor $\Phi(t)$:

$$\frac{\partial\Phi(t)}{\partial\lambda} = \frac{\partial\Phi(t)}{\partial\rho} = \frac{-te^{-(\rho+\lambda)t}}{[1-e^{-(\rho+\lambda)t}]^2} < 0; \qquad \frac{\partial\Phi(t)}{\partial t} = \frac{-(\rho+\lambda)e^{-(\rho+\lambda)t}}{\left[1-e^{-(\rho+\lambda)t}\right]^2} < 0$$

The difference between valuation and reservation price is lower when the probability of exiting the market at the end of each period is higher; when the time preference rate is higher, i.e. when costs of postponing the purchase are higher; and finally when the period of search is longer.

Figure 3.2 illustrates the boundaries of valuations, prices and reservation prices in the benchmark in any given search round.

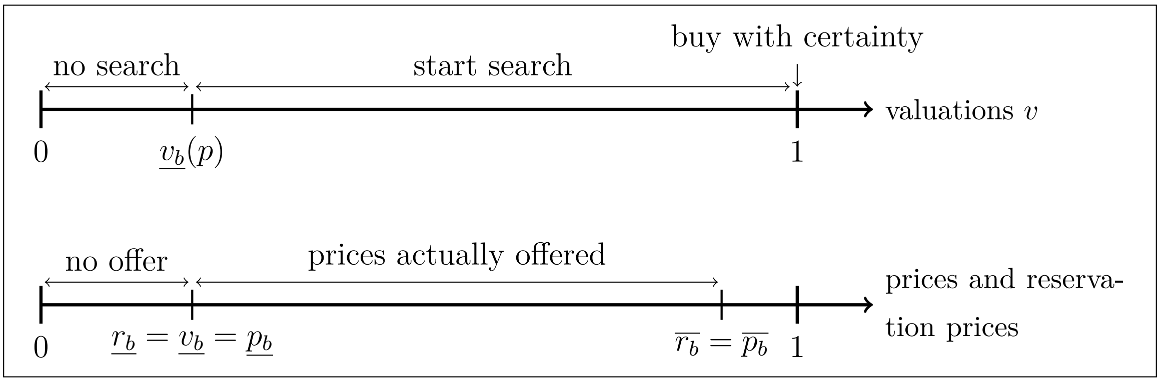


Figure 2.1: Valuations, Prices and Reservation Prices in the Benchmark

Distribution of the Reservation Prices

Equation (2.2) describes a bijective relation between valuations and reservation prices, and therefore implies a distribution $h_b(_b)$ of the reservation prices for $r_b \in [\underline{r_b}, \overline{r_b}]$. We define $r_b(v)$ as the reservation price associated to the uniformly distributed valuation v. The cumulative distribution function of reservation prices is:

$$\forall r \mid r \in [\underline{r_b}; \overline{r_b}], \quad H_b(r) = P\left[\tilde{r} \le r_b\right] = P\left[v_b(\tilde{r}) \le v_b(r)\right] = r + \frac{e^{-(\rho+\lambda)t}}{1 - e^{-(\rho+\lambda)t}} \int_{\underline{p_t}}^{r} F(p)dp$$

Using Leibniz integral rule, we deduce the probability density function:

$$h_b(r) = \begin{cases} 1 + \frac{e^{-(\rho+\lambda)t}}{1-e^{-(\rho+\lambda)t}} \cdot F(r) & \forall r \in [\underline{r_b}; \overline{r_b}] \\ 0 & \text{else} \end{cases}$$

As there is no higher valuation than $\overline{r_b}$ and no price set below $\underline{r_b}$, the condition $r \in [\underline{r_b}; \overline{r_b}]$ is fulfilled. The distribution of the reservation prices depends on the discount rate ρ, the search period length t, the probability of exiting the market λ and the cumulative density function of prices F, of which we know the boundary values: $F(\underline{r_d}) = 0$; $F(\overline{r_d}) = 1$.

We assume that the number of consumers is sufficiently high so that by the law of large numbers, the distribution of the reservation prices addressing each individual, representative shop i is:

$$h_b^i(r) = h_b(r)/n_b \ .$$

Demand Function

Consider the initial cohort of consumers, with mass 1. As it starts searching at time 0, we define it as cohort 0. The number of consumers with reservation price r looking up a given shop i is equal to h_b^i in this first round. All consumers with reservation above the shop's price p_i will buy. The demand addressed at shop i from the first consumers cohort in the first round is noted as $D_b^{0,i}$, where the first superscript refers to the cohort, the second to the shop; the first subscript refers to the benchmark situation and the second to the search round:

$$D_{b,1}^{0,i} = \int_{p_i}^{\overline{r_b}} h_b^i(r)\ dr$$

In the second round, only the consumers who have not bought in the first round can look up that shop (fraction $[1 - F(r)]$ for any reservation price r); their number is reduced also because a fraction of them exited the market during the first search round, which has length t; the remaining share is $e^{-\lambda t}$. The corresponding demand addressed at the representative shop i by the first cohort in its second search round is:

$$D_{b,2}^{0,i} = e^{-\lambda t} \int_{p_i}^{\overline{r_b}} [1 - F(r)]\, h_b^i(r)\ dr$$

Similarly for the third round:

$$D_{b,3}^{0,i} = e^{-2\lambda t} \int_{p_i}^{\overline{r_b}} [1 - F(r)]^2\, h_b^i(r)\ dr$$

And for the k-th round, $k \geqslant 2$:

$$D_{b,k}^{0,i} = e^{-(k-1)\lambda t} \int_{p_i}^{\overline{r_b}} [1 - F(r)]^{(k-1)}\, h_b^i(r)\ dr$$

Summing up all search rounds from 1 to infinity leads to the discounted demand from the first cohort (discounted down to period 0, before the start of the first round):

$$\begin{aligned} D_b^{0,i} &= \sum_{k=1}^{\infty} e^{-k\rho t} D_{b,k}^{0,i} = \sum_{k=1}^{\infty} e^{-[k\rho + (k-1)\lambda]t} \int_{p_i}^{\overline{r_b}} \Big[1 - F(r)\Big]^{(k-1)} h_b^i(r)\ dr \\ &= \frac{1}{n_b} \cdot \frac{e^{-\rho t}}{1 - e^{-(\rho+\lambda)t}} \cdot (\overline{r_b} - p_i) \end{aligned}$$

The same result is valid for any cohort j; however, it has to be scaled as the following cohorts have a mass of λ instead of 1.

$$D_b^{j,i} = \lambda \cdot \frac{1}{n_b} \cdot \frac{e^{-\rho t}}{1 - e^{-(\rho+\lambda)t}} \cdot (\overline{r_b} - p_i)$$

At each point in time, a share λ of the consumers is indeed "renewed" and a new cohort starts searching. The discounted demand function of firm i sums up the demand of all cohorts:

$$D_b^i(p_i) = D_b^{0,i} + \int_0^\infty e^{-\rho j} \cdot D_b^{j,i}(p_i)\ dj$$
$$= \frac{1}{n_b} \cdot \frac{e^{-\rho t}}{1 - e^{-(\rho+\lambda)t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot (\overline{r_b} - p_i)$$

The demand function for firm i is linear in p_i. It is a function of the number of shops, but not of the prices of rivals.

2.3.2 Firms' Decision

Firms who find it profitable, i.e. who can collect profits equal to or higher than zero, run a traditional shop. Each firm i bears unit costs k_t^i, which are uniformly distributed between 0 and 1, and maximizes the expected discounted profits π_b^i with respect to the price p^i.

$$\pi_b^i = (p^i - k_t^i) \cdot \frac{1}{n_b} \cdot \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot (\overline{r_b} - p_i)$$

The maximizing price is $p^*(k_t^i) = (\overline{r_b} + k_t^i)/2$. Firms can participate in the traditional market only if their price is maximally equal to the highest reservation value $\overline{r_b}$ defined in (2.6):

$$\frac{\overline{r_b} + k_t^i}{2} \leqslant \overline{r_b} \quad \Leftrightarrow \quad k_t^i \leqslant 1 - \Phi(t) \int_{\underline{p_b}}^{\overline{p_b}} F(p)dp < 1$$

We denote the maximal unit costs for which a firm can participate in the market as $\overline{k_t}$:

$$\overline{k_t} = \overline{r_b} = 1 - \Phi(t) \int_{\underline{p_b}}^{\overline{p_b}} F(p)dp$$

This value is strictly smaller than 1; it depends positively on the transaction time t, on the exit probability λ, and on the discount rate ρ. The firm bearing this boundary costs $\overline{k_t}$ offers the highest price, $\overline{p_b}$:

$$\overline{p_b} = p^*(\overline{k_t}) = \overline{r_b} = 1 - \Phi(t) \int_{\underline{p_b}}^{\overline{p_b}} F(p)dp$$

The lowest price is offered by the firm with the lowest traditional transaction unit costs, i.e. zero; it amounts to: $\underline{p_b} = \overline{r_b}/2$. These two prices, $\overline{p_b}$ and $\underline{p_b}$, set the boundaries of prices offered in the benchmark case. We still need to define the distribution of these prices.

Using the fact that unit costs are uniformly distributed,[4] we deduce the following distribution for the prices comprised between $\underline{p_b}$ and $\overline{p_b}$:[5]

$$F(p) = P[\tilde{p}_i \leqslant p \mid k_t^i \leqslant \overline{r_t}] = \frac{2p - \overline{r_b}}{\overline{r_b}}$$

[4] Especially, we are using the two probabilities $P[k_i \leqslant \overline{r_b}] = \overline{r_b}$ and $P[k_t^i \leqslant 2p - \overline{r_b}] = 2p - \overline{r_b}$.

[5] It is easily verified that $F(\underline{p_t}) = \frac{\overline{r_t} - \overline{r_t}}{\overline{r_t}} = 0$ and $F(\overline{r_t}) = \frac{2\overline{r_t} - \overline{r_t}}{\overline{r_t}} = 1$.

We compute the explicit solution for $\overline{r_b}$. Let us call I_F the integral $\int_{\underline{p_b}}^{\overline{p_b}} F(p)dp$;

$$I_F = \int_{\underline{p_b}}^{\overline{r_b}} \frac{2p - \overline{r_b}}{\overline{r_b}} \, dp = \frac{1 - \Phi(t) I_F}{4}$$

We can deduce the value of the integral: $I_F = 1/(4 + \Phi(t))$, and consequently the explicit formula for the lowest reservation price for which purchase is possible:

$$\overline{r_b} = \frac{4}{4 + \Phi(t)} < 1$$

And finally, we can reformulate the distribution function of prices in the benchmark:

$$F(p) = \frac{[4 + \Phi(t)] \cdot p - 2}{2} \quad \forall p \in \left[\underline{p_b}; \overline{p_b}\right] \tag{2.7}$$

Using this explicit solution for the distribution of prices, we can compute the explicit relationship between valuation and reservation price from (2.4):[6]

$$v_b(r_b) = \frac{4 + \Phi(t)}{4} \cdot \Phi(t) \cdot r_b^2 + [1 - \Phi(t)] \cdot r_b + \frac{\Phi(t)}{4 + \Phi(t)} \tag{2.8}$$

The equilibrium price setting strategy reads:

$$p^*(k_t^i) = \frac{4}{2[4 + \Phi(t)]} + \frac{k_i}{2} \tag{2.9}$$

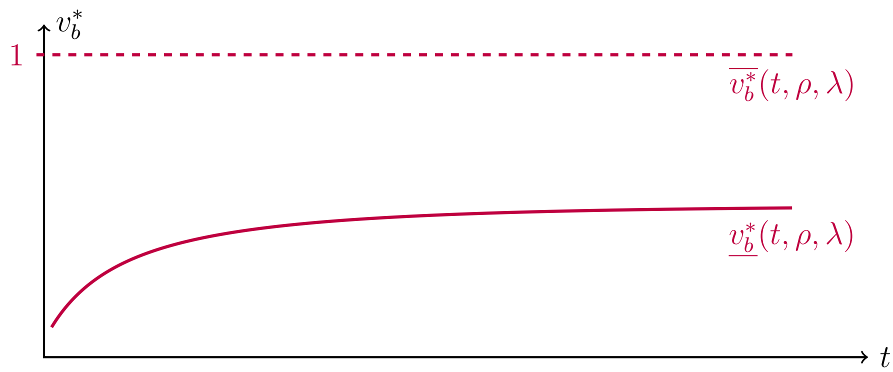


Figure 2.2: Minimal and maximal valuations in the benchmark ($\lambda = 0.3; \rho = 0.3; t \leqslant 5$)

As illustrated in Figure (2.2), the minimal valuation for which a consumer can buy increases with the period length t. Similarly, it increases with the time preference ρ and the probability of exit driven by λ: these parameters restrict the access of consumers to the market.

[6]It can be verified, using the solutions for $\underline{r_b}$ and $\overline{r_b}$ that $v_b(\underline{r_b}) = 2/\left[4 + \Phi(t)\right] = \underline{r_b} = \underline{p_b}$ and $v_b(\overline{r_b}) = 1$.

The price set by the active firm with the highest unit costs is twice as high as the price set by the firm with zero unit costs. Prices increase with the length of the periods t (see Figure (2.3)), the time preference ρ and the exit probability driven by λ, and are ceiled by the highest reservation price. This increase explains why these parameters have a restricting effect on the access of consumers to the market.

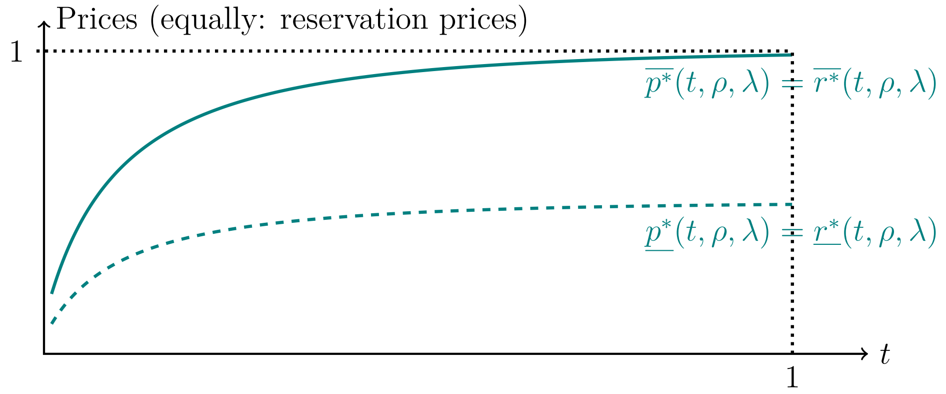


Figure 2.3: Minimal and maximal (reservation) prices ($\lambda = 0.3; \rho = 0.3; t \leqslant 5$)

The overall shape of the reservation prices as function of the time preference is also increasing (see Figure (2.4)). A higher time preference indicates a stronger preference for present-time consumption: consumers are less patient, and are willing to accept higher prices in order to shorten their search: both the reservation prices and the amplitude between minimal and maximal reservation prices increase with the time preference ρ.

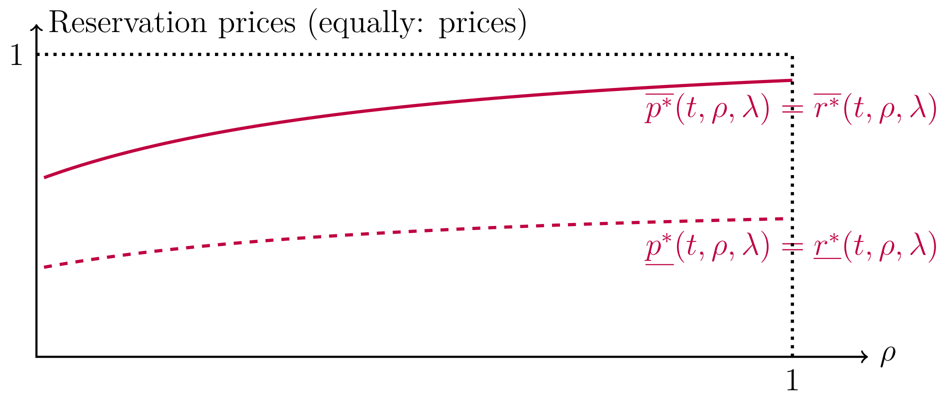


Figure 2.4: Reservation prices as function of the time preference ($\lambda = 0.3; \rho \leqslant 1; t = 1$)

In order to compute the equilibrium demand and profits functions, we need a last information: the explicit formula for the number of firms active in the market, n_b. Active firms are those offering a price between the maximum and minimum prices:

$$\overline{p_b} = \overline{r_b} = \frac{4}{4+\Phi(t)} \quad ; \qquad \underline{p_b} = \underline{r_b} = \frac{2}{4+\Phi(t)}$$

which corresponds to all firms with unit costs comprised between zero and $\overline{k_t} = \overline{r_b} = \frac{4}{4+\Phi(t)}$. As unit costs are uniformly distributed on $[0;1]$, the number of firms active in equilibrium reads:

$$n_b^* = \overline{k_t} = \frac{4}{4+\Phi(t)}$$

The number of firms increases with the period length t (see Figure (2.5)) as well as with the time preference ρ and the probability of exit driven by λ. Even though they restrict the access of consumers to the market, these parameters have a positive effect on the market structure: due to the possibility of setting higher prices, more firms enter the market.

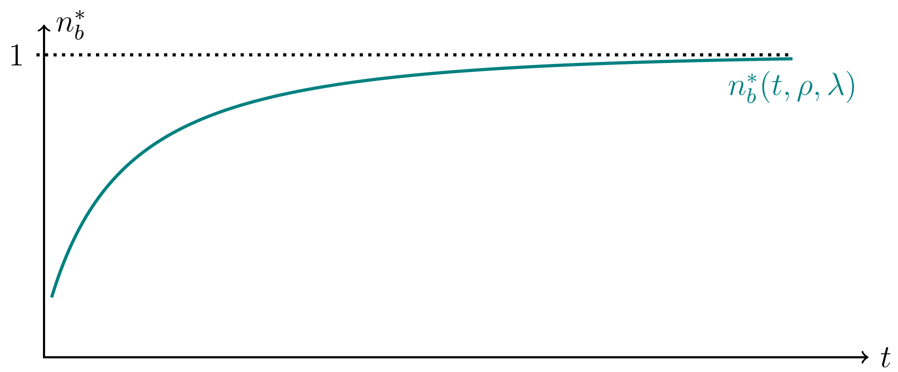


Figure 2.5: Number of firms in the benchmark ($\lambda = 0.3; \rho = 0.3; t \leqslant 5$)

These results lead to the equilibrium demand functions depicted in Figure (2.6).

$$D_b^*(k_t^i) = \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot \frac{4 - [4+\Phi(t)]k_i}{8}$$

Remark that because we consider the expected discounted demand aggregated until eternity, the demand functions might well be over 1. While the demand addressed to the firms with the highest unit costs is close to zero, the firms with the lowest costs see their demand decrease as the period length augments.

It is striking that the individual demand functions decrease with the period length t, while the number of firms augments with it. The aggregated demand amounts to:

$$\int_0^{\overline{k_t^i}} D_b^*(k_t^i)k_t^i = \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot \frac{1}{2[4+\Phi(t)]} = \frac{(\lambda+\rho)e^{\lambda t}}{4\rho\,(4e^{(\lambda+\rho)t} - 3)}$$

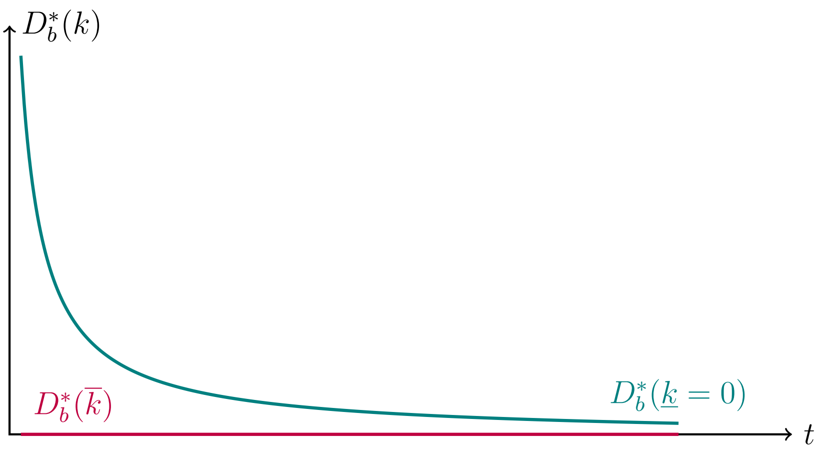


Figure 2.6: Demand for the firms with highest and lowest unit costs ($\lambda = 0.3; \rho = 0.3; t \leqslant 3$)

This aggregated demand is always monotonously decreasing in t,[7] as depicted in Figure (2.7): the negative impact of t on the individual demand functions dominates the positive effect of the increase in the number of firms.

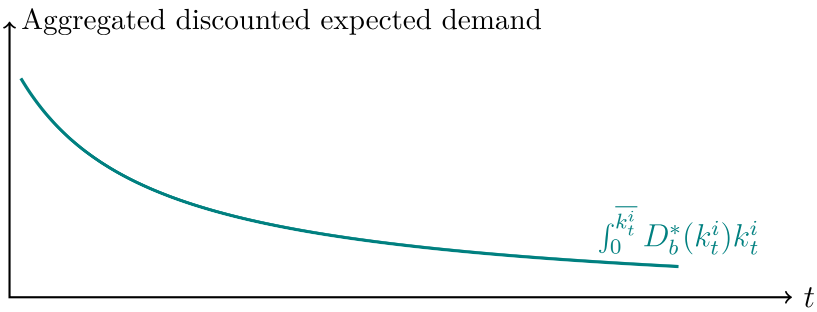


Figure 2.7: Aggregated demand ($\lambda = 0.3; \rho = 0.3; t \leqslant 3$)

As the time preference ρ and the parameter determining the probability of exit λ have a different impact on the aggregated demand functions, it is worth having a look at them separately. Time preference has a severe negative impact on the aggregated demand addressed to a firm i over time (see Figure (2.8)).

On the contrary, the probability of exit has a modest positive impact on the aggregated demand addressed to a firm i over time. As this probability increases, there is a growing threat for the consumers to exit the market before having bought the good, so that they rather consider a higher level of reservation prices.

Finally, the equilibrium expected discounted profits read:

$$\pi_b^*(k_t^i) = \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot \frac{(4 - [4 + \Phi(t)]k_i)^2}{16[4 + \Phi(t)]}$$

[7] $\partial \left[\int_0^{\overline{k_t^i}} D_b^*(k_t^i)k_t^i\right] / \partial t = -\frac{(\lambda+\rho)e^{\lambda t}\left(4\rho e^{t(\lambda+\rho)}+3\lambda\right)}{4\rho\left(3-4e^{t(\lambda+\rho)}\right)^2} < 0$

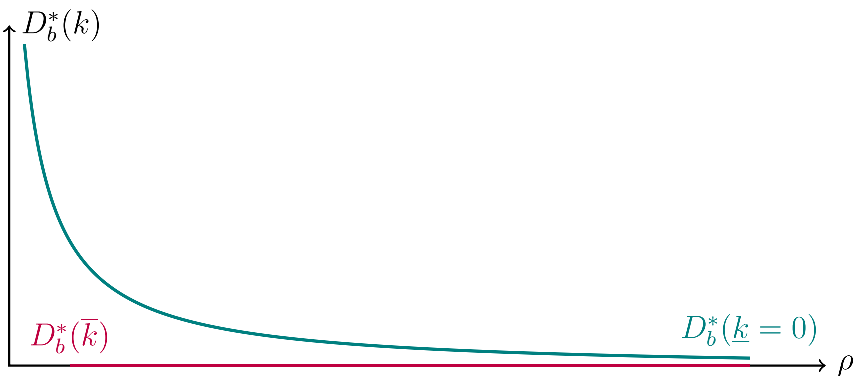


Figure 2.8: Demand for the firms with highest and lowest unit costs ($\lambda = 0.3; \rho \leqslant 1; t = 1$)

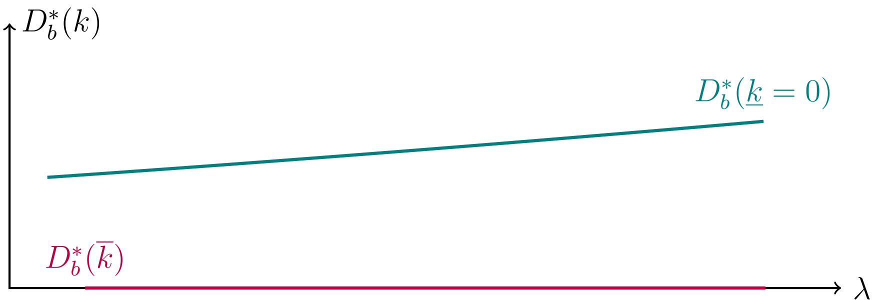


Figure 2.9: Demand for the firms with highest and lowest unit costs ($\lambda \leqslant 1; \rho = 0.3; t = 1$)

Profits are quadratically decreasing with the unit costs. This becomes apparent in the curvature of the profit curve in Figure (2.10).

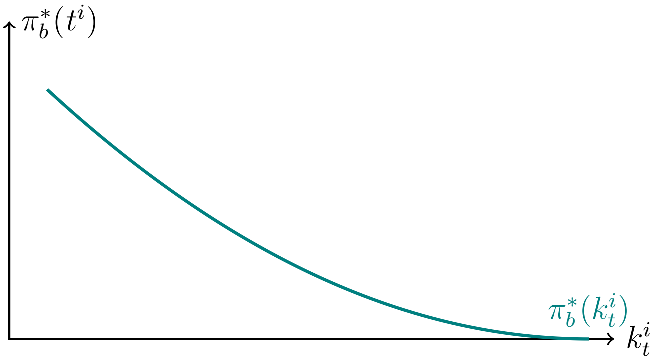


Figure 2.10: Profits depending on the unit costs ($\lambda = 0.3; \rho = 0.3; t = 1; 0 \leqslant k_t^i \leqslant \overline{k_t}$)

Similarly, the profit functions are negatively affected by the period length t and the time preference ρ: the negative effect on the equilibrium demand dominates the positive impact on the equilibrium price (see Figures (2.11) and (2.12)).
In the case of the parameter determining the exit probability, λ, price, demand and correspondingly profits are influenced positively (see Figure (2.13)). Whereas the mathematical position of time preference and exit probability is mostly similar, we see here

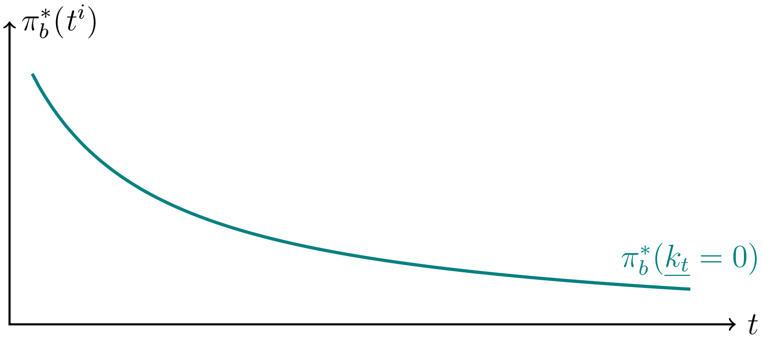


Figure 2.11: Profits in the benchmark ($\lambda = 0.3; \rho = 0.3; t \leqslant 3$)

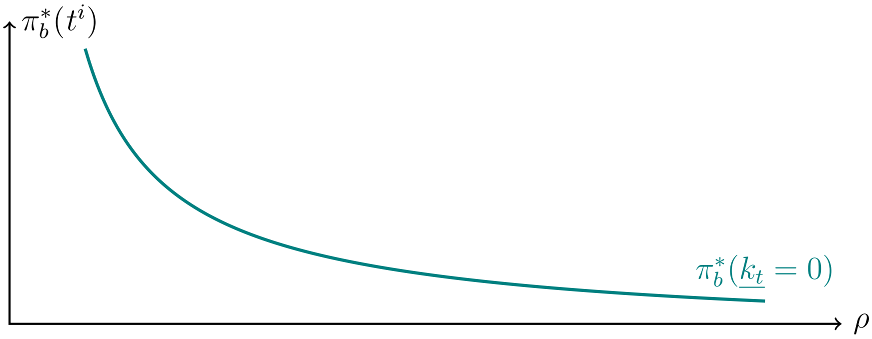


Figure 2.12: Profits in the benchmark ($\lambda = 0.3; \rho \leqslant 1; t = 1$)

the difference in their roles: while the discounting effect restricts demand and profits, the threat of exit rather animates consumers to buy and enhances firms' profits.

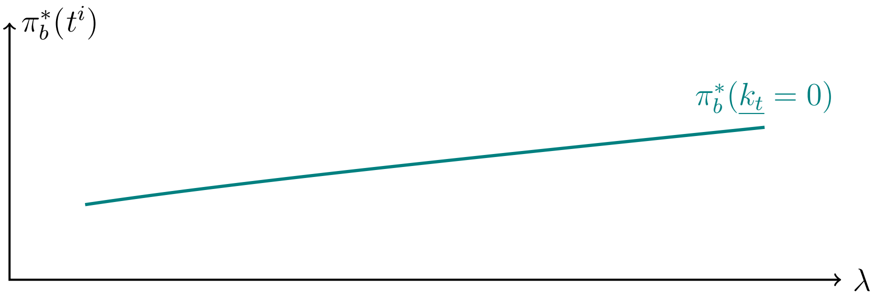


Figure 2.13: Profits in the benchmark ($\lambda \leqslant 1; \rho = 0.3; t = 1$)

2.3.3 Welfare in the Benchmark Case

Firms' surplus FS_b is the sum of the expected discounted profits of all firms active in the market:

$$FS_b = \int_0^{\overline{k_t}} \pi_b^*(k_t^i)\ dk_t^i = \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot \frac{4}{3\,[4+\Phi(t)]^2} = \frac{4(\lambda+\rho)\left(1 - e^{(-(\lambda+\rho)t)}\right)}{3\rho\left(4 - 3e^{(-(\lambda+\rho)t)}\right)^2}$$

Consumers' surplus depends on whether they exit the market before they could buy, on how long they search and which acceptable price they find. In a first step, we consider the expected discounted surplus for an individual consumer with reservation price r_b. In any period $j \geq 1$, this consumer can buy and realize the expected surplus $\int_{\underline{p_b}}^{r_b} [v(r_b) - p] f(p)\, dp$, under the condition that he has not been successful until then (probability $[1 - F(r_b)]^{j-1}$) and that he has not exited the market until the end of period j (probability $e^{-\lambda j t}$). The density function f can be deduced from the cumulative distribution function in (2.7):

$$f(p) = \frac{4 + \Phi(t)}{2} \quad \forall p \in \left[\underline{p_t}; \overline{p_t}\right]$$

The surplus of this consumer is the sum of the discounted expected surpluses in any period:[8]

$$\begin{aligned} CS_b^i &= \sum_{j=1}^{\infty} e^{-(\rho+\lambda)jt} \cdot [1 - F(r_b)]^{j-1} \cdot \int_{\underline{p_b}}^{r_b} [v_b(r_b) - p] f(p)\, dp \\ &= \frac{[4 + \Phi(t)] \cdot r_b - 2}{2} \cdot \left[v_b(r_b) - \frac{r_b + \underline{p_b}}{2} \right] \cdot \frac{e^{-(\rho+\lambda)t}}{1 - e^{-(\rho+\lambda)t} \cdot [1 - F(r_b)]} \end{aligned}$$

Integrating over the mass of consumers actually buying, i.e. with reservation prices between $\underline{r_b}$ and $\overline{r_b}$ and with valuations between $\underline{v_b}$ and 1, we deduce the overall consumers surplus:[9]

$$\begin{aligned} CS_b &= \int_{\underline{r_b}}^{\overline{r_b}} CS_b^i(r_b) h_b(r_b)\, dr_b = \int_{\underline{r_b}}^{\overline{r_b}} CS_b^i(r_b) \cdot \left[1 + \Phi(t) \cdot \frac{[4 + \Phi(t)] \cdot r_b - 2}{2} \right] dr_b \\ &= \frac{4e^{t(\lambda+\rho)} - 1}{6\left(3 - 4e^{t(\lambda+\rho)}\right)^2} \end{aligned} \tag{2.10}$$

The impact of the exogenous parameters on these welfare results is depicted below. The length of time necessary for each single search procedure, t, has a negative impact on consumers' surplus, but not necessarily on firms' profits (see figure 2.14). This laps of time relaxes the price competition among the shops: the higher it is, the higher are consumers' search costs, so that their reservation price increases. The exit probability λ is also negative for consumers, as it directly reduces the probability of successfully finding a product fitting their reservation value. Because a higher exit probability λ also implies a faster replacement of consumers, i.e. a more frequent appearance of consumers with higher reservation prices, it is positive for firms: for a constant volume of sales, they

[8] Notice the asymmetry between the time depending superscripts of the first and second factors (j vs. $j-1$). This is due to the following consideration: in a given period j, given the considered consumer has not found any price below his reservation price in the prior $j-1$ search rounds (an eventuality with probability $[1 - F(r_b)]^{j-1}$), he might find a fitting price and will have to discount over j periods the utility resulting from the positive difference between this price and his reservation price.

[9] This computation is simpler when applying first the integrals and the summation over j only afterwards.

can target consumers with a higher willingness to pay (see figure 2.15. Finally, the time preference of consumers respectively discount factor of firms ρ has a negative impact for both consumers and firms (see 2.16) as it delays as well consumption as the realization of profits.

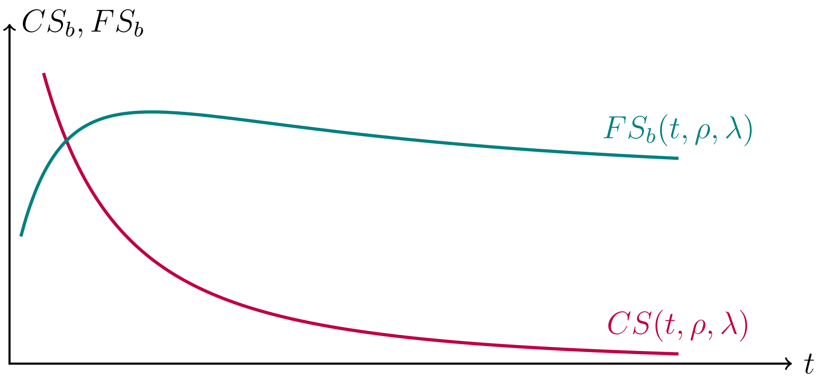


Figure 2.14: Firms and Consumers Surplus in the Benchmark ($\lambda = 0.3; \rho = 0.3; t \leqslant 3$)

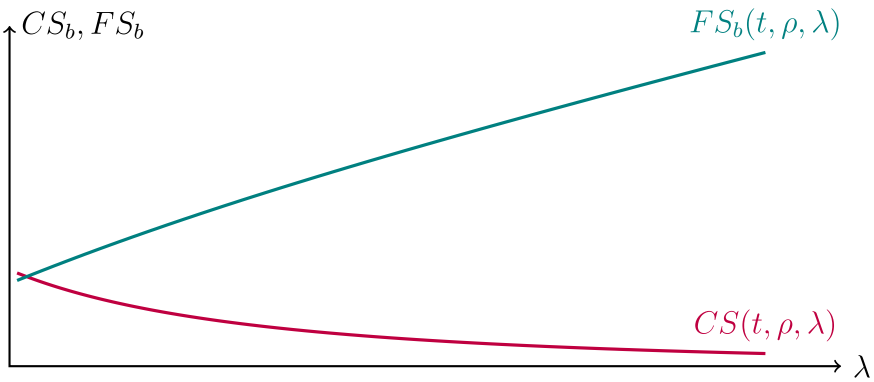


Figure 2.15: Consumers Surplus in the Benchmark ($\lambda \leqslant 1; \rho = 0.3; t = 1$)

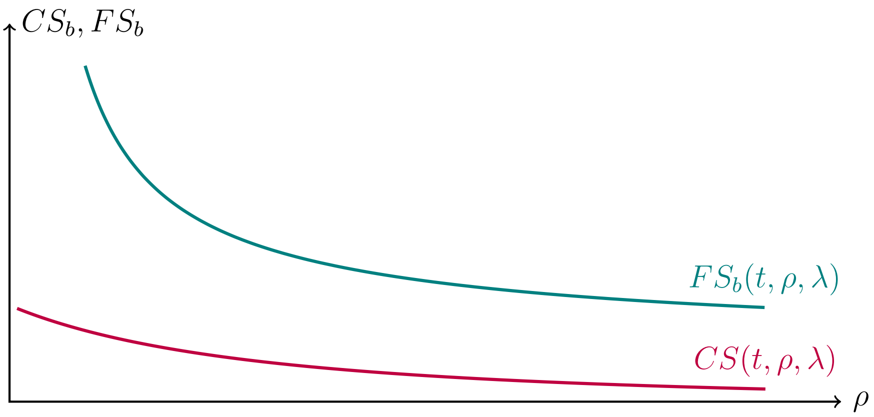


Figure 2.16: Consumers Surplus in the Benchmark ($\lambda = 0.3; \rho \leqslant 1; t = 1$)

2.4 Online Expansion

2.4.1 Consumers Decision

In the case when traditional and online shops coexist, consumers can choose among the channels. We assume than they know the distribution of prices $F(p)$ among a number n_t of traditional shops and the distribution of prices $G(p)$ among a number n_o of online shops. Consumers build expectations on each channel and choose a given search channel at the beginning of their search; they do not switch the channel during their search.

Consumers choosing the traditional channel observe the prices p_t in traditional shops. Sampling the offer in a traditional shop takes time t. In case they decide to buy, they get the good at the end of the period t.

Consumers choosing the online channel browse the internet for a fitting offer p_o. Sampling the offer in an online shop takes only time $\tau < t$, however, consumers deciding to buy online get the good only after the delivery time lag $T \geqslant t$.

In the Traditional Channel

We consider here first the traditional channel and then the online channel, in order to determine the conditions of the consumers whether to stick to the one or not the other channel. As the traditional channel is quite similar to the benchmark, results are displayed without reiterating the explanations.

In the traditional channel, consumers' maximization satisfies the recursive relation:

$$V^t(y_t^{k+1}) = \max\left\{0, y_t^k, e^{-(\rho+\lambda)t}\left[V^t(y_t^k) \cdot \int_{p_t^k}^{\overline{p_t}} f(p)dp + V(y_t^{k+1}) \int_{\underline{p_t}}^{p_t^k} f(p)dp)\right]\right\}$$

where V^t is the value function in the traditional channel, y_t^k is the best reward previously collected during the search and y_t^{k+1} is the updated best reward after the search round $k+1$.

Similarly to the benchmark case, consumers' stopping rule in the traditional channel reads:

$$v(r_t) \leqslant v_t(r_t) = r_t + \Phi(t) \int_{\underline{p_t}}^{r_t} F(p)dp \tag{2.11}$$

$$\text{with} \qquad \Phi(t) = \frac{e^{-(\rho+\lambda)t}}{1-e^{-(\rho+\lambda)t}} \tag{2.12}$$

The function $v_t(r_t)$ depicts the critical valuation for going on searching in traditional shops. The first consumer susceptible of buying has a reservation price just above the smallest traditional price:

$$\underline{p_t} < \underline{r_t} < \underline{v_t} \tag{2.13}$$

The strict inequality signs are here necessary; a hypothetical consumer with valuation and reservation price $\underline{v_t} = \underline{p_t} + \Phi(t) \int_{\underline{p_t}}^{\underline{p_t}} F(p)dp = \underline{p_t}$ would never achieve a strictly positive utility in traditional shops and therefore turn to online shops.

The first consumer who would accept any price in $[\underline{p_t}; \overline{p_t}]$ has valuation:

$$v_t(\overline{p_t}) = \overline{p_t} + \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp$$

Be $\overline{r_t}$ the reservation price of the consumer with valuation $v = 1$. $\overline{r_t}$ is the unique and unambiguous solution of the equation:

$$\overline{r_t} + \Phi(t) \int_{\underline{p_t}}^{\overline{r_t}} F(p)dp = 1 \quad \Rightarrow \quad \overline{r_t} < 1 \tag{2.14}$$

As $\overline{r_t}$ is the highest price a consumer would accept, no firm can offer a price above it (it would sell nothing); the highest price is maximally equal to $\overline{r_t}$: $\overline{p_t} \leqslant \overline{r_t}$.

In the Online Channel

The decision problem of consumers is basically similar in both channels; with the difference that in the online channel, consumers face the distribution of prices $G(p)$, and have to bear a time discount for the search time τ as well as for the time of delivery T. Their maximization satisfies the recursive relation:

$$V^o(y_o^{k+1}) = \max\left\{0, e^{-(\lambda+\rho)T} y_o^k, e^{-(\lambda+\rho)(\tau+T)} \left[V(y_o^k) \cdot \int_{p_o^k}^{\overline{p_o}} g(p)dp + \int_{\underline{p_o}}^{p_o^k} V^o(y_o^{k+1}) g(p)dp\right]\right\}$$

Because the utility discounting linked to the time of delivery affects equally the two possibilities of stopping the search procedure or going on with it, it simplifies away when comparing them. The resulting stopping rule in the online channel is derived from the indifference condition:

$$e^{-(\rho+\lambda)T}(v-r_o) = e^{-(\rho+\lambda)(T+\tau)}\left[\int_{\underline{p_o}}^{r_o}\begin{bmatrix}\text{expected utility}\\ \text{if ordering}\\ \text{from next seller}\end{bmatrix} g(p)dp + \int_{r_o}^{\overline{p_o}}\begin{bmatrix}\text{expected utility}\\ \text{if next seller's}\\ \text{price is too high}\end{bmatrix} g(p)dp\right]$$

The stopping rule in the online channel can be formulated in a shape similar to the one in the traditional channel (2.12). Consumers go on searching in online shops if:

$$v(r_o) \leqslant v_o(r) = r_o + \frac{e^{-(\rho+\lambda)\tau}}{1-e^{-(\rho+\lambda)\tau}}\int_{\underline{p_o}}^{r_o} G(p)dp = r_o + \Phi(\tau)\int_{\underline{p_o}}^{r_o} G(p)dp \tag{2.15}$$

Consumers with lower valuation will browse more online offers. The lowest reservation price for which buying is possible, $\underline{r_o}$, is equal to $\underline{p_o}$, the corresponding critical valuation is $v_o(\underline{p_o}) = \underline{p_o}$. As $\tau < t$ and $\Phi(t)$ decreases in t (see above), we have: $\Phi(t) < \Phi(\tau)$; the cumulative density function G is more strongly weighted for the definition of the online reservation price, than F for the reservation price in traditional shops.

Comparison of the two reservation prices

Because F and G are still unknown, it is not possible to solve analytically for the valuation with equal online and traditional reservation prices (i.e. v such that $r_t(v) = r_o(v)$). The consumer with the lowest reservation price for which buying is possible, $\underline{r} = \min\{\underline{p_t}, \underline{p_o}\}$ has valuation $\min\{\underline{p_t}, \underline{p_o}\}$; he has no choice about where to buy.

Let us compare the respective reservation prices in traditional and online shops of one and the same consumer with valuation v:

$$v = r_o + \Phi(\tau)\int_{\underline{p_o}}^{r_o} G(p)dp = r_t + \Phi(t)\int_{\underline{p_t}}^{r_t} F(p)dp$$

As $\Phi(t)$ is declining in t and $t > \tau$, we have $\Phi(t) < \Phi(\tau)$. The integrals are increasing in the respective reservation prices. We expect that the reservation price in traditional shops is higher: $r_t > r_o$, as search costs are lower in online shops.

Indifference between search in online or traditional shops

As mentioned in the introduction, we assume that consumers choose a given search channel at the beginning of their search and then stick to it. They prefer online search if their discounted expected reward from online search, where they would get satisfied with a price of maximally r_o, is higher than their expected reward from sampling offers in traditional shops, where they would get satisfied with a price of maximally r_t. In a first step, we compute the respective value functions. Be V^t the discounted expected value of a search in traditional shops:

$$V^t = \sum_{j=1}^{\infty} e^{-(\rho+\lambda)jt(}[1-F(r_t)]^{(j-1)} \int_{\underline{p_t}}^{r_t} (v-p)f(p)dp$$
$$= \frac{e^{-(\rho+\lambda)t} \int_{\underline{p_t}}^{r_t} (v-p)f(p)dp}{1-e^{-(\rho+\lambda)t}[1-F(r_t)]} \tag{2.16}$$

Be V^o the discounted expected value of a search in online shops:

$$V^o = e^{-(\rho+\lambda)T} \sum_{j=1}^{\infty} e^{-(\rho+\lambda)j\tau}[1-G(r_o)]^{(k-1)} \int_{\underline{p_o}}^{r_o} (v-p)\ dG(p)$$
$$= \frac{e^{-(\rho+\lambda)(\tau+T)} \int_{\underline{p_o}}^{r_o} (v-p)\ dG(p)}{1-e^{-(\rho+\lambda)\tau}[1-G(r_o)]} \tag{2.17}$$

We assume that there is a critical value $\overline{v}$ from which on consumers will turn to traditional shops:

$$\forall v > \overline{v}, \quad V^t > V^o$$

The consumer with valuation $\overline{v}$ has the highest online reservation price $\overline{r_o}$ among all consumers turning to online shops. Online shops with a higher price than $\overline{r_o}$ would meet no demand; therefore we have $G(\overline{r_o}) = 1$. We denote as $\underline{r_t}$ the reservation price in traditional shops of the indifferent consumer; it is the lowest reservation price among all consumers buying from traditional shops. This indifferent consumer expects a positive utility when buying in traditional shops: if $\underline{r_t}$ were the lowest price in traditional shops, it would be evident that the utility from buying in traditional shops is maximally zero and turning to online shops would be more attractive; so that $F(\underline{r_t}) > 0$. Compute the critical value $\overline{v}$:

$$\frac{\int_{\underline{p_t}}^{\underline{r_t}} (\overline{v}-p)\ dF(p)}{1-e^{-(\rho+\lambda)t}[1-F(\underline{r_t})]} = \frac{e^{-(\rho+\lambda)T} \int_{\underline{p_o}}^{\overline{r_o}} (\overline{v}-p)\ dG(p)}{1-e^{-(\rho+\lambda)\tau}[1-G(\overline{r_o})]} \Leftrightarrow$$

$$\overline{v} = \frac{\int_{\underline{p_t}}^{\underline{r_t}} p\ dF(p) + e^{-(\rho+\lambda)T} \int_{\underline{p_o}}^{\overline{r_o}} p\ dG(p)\Big(1-e^{-(\rho+\lambda)t}[1-F(\underline{r_t})]\Big)}{F(\underline{r_t}) - e^{-(\rho+\lambda)T}\Big(1-e^{-(\rho+\lambda)t}[1-F(\underline{r_t})]\Big)}$$

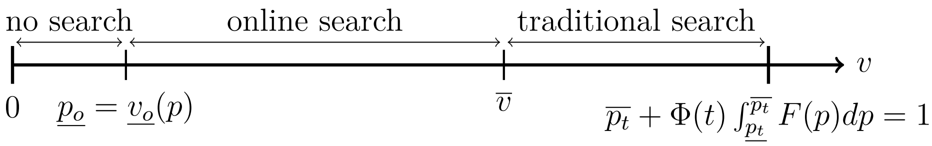
Expected Consumers' Behavior

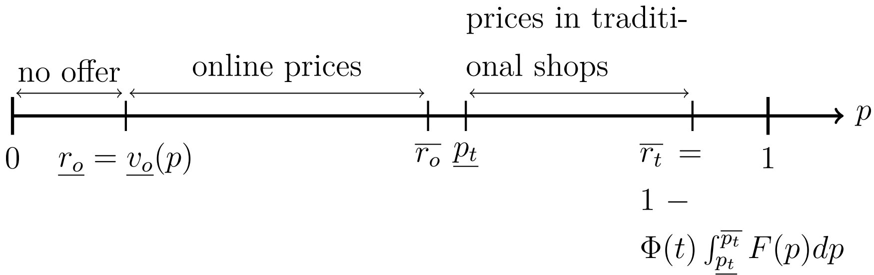
We expect the following behavior:

- in the first round, consumers with a reservation price above $\underline{p_o}$ will start a search process.

- Consumers with low valuations ($\overline{v}$ or lower) will search online, consumers with high valuation ($\overline{v}$ or higher) will search in traditional shops as they expect to find a satisfying good quickly and to bear less time costs when buying in a traditional shop, because of the delivery time lag in online shops.

- The maximal price $\overline{p_t}$ is maximally equal to the highest reservation price $\overline{r_t} = 1 - \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp$, as a firm with a higher price would not sell. We assume that firms participate in the market as soon as they have the perspective to cover their costs, i.e. to reach zero or positive profits. As there are firms with unit costs higher than the maximal reservation price (the maximal unit costs are 1, while the maximal reservation price is $1 - \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp \leqslant 1$), there will be a firm that can just achieve zero profits while offering the maximum reservation price as a price. Consumers with the maximum valuation, 1, have a reservation price of $\overline{r_t} = \overline{p_t} = 1 - \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp$, turn to traditional shops and buy with certainty right in the first period.





Distribution of the Reservation Prices

Equations (2.12) and (2.15) imply distributions $h_t(r)$ and $h_o(r)$ of the reservation prices for $r_t \in [\underline{r_t}, \overline{r_t}]$ and $r_o \in [\underline{r_o}, \overline{r_o}]$, respectively. $\underline{r_o}$ is the lowest online reservation price for which buying is possible; $\overline{r_o}$ is the highest reservation price among the consumers addressing online shops, it is the online reservation price of the consumer indifferent between both channels. $\underline{r_t}$ is the lowest reservation price among the consumers addressing traditional shops, it is the traditional reservation price of the consumer indifferent between both channels; $\overline{r_t}$ is the traditional reservation price of the consumer with maximal valuation $\overline{v} = 1$, who is addressing traditional shops and will buy with certainty in the first round of search. We define $r_t(v)$ as the reservation price associated to valuation v when buying in a traditional shop and $r_o(v)$ as the reservation price associated to valuation v when buying in an online shop. Because consumers' behavior is endogenous, all consumers addressing

a given shop type do so non-randomly, so we don't have to consider some hypothetical conditional probabilities but can cut the distribution of the consumers.
For the traditional channel, we compute following distribution function and, using Leibniz' integral rule for the upper boundaries, the corresponding probability density function:

$$\forall r \; |r > r_t(\overline{v}) \quad H_t(r) = \begin{cases} r + \frac{e^{-(\rho+\lambda)t}}{1-e^{-(\rho+\lambda)t}} \int_{\underline{p_t}}^{r} F(p)dp & \forall r \mid v_t(r) \geqslant \overline{v} \\ 0 & \forall r \mid v_t(r) \leqslant \overline{v} \end{cases}$$

$$h_t(r) = \begin{cases} 1 + \frac{e^{-(\rho+\lambda)t}}{1-e^{-(\rho+\lambda)t}} \cdot F(r) & \forall r \mid v_t(r) \geqslant \overline{v} \\ 0 & \forall r \mid v_t(r) \leqslant \overline{v} \end{cases} \tag{2.18}$$

For online shops, where the minimum price is equal to the minimum reservation price ($\underline{p_o} = \underline{r_o}$) we get in a similar way:

$$H_o(r) = \begin{cases} r + \frac{e^{-(\rho+\lambda)\tau}}{1-e^{-(\rho+\lambda)\tau}} \int_{\underline{r_o}}^{r} G(p)dp & \forall r \mid v_o(r) \leqslant \overline{v} \\ 0 & \forall r \mid v_o(r) \geqslant \overline{v} \end{cases}$$

$$h_o(r) = \begin{cases} 1 + \frac{e^{-(\rho+\lambda)\tau}}{1-e^{-(\rho+\lambda)\tau}} \cdot G(r) & \forall r \mid v_o(r) \leqslant \overline{v} \\ 0 & \forall r \mid v_o(r) \geqslant \overline{v} \end{cases} \tag{2.19}$$

The distribution of the reservation prices depends on the discount rate ρ, the respective search period length t or τ, the delivery time T, the probability of exiting the market λ and the respective cumulative density function of prices F or G.

The traditional (respective online) shops cannot set prices higher than the maximal reservation price $\overline{r_t}$ (respective $\overline{r_o}$), as they would meet no demand. We have therefore: $F(\overline{r_t}) = 1$ (respectively $G(\overline{r_o}) = 1$).

We assume that the number of consumers is sufficiently high so that by the law of large numbers, the distribution of the reservation prices addressing each traditional shop is:

$$h_t^i(r) = h_t(r)/n_t \tag{2.20}$$

and the distribution of the reservation prices addressing each online shop is:

$$h_o^i(r) = h_o(r)/n_o \tag{2.21}$$

Demand functions

As in the benchmark case, considering successive cohorts of consumers and summing up their successive search rounds leads to the discounted expected demand functions for each channel, which are linear in the prices. However, here, we additionally take into account the segmentation of demand into the two channels.

$$D_t^i(p_i) = \frac{1}{n_t} \cdot \frac{\Phi(t)}{e^{-\lambda t}} \cdot \frac{1-\overline{v}}{\overline{r_t} - \underline{r_t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot (\overline{r_t} - p_i) \tag{2.22}$$

$$D_o^i(p_i) = \frac{1}{n_o} \cdot \frac{\Phi(\tau)}{e^{-\lambda \tau}} \cdot \frac{\overline{v} - \underline{v_o}}{\overline{r_o} - \underline{r_o}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \cdot (\overline{r_o} - p_i) \tag{2.23}$$

These are functions of the number of shops in the considered channel, but not on the prices of rivals.

2.4.2 Firms Decision

Firms run a traditional shop and / or an online shop. There are many potential sellers on the market; $n_t \leqslant n$ sellers offer a traditional shop and $n_o \leqslant n$ sellers offer an online shop. Their transaction costs for the traditional shop k_t as well as for an online shop k_o are uniformly distributed on $[0, 1]$. We assume that there is no correlation between these distributions, as the technological and managerial abilities required for a traditional shop or an online shop are quite different.

Each seller i sets price p_t^i in the traditional shop and / or p_o^i in the online shop, to maximize the expected discounted profits. We assume that stock-keeping is not possible. For simplicity, the firms' discount rate is assumed to be ρ, as for the consumers. Each firm i earns profits π_t^i from the traditional, and/ or π_o^i from the online shop:

$$\begin{aligned}
\pi^i &= \max(\pi_t^i; 0) + \max(\pi_o^i; 0) \\
\pi_t^i &= (p_t^i - k_t^i) \cdot D_t^i(p_t^i), \qquad \forall p_t^i - k_t^i \geqslant 0 \\
&= (p_t^i - k_t^i) \cdot (\overline{r_t} - p_i^t) \cdot \frac{1}{n_t} \cdot \frac{e^{-\rho t}}{1 - e^{-(\rho+\lambda)t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \\
\pi_o^i &= (p_o^i - k_o^i) \cdot D_o^i(p_o^i), \qquad \forall p_o^i - k_o^i \geqslant 0 \\
&= (p_o^i - k_o^i) \cdot (\overline{r_o} - p_o^i) \cdot \frac{1}{n_o} \cdot \frac{e^{-\rho \tau}}{1 - e^{-(\rho+\lambda)\tau}} \cdot \left[1 + \frac{\lambda}{\rho}\right]
\end{aligned}$$

We assume that due to the law of large numbers, $\overline{r_o}$ is constant with respect to the prices p_o^i, p_t^i, so that the two optimizations are not directly correlated.

Price optimization in the traditional shop

$$\frac{\partial \pi_t^i}{\partial p_t^i} = (\overline{r_t} - 2p_t^i + k_t^i) \cdot \frac{1}{n_t} \cdot \frac{e^{-\rho t}}{1 - e^{-(\rho+\lambda)t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_t^i = \frac{\overline{r_t} + k_t^i}{2}$$

The corresponding equilibrium traditional firm's demand is:

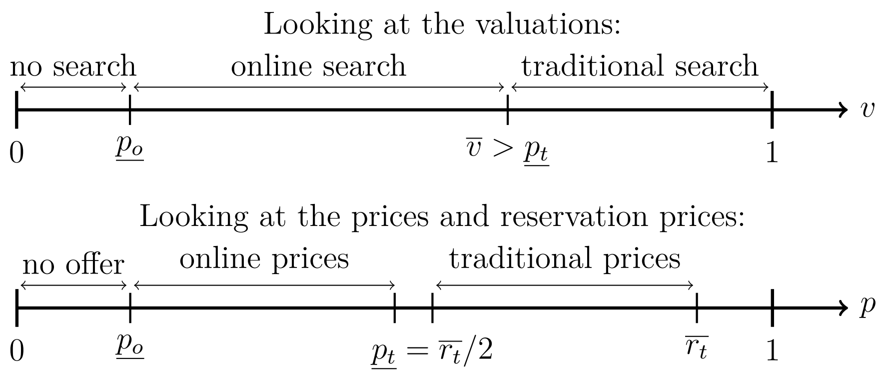
$$D_t^i(k_t^i) = \frac{\overline{r_t} - k_t^i}{2} \cdot \frac{1}{n_t} \cdot \frac{\Phi(t)}{e^{-\lambda t}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \tag{2.24}$$

Firms can participate in the traditional market only if their price is maximally equal to the highest reservation value, $\overline{r_t} = 1 - \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp$ (see (2.6)).

$$\begin{aligned} \frac{\overline{r_t} + k_t^i}{2} \leqslant \overline{r_t} \quad \Leftrightarrow \quad & \frac{k_t^i}{2} \leqslant \frac{\overline{r_t}}{2} \\ & k_t^i \leqslant \overline{r_t} \qquad \text{or equivalently} \\ & k_t^i \leqslant 1 - \Phi(t) \int_{\underline{p_t}}^{\overline{p_t}} F(p)dp := \overline{k_t} < 1 \end{aligned} \tag{2.25}$$

The highest traditional transaction cost for which a firm can be active in the market, $\overline{k_t}$, is strictly smaller than 1; is depends positively on the transaction time t, on the exit probability λ, and on the discount rate ρ.
The lowest price is offered by the firm with the lowest traditional transaction unit costs (0); it amounts to:

$$\underline{p_t} = \overline{r_t}/2$$



The distribution $F(p)$ of the prices comprised between $\underline{p_t}$ and $\overline{r_t}$ can now be computed. It reads:[10]

$$F(p) = P[\tilde{p}_i \leqslant p \mid k_t^i \leqslant \overline{r_t}] = \frac{2p - \overline{r_t}}{\overline{r_t}} \qquad \text{where } \overline{r_t} = 1 - \Phi(t) \int_{\underline{p_t}}^{\overline{r_t}} F(p)dp$$

[10] We can check that: $F(\underline{p_t}) = [\overline{r_t} - \overline{r_t}]/\overline{r_t} = 0$ and $F(\overline{r_t}) = [2\overline{r_t} - \overline{r_t}]/\overline{r_t} = 1$

We can now compute the explicit solution for the highest reservation price $\overline{r_t}$.[11]

$$\overline{r_t} = \frac{4 + \Phi(t)}{4 + \Phi(t)} < 1 \tag{2.26}$$

This solution for the maximal reservation price yields following results for the highest and lowest prices in the traditional channel:

$$\overline{p_t} = \overline{r_t} = \frac{4}{4 + \Phi(t)} \tag{2.27}$$

$$\underline{p_t} = \frac{2}{4 + \Phi(t)} \tag{2.28}$$

As depicted in Figure 2.17, these prices are monotonously increasing with the length of a search period t.

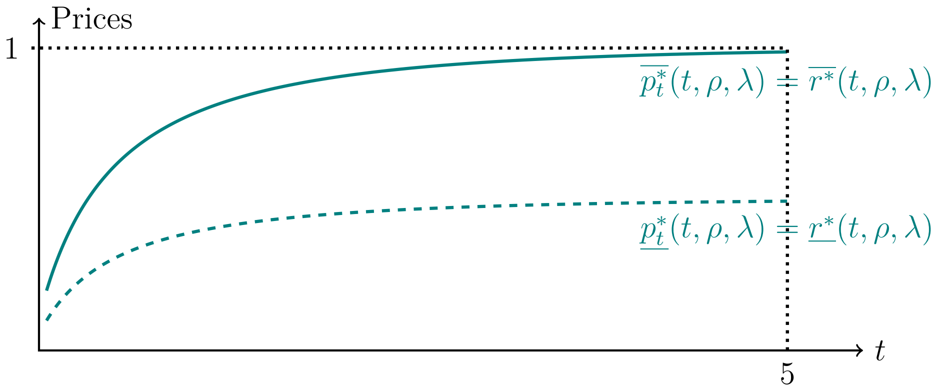


Figure 2.17: Minimal and maximal prices ($\lambda = 0.3; \rho = 0.3; t \leqslant 5$)

And finally, we can reformulate the distribution function of prices in traditional shops:

$$F(p) = \frac{\left[2 + \Phi(t)\right] - \left[4 + \Phi(t)\right](1 - p)}{2} \tag{2.29}$$

[11] Let us call I_F the integral $\int_{\underline{p_t}}^{\overline{p_t}} F(p)dp$;

$$I_F = \int_{\underline{p_t}}^{\overline{p_t}} \frac{2p - \overline{r_t}}{\overline{r_t}}\, dp \quad \text{where} \quad \underline{p_t} = \frac{\overline{r_t}}{2} \quad \text{and} \quad \overline{p_t} = \frac{\overline{r_t} + \overline{k_t}}{2} = \overline{r_t}$$

$$I_F = \left[\frac{p^2 - p \cdot (\overline{r_t})}{\overline{r_t}}\right]_{\underline{p_t}}^{\overline{r_t}} = \frac{\overline{r_t}}{4} = \frac{1 - \Phi(t) I_F}{4}$$

$$I_F \left(\frac{4 + \Phi(t)}{4}\right) = \frac{1}{4}$$

$$I_F = \frac{1}{4 + \Phi(t)}$$

$$\overline{r_t} = 1 - \Phi(t) I_F = \frac{4 + \Phi(t) - \Phi(t)}{4 + \Phi(t)}$$

This is a linear function increasing (in its domain of definition which is delimited by the minimal and maximal prices) from 0 to 1.[12]

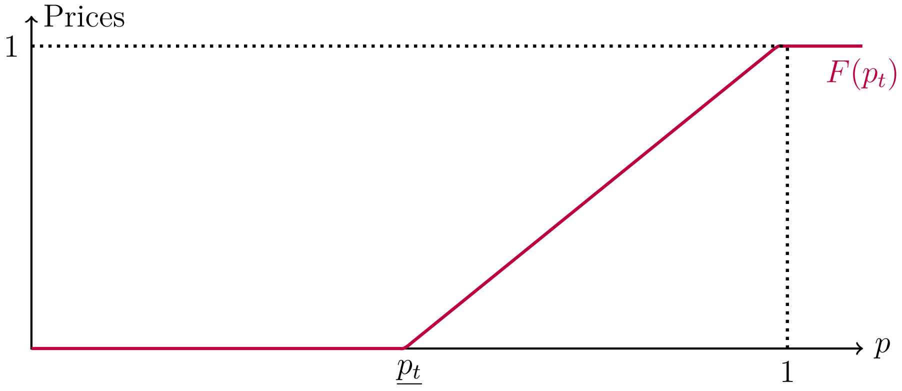


Figure 2.18: Distribution of prices in the traditional channel ($\lambda = 0.3; \rho = 0.3; t = 5$)

Price optimization in the online shop

The maximization of profits in the online channel:

$$\begin{aligned}\pi_o^i &= (p_o^i - k_o^i) \cdot D_o^i(p_o^i), \qquad \forall p_o^i - k_o^i \geqslant 0 \\ &= (p_o^i - k_o^i) \cdot (\overline{r_o}(p_o^i, p_t^i) - p_o^i) \cdot \frac{1}{n_o} \cdot \frac{e^{-\rho\tau}}{1 - e^{-(\rho+\lambda)\tau}} \cdot \left[1 + \frac{\lambda}{\rho}\right]\end{aligned}$$

leads to the equilibrium price:

$$p_o^i = \frac{\overline{r_o} + k_o^i}{2}$$

and the equilibrium demand function for online firm's:

$$D_o^i(k_o^i) = \frac{\overline{r_o} - k_o^i}{2} \cdot \frac{1}{n_o} \cdot \frac{\Phi(\tau)}{e^{-\lambda\tau}} \cdot \left[1 + \frac{\lambda}{\rho}\right] \tag{2.30}$$

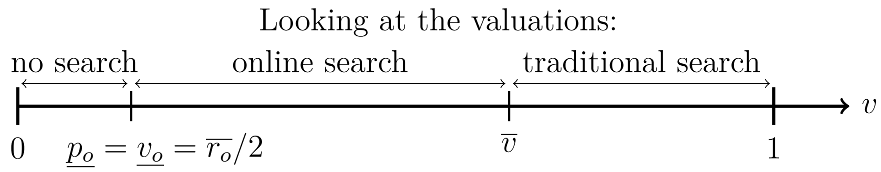
The equilibrium profits read:

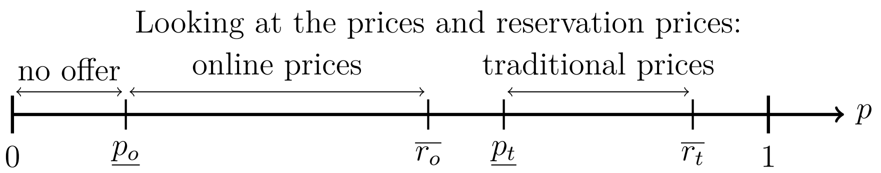
$$\pi_o^i = \left[\frac{\overline{r_o} - k_o^i}{2}\right]^2 \cdot \frac{1}{n_o} \cdot \frac{e^{-\rho\tau}}{1 - e^{-(\rho+\lambda)\tau}} \cdot \left[1 + \frac{\lambda}{\rho}\right]$$

and are positive for costs below the maximal reservation price: $k_o^i \leqslant \overline{r_o}$. The firms endowed with the maximal costs set the maximal price in the online channel $\overline{p_o} = \overline{r_o}$. As the smallest online unit transaction costs are $\min_i k_o^i = 0$, the smallest price will be $\underline{p_o} = \overline{r_o}/2$.

[12] We can check that $F(\underline{p_t}) = \frac{[2+\Phi(t)]-4-\Phi(t)+2}{2} = 0$ and $F(\overline{p_t}) = \frac{[2+\Phi(t)]-4-\Phi(t)+4}{2} = 1$.

The consumer with the smallest valuation who will be able to buy online has reservation price and valuation $\underline{v_o} = \underline{p_o} = \overline{r_o}/2$. In any case, his utility will be zero, so that the costs of discounting do not matter any more for this particular consumer, arising the question of whether we should exclude this consumer and consider only valuations above the minimum online price.





There might be a gap or an overlapping in the prices set online and in traditional shops, as the exact position of $\underline{p_t}$ is still unknown. The distribution $G(p)$ of the prices comprised between $\underline{v_o}$ and $\overline{r_o}$ can now be computed:

$$G(p) = P[\tilde{p}_i \leqslant p | k_i \leqslant \overline{r_o}] = P\left[\frac{\overline{r_o} + k_o^i}{2} \leqslant p\right] \cdot \frac{1}{\overline{r_o}} = \frac{2p - \overline{r_o}}{\overline{r_o}}$$

As in the case of the online shops, the price distribution increases linearly from 0 to 1.[13] The (implicit) distribution of the prices in online shops is:

$$G(p) = \frac{2p - \overline{r_o}}{\overline{r_o}} \qquad \text{where } \overline{r_o} = \overline{v} - \Phi(\tau) \int_{\underline{p_o}}^{\overline{r_o}} G(p)\ dp$$

We can now solve for the explicit value of $\overline{r_o}$. Let us call I_G the integral $\int_{\underline{p_o}}^{\overline{r_o}} G(p)\ dp$:

$$I_G = \int_{\underline{p_o}}^{\overline{r_o}} \frac{2p - \overline{r_o}}{\overline{r_o}}\ dp = \frac{\overline{r_o}}{4} \qquad \text{whith} \quad \underline{p_o} = \frac{\overline{r_o}}{2}$$

It can be verified that the maximal reservation price is below 1:

$$\overline{r_o} = \frac{4\overline{v}}{4 + \Phi(\tau)} \quad < \quad 1 \tag{2.31}$$

Consequently, we obtain for the maximum and minimum online prices:

$$\overline{p_o} = \overline{r_o} = \frac{4\overline{v}}{4 + \Phi(\tau)} \tag{2.32}$$

$$\underline{p_o} = \frac{\overline{r_o}}{2} = \frac{2\overline{v}}{4 + \Phi(\tau)} \tag{2.33}$$

And eventually, we compute the explicit distribution of prices in online shops:

$$G(p) = \frac{2p - \overline{r_o}}{\overline{r_o}} = \frac{\big[2 + \Phi(\tau)\big] \cdot \overline{v} - \big[4 + \Phi(\tau)\big](\overline{v} - p)}{2\overline{v}} \tag{2.34}$$

[13] We can check that $G(\underline{v_o}) = [2 \cdot \frac{\overline{r_o}}{2} - \overline{r_o}]/\overline{r_o} = 0$ and $G(\overline{r_o}) = 2\overline{r_o} - \overline{r_o}/\overline{r_o} = 1$.

2.4.3 Consumers' Surpluses and Indifference Condition

The consumers build expectations upon each channel and choose one of them depending on their expected surplus in each case. We will therefore solve for the indifferent consumers' expected surpluses in each channel, before we turn to the solution for the critical valuation $\overline{v}$.
The indifferent consumer's surplus in the traditional channel can be computed following the same logic as in the benchmark, with a decisive difference: here, the traditional channel does not address all consumers (with valuations between 0 and 1) but only those with valuations above the critical value $\overline{v}$.

The surplus of a consumer with reservation price r_t in the traditional channel is the sum of the discounted expected surpluses in any period, as computed in (2.16). Using the relationship between valuation and reservation price in (2.12)[14] and inserting the value for $\Phi(t)$, we can formulate the result as a function of the indifferent consumer's valuation $\overline{v}$.

$$CS_t^i = \sum_{j=1}^{\infty} e^{-(\rho+\lambda)jt} \cdot [1 - F(r_t)]^{j-1} \cdot \int_{\underline{p_t}}^{r_t} [\overline{v} - p] f(p)\ dp$$
$$= \frac{\left[1 - e^{(\lambda+\rho)t} + \sqrt{3 + e^{2(\lambda+\rho)t} + 4e^{(\lambda+\rho)t}(\overline{v} - 1) - 3\overline{v}}\right]^2}{4e^{(\lambda+\rho)t} - 3}$$

Consumers' surplus in the online channel is structurally similar, given the price distribution in the online channel, and addresses all consumers with valuations below the critical value $\overline{v}$.

The surplus of the indifferent consumer with reservation price r_o in the online channel is the sum of the discounted expected surpluses in any period, as computed in (2.17). Using the relationship between valuation and reservation price in (2.15) and inserting the value for $\Phi(t)$, we can formulate the result as a function of the indifferent consumer's valuation $\overline{v}$.

$$CS_o^i = e^{-(\rho+\lambda)T} \sum_{j=1}^{\infty} e^{-(\rho+\lambda)j\tau} \cdot [1 - G(r_o)]^{j-1} \cdot \int_{\underline{p_o}}^{r_o} [\overline{v} - p] f(p)\ dp$$
$$= \frac{e^{-(\lambda+\rho)T}\overline{v}}{4e^{(\lambda+\rho)\tau} - 3}$$

The graphical solution for the indifference valuation is depicted in Figure (2.19). Consumers' surplus in the traditional channel is convex in the valuation, while consumers'

[14]When inserting the distribution function of prices (2.7) into the stopping rule (2.12), we get a quadratic relationship between the valuation v and the reservation price r_t: $v = r_t[1 - \Phi(t)] + r_t^2 \frac{4+\Phi(t)}{4}\Phi(t) + \frac{\Phi(t)}{4+\Phi(t)}$. As only as positive relation between valuation and reservation price makes economic sense, the solution is easily identified:
$r_t = \frac{2\Phi(t)-2}{\Phi(t)[4+\Phi(t)]} + \frac{2}{\Phi(t)[4+\Phi(t)]} \cdot \sqrt{[1 + \Phi(t)]^2 + \Phi(t)[4 + \Phi(t)](v - 1)}$.

surplus in the online channel is linear, so that there are, mathematically, two solutions. The relevant domain is, however, restricted by the boundaries on valuation; v is between 0 and 1 and in the traditional channel, the indifferent consumer has a valuation strictly higher than the minimal price in the traditional channel. When eliminating the non-relevant domains (hatched areas in the graph), only one feasible solution remains. Up to this solution, the consumer's surplus is higher in the online channel; for valuations above $\overline{v}$, it is higher in the traditional channel.

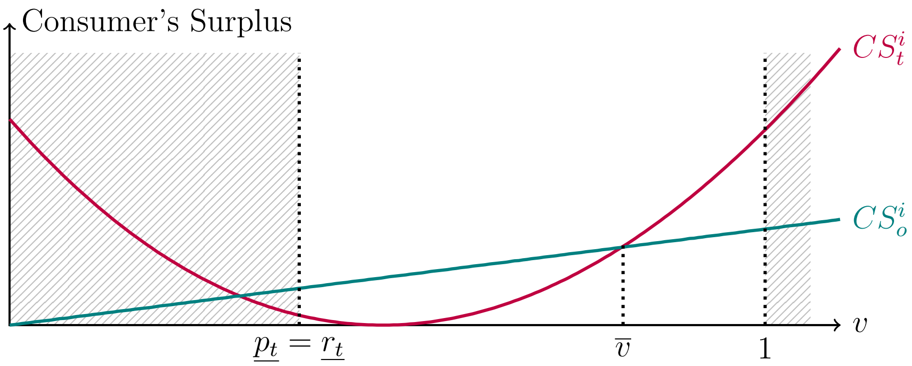


Figure 2.19: Distribution of prices in the traditional channel ($\lambda = 0.3; \rho = 0.3; \tau = 1; t = 5; T = 5$)

There is an explicit formula for this unique solution:

$$\begin{aligned}\overline{v} =\Big[& 36e^{(\lambda+\rho)T} + 54e^{2(\lambda+\rho)T} - 102e^{(\lambda+\rho)(t+T)} + 72e^{2(\lambda+\rho)(t+T)} + 90e^{(\lambda+\rho)(2t+T)} - 24e^{(\lambda+\rho)(3t+T)} \\ & - 126e^{(\lambda+\rho)(t+2T)} - 48e^{(\lambda+\rho)(\tau+T)} + 96e^{2(\lambda+\rho)(\tau+T)} + 13e^{(\lambda+\rho)(\tau+t+T)} + 128e^{2(\lambda+\rho)(\tau+t+T)} \\ & - 120e^{(\lambda+\rho)(\tau+2t+T)} + 32e^{(\lambda+\rho)(\tau+3t+T)} - 144e^{(\lambda+\rho)(\tau+2T)} + 336e^{(\lambda+\rho)(\tau+t+2T)} \\ & - 192e^{(\lambda+\rho)(\tau+2t+2T)} - 224e^{(\lambda+\rho)(2\tau+t+2T)} \\ & + 2e^{(\lambda+\rho)T}(-1+e^{(\lambda+\rho)t})^{3/2}(-3+4e^{(\lambda+\rho)t})(-3+4e^{(\lambda+\rho)\tau})\sqrt{-3+e^{(\lambda+\rho)t}-6e^{(\lambda+\rho)T}+8e^{(\lambda+\rho)(T+\tau)}}\Big] \\ & \Big/ \Big[\Big(3-4e^{(\lambda+\rho)t}\Big)^2\Big(1+3e^{(\lambda+\rho)T}-4e^{(\lambda+\rho)(T+\tau)}\Big)^2\Big]\end{aligned}$$

This indifference valuation is increasing in the search time in the traditional channel t: as it gets longer, the online channel gets more attractive. It is decreasing in the online delivery time T: as this delivery time gets longer, the traditional shopping becomes more attractive. This behavior is illustrated in Figures (2.21) and (2.20).

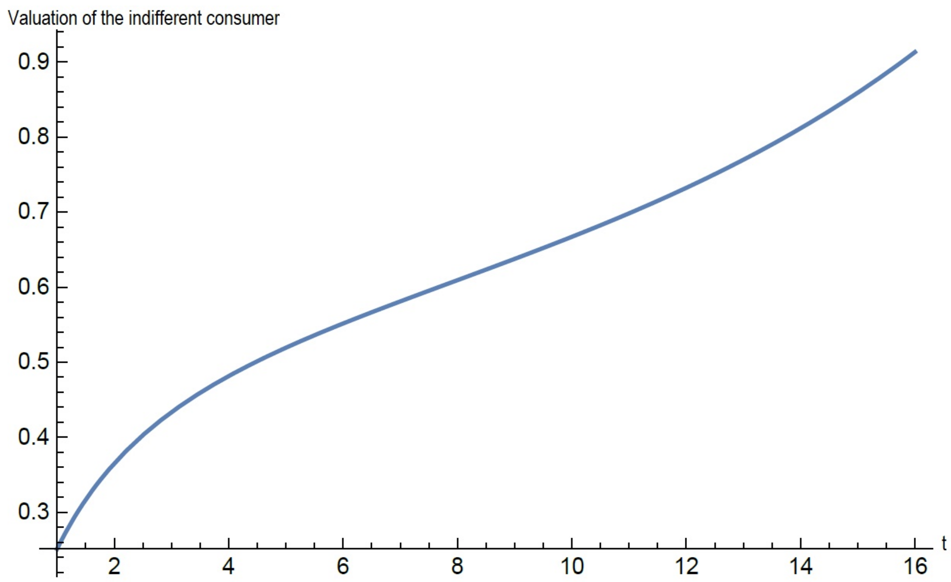


Figure 2.20: Valuation of the indifferent consumer as a function of t (with $\lambda = 0.1, \rho = 0.1, \tau = 1$ and $T = 21$)

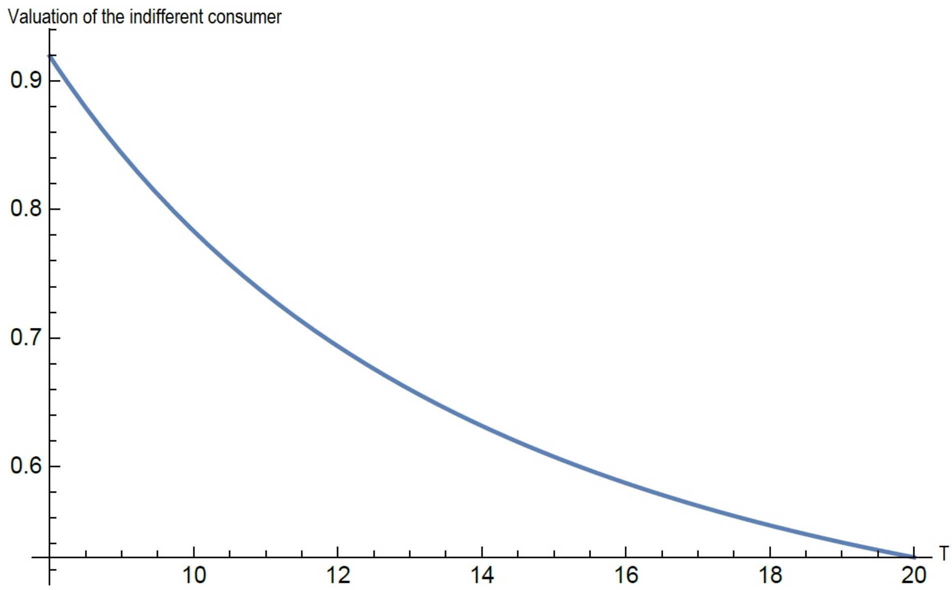


Figure 2.21: Valuation of the indifferent consumer as a function of T (with $\lambda = 0.1, \rho = 0.1, \tau = 1$ and $t = 5$)

2.4.4 Intermediary summary

Table (2.1) displays the equilibrium results of the game. These solutions yield some simplifications: we observe that the respective minimum and maximum prices coincide with the corresponding reservation prices. These prices are below the valuations: all consumers can, in the channel of choice, find a price lower than or equal to their reservation price. While the segmentation of the consumers with respect to their valuation is a partition into disjoint subsets ($[0; \underline{v_o}]$; $(\underline{v_o}; \overline{v}]$; $(\overline{v}; 1]$), the prices in the two channels are not necessarily ordered: some of the higher prices in the online channel might be higher than some of the lower prices in the traditional channel (see Figure (2.22)). This property of our model is consistent with the empirical observation that prices in the online channel are mostly (not: always) lower but still volatile. And indeed, depending on the ratio between periodical search time in both channels, there might or might not be an overlapping of the equilibrium price ranges in the two channels (see Figure 2.23).

	no search	online shopping	traditional shopping
Range of valuations	$0 - \underline{v_o}$	$\underline{v_o} - \overline{v}$	$\overline{v} - 1$
Range of reservation prices	$0 - \underline{r_o}$	$\underline{r_o} - \overline{r_o}$	$\underline{r_t} - \overline{r_t}$
Prices	-	$\underline{p_o} - \overline{p_o}$	$\underline{p_t} - \overline{p_t}$
Values	$\underline{r_o} = \underline{v_o}$	$\underline{r_o} = \underline{v_o} = \underline{p_o} = \frac{2\overline{v}}{4+\Phi(\tau)}$ $\overline{p_o} = \overline{r_o} = \frac{4\overline{v}}{4+\Phi(\tau)}$	$\underline{p_t} < \overline{v} < \overline{p_t}$ with: $\underline{p_t} = \underline{r_t} = \frac{2}{4+\Phi(t)}$ $\overline{p_t} = \overline{r_t} = \frac{4}{4+\Phi(t)} < 1$

Table 2.1: Range of the different variables

The striking effect of the expansion to the online channel is that more consumers can access the market. Figure (2.24) displays a comparison of the lowest valuations for which consumers can access the market. In the benchmark (intermediary blue line with minimal slope), this minimal valuation is quite high, much higher than in the full game with online expansion (gray line at the bottom). Interestingly, when the search time in both channels are close to each other (starting points of the lines), the lowest valuation for which consumers access the market in the benchmark corresponds to the indifference valuation in the full game (orange, steeper line below the boundary reference line of 1). The difference between those lines can be considered as firms opportunity costs from the online expansion: these are consumers who would have bought anyway, but who grasps the opportunity of searching online for a lower price. The difference between the two lower lines, the lowest valuations in each game, is the net gain of market share due to the online expansion: these are consumers who would not have bought in the benchmark, but

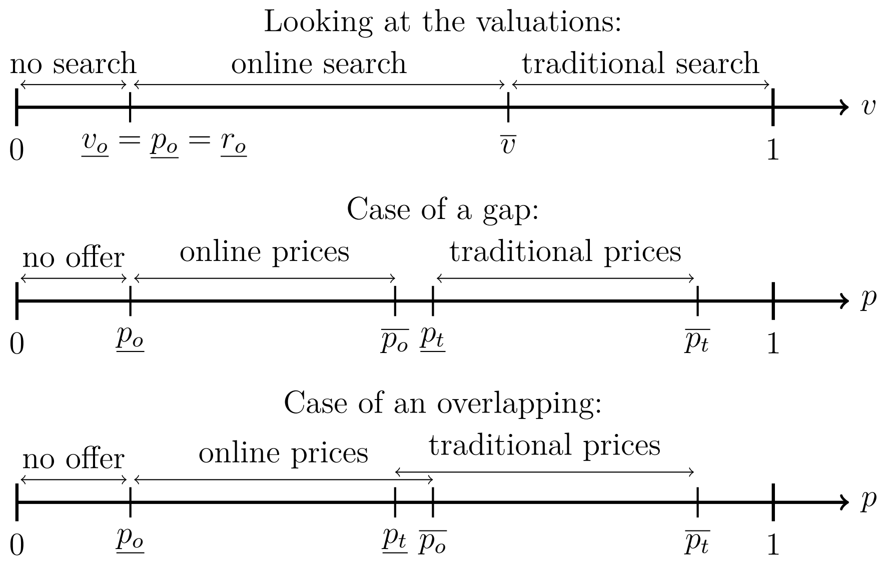


Figure 2.22: Equilibrium constellations

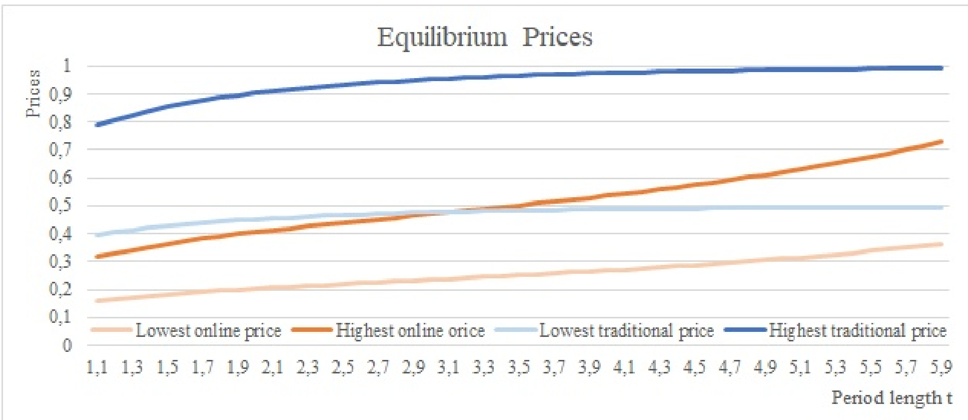


Figure 2.23: Comparison of the equilibrium prices (with $\lambda = 0.3, \rho = 0.3, \tau = 1, T = 6$)

who can participate in the online market.

The question of how many firms are active in both models is differentiated. On one hand, there is no decrease in the number of firms active in the traditional market: because it is determined by the highest reservation price, it is equal to the number of firms active in the benchmark. On the other hand, the number of firms in the online channel is limited by the low level of the online reservation prices. This model does not yield the threat that the online channel could cannibalize the traditional one.

A further interesting aspect is that the market segmentation obtained via the expansion to the online channel has no impact on the prices in traditional shops, there are equal to the prices in the benchmark. The reasons for this phenomenon is that in both cases, firms focus at the consumers with high valuations and set their prices so that the consumers with the highest willingness to pay can just afford the highest price in the market. This result is an idiosyncrasy of the model and is linked with the simplifying assumption that

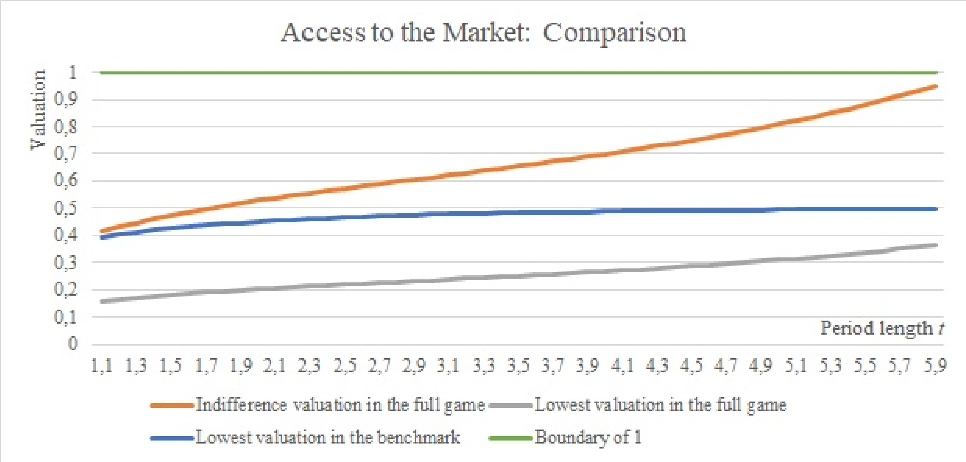


Figure 2.24: Comparison of the minimal valuations (with $\lambda = 0.3, \rho = 0.3, \tau = 1, T = 6$)

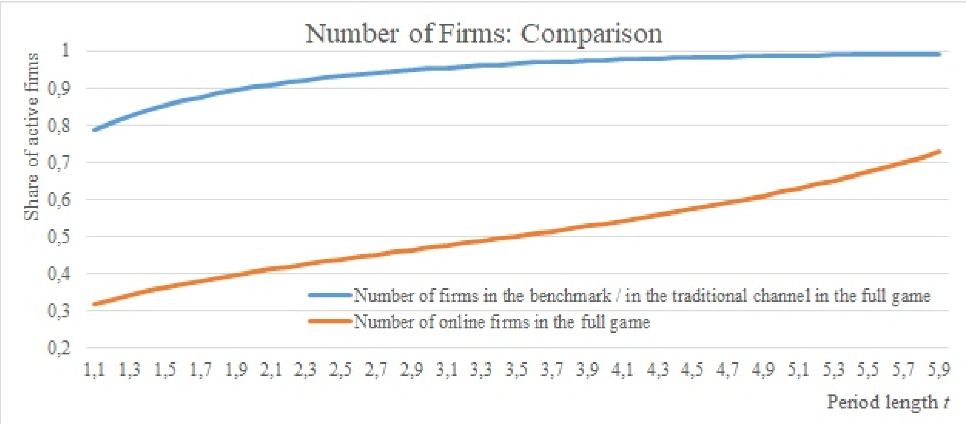


Figure 2.25: Comparison of the minimal valuations (with $\lambda = 0.3, \rho = 0.3, \tau = 1, T = 6$)

individual price decisions are negligible and do not affect the overall indifference condition. This simplifying assumption might be lifted in a numerical solution of the model, as many of the final analytic results prove to be daring already in this simplified version.

In order to assess these results, we will, in the next section, compare the welfare in the full game to the welfare in the benchmark.

2.5 Welfare Considerations

2.5.1 Firms' Surplus

As in the benchmark case, firms' surpluses are obtained by aggregating firms' expected discounted profits over all firms active in the market, i.e. which unit costs below the maximum equilibrium price in their channel. We turn separately to each channel. The aggregated profits in the traditional channel FS_t read:

$$FS_t = \int_0^{\overline{k_t}} \pi_t^*(k_t^i)\ dk_t^i = \frac{2(\lambda+\rho)\cdot[4+\Phi(t)]\cdot(1-\overline{v})\cdot\left[1-e^{(-(\lambda+\rho)t)}\right]}{3\rho\cdot[4-3e^{(-(\lambda+\rho)t)}]^2}$$

The aggregated profits in the traditional channel reach a peak for a low value of t (see Figure (2.26)). This is linked with the two sides of the search period: a short period might set incentives for consumers to search longer, but it decreases the reservation prices, which sets an opposite incentive to reduce the searching behavior; finally, it increases the eventuality that a consumer exits the market within a given time span. As comparison,

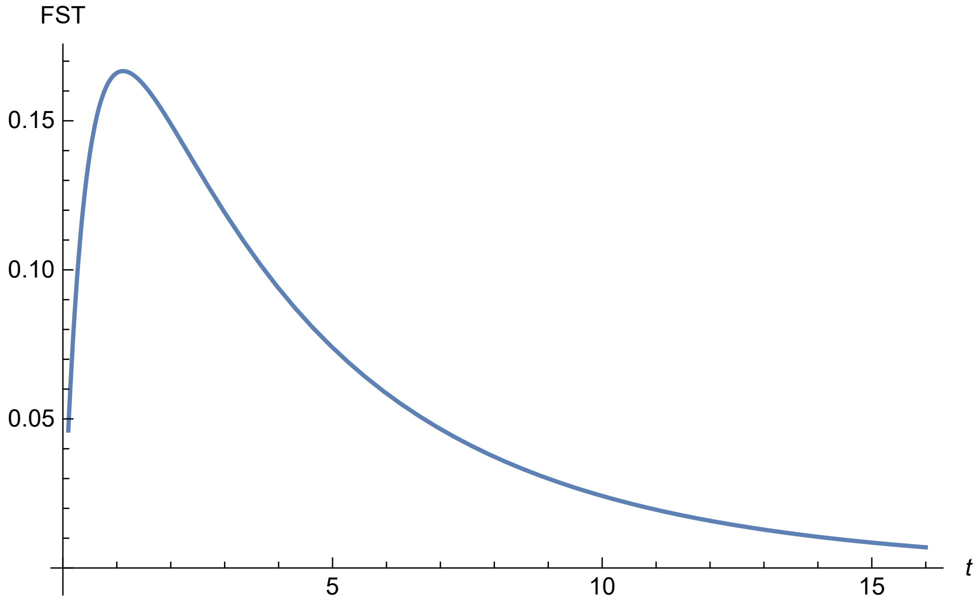


Figure 2.26: Aggregated profits in the traditional channel (with $\lambda = 0.1, \rho = 0.1, \tau = 1$)

we have plotted in Figure (2.27) the corresponding graph in the benchmark. Even though the overall shape is comparable, the firms' surplus in the benchmark are less volatile with respect to the search time t, and the level of firms surpluses is higher.

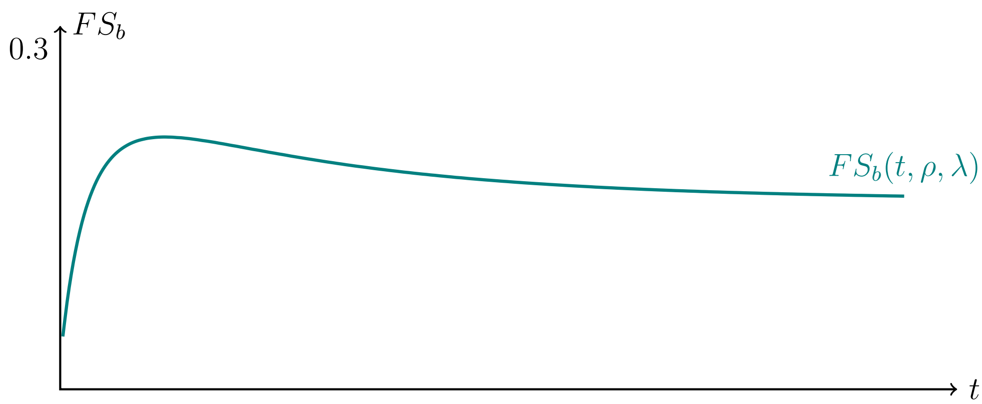


Figure 2.27: Firms Surplus in the Benchmark ($\lambda = 0.1; \rho = 0.1; t \leqslant 16$)

The aggregated profits in the traditional channel FS_o read:

$$FS_o = \int_0^{\overline{k_o}} \pi_o^*(k_o^i)\ dk_o^i = \frac{2(\lambda+\rho)\cdot[2+\Phi(t)]\cdot\left[1-e^{(-(\lambda+\rho)t)}\right]\cdot\overline{v}}{3\rho\cdot[4-3e^{(-(\lambda+\rho)t)}]^2}$$

These aggregated profits in the online channel are negatively affected by longer online delivery time, and positively by an increased search length in the rival channel (see Figures (2.30) and (2.29)). Similarly to the picture in the traditional channel, the search time leads to a peak in aggregated profits for a low value (see Figure (2.28)).

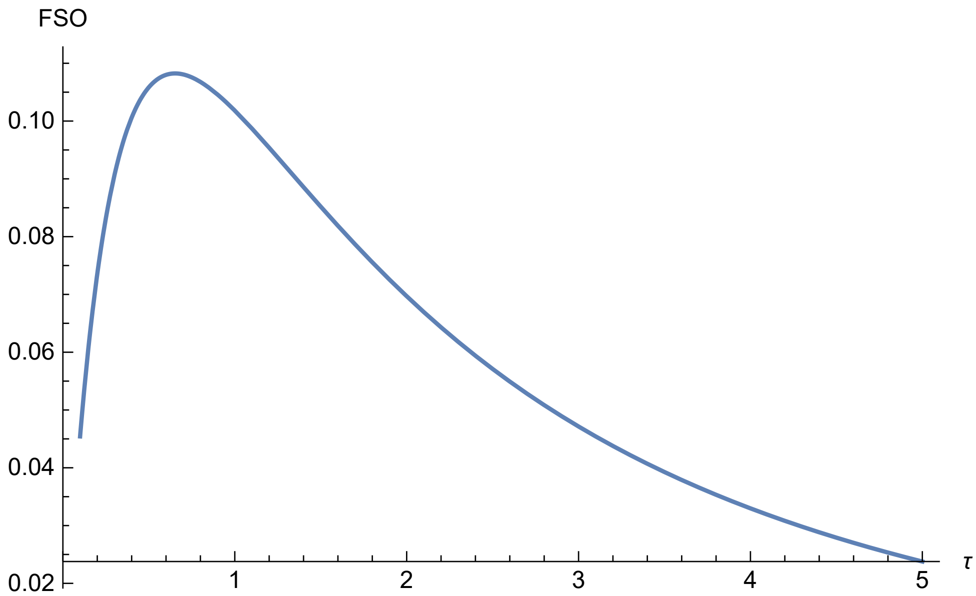


Figure 2.28: Aggregated profits in the online channel (with $\lambda = 0.1, \rho = 0.1, t = 5, T = 10$)

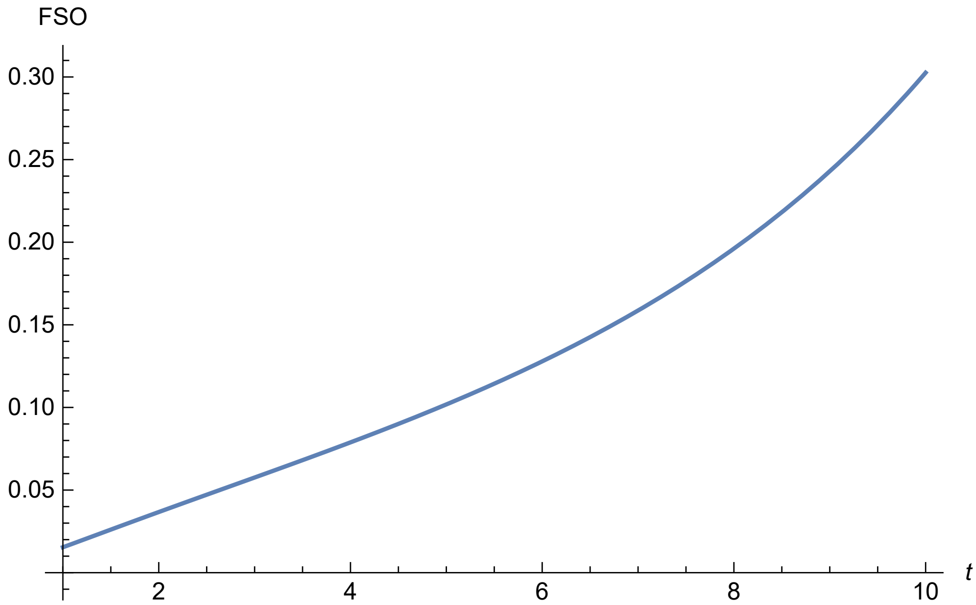


Figure 2.29: Aggregated profits in the online channel (with $\lambda = 0.1, \rho = 0.1, \tau = 1, T = 10$)

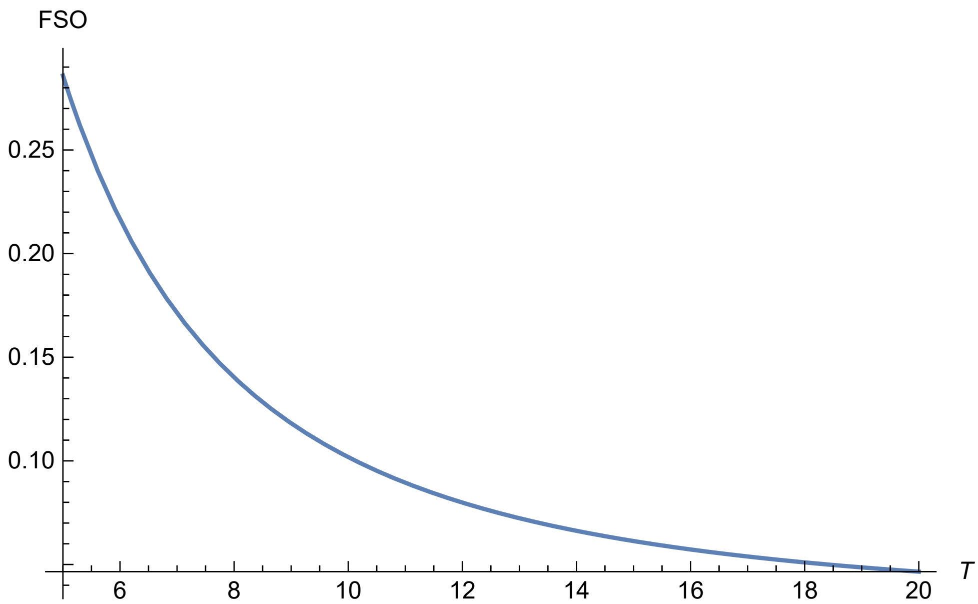


Figure 2.30: Aggregated profits in the online channel (with $\lambda = 0.1, \rho = 0.1, \tau = 1, t = 5$)

The overall firms surplus is the aggregation of the profits in the two channels.

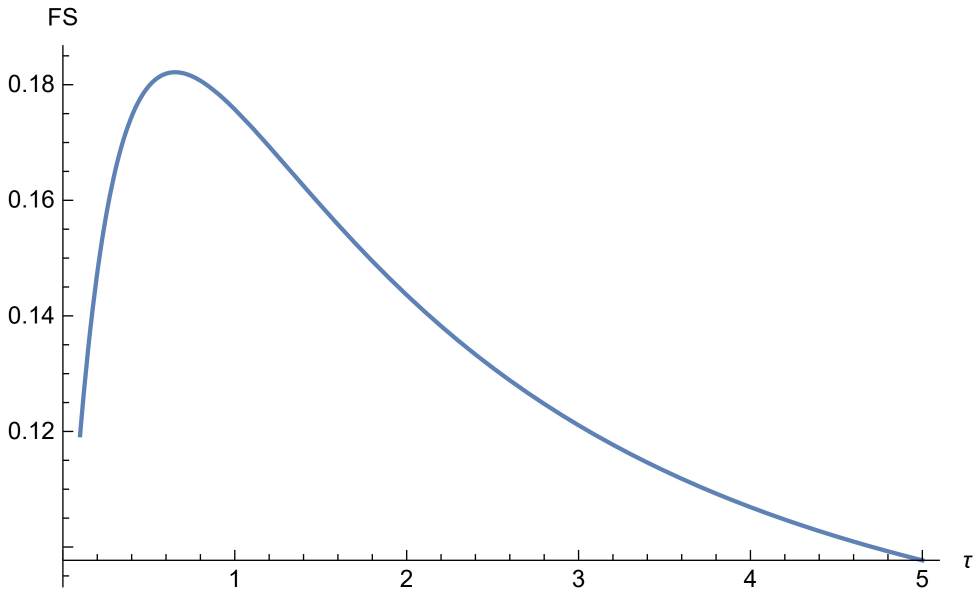


Figure 2.31: Aggregated firms profits (with $\lambda = 0.1, \rho = 0.1, t = 5, T = 10$)

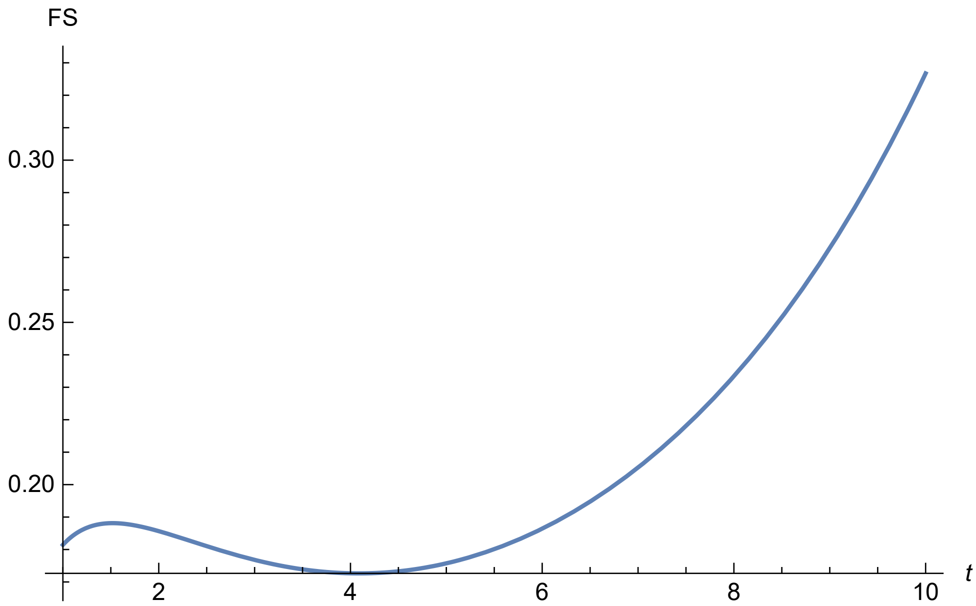


Figure 2.32: Aggregated firms profits (with $\lambda = 0.1, \rho = 0.1, \tau = 1, T = 10$)

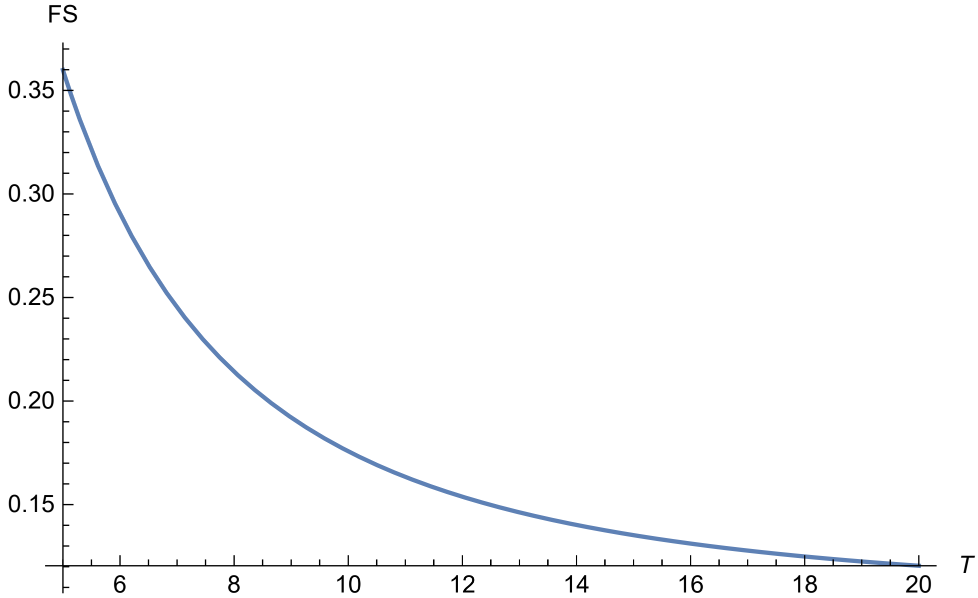


Figure 2.33: Aggregated firms profits (with $\lambda = 0.1, \rho = 0.1, \tau = 1, t = 5$)

2.5.2 Consumers' Surplus

Consumers' surplus is the aggregated surplus over all periods and of consumers with valuations above $\overline{v}$. The respective formula for the online and the traditional channel read:

$$CS_t = \int_{\overline{v}}^{1} \left[\sum_{j=1}^{\infty} e^{-(\rho+\lambda)jt} \cdot [1 - F(r_t)]^{j-1} \cdot \int_{\underline{p_t}}^{r_t} [v_t(r_t) - p] f(p) \, dp \right] \, dv$$

$$CS_o = e^{-(\rho+\lambda)T} \int_{\underline{v_o}}^{\overline{v}} \left[\sum_{j=1}^{\infty} e^{-(\rho+\lambda)j\tau} [1 - G(r_o)]^{(k-1)} \int_{\underline{p_o}}^{r_o} (v - p) \; g(p) dp \right] \, dv$$

Because of their complexity, the resulting formulas are not reproduced here.
A reduction of the search time in the online channel τ has, for low values, a positive impact on consumers' surplus (see Figures (2.34) and (2.35)). However, we observe that for higher values, a decrease in τ leads to a decrease in consumers' surplus in the traditional channel: when the search time in both channels are close to each other, the discrimination between the two channels enables firms to better exhaust consumers' willingness to pay in each channel, without improving the overall consumers' surplus.

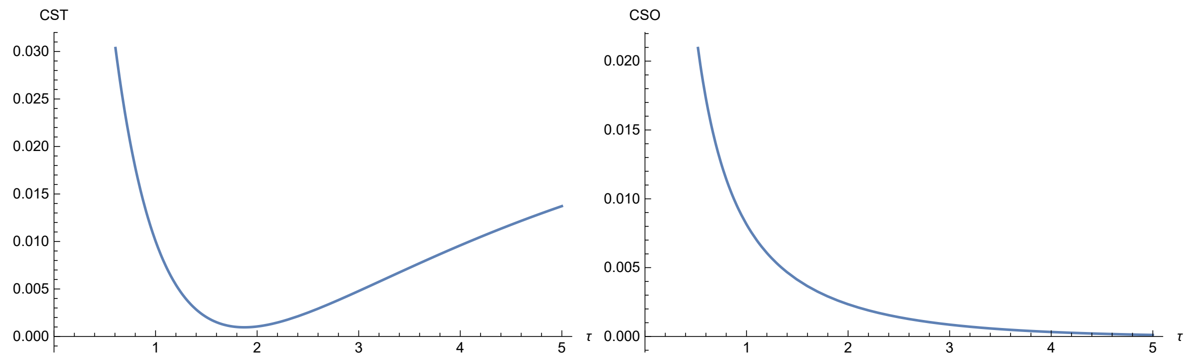


Figure 2.34: Consumers' surpluses as functions of τ (with $\lambda = 0.1, \rho = 0.1, t = 5, T = 16$)

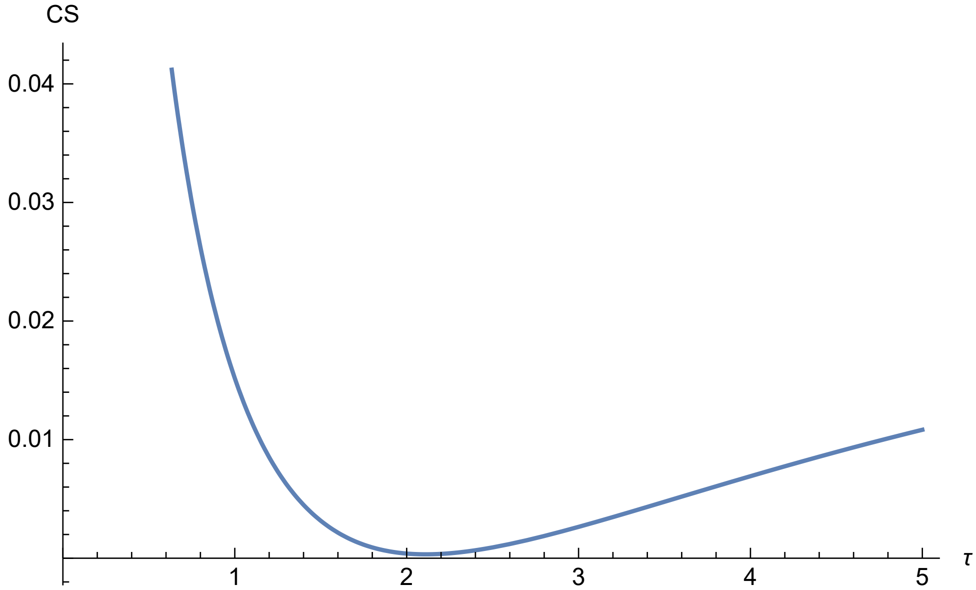


Figure 2.35: Overall Consumers' surplus as function of τ ($\lambda = 0.1, \rho = 0.1, t = 5, T = 16$)

The reduction of the search time in the traditional channel is beneficial for consumers' surplus in the traditional channel and negative in the online channel (see Figures (2.36) and (2.37)). It increases the probability of exit and, as mentioned above, does not necessarily enhance the searching behavior of consumers.

Figure (2.38) displays the comparable graph for the benchmark model. Here too, the overall shape is similar but not identical: as in the traditional channel in the full game, consumers' surplus increases when the search time t decreases. Depending on the value of t, the consumers' surplus might be higher or lower in the benchmark.

With respect to the delivery time in the online channel T, we also observe conflicting effects (see Figures (2.39) and (2.40)). Even though shorter delivery times are beneficial for overall consumers' surplus, there might have a negative impact on the consumers' surplus in the traditional channel - simply because of the induced increase in the indifference valuation and of the migration of consumers from the traditional to the online channel.

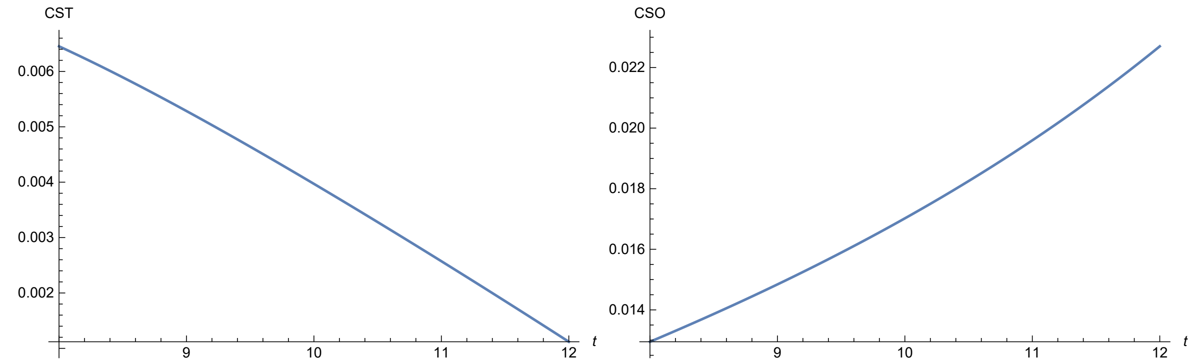


Figure 2.36: Consumers' surpluses as functions of t (with $\lambda = 0.1, \rho = 0.1, \tau = 1, T = 16$)

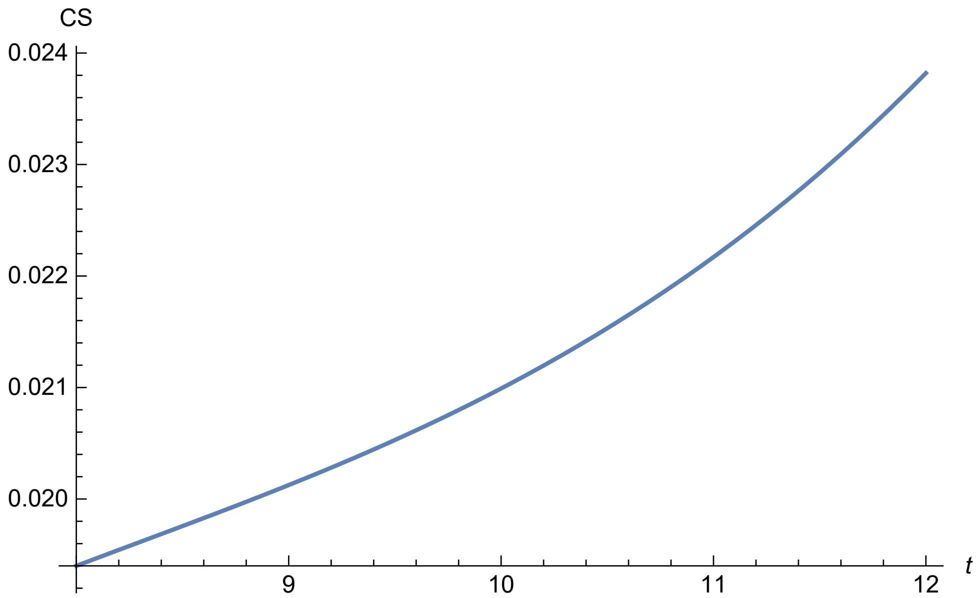


Figure 2.37: Overall Consumers' surplus as function of t ($\lambda = 0.1, \rho = 0.1, \tau = 1, T = 16$)

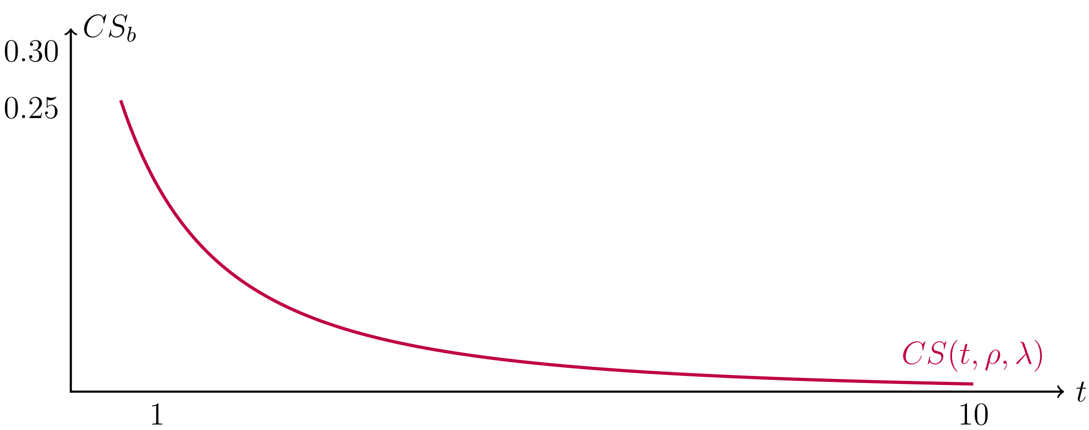


Figure 2.38: Consumers Surplus in the Benchmark ($\lambda = 0.1; \rho = 0.1; t \leqslant 12$)

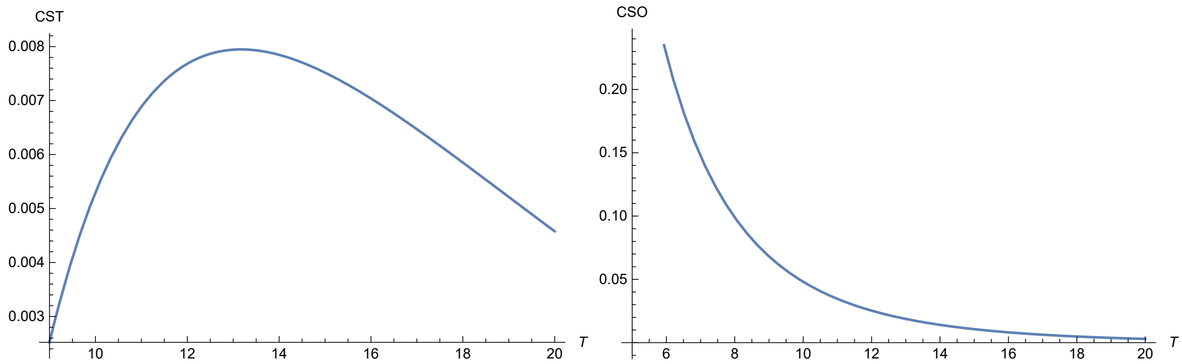


Figure 2.39: Consumers' surpluses as functions of T (with $\lambda = 0.1, \rho = 0.1, \tau = 1, t = 5$)

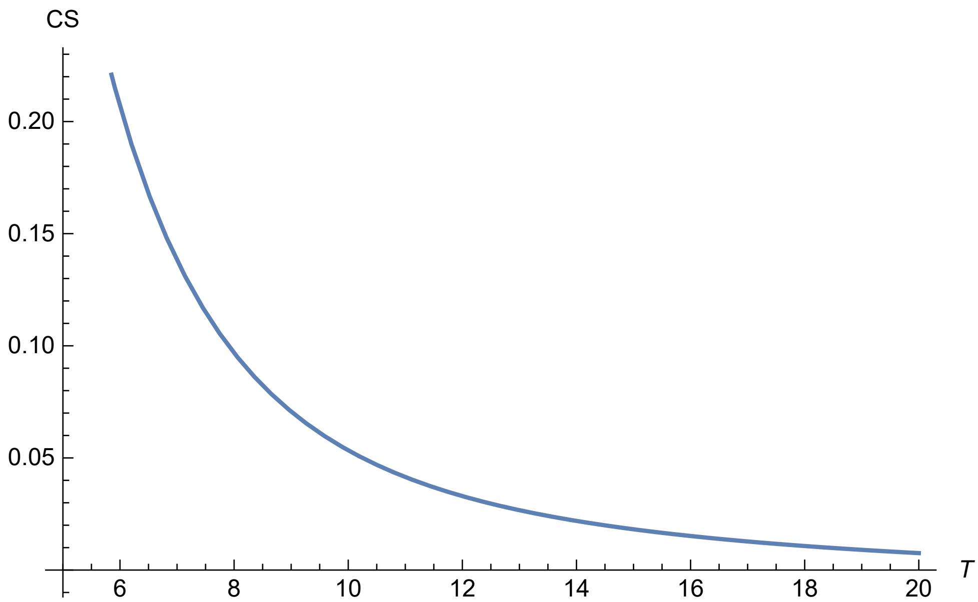


Figure 2.40: Overall Consumers' surplus as function of T ($\lambda = 0.1, \rho = 0.1, \tau = 1, t = 5$)

2.5.3 Overall Welfare

The aggregation of firms' and consumers' welfare yields clear results concerning the impact of the different time factors. The search time in the traditional channel has proved to have a mainly negative impact on firms' profits and consumers' surplus, and indeed the impact of a reduction in this search time parameter on welfare is negative, with an exception for the domain when the search times in both channels are close to each other (see Figure (2.43)).

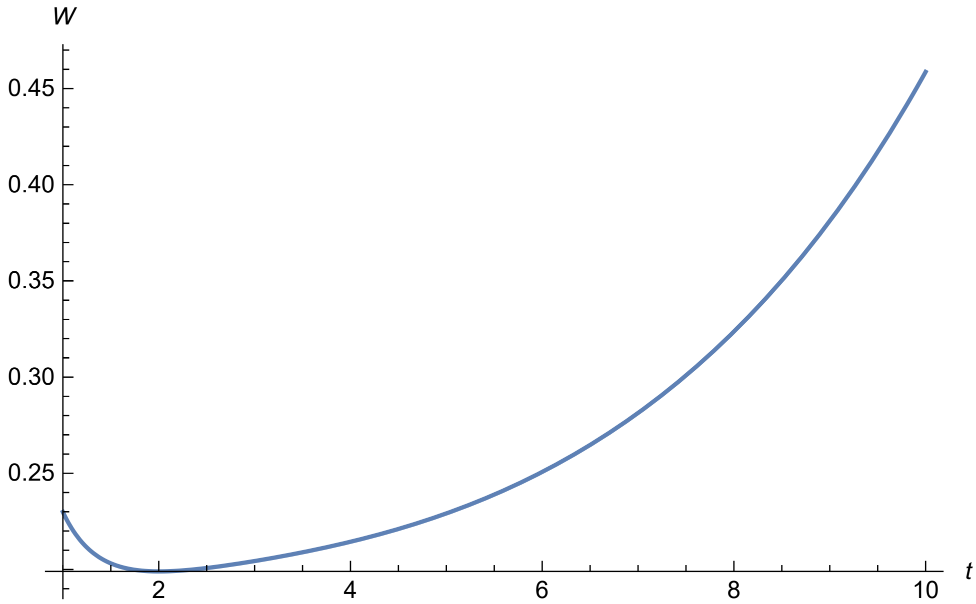


Figure 2.41: Overall welfare as function of t (with $\lambda = 0.1, \rho = 0.1, \tau = 1, T = 10$)

Figure (2.42) represents the comparable welfare graph for the benchmark model. Here, the overall shape is different: welfare increases when the search time t decreases. The negative impact of a reduction in the traditional search time t on the outcome of the online channel dominates the overall results.

A reduction in the search time in the online channel, τ, which had in parts conflicting effects, proves to have an overall positive impact on welfare (see Figure (2.41)). Last, a reduction in the delivery time is positive for overall welfare (see Figure (2.44)).
The welfare impact of the expansion to the online channel is not directly measurable, as it also depends on the level of the online search and delivery time. However, we can conclude that when the search time in the traditional channel is sufficiently distinct from the delivery time in the online channel, overall welfare is improved. This is due to a shift of welfare from the firms towards the consumers. It is especially noticeable that a far larger number of consumers has access to the market.

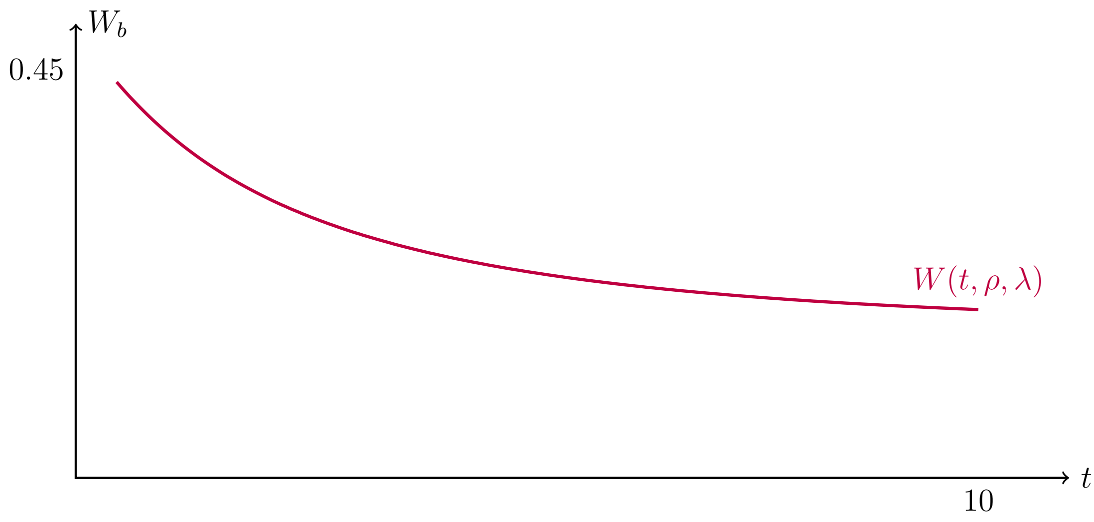


Figure 2.42: Welfare in the Benchmark ($\lambda = 0.1; \rho = 0.1; t \leqslant 10$)

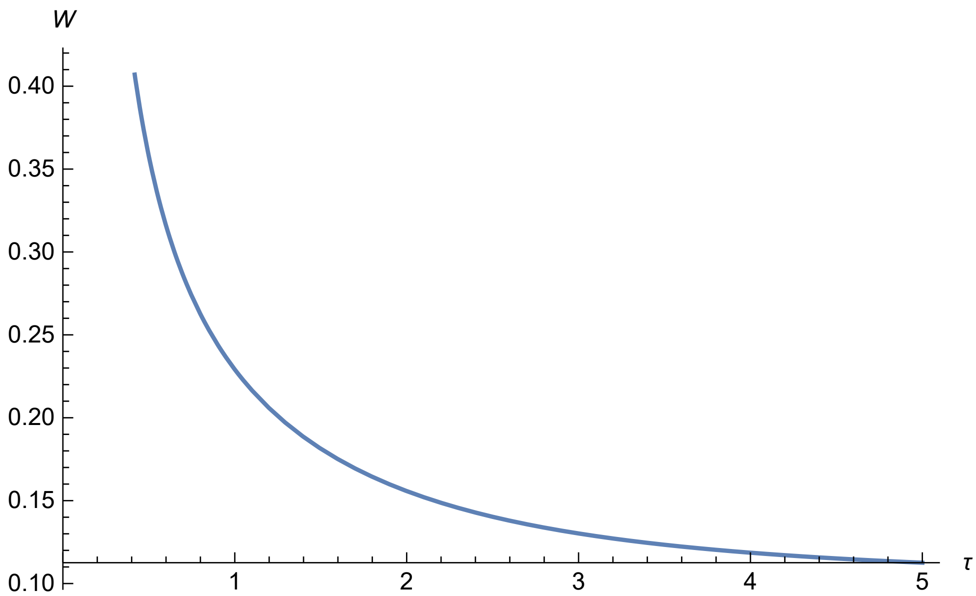


Figure 2.43: Overall welfare as function of τ (with $\lambda = 0.1, \rho = 0.1, t = 5, T = 10$)

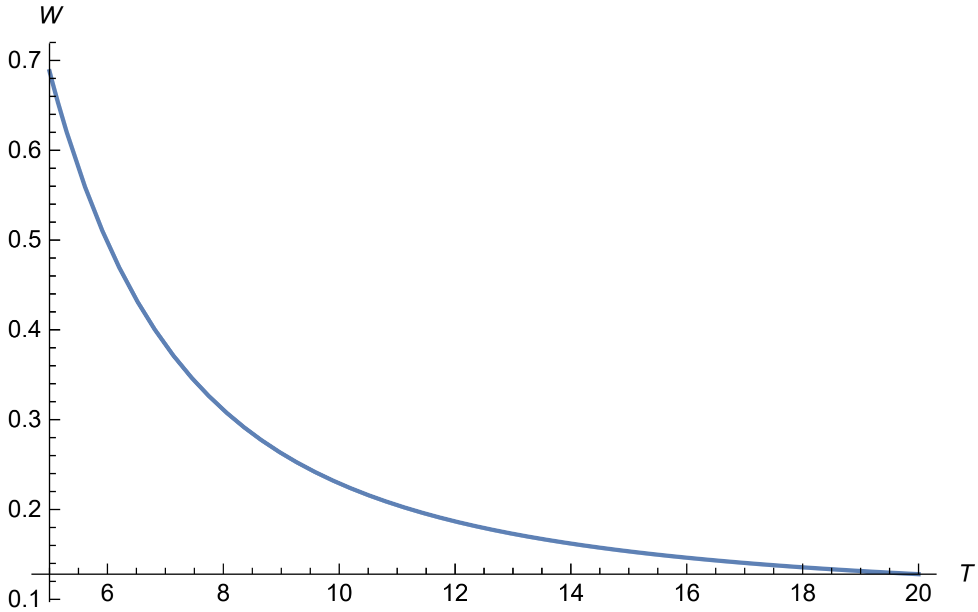


Figure 2.44: Overall welfare as function of T (with $\lambda = 0.1, \rho = 0.1, \tau = 1, t = 5$)

2.6 Conclusion: Impact of the Online Expansion

This model is in the tradition of classical search models. It is appropriate to describe markets where a large number of firms is potentially active in the traditional as well as in the online markets, and, in this regard, it is adequate to handle the COVID-19-lockdown situation mentioned in the opening of this chapter, when many firms had to switch at short notice and expand to the online channel. Even though it is not appropriate for describing phenomena of market power concentration in e-commerce, it accounts for many empirical observations.

First, it illustrates the persisting of price heterogeneity in the online channel. This heterogeneity is linked as well to the heterogeneity in unit costs as in the heterogeneity in reservation prices, which is the central element emphasized also in obfuscation models.

Second, it hints at the fact that the lower levels of unit costs in the online channel might not be inherent to that channel, but instead induced by a lower level of reservation prices.

Third, it outlines a strong stability of the traditional channel. Neither the level of prices nor the reservation prices of the consumers with the highest valuations are not influenced by the introduction of the online channel. This is in line with empirical results on the stability of the higher-price segments of markets, independently of the introduction of e-commerce.

Instead, the introduction of the online channel give access to the market to consumers with lower reservation prices, and enables consumers with intermediate valuations to get a higher surplus from the online channel, than they would get from the traditional one in the benchmark.

Consequently, the welfare impact of the introduction of the online channel is positive in this model. Welfare is increased, and partly shifted from the firms to the consumers. In this optimistic model, e-commerce is a chance for lesser endowed consumers and no existential threat for traditional commerce.

While this model focuses on searching behavior, daily experience leads to the question of who is actually searching whom. The rise of artificial intelligence and big data enables firms to track and target consumers in a wholly new dimension. The next chapter focuses therefore on the inverse question: what happens when firms search for consumers, instead of the other way around?

Chapter 3

Getting Searched

3.1 Introduction and Literature Review

E-commerce has initially raised the expectations that it would at last be the realization of the perfect competition commonly assumed in economic theory - efficient, frictionless markets under perfect information (Brynjolfsson and Smith, 2000). It was soon clear that the expectations about a lower level of prices, homogeneity in price, and higher price elasticity are at most only partially fulfilled (Smith et al., 2001). The question whether e-commerce is beneficial to consumers and/or to firms is still highly controversial.

Consumers acceptance of the online channel is often modeled in a self-evident way as an actual *reluctance against* the online channel. Distrust against a new, unfamiliar, not yet mature technology which is, on top, difficult to understand, is not surprising; just remember the quotation attributed to the German Kaiser Wilhelm II about the first cars: "I do believe in the horse. The automobile is no more than a transitory phenomenon". But while such form of distrust is natural in pioneer literature,[1] it is quite surprising to observe it also in later papers, as for example in the qualitative study in Kacen et al. (2013), where disadvantages as uncertainty about handling, exchange, authenticity or lacking the shopping experience and client counseling are said to overweight the benefits in term of variety of choice or lower price. The rise of e-commerce contradicts this posture: the overall increase in the volume of online sales, amounting to more than 4.25 trillion US dollars in 2020,[2] is speaking for itself: consumers adopt e-commerce. Estimations of considerable consumers' surplus from the Internet, such as those listed in Goldfarb and Tucker (2019) (section 8.4), suggests that consumers maximizing their utility when choosing the online channel.

[1] To give an example from a still frequently cited paper, consider Liang and Huang (1998): the formulation of the questionnaire in it feels nowadays biased against online commerce

[2] See `https://www.statista.com/statistics/379046/worldwide-retail-e-commerce-sales/`, visited 2021-12-12.

A major concern linked to the rise of e-commerce is that local stores would get "endangered by extinction", as put in Güsken et al. (2020). The authors formulated concerns that owner-managed retail stores would miss the skills and the knowledge necessary for the implementation of multichannel strategies, and would not be able to compete against the rising e-commerce. Expanding to the online channel is, however, no panacea, as this new channel might offset the old one: online success might cannibalize local sales (Bar-Gill and Reichman (2021)).

This concern of cannibalism is at the core of our analysis, but we must first delimit our perspective from neighboring topics. The eventuality of a cannibalistic behavior of firms has already been testified for the case when a producer introduces direct marketing additionally to its local retailer supply-chain (see e.g. Chiang et al. (2003), where a preference parameter for direct sales is introduced, which might be re-interpreted as preference for online sales; or Tsay and Agrawal (2004), where multiple channel strategies are considered): direct marketing might or might not be beneficial to the interested parties. However, we do not consider a chain of producer and retailer, but rather a firm established as a local store, considering the question of whether to adopt the online channel. The danger of cannibalistic behavior has also been mentioned with respect to information externalities: consumers might inspect a product in a local store, and browse subsequently the Internet for the best price, a behavior called *showrooming*. Inversely, consumers might get information about a product online (from experts or fellow consumers assessments) and buy in a local store, a counterpart called *webrooming* behavior, and that is none less widespread (Arora and Sahney, 2017). Both showrooming and webrooming are, basically, free-riding behaviors. Here again, depending on the level of search costs, information externalities might or might not be beneficial to the retailers. Webrooming has been put forward as an instrument to attract consumers in local stores (Kim et al., 2022), which leads us back to the topic of local stores as endangered species.

The importance accorded to information and search is put into perspective when questioning the actual searching behavior of consumers. It is now a well-known and widely discussed paradoxon that, while the online availability of information on products leads to lower search costs, consumers' search behavior remains lower than optimal (see e.g. Johnson et al. (2004) or the literature discussion in De los Santos et al. (2012)). The first explanatory factor for this phenomenon, basing on Stigler (1961), is consumers' impatience: because of their high time preference, the marginal costs of searching soon offset the discounted marginal benefit of searching. This approach still inspires a strand of literature examining the nature,[3] geographical development,[4] and branch specificity[5] of searching costs. A second, now central factor is the use of data analysis and technological opportunities to track, profile and target consumers, with the ultimate aim to present them with the products they want to buy before they even start to search, as formulated in the method for "anticipatory shipping" patented by Amazon.[6] It seems only legitimate to shift the focus: in front of the mass of product and retailers alternatives in the Internet, it is not (solely) the searching behavior of consumers which is decisive, but the attention seeking behavior of firms. E. Calvano and M. Polo outlined that getting consumers attention is no less than a way to relax competition: *"[m]any websites and apps are in the business of harvesting and reselling human attention. (...) New technologies that use data to profile users and follow them as they traverse the internet (e.g. cookies) scale down competition for attention at the individual level"* (Calvano and Polo, 2021).[7] In our model, we explore this perspective and turn the tables: we assume no searching behavior of consumers, but confer a determinant role to the visibility achieved by online retailers.

This perspective as well as our analysis methods are quite distant from the existing literature on the topic. The decision for the online or the local channel is at the core of a number of behavioral studies, resulting in less (Kollmann et al. (2012), Schröder and Zaharia (2008)) or more (Chang et al., 2005) complex typologies of consumers, accessorily of products and websites. Beyond these bevioral approaches, the existing literature on e-commerce is mainly quantitative - and indeed, digital technologies and the access to never ending sources of data about individual buying behavior are attractive not only to firms, but also to researchers. We complement these approaches with a purely theoretical one and examine the findings of behavioral or quantitative papers by developing a brand-new model. This model relies on an established working-horse: the Salop circle (Salop, 1979) is appropriate to capture competition among local stores. We add a second layer: above the circular city, there is a "cloud", where online retailers are gathered. The competitive feature most relevant to local stores is the distance to them; the competitive feature most relevant to online retailers is their visibility for consumers. We are not aware of any other model explicitly integrating virtual space in a location model, and modeling the relations-

[3] An interesting example is Dutta and Das (2017), which differentiates impatience by assessing the respective role of cognitive factors (education and internet experience) and monetary factors (income and internet costs).

[4] E.g.Taiwan in Liang and Huang (1998) or Beijing in Clemes et al. (2014).

[5] E.g. Park et al. (2009) for health information; Klein and Ford (2003) for the automobile branch; or Dutta and Das (2017) for laptops and mobile phones.

[6] Amazon Technologies Inc. Reno NV (2013), available under `https://patents.google.com/patent/US8615473B2/en` (visited 2021-12-12).

[7] As a complement, let us mention that this individual tracking even makes the eventuality of first-degree price discrimination less theoretical, as firms can even gather information on the individual willingness to pay.

hip between real and virtual stores with the classical instruments of microeconomics.

The structure of this analysis is as follows. After depicting the underlying assumptions (section 2), we summarize the benchmark situation when the online channel is inexistent (section 3). We then consider first consumers' (section 4), then firms' optimal decisions (section 5), and dive into the dynamics of the two possible constellations: the first one when some, but not all firms decide to become active as online retailers, which will prove to be a transitional state (section 6); and the second one when all firms are multichannel (section 7). We will then solve the model for the endogenous market structure, and compare the resulting welfare to that in the initial benchmark (section 8). Then, after a look at an interesting limiting case (section 9), we will conclude with a discussion of the question whether it is, or not, justified to fear a cannibalistic behavior of firms expanding to the online channel (section 10).

3.2 Model Assumptions

In this model, locally established firms consider the option of acting, additionally, as an online retailer.[8] The n_l local stores are distributed equidistantly around a Salop circular city of unit length, so that the distance between any two local stores is $1/n_l$.[9] Local stores bear symmetric fix costs f, which represent market entry costs, but, for simplification, no unit costs. Among the local stores, each firm is in competition only with its immediate neighbours. In the following, we will focus on a representative firm i located at x_i, which immediate neighbours are labelled $i-1$ and $i+1$, located respectively at $x_i - 1/n_l$ and $x_i + 1/n_l$, as depicted in Figure 3.1.

Above this circular city, in a "world-wide-web cloud" accessible equally to all consumers, there is a number $n_o \leq n_l$ of online retailers, corresponding to those, among the firms, who decided to participate actively in the online channel (each of them running one, and only one online retailer; we elude the question of whether this retailer is an actor in an e-commerce platform and / or an online store of its own). They compete with the local stores with respect to prices, and among each other with respect to their visibility. Online retailers can improve their visibility via advertising, tracking and profiling technologies, selling via many platforms and / or via the own website, search engine optimization etc.;

[8] For clarity, we will use the term of "store" only for local, brick-and-mortar store, while the term of "retailer" will be reserved for the online channel.

[9] In a symmetrical equilibrium, under our assumption of quadratically increasing transportation costs, the optimal distribution of firms around the Salop circle is the uniform one, as substantiated in Economides (1989). A full discussion of the optimality conditions can be found in Gong et al. (2016).

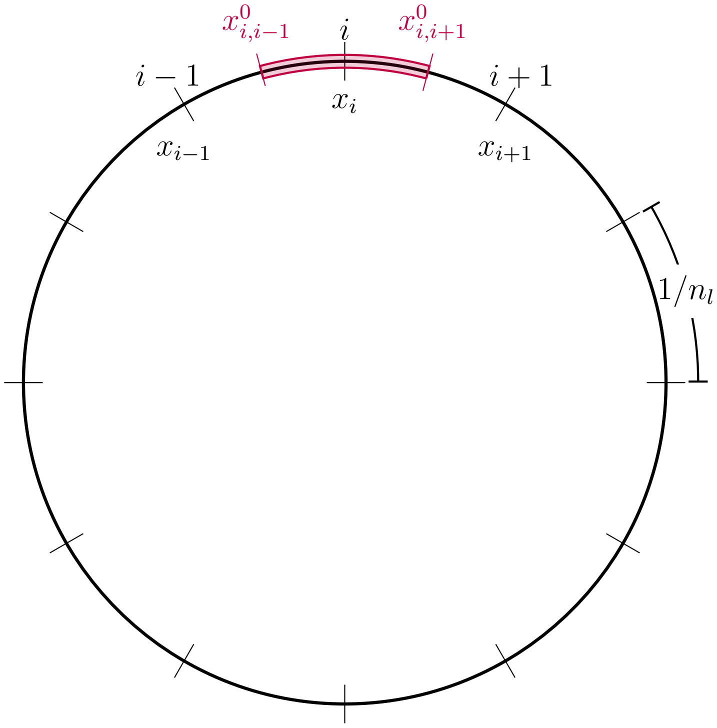


Figure 3.1: Local stores as a Salop circular city

these measures generate costs w_i. For simplicity, we consider that the unit costs are negligible, as in the local channel.

Consumers are distributed uniformly around the circle; their mass is normalized to one. Each consumer buys exactly one unit of a homogeneous good, whether in a local store or from an online retailer. We assume that their utility from consumption, and as a consequence their reservation price, is high enough for the market to be covered.

Consumers buying from a local store i incur, additionally to the mill price p_l^i, transportation costs increasing quadratically in the distance to the firm and weighted with the transportation parameter t. The overall costs of a consumer located at position x amount to $p_l^i + t(x - x_i)^2$. Consumers prefer the store for which this overall price is the lowest, which is either of the two stores located directly next to them. The consumers being indifferent between store i and each of its neighbors are located respectively at position $x^0_{i,i-1}$ and $x^0_{i,i+1}$, as represented on Figure 3.1. All the consumers in between will, if they decide to buy locally, opt for store i.

Alternatively, consumers can buy from an online retailer. Consumers reluctance against the online channel is captured by the parameter A, which represents all barriers to online consumption (distrust, delivery delay, lacking physical examination...) and is handled as an additional cost parameter for consumers. A can be considered as a technology parameter that decreases as the usability of e-commerce improves, i.e. as the related state-of-the-art technologies get safer, more efficient and more enjoyable. These technologies are varied: content management systems for the creation of user-friendly online shops; diversification of the selling platforms; safe payment systems; efficient transaction processing and shipping facilities; or personalized product recommendations.[10] For simplicity, we handle these various aspects as one index, which is symmetric among firms. When buying from an online retailer i, consumers bear costs equal to the sum of A and the retailer's price p_o^i. As depicted above, we don't assume any searching behavior of consumers: they pick one online retailer at random with a probability linked to its online visibility, and compare it to their preferred brick-and-mortar store.

In the following, we will first consider the benchmark situation in which no firm is active online, i.e. with only local stores. We will then turn successively to the optimal decisions of consumers and of firms, for any number of online retailers $1 \leq n_o \leq n_l$, and analyze in more details both the asymmetric case (where single-channel and multichannel firms coexist) and the symmetric one (where all firms are multi-channel). In the next step, we will endogenize the structure of the markets (i.e. the number of firms in each channel) and characterize the outcome in terms of welfare. The model beeing then extensively solved, we will turn back to our initial question: are firms expanding to the online channel harming their local retail in a cannibalistic way?

3.3 Initial Benchmark: Inexistence of Online Shopping

In the initial benchmark situation, there is no online retailer; firms are running only their local stores. It is a standard Salop model with n_l firms.

3.3.1 Benchmark Equilibrium

A consumer is indifferent between buying from store i located at x_i, or from its neighbour $i+1$ at x_i+1/n_l, if she is located at the position $x^0_{i,i+1}$ defined by the indifference condition:

$$p^i + t(x^0_{i,i+1} - x_i)^2 = p^{i+1} + t(x^0_{i,i+1} - x_i - 1/n_l)^2$$

This condition yields the following solution:

$$x^0_{i,i+1} = x_i + \frac{1}{2n_l} - \frac{n_l}{2t} \cdot \left(p^i - p^{i+1}\right)$$

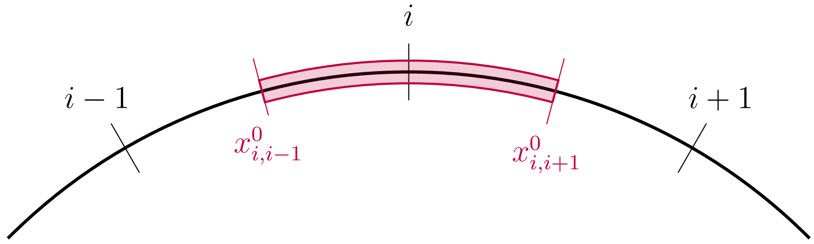


Figure 3.2: Benchmark demand function for firm i

All consumers located between x_i and $x^0_{i,i+1}$ will prefer store i to store $i+1$. The corresponding area of potential demand is marked in purple in Figure 3.2. We use here the cautious qualification of "potential", because the consumers in this region still have to compare store i to the online channel and will separate in local consumption (at store i) and online consumption. Similar considerations for the choice between store i and its neighbor on the other side, firm $i-1$, lead to the following demand function for firm i:

$$D_i(p^i, p^{i-1}, p^{i+1}) = \frac{1}{n_l} - \frac{n_l}{2t} \cdot (2p^i - p^{i+1} - p^{i-1})$$

The maximization of the profits:

$$\pi^i = p^i \cdot D_i(p^i, p^{i-1}, p^{i+1}) - f$$

with respect to the price leads to the equilibrium prices:

$$p^* = t/n_l^2$$

and to an equilibrium distribution of the consumers in equal shares among the firms. The equilibrium profits read:

$$\pi^* = t/n_l^3 - f$$

which yields the endogenous structure of the market: under the condition of free market entry, the number of firms in the market is the maximum number for which the equilibrium profits are still positive. As this number is an integer, we have to use a floor function:

$$n_l^* = \lfloor \sqrt[3]{t/f} \rfloor \tag{3.1}$$

Remark that when the transport costs parameter tends to a negligible value, we are in the case of a polypoly under perfect competition.

[10] An overview of the different aspects of digital economics can be found in Goldfarb and Tucker (2019), p.3 sq.

3.3.2 Welfare in the Benchmark Case

In equilibrium, welfare Wel is driven by consumers' surpluses: with free market entry, firms expect zero profit. Consumers' surpluses CS depend on the sum of mill price and transportation costs. Be $\overline{u}$ the individual gross utility from consumption:

$$CS = \sum_{i=1}^{n_l} \int_{x_{i-1}}^{x_{i+1}} \overline{u} - p^i - t(x - x_i)^2 \; dx$$

In a symmetric pattern, all consumers located at a distance of maximally $1/(2n_l)$ from any firm will buy from this firm. Normalizing their coordinates from 0 to $1/n_l$ and the location of the firm to $1/(2n_l)$, we compute the value of consumer surplus in symmetry:

$$CS^* = n_l \cdot \int_0^{1/n_l} \left(\overline{u} - \frac{t}{n_l^2} - t \left[x - \frac{1}{2n_l} \right]^2 \right) \; dx = \overline{u} - \frac{13}{12} \cdot \frac{t}{n_l^2}$$

Consumer surplus declines as the transportation costs parameter t increases, i.e. as transportation gets more costly, and as the number of firms decreases, i.e. as the average distance a consumer has to overcome increases. Inserting the endogenous market structure leads to the equilibrium benchmark welfare:

$$Wel^* = \overline{u} - \frac{13}{12} \cdot \sqrt[3]{tf^2}$$

and the stability of the market is given for any individual gross utility $\overline{u}$ above $\frac{13}{12} \cdot \sqrt[3]{tf^2}$.

Welfare is negatively influenced as well by transportation costs t as by fix costs f, whereby the impact of this last exogenous parameter is stronger and there is an interdependence between the two parameters, as illustrated in Figure (3.3).

3.4 Consumers' Decision

Consumers located between $x^0_{i,i-1}$ and $x^0_{i,i+1}$ compare the costs of buying from the local store i or from one given online retailer λ, chosen with probability w_λ/W, where W is the sum of the visibility parameters of all stores:

$$W = \sum_{k=1}^{n_o} w_k$$

We focus on the case when no channel is absolutely dominating the other one, i.e. when there is, for the consumers, an actual trade-off between the two channels, depending on their location.

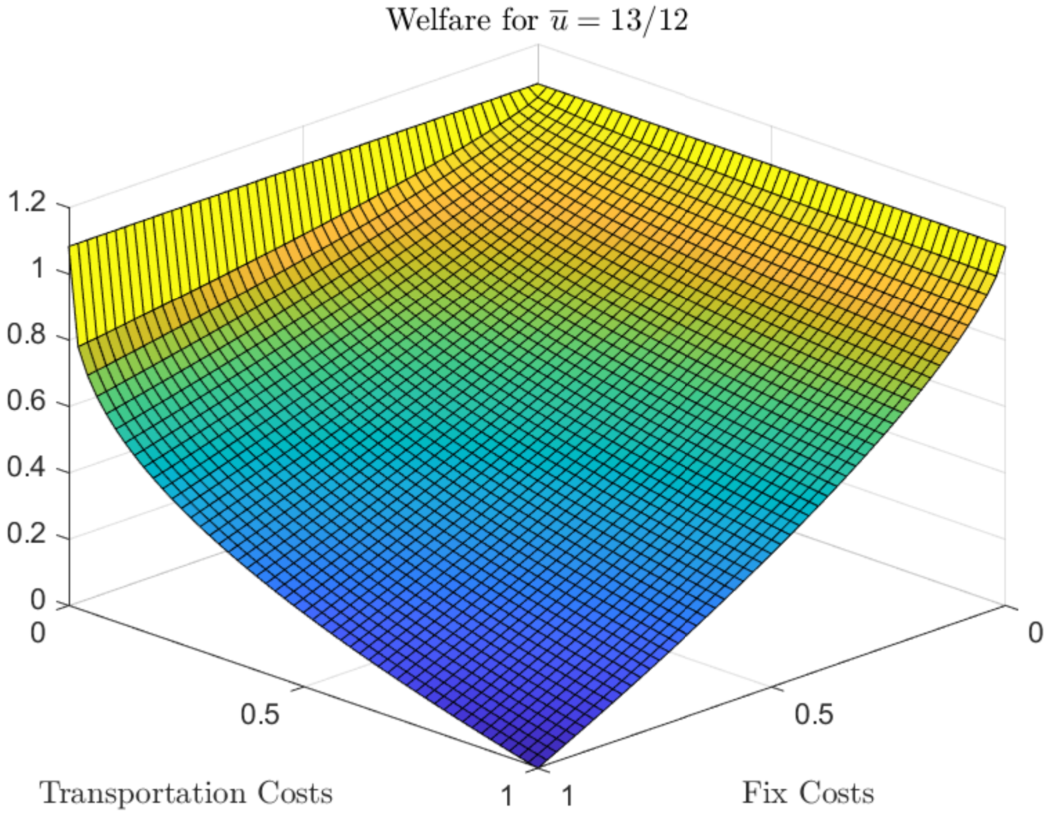


Figure 3.3: Welfare in the benchmark equilibrium

3.4.1 Demand addressed to online retailers

Online retailers potentially address all consumers; their scope is limited only by their price setting and their relative visibility. Let us consider a consumer located at x, whose preferred local store is the store i, located at x_i and offering the price p_l^i.[11] We consider the deliberation of this consumer whether to buy from the local store i or online. She looks up the online retailer λ offering price p_o^λ with probability w_λ/W, compares the overall costs to those in the local store i, and decides to buy online if the costs are lower, i.e. if:

$$A + p_o^\lambda \quad \leq \quad p_l^i + t(x_i - x)^2 \ .$$

The further away the local store, the more plausible it is that the consumer will turn to e-commerce. The decision can be reformulated depending on the distance: the consumer prefers to buy online if:

$$|x_i - x| \geq \sqrt{\frac{A + p_o^\lambda - p_l^i}{t}} \tag{3.2}$$

[11] In this section, we consider prices both from a local store and from an online retailer; to distinguish them clearly, we introduce a subscript referring to the channel, o for online and l for local.

Those consumers located at some distance of the local stores will rather turn to online retailers. Such consumers are located around the point of indifference between two local stores.

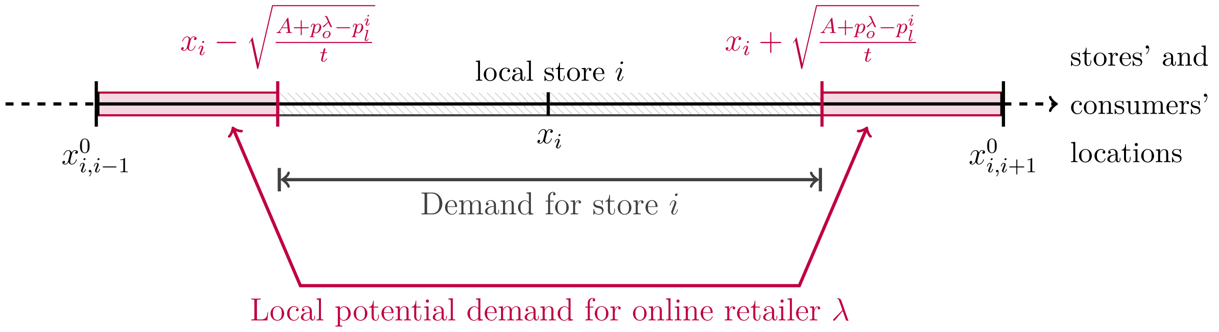


Figure 3.4: Potential demand for the online retailer i around local store λ

Figure 3.4 illustrates in purple the area around the local store i in which consumers, if they pick the online retailer λ (an event with probability w_λ/W), will prefer to buy from this online retailer rather than from i. Here again, because of this condition, we speak from a "potential" demand. This figure illustrates only the competition between one of the local stores and one of the online retailers; the situation is similar for any pair of local and online retailers. The purple area capturing this potential demand amounts to:

$$|x^0_{i,i+1} - x^0_{i,i-1}| - 2\sqrt{\frac{A + p_o^\lambda - p_l^i}{t}} \tag{3.3}$$

The potential demand for the representative online retailer λ around any of the n_l local stores is computed in a similar way. The sum of these local potential demand areas over the complete Salop circle, weighted with the probability w_λ/W that λ gets indeed picked by the consumers in these areas, yields the expected demand for the online retailer λ:

$$\mathrm{E}D_o^\lambda = \sum_{i=1}^{n_l}\left(\frac{w_\lambda}{W}\cdot\left[|x^0_{i,i+1} - x^0_{i,i-1}| - 2\sqrt{\frac{A + p_o^\lambda - p_l^i}{t}}\right]\right)$$

As long as the value in the square brackets is comprised between 0 and $|x^0_{i,i+1}-x^0_{i,i-1}|$, there is no overlapping between the different demand areas and consumers' preferences about where to buy are transitive. The sum of all distances between the indifferent consumers is the circumference of the Salop circle: 1. The correct formulation reads therefore:

$$\mathrm{E}D_o^\lambda = \frac{w_\lambda}{W}\cdot\left[1 - 2\sum_{i=1}^{n_l}\sqrt{\frac{A + p_o^\lambda - p_l^i}{t}}\right] \tag{3.4}$$

$$s.t.:\ \forall\lambda, i,\ 0 \leq 2\sqrt{\frac{A + p_o^\lambda - p_l^i}{t}} \leq |x^0_{i,i+1} - x^0_{i,i-1}|$$

The restriction on the expected demand is considered implicitly in a first step. The explicit formulation of these restrictions in equilibrium is the topic of appendix B and will be considered in due place.

3.4.2 Demand addressed to local stores

The expected demand for the local store i, which arises in the region between the two indifferent consumers $[x^0_{i,i-1}; x^0_{i,i+1}]$, corresponds to the number of consumers for whom the costs from buying online are higher. A consumer located at $x \in [x^0_{i,i-1}; x^0_{i,i+1}]$ will compare the local store with some online retailer λ, chosen with probability w_λ/W, and buy from the local store if:

$$p^i_l + t(x_i - x)^2 \leq A + p^\lambda_o \quad \Leftrightarrow \quad p^i_l + t(x_i - x)^2 - A \leq p^\lambda_o$$

Be $F(p^\lambda_o)$ the distribution of the prices in the online retailers. The preceding inequality is fulfilled with frequency $1 - F[p^i_l + t(x_i - x)^2 - A]$, i.e. F can be linked to the probability that a consumer buys online. Aggregating over all eligible online retailers, weighted with the corresponding probabilities, we can deduce the pointwise expected demand at x:

$$\mathrm{E}^{point} D^i_l(x, p^i_l) = \sum_{\lambda=1}^{n_o} \left[\frac{w_\lambda}{W} \cdot \Big(1 - F[p^i_l + t(x_i - x)^2 - A]\Big) \right]$$

The expected demand function for firm i results from the integration of this pointwise expected demand over all consumers locations between the two indifferent consumers:

$$\mathrm{E}D^i_l(p^i_l) = \int_{x^0_{i,i-1}}^{x^0_{i,i+1}} \left[1 - \sum_{\lambda=1}^{n_o} \left(\frac{w_\lambda}{W} \cdot F[p^i_l + t(x_i - x)^2 - A] \right) \right] dx \tag{3.5}$$

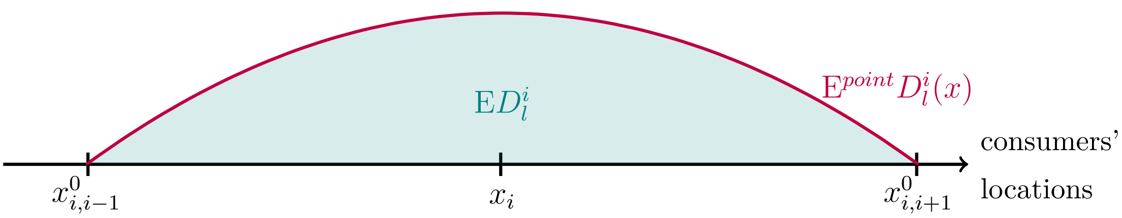


Figure 3.5: Expected demand function for the local store i

Figure 3.5 represents the evolution of the locally expected demand for a local store i: as the distance between the consumers and the firm increases, the probability that they turn to this store i decreases.

The expected demand function (3.5) can be simplified: as the prices set by the different online retailers are not random but known to the local store, the distribution function simplifies into a Heaviside step function for each online retailer. If the distance to the local firm is below the critical value computed in the precedent part, $\sqrt{(A + p^o_\lambda - p^i_l)/t}$, the cumulated probability that the costs of buying from the online retailer are lower is constantly zero; above it, it is constantly one. We focus here, again, upon a representative local store i and some online retailer λ. We introduce the tie-break rule assumption that a consumer located exactly at this critical distance from the firm, and being indifferent between purchasing online or in a local store, will choose the latter.

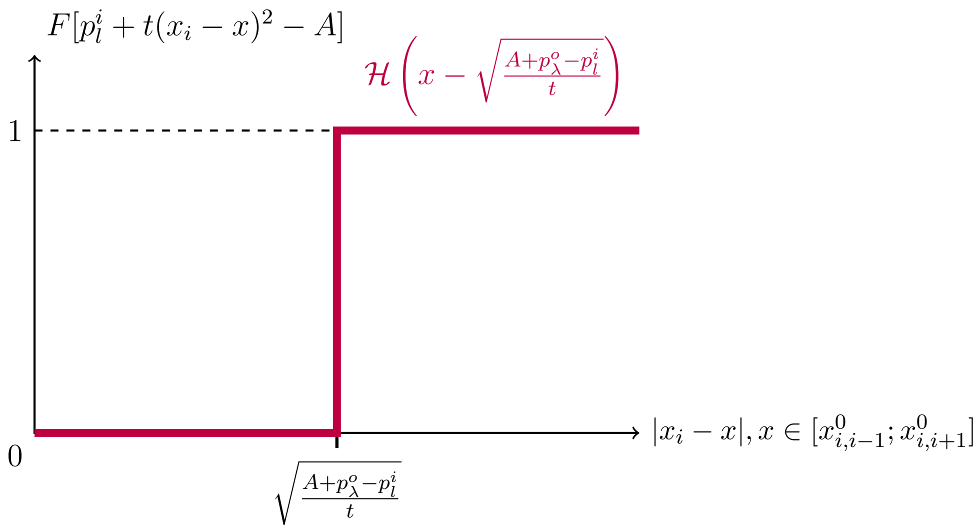


Figure 3.6: Heaviside shape of $F[p^i_l + t(x_i - x)^2 - A]$

This Heaviside function is illustrated in Figure 3.6: at locations close to the local store i, consumers do not buy online; at locations beyond the critical value, they always do.

Equation (3.5) can be transformed into (see appendix A on p. 151):

$$\mathrm{E}D^i_l(p^i_l, p^1_o, p^2_o, ..., p^{n_o}_o) = \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot 2\sqrt{\frac{A + p^\lambda_o - p^i_l}{t}} \right) \tag{3.6}$$

Remark that local stores are competing directly with the online channel; the price decisions or the neighboring local stores have no direct impact on their expected demand functions (but, via the location of the consumers indifferent between two local stores, they have an impact on the conditions for non overlapping of the different demand areas, as computed in appendix B). Depending on the exogenous parameters determining the transportation costs, t, and the reluctance against the online channel, A, it might be less or more attractive for firms to be active as online retailers. The decision of firms is topic of the next section.

3.5 Firms' Decision

In our model, established local firms can decide to expand to the online channel, so that the number of local stores is higher than or equal to the number of online retailers: $n_l \geq n_o$. A firm running only a local store will compete with similar, single-channel firms and with multichannel firms running both channels, but there is no firm running only an online retailer. We denote all considered variables with a subscript referring to the type of the referred firm (s for single channel, m for multichannel). For multichannel firms, a further subscript refers to the channel (it is unchanged: o for online, l for local) if a disambiguation is necessary (i.e. for prices, expected demand and channel-specific expected profits, but not, for example, for the visibility expenditure, as they are in essence specific to the online channel).

We further adapt the notation to the perspective of firms. In the preceding section, λ referred to any online retailer, because consumers do not consider whether the two shops they look up, local and online, belong to the same firm. This consideration is, however, of central interest from the point of view of the firm. While the undifferentiated λ avoided redundancy, we now need to differentiate λ into superscripts i and j to account for the perspective of multichannel firms. In each channel, a given, multichannel firm i is confronted to the competition by the other channel, where one shop belongs to the same firm (index i) and the others to rivals (index j).[12]

3.5.1 Optimization by a single-channel firm

Be i a representative firm running only a local store, in which it sets price p_s^i and attracts the expected demand $\mathrm{E}D_s^i$; it has a one-part expected profit function:

$$\mathrm{E}\pi_s^i(p_s^i) = p_s^i \cdot \mathrm{E}D_s^i(p_s^i) - f$$

The individual channel specific demand function $\mathrm{E}D_s^i$ stems from equation (3.6). As the online retailers are symmetric, all run by multichannel firms, they will, in equilibrium, set the same prices $p_{m,o}^*$. We anticipate this symmetry without any distorting consequence

[12] As this taxonomy with the different index pairs i, j, s, m and l, o becomes quite long, we avoid redundancy whenever possible. For example, the symmetric fix costs in the local channel are denoted only as f; the visibility costs incurring only in the online channel, as w^i, w^j.

for the optimization of the single-channel firm, and reformulate its demand function as:

$$\mathrm{E}D_s^i(p_s^i) = 2\sqrt{\frac{A + p^*_{m,o} - p_s^i}{t}}$$

The maximization of the profit function with respect to the price set in the local store leads to the price reaction function:

$$p_s(p^*_{m,o}) = \frac{2(A + p^*_{m,o})}{3} \tag{3.7}$$

As the presence of online retailers makes the competition fiercer, we expect this price to be lower than the benchmark price $p^* = t/n_l^2$, for a similar number of local stores.

The optimal price does not depend on the number or prices of other local stores, as each local store competes directly only with online retailers. As mentioned above, the impact of their neighboring stores becomes visible only in the restrictions on the equilibrium results (no infringement of the different areas of expected demand).[13] Whether the neighboring local stores belong to single or multichannel firms will, indeed, have an impact on their respective price decisions and on these restrictions; but the neighbors' price do not show up in the formula for the optimal decision of our representative single-channel firm.

The single-channel optimal demand and profit functions of a single channel firm, depending on the online price $p^*_{m,o}$, can be expressed as:

$$\mathrm{E}D_s^*(p^*_{m,o}) = 2\sqrt{\frac{A + p^*_{m,o}}{3t}} \tag{3.8}$$

$$\mathrm{E}\pi_s^*(p^*_{m,o}) = \frac{4(A + p^*_{m,o})}{3} \cdot \sqrt{\frac{A + p^*_{m,o}}{3t}} - f \tag{3.9}$$

This profit is non-negative if the online price is above a critical threshold:

$$p^*_{m,o} \geq \underline{\underline{p^*_{m,o}}} \qquad \text{where} \qquad \underline{\underline{p^*_{m,o}}} = 3 \cdot t^{1/3} \cdot (f/4)^{2/3} - A \tag{3.10}$$

The value $\underline{\underline{p^*_{m,o}}}$ is the lowest price for which single-channel can participate in the market. In case this value is negative, it is not realizable as a price. In case it is positive and the optimal price of online retailers $p^*_{m,o}$ is lower, single-channel firms cannot participate in the market: the underlying assumptions on the market structure and on the exogenous parameters A, f, t do not lead to a stable equilibrium.

[13] See appendix B, p. 153.

3.5.2 Optimization by a multichannel firm

The expected profit function of a representative multichannel firm i is the sum of the profits in both channels:

$$\mathrm{E}\pi_m^i(p_{m,l}^i, p_{m,o}^i, w^i) = p_{m,l}^i \cdot \mathrm{E}D_{m,l}^i(p_{m,l}^i, p_{m,o}^i, w^i) - f + p_{m,o}^i \cdot \mathrm{E}D_{m,o}^i(p_{m,l}^i, p_{m,o}^i, w^i) - w^i$$

For the sake of keeping the notation simple, we use the fact that the optimal decisions taken by the $n_o - 1$ multichannel rivals on prices and visibility expenditure are symmetric, and denote them as $p_{m,l}^*$; $p_{m,o}^*$ and w^*; while the symmetric, optimal price decisions taken by the $n_l - n_o$ single channel rivals are denoted p_s^*. We thus reformulate the demand function in the local channel (3.6) as:

$$\mathrm{E}D_{m,l}^i(p_{m,l}^i, p_{m,o}^i, w^i) = \frac{2 \cdot \left[w^i \sqrt{\frac{A + p_{m,o}^i - p_{m,l}^i}{t}} + (n_o - 1)w^* \sqrt{\frac{A + p_{m,o}^* - p_{m,l}^i}{t}} \right]}{w^i + (n_o - 1)w^*}$$

With probability w^i/W, where $W = w^i + (n_o - 1)w^*$, a representative local store of a multichannel firm is confronted to the firm's own online retailer; in such a case it gathers the expected demand $2\sqrt{(A + p_{m,o}^i - p_{m,l}^i)/t}$. With the complementary probability $(n_o - 1)w^*/W$ it is confronted to a rival's online retailer, and attracts the expected demand $2\sqrt{(A + p_{m,o}^* - p_{m,l}^i)/t}$.

For the online channel, we reformulate the demand function (3.4) as:

$$\mathrm{E}D_{m,o}^i(p_{m,l}^i, p_{m,o}^i, w^i) = \frac{w^i \left[1 - 2\sqrt{\frac{A + p_{m,o}^i - p_{m,l}^i}{t}} - 2(n_o - 1)\sqrt{\frac{A + p_{m,o}^i - p_{m,l}^*}{t}} - 2(n_l - n_o)\sqrt{\frac{A + p_{m,o}^i - p_s^*}{t}} \right]}{w^i + (n_o - 1)w^*}$$

A representative online retailer of a multichannel firm attracts an expected share w^i/W of the demand that has not been attracted by any of the local stores, therefore the subtractive shape. The different terms subtracted in the numerator correspond to the different types of local stores: firm i's own local store (price $p_{m,l}^i$); the stores of the $n_o - 1$ other multichannel firms (price $p_{m,l}^*$), and at last the stores of $n_l - n_o$ single-channel firms (price p_s^*).

These expected demand functions of individual stores are increasing in the price of the other channel. This is consistent with the existence of price competition between the channels. And here again, the prices of the competitors in the same channel prove to have no impact.

Firm i maximizes simultaneously with respect to the local price $p_{m,l}^i$, to the online price $p_{m,o}^i$ and to the expenditure for visibility, w^i. Inserting the price of the single channel firms (3.7) into the system of first order conditions leads to the results:

$$\frac{\partial \mathrm{E}\pi_m^i}{\partial p_{m,l}^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_{m,l}^* = \frac{2(A + p_{m,o}^*)}{3} + \frac{p_{m,o}^*}{3n_o} \tag{3.11}$$

$$\frac{\partial \mathrm{E}\pi_m^i}{\partial p_{m,o}^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad \sqrt{t} = \frac{\left[2A + 5p_{m,o}^*\right](n_l - n_o)}{\sqrt{3}\sqrt{A + p_{m,o}^*}} + \frac{(2A + 5p_{m,o}^* + p_{m,o}^*/n_o) \cdot (n_o - 1)}{\sqrt{3}\sqrt{A + p_{m,o}^* - p_{m,o}^*/n_o}} \tag{3.12}$$

$$\frac{\partial \mathrm{E}\pi_m^i}{\partial w^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad w^* = \frac{n_o - 1}{n_o} \cdot p_{m,o}^* \left[\frac{1 - 2(n_l - n_o)\sqrt{\frac{A+p_{m,o}^*}{3t}} - 2n_o\sqrt{\frac{A+p_{m,o}^*}{3t} - \frac{p_{m,o}^*}{3n_o t}}}{n_o}\right] \tag{3.13}$$

In the reaction function (3.11), the positive impact of the competitors' prices on the own price, typical for price competition, becomes obvious. The price set in the local channel by multichannel firms is evidently higher than the one set by single-channel firms (compare (3.11) to (3.7): $\frac{2(A+p_{m,o}^*)}{3} + \frac{p_{m,o}^*}{3n_o} \geq \frac{2(A+p_{m,o}^*)}{3}$): the diversification of sale channels enables firms to set higher prices.

Equation (3.12) has no simple explicit solution, but can be approximated numerically.

Last, the expression in square brackets in the formula for w^* corresponds to the equilibrium expected demand for an online retailer. Through the multiplication with the price, w^* is directly indexed to the expected profits: the share of profits spent on visibility expenditure is considerable, as it is equal to the share of rivals in the online market ($[n_o - 1]/n_o$).

In the next step, we compute the equilibrium performance of multichannel firms and test the non-negativity conditions on the resulting prices, expected demands and profits.

The equilibrium price (3.11) set by multichannel firms in the local channel is obviously positive for any non-negative value of their equilibrium price in the online channel. The implicit solution for the price in the online channel hinders an exact computation of non-negativity conditions, but a flash-forward to the comparative statics below reveals that this non-negativity condition might be effectively binding in case the reluctance parameter A is very high[14] or the transportation costs t are very low[15]. This is quite intuitive: in constellations conferring a comparative advantage to the local channel, multichannel firms have an incentive to lower the prices of their online retailers and the non-negativity might become binding. In the extreme, the online channel collapses and the market structure is as described in the initial benchmark case above. An explicit solution of this non-negativity condition is possible in the parity case, when all firms are multichannel firms

[14]See Figure 3.10 on p. 74 for a numerical illustration and appendix D, p. 162 for the mathematical proof.

[15]See Figure 3.18, p. 81 for a numerical illustration and appendix D.2, p. 170 for the mathematical proof.

(see below, solution (3.26) on p. 88).

The equilibrium expected demand functions simplify as:

$$\mathrm{E}D^*_{m,l}(p^*_{m,o}) = 2\sqrt{\frac{A + p^*_{m,o}}{3t} - \frac{p^*_{m,o}}{3n_o t}} \tag{3.14}$$

$$\mathrm{E}D^*_{m,o}(p^*_{m,o}) = \frac{1}{n_o} \cdot \left[1 - 2(n_l - n_o)\sqrt{\frac{A + p^*_{m,o}}{3t}} - 2n_o\sqrt{\frac{A + p^*_{m,o}}{3t} - \frac{p^*_{m,o}}{3n_o t}}\right] . \tag{3.15}$$

Again, it is quite straightforward that the non-negativity condition is not binding in the local channel.[16] The expected demand function in the online channel declines with the equilibrium price $p^*_{m,o}$. We can compute a value from which on it turns negative.[17] However, this global non-negativity condition hides heterogenous situations, as areas of potential demand for a given online shop might be delimited by the demand areas of multichannel and / or of single-channel local stores. The global non-negativity conditions does not account for the fact that the areas of potential demand for an online shop are broader when confronted to multichannel firms, who set a higher price in their local store, and smaller when confronted to single-channel stores.[18] This is the point when the restriction in (3.4) kicks in. For the expected demand for an online retailer, we have to consider, instead of a unique, global non-negativity condition, a bundle of non-negativity conditions, some of them being more restrictive than the others. This is done in appendix B, from p. 153 on and yields the necessary condition (B.3):

$$p^*_{m,o} \leq \frac{6n_o^2 t}{n_l^2} - \frac{2\sqrt{3}n_o\sqrt{An_l^2 t + 3t^2(n_o^2 - 1)}}{n_l^2}$$

as well as the two following configuration-specific conditions, depending on the relative number of multichannel local stores and their distribution among the single channel ones. (B.1) applies as soon as two single-channel stores are neighbors:

$$p^*_{m,o} \leq \frac{3}{4n_l^2} - A$$

[16]For any number of online retailers $n_o \geq 1$, the factor $1 - 1/n_o$ is positive and the expression in (3.14) in the square root $\frac{A+(1-1/n_0)p^*_{m,o}}{3t}$ is strictly positive.

[17]This value $\overline{p^*_{m,o}}$ reads:

$$\overline{p^*_{m,o}} = \frac{4An_l(2n_o - n_l) + 3t}{4\left(n_o - 2n_o n_l + n_l^2\right)} + \frac{3n_o(n_o-1)t - \sqrt{3}(n_l - n_o)\sqrt{n_o t\left(4A\left(n_o - 2n_o n_l + n_l^2\right) + 3(n_o - 1)t\right)}}{2\left(n_o - 2n_o n_l + n_l^2\right)^2}$$

Remark that the denominator is not problematic, as there is no solution to $n_o - 2n_o n_l + n_l^2 = 0$ over the integers when n_l is at least 2.

[18] There is no such heterogeneity when looking at the expected demand of local store, whether multi-channel or single-channel, as it is confronted to symmetric online retailers: all areas of expected demand are symmetric.

and the less restrictive (B.2) applies as soon as two multichannel stores are neighbors:

$$p^*_{m,o} \leq \left[\frac{3}{4n_l^2} - A\right] \cdot \frac{n_o}{n_o - 1}$$

These conditions will be especially helpful later on, when computing the sign of derivatives for the comparative statics. We procede with the expected profits of a multichannel firm in each channel, in equilibrium.

The equilibrium expected demand function (3.14) yields the reformulation for the expected profits in the local channel:

$$\mathrm{E}\pi^*_{m,l} = \left[\frac{2(A + p^*_{m,o})}{3} + \frac{p^*_{m,o}}{3n_o}\right] \cdot 2\sqrt{\frac{A + p^*_{m,o}}{3t} - \frac{p^*_{m,o}}{3n_o t}} - f \tag{3.16}$$

These equilibrium profits are always lower than those in the single-channel local store (3.9) (see appendix C, p. 159, for the exhaustive proof). This difference in profits might explain the fear for a cannibalistic behavior of firms: it supports the observation that digitization leads to lower profits in the local stores. Focusing only at the performance in local stores is, however, only part of the story: to evaluate whether there is a phenomenon of cannibalization, we need to look at the overall results. The difference in profits in the local channel hints indeed at the effect of the coordination between the two channels by multichannel firms: they renounce to a share of profits in the local channel, in order to enhance their performance in the online one.

The expected profits in 3.16 are increasing in the online price. The corresponding gross profits, before deduction of the fix costs f, are positive. The minimal level of fix costs for which the expected profits 3.16 might turn negative is:

$$\underline{f} = \frac{4A}{3} \cdot \sqrt{\frac{A}{3t}}$$

For higher fix costs, the expected profits in the online channel will increase from negative to positive values as the online price $p^*_{m,o}$ increases, as depicted below in Figure 3.7. The price from which on the expected profits in the local channel are positive reads:

$$\underline{p^*_{m,o}} = \frac{n_o\,(4A^2 + 2A(1 - 2n_o)S + S^2)}{2(n_o - 1)(2n_o + 1)S} \qquad \text{where}$$

$$S = \sqrt[3]{3f(n_o-1)\left(\sqrt{3t}\sqrt{(2n_o+1)\,(27f^2t(n_o-1)^2(2n_o+1) - 16A^3)} + 9ft(2n_o+1)(n_o-1)\right) - 8A^3}$$

Remark that for $f \geq \underline{f}$, the help variable S is always real and positive.

We now turn to the online channel. The equilibrium expected profits of multichannel firms in this channel amount to:

$$\mathrm{E}\pi^*_{m,o} = p^*_{m,o} \cdot \frac{1}{n_o^2} \cdot \left[1 - 2(n_l - n_o)\sqrt{\frac{A + p^*_{m,o}}{3t}} - 2n_o\sqrt{\frac{A + p^*_{m,o}}{3t} - \frac{p^*_{m,o}}{3n_o t}}\right] \tag{3.17}$$

The expected profits in the online channel are equal to a share of the visibility expenditures: $\mathrm{E}\pi^*_{m,o} = w^*/(n_o - 1)$. This implies that each online retailer spends as much in visibility expenditures on each online competitor, as it gathers profits. The expected online profits are concave in the equilibrium online price $p^*_{m,o}$, as depicted in Figure 3.7. Excluding negative prices, the non-negativity condition for these profits is driven by the sign of the demand, as previously analyzed; the expected profits in the online channel are, like the expected demand, positive below $\overline{p^*_{m,o}}$.

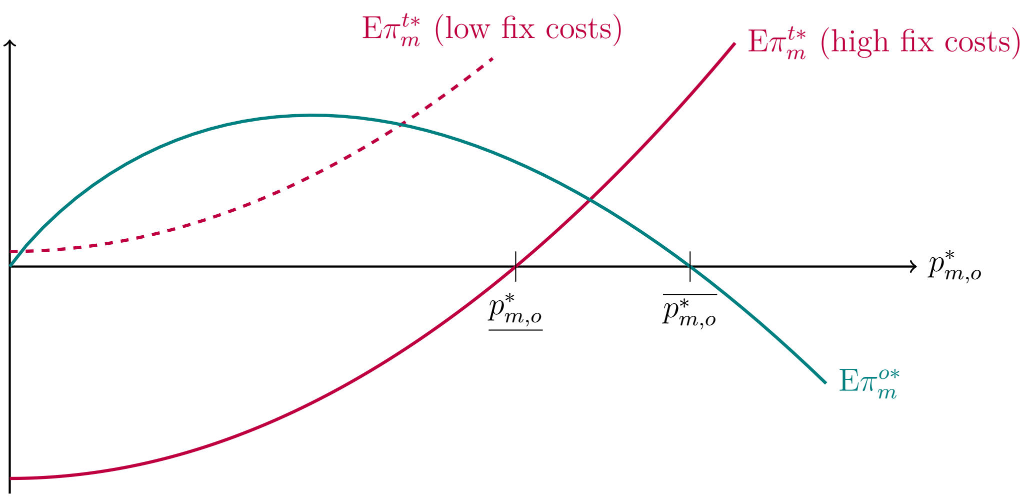


Figure 3.7: Decomposition of the profits of multichannel firms

The overall expected profits of multichannel firms amount to:

$$\mathrm{E}\pi^*_m = \frac{p^*_{m,o}}{n_o^2} \cdot \left[1 - 2(n_l - n_o)\sqrt{\frac{A + p^*_{m,o}}{3t}}\right] + \frac{4}{\sqrt{t}} \cdot \left[\frac{An_o + (n_o - 1)p^*_{m,o}}{3n_o}\right]^{3/2} - f \tag{3.18}$$

The overall shape of these expected profits is not clear at first sight: it depends on the exogenous variables and will be analyzed in the upcoming comparative statics. As we do not exclude the eventuality of cross-subsidy between the channels, especially the possibility that firms take into account negative profits in the local channel to enhance their overall expected profits, we cannot formulate a simple closed-form formula for the non-negativity condition. One case is, however, quite straightforward: when the equilibrium online price is between the two boundary prices $\underline{p^*_{m,o}}$ and $\overline{p^*_{m,o}}$ as defined in Figure 3.7, there is no cross-subsidy and the multichannel firms expect positive profits in each channel.

In the following section, we will analyze the impact of the different exogenous variables (reluctance parameter A, transportation costs t and number of firms in each channel n_l, n_o) on these equilibrium results. When analyzing the incentives of firms to participate in the online channel, it will show that all established firms have an incentive to expand to the online channel; this case of symmetry between $n = n_l = n_o$ multichannel firms yields many simplifications and enables us to endogenize the structure of the market and measure the corresponding welfare. We therefore turn first to the dynamics of the general case ($n_l \geq n_o$) in section 3.6, then take a closer look at the dynamics of the symmetry case ($n = n_l = n_o$) in section 3.7, before we turn to the endogenous market structure and the welfare analysis in section 3.8.

In the next part, we inspect the influence of the reluctance parameter A, the transportation costs t and the number of firms n in a more detailed way. At this stage, the level of the fix costs f is not yet relevant; it will get a more prominent role when it comes to defining the endogenous structure of the market.

3.6 Dynamics of the Transitional Asymmetry Case

The dynamics of this transitional state are interesting in two different perspectives: first, they offer an insight at the impact of an asymmetric structure of the market, with less online retailers than local stores. Second, they bring the transitory essence of this case forward, with the proof that all single-channel firms always have an incentive to become active in the online channel.

A small technical prolegomenon and some caveats are necessary to explain the methodology in this section. In the following, only numerical illustrations are displayed; the analytical proofs are relegated to the appendix D from p. 161 on and referred to in due place. Unless otherwise mentioned, the numerical illustrations rely on the assumptions $A = 1; t = 100; f = 0.0001; n_o = 2$ and $n_l = 6$. The domain of the functions might be restricted, as a result of the non-negativity conditions for the transitional case discussed in the precedent section (see p. 66 and p. 69 et sqq.): each local store and each online retailer must have a positive expected demand (see conditions (3.10), (B.1), (B.2) and (B.3)). This explains why in our numerical example, some functions look truncated: from the truncation point on, they would violate one of these conditions.[19]

3.6.1 Reluctance Parameter A

The reluctance parameter A captures the loss of utility incurred by consumer when shifting to the online channel. It is modelized in the fashion of a cost parameter increasing

[19] In the comparative statics with respect to the reluctance parameter A, the functions are truncated for high values: above $A = 1.877$, the non-negativity condition (B.3) on the demand for the online channel is violated. In the comparative statics with respect to the transportation cost parameter t, the functions are truncated for low values: values below $t = 54$ are not admissible for the same reason.

consumers' overall costs from buying online $A + p^o$. We consider a scenario where the acceptance of the online channel increases, i.e. a decrease in A. It is not surprising that such a decrease is beneficial to the online channel: as consumers' acceptance is enhanced, the expected demand for online retailers increases while that for local stores decreases. Expected demand migrates from the local to the online channel (see appendix, results (D.12), (D.13) and (D.14)). Figure 3.8 illustrates this migration. The variations in individual expected demand are not symmetric, simply because there are more local stores: each online retailer "inherits" more than the demand migrating away from one local store. We refer to this phenomenon as asymmetry in the migration effect.

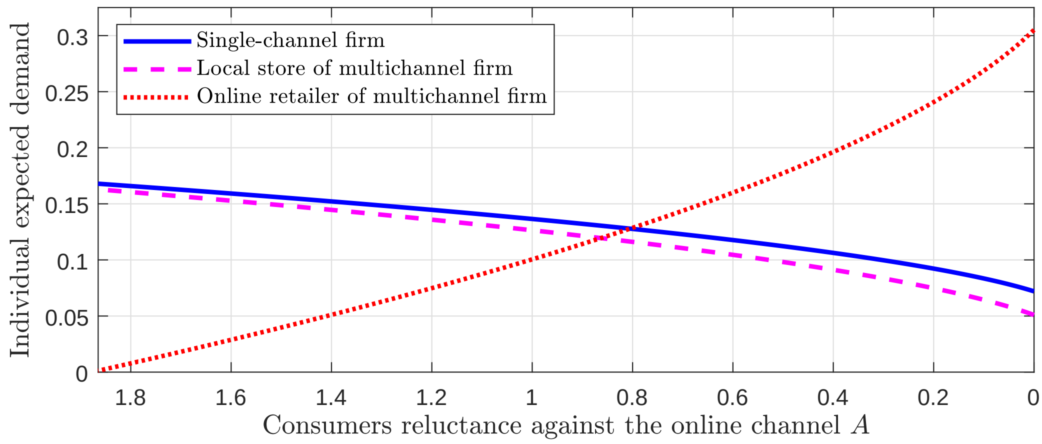


Figure 3.8: Migration of the expected demand as the reluctance parameter A decreases

For low levels of the reluctance parameter A, the comparative advantage of the online channel enhances the competition among online retailers, which translates into a higher level of visibility. This phenomenon is pictured in Figure 3.9 and discussed in the appendix on p. 167. The level of visibility expenditure per unit of expected demand is concave in A, similarly to the equilibrium online price (see below, in the discussion of Figure 3.10).

The mathematical discussion (see appendix, p. 167 et sqq.) of the variation of overall visibility expenditures leads to the result that visibility expenditures increase as consumers' reluctance decrease, with one exception: when the number of stores in each channel is very low. This exception is however not consistent with the assumption of free market entry; so that the positive relationship between decreasing hurdle for the online channel, i.e. decreasing A, and enhanced visibility expenditures holds in the case of free market entry.

Figure 3.9 illustrates the effect of a decrease in the reluctance parameter A on the competition among online retailers, which is related only to their visibility. There is a further dimension of competition in our model: the competition between stores of the two chan-

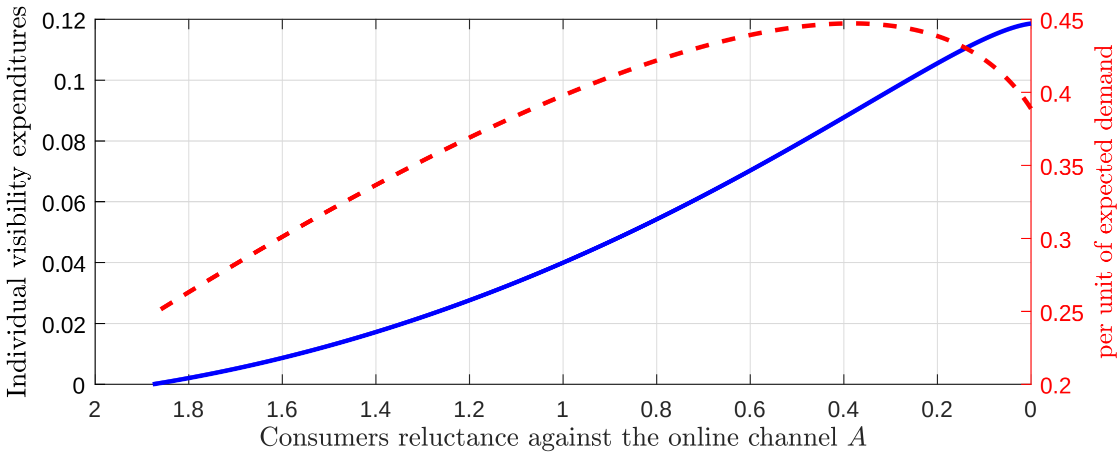


Figure 3.9: Variation of the visibility expenditures with the reluctance parameter A

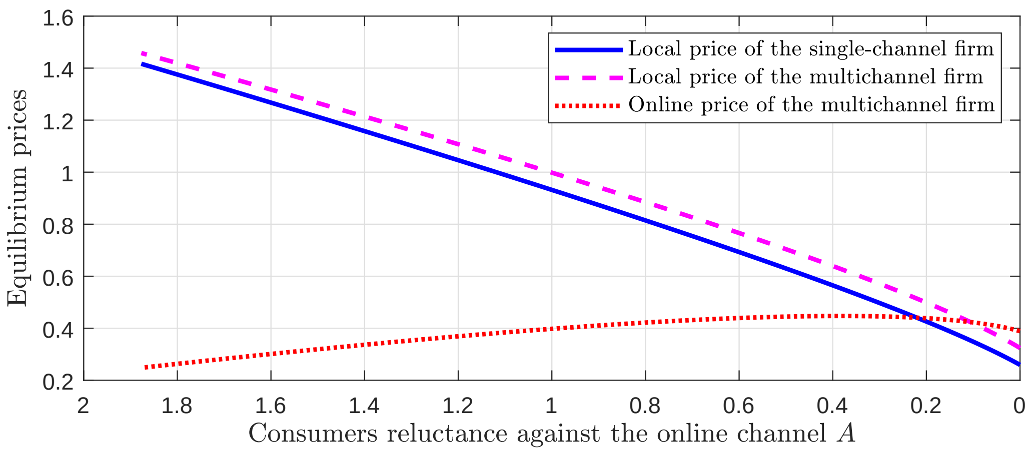


Figure 3.10: Variation of the equilibrium prices with the reluctance parameter A

nel, which is related to prices.

The impact of A on the evolution of prices is summarized in Figure 3.10. First, remark that in the local channel, multichannel firms can afford to set higher prices than single-channel ones, as remarked on p. 68: this is a benefit of their coordination of the two channels. Second, we observe that enhanced competition has a negative effect on the prices in the local channel: the prices of local stores decrease as these stores get exposed, via the decrease in A, to a fiercer price competition with the online channel (see appendix, results (D.8) and (D.9)).

Third, the variations in the price set by traditional retailers are more important: these curves are steeper. This is simply linked to the fact that the prices in the traditional channel depend, on the one hand, on A, and on the other hand on the price in the online channel (see equilibrium results (3.7) and (3.11)), and that the variations in A dominate (see appendix, results (D.10) and (D.11)).

Fourth, the equilibrium online price follows an inverted-u-shaped curve. The reason is that it underlies partially conflicting effects.[20, 21] On the one hand, a decrease in A confers a comparative advantage to the online channel, which allows for a higher level of prices. Remember that prices are, indeed, the key factor for the competition among traditional stores and between the channels, but not among online retailers. Even though a decrease in A means a fiercer competition among online retailers, they compete among each other only with respect to their visibility. On the other hand, a negative incentive is at work, but only for low levels of the reluctance parameter A: in this region, the sharper convexity of the demand function enhances the price elasticity of demand. Depending on the parameter constellation, these last two effects might overweight the two preceding ones and lead, for low levels of the parameter A, to a price decrease, as is the case in our numerical example.

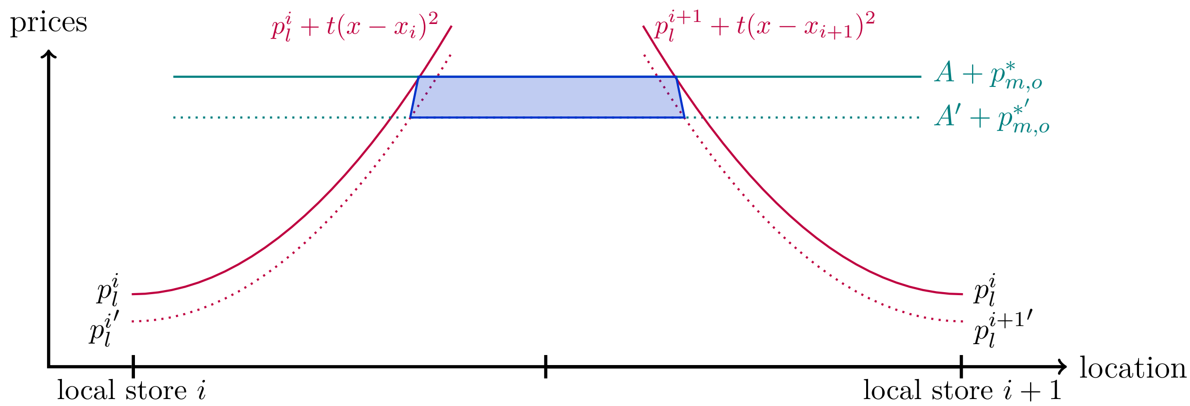
This higher price elasticity is linked to the effect of the transportation costs on consumers' decision about the channel. When considering the question whether to buy online or in a local store, each consumer compares, depending on the own location x, the overall costs from buying online $(A + p^*_{m,o})$ with the overall costs from buying in the next local store i $(p^t_i + t(x - x_i)^2)$. This decision is illustrated in Figure 3.11.

The same pattern can be observed between any two local stores, so that it is sufficient to focus at an arc of the Salop circle between two local stores. Here, for convenience, the arc is straightened: the abscissae are the locations of stores and consumers. The axis of ordinates depicts the overall costs consumers bear when buying. The full teal line represents the overall costs from buying online; it is higher in the upper graph (case 1), and lower in the second (case 2). A higher level of reluctance implies lower overall costs $A+p^*_{m,o}$.[22] The dotted teal lines represent respectively the level of these overall costs after a marginal decrease in the reluctance parameter A. The red curves depict the overall price from buying from the local channel, and increases quadratically with the distance between consumers and the local store. We elude here the question whether the two local stores belong to single-channel or multichannel firms and whether they are symmetric or not, as the different situations lead to the same qualitative outcome. Here again, the dotted line

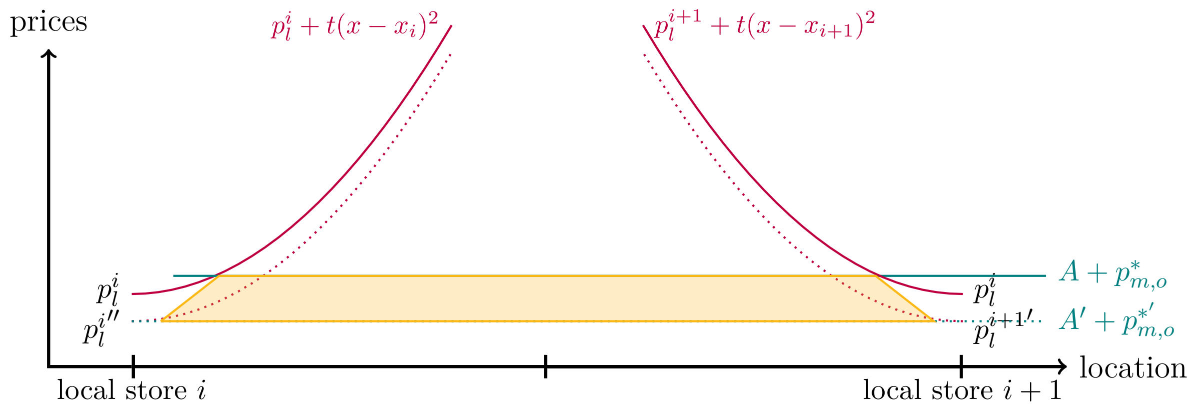
[20]See the extensive discussion of the derivative $\partial p^*_{m,o}/\partial A$ in (D.1), in the appendix from p. 161 on.

[21]These effects are also at work in the evolution of the prices in local stores: but because their evolution is dominated by the decrease in A, these effects are difficult to spot on the graph: they can at most be detected in the steeper slope for low levels of A, related to the higher price elasticity of demand described below.

[22] See appendix, result D.3 on p. 164: $\frac{\partial A + p^*_{m,o}}{\partial A} = 1 + \frac{\partial p^*_{m,o}}{\partial A} > 0$.



Case (1): high reluctance



Case (1): low reluctance

Figure 3.11: Price elasticity of demand and the reluctance parameter A

refers to the impact of a marginal decrease in the reluctance parameter. This impact on the local prices is lower than the impact on the overall online price: the vertical distance between the local prices is lower than between the full and the dotted teal lines.[23] In each case, all consumers between the intersections of the red curves with the respective teal line will buy from the online channel; and all consumers outside these intersections, from the local channel. For lower levels of A, the distance between the dotted and full red lines increase, which implies a higher price elasticity of demand: a similar change in price implies a stronger variation of demand for lower levels of the reluctance parameter A. For example, at the high level of reluctance A, a marginal decrease in reluctance would lead

[23] See appendix, for the two possible local prices D.8 and D.9:

$$\frac{\partial p_s^*}{\partial A} = \frac{2}{3}\left[1 + \frac{\partial p_{m,o}^*}{\partial A}\right] = \frac{2}{3}\left[\frac{A + \partial p_{m,o}^*}{\partial A}\right] < \frac{\partial (A + p_{m,o}^*)}{\partial A}$$

$$\frac{\partial p_{m,l}^*}{\partial A} = \frac{1}{3n_o}\left[2n_o + (2n_o + 1)\frac{\partial p_{m,o}^*}{\partial A}\right] = \left[1 + \frac{\partial p_{m,o}^*}{\partial A}\right] - n_o - (n_o - 1)\frac{\partial p_{m,o}^*}{\partial A}$$

$$= \left[\frac{\partial (A + p_{m,o}^*)}{\partial A}\right] - n_o - (n_o - 1)\frac{\partial p_{m,o}^*}{\partial A} < \frac{\partial (A + p_{m,o}^*)}{\partial A}$$

(from the dotted to the full line) to a small expansion of the area of potential demand for the online retailers, marked in blue. In contrast, for the low level A', a marginal decrease would lead to a higher increase in the area of potential demand for the online retailers (marked in yellow). As the variation of demand depends on the square of the distance between consumer and local store, it reaches its maximum when the intersection between the overall costs in the two channels is close to the location of the considered local store: the demand elasticity reaches its maximum for the lowest realizable levels of the reluctance parameter A.[24]

Finally, we turn to the combination of demand, visibility and price variations and observe the impact of the reluctance parameter A on individual firms profits, as illustrated in Figure 3.12.

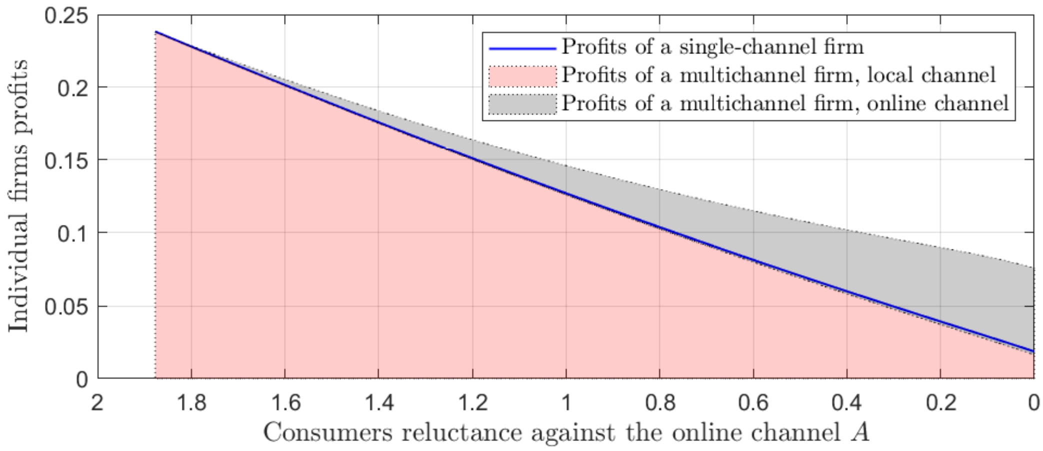


Figure 3.12: Variation of the equilibrium profits with the reluctance parameter A

To facilitate the comparison and readability of the graph, we plotted the profits of the multichannel firms as two stacked areas, and the profits of the single-channel firm as a curve. The evolution of price and demand in the two kinds of local stores are very similar; it is therefore not surprising that the evolutions of their profits are similar too. The profits of the single-channel firm are slightly higher than those of the multichannel firm, but the difference is so minimal that it is covered by the curve of the local, single-channel curve itself. The explanation for this difference is the concessions of multichannel firms to the interests of their own online retailer: a slightly higher price, implying a slightly lower demand than for single-channel local stores, leads to slightly lower profits. Remark that this concession also benefit all other online retailers. Multichannel firms' price decisions

[24] Figure 3.11 depicts the convexity of the right-hand side and left-hand side demand for the online channel around two local stores. The convexity of the aggregated expected demand for one online retailer is illustrated in the appendix, in Figure D.2 on p. 164, for our numerical example.

impact not solely their own stores, they always also impact the overall balance between the two channels.

Local stores' profits decline as consumers acceptance of the online channel increases (see appendix, results (D.16) and (D.17)). This is consistent with the fact that a lower reluctance parameter A confers a comparative advantage to the online channel. What is more surprising is the modesty of the increase in the profits of an individual online retailer. This increase is curbed by the visibility expenditures, that represent a share of the profits equal to the share of the rivals in the online channel, $(n_o - 1)/n_o$ (see above, equation (3.13)). Correspondingly, the variation in these profits is proportional to the variations in the visibility expenditures (see appendix, result (D.16)). When departing slightly from our basic numerical example and considering the case when there is only one online retailer who can pass on visibility expenditures, this one online retailers gathers fastly expanding profits as the reluctance parameter A decreases, as illustrated in Figure 3.13.

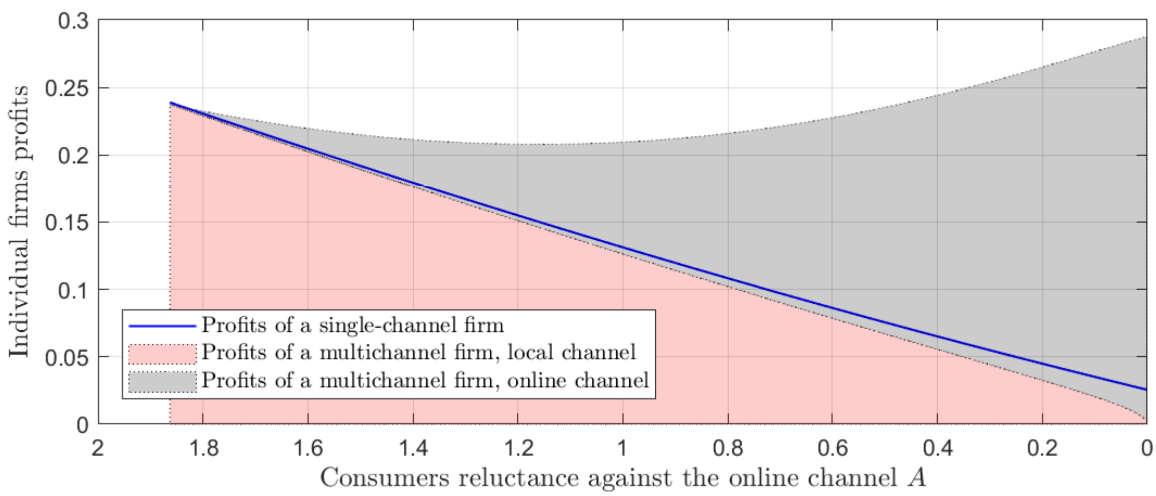


Figure 3.13: Variation of the profits with A, case with only one online retailer and no visibility competition, all other parameters unchanged

As the asymmetry between the numbers of local and online retailers is the essential characteristic of the transitory case, it is worth looking at the aggregated results.
Figure 3.14 depicts the distribution of the expected demand between the different types of stores and illustrates the migration of demand from the local to the online channel. Multichannel firms benefit from this migration: as the acceptance of the online channel increases, they dominate, in terms of demand, their single-channel competitors. The change in the distribution of profits is illustrated in Figure 3.15. Here again, we see that single-channel firms suffer from a decrease in the reluctance A: their share of profits decreases.

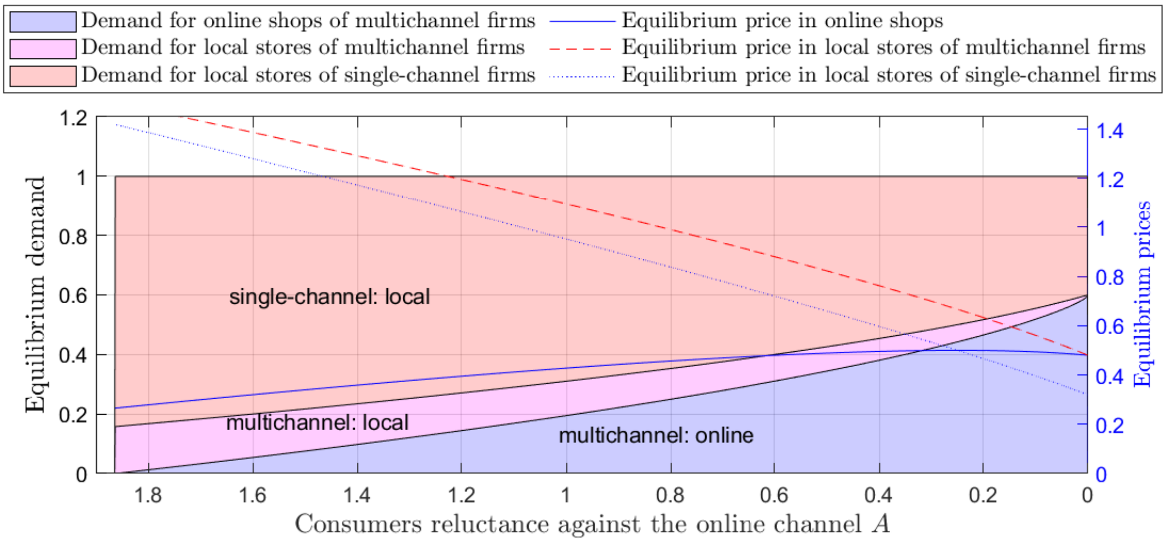


Figure 3.14: Aggregated demands as the reluctance parameter A decreases

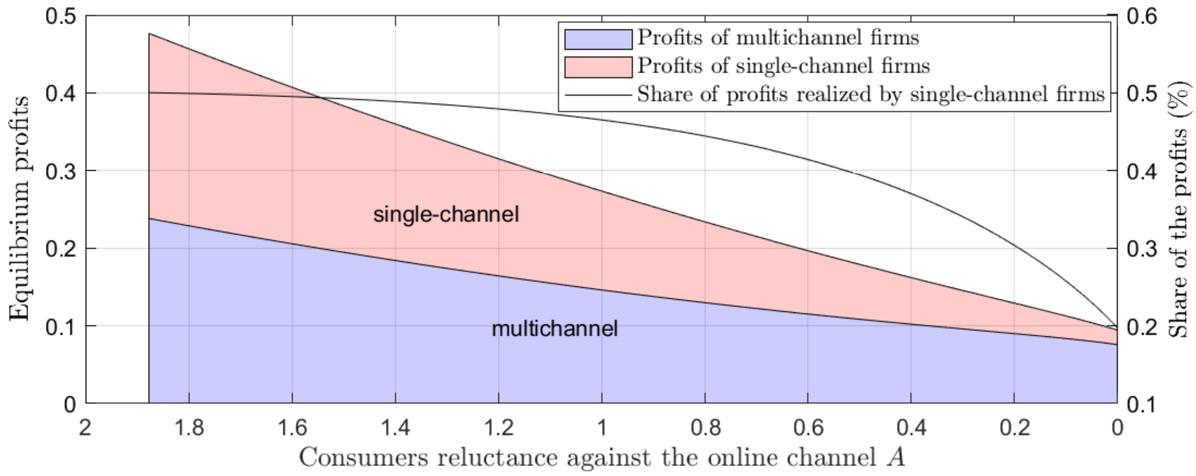


Figure 3.15: Evolution of channels' profits as the reluctance parameter A decreases

3.6.2 Transportation Costs t

As in the Hotelling or Salop basic models, the transportation costs t affecting the local channel have the effect of relaxing competition, in first place among the neighbouring local stores: if the transportation costs are higher, firms can set higher prices in the local channel. At the same time, transportation costs affect the balance between the two channels, as they affect the consumers decision whether to buy local or online. Increasing transportation costs have therefore two distinct impacts: on the one hand, they relax the price competition among local stores; on the other hand, they confer an advantage to the online channel.

This second impact is visible in Figure 3.16: as the transportation costs increase, demand shifts to the online channel. The expected demand for online retailers increases

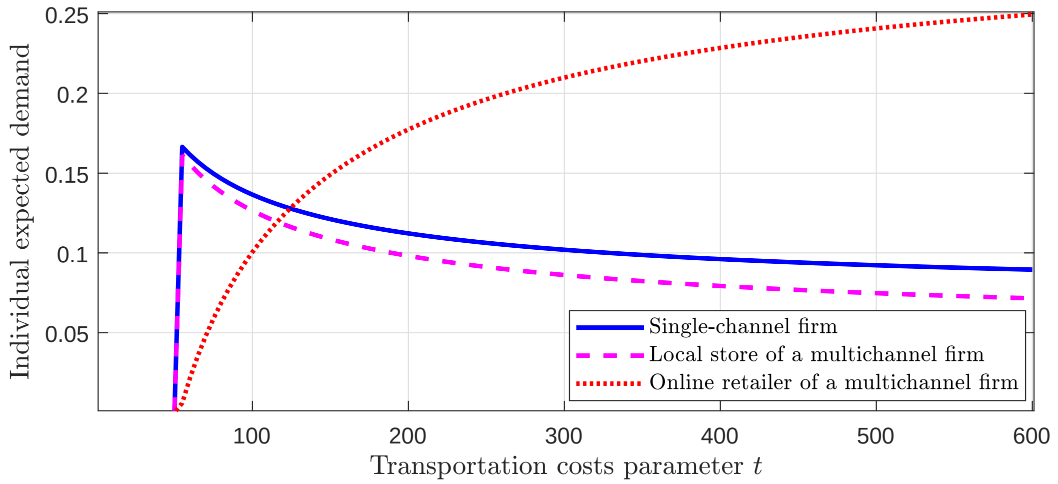


Figure 3.16: Migration of the demand as the transportation costs t increase

(see appendix, result (D.28)), and the expected demand for local stores, whether from single-channel or from multichannel firms, decreases (see appendix, results (D.26) and (D.27)).

The enhanced demand for online retailers animates the competition among them: as they do not compete directly with respect to their prices against each other, but rather with respect to their visibility, this effects translates into a higher level of visibility expenditures, as well absolutely as relatively to the expcted demand in the online channel (see Figure 3.17 and appendix, results (D.24)).

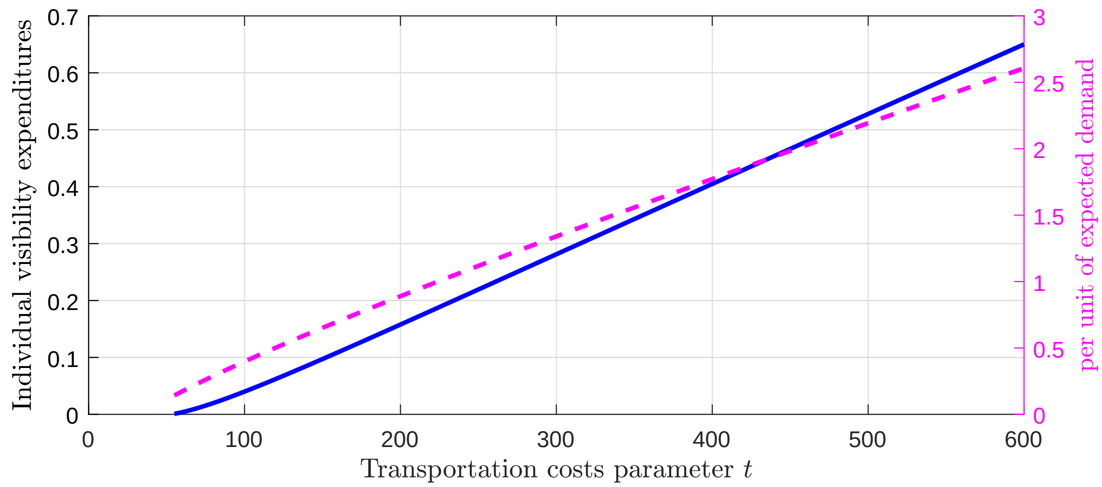


Figure 3.17: Increase of the visibility expenditures with the transportation costs t

The relaxation of the price competition when the transportation costs increase translates into a higher level of prices: all prices increase, in both channels, as illustrated in Figure

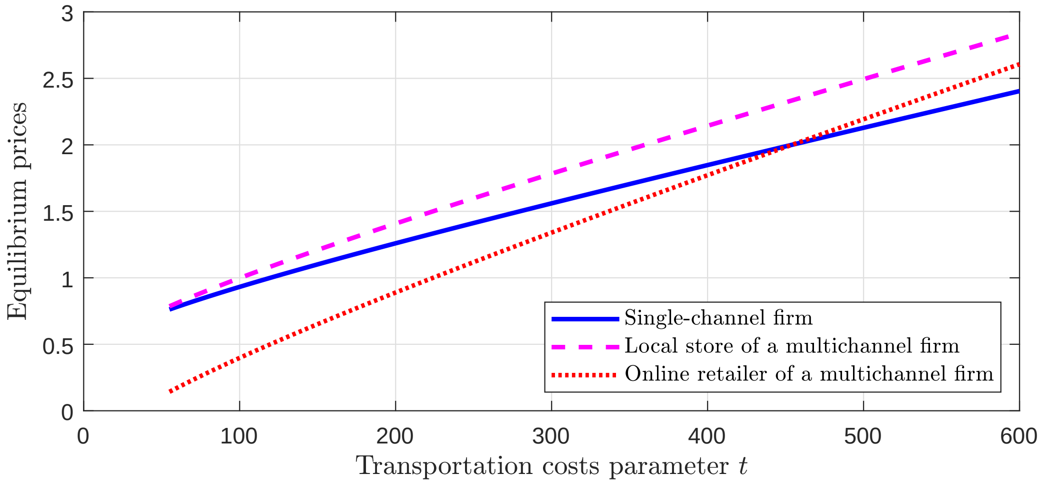


Figure 3.18: Increase of the prices with the transportation costs t

3.18 (see appendix, result (D.19), (D.21) and (D.22)). Even though the visibility expenditure per unit increases, the relaxation of price competition also leads to an increase in the net price in online retailers (see appendix, results (D.25)). The difference in the augmentation of the three prices illustrated in Figure 3.18 holds true for any parameter constellation: the price in online retailers increases at the highest pace, while the price in the local store of single-channel firms augments at the lowest pace (see appendix, result (D.23)). This is due to the different effects of the prices on the demands for the three kinds of stores: online retailers benefit from the migration effect so that their increased price only mitigates the increase of the demand. The local stores of multichannel firms are indeed affected by a reduction of the local demand - but some of the buyers lost in the local channel will become buyers from the same firm in the online channel. In contrast, nothing mitigates the migration of buyers from single-channel local stores.

As both the effects on prices and expected demand are positive for the online retailers, it is not surprising that their expected profits increase with the transportation costs (see appendix, result (D.31)). In the local channel, the two effects are conflicting. The local stores of multichannel firms, however, have slightly different incentives than those of single-channel firms. They always set a higher price than their single-channel counterparts, and lose more of the expected demand to the online channel - which is, partially, recovered by the same firm's online retailer, so that this demand does not, at the level of the firm, get completely lost. Both the positive price effect and the negative migration effect affect them more strongly (see Figures 3.16 and 3.18) and result in a slightly lower level of profits. But as illustrated in Figure 3.19, the profits of local stores of single-channel and multichannel firms are quite similar in shape and close to each other.

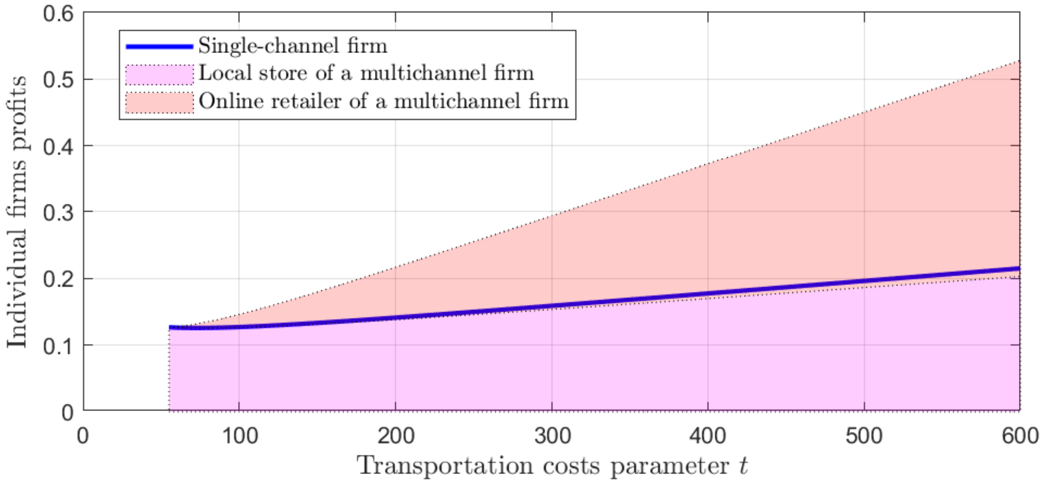


Figure 3.19: Evolution of profits as the transportation costs parameter t increases

In Figure 3.19, the profits in the local channel seem to be mostly increasing as the transportation costs parameter increases. Depending on the parameter constellation, there might be an exception. It is the case in our example and becomes visible when zooming into a specific region of the graph. When the demand for the online channel is in the fledgling stages, the profits in the local channel turn out to be declining. This is observable in Figure 3.20, which is an detail of Figure 3.19, focusing at the values of the transportation costs parameter t between 55 and 80.

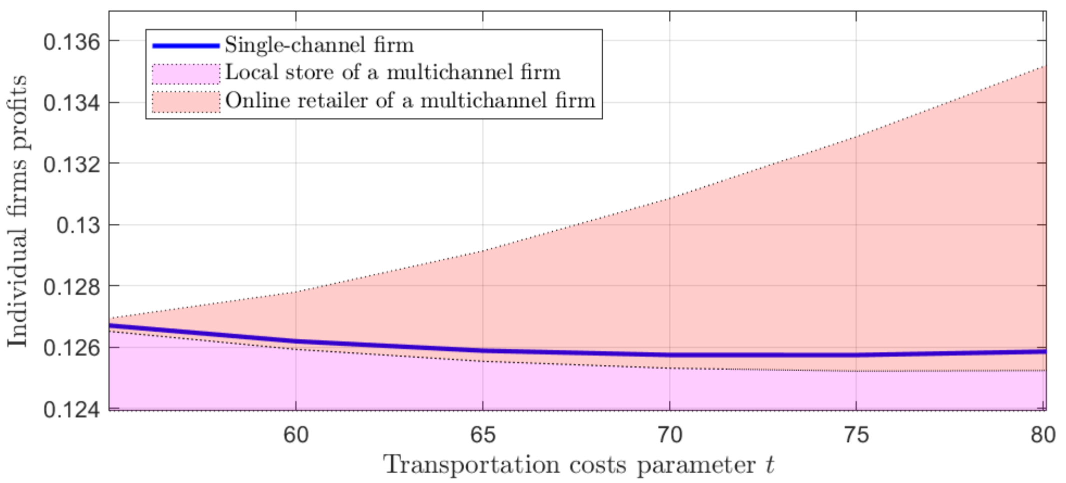


Figure 3.20: Evolution of stores' profits: detail

The reason is that in this specific region, the migration effect is maximized and dominates the price effect. The analytical proof is given in the appendix (analysis of result (D.29), p. 172): when the demand for the online retailer is close to zero, both migration effect and price effect are maximized, but then and only then, the migration effect might dominate.

To understand this steep increase of the migration effect when the demand for the online channel is low, we need to look more closely at the mechanism behind the migration effect. Again, like in the case of the inverted-u-shape of the online price as A decreases in the preceding subsection, this phenomenon is linked to the shape of the transportation costs.

Figure 3.21 illustrates the consumers decision between local and online channel when the transportation costs parameter t increases from some value t_1 to t_2. It is constructed similarly to the graphs in Figure 3.11. The lower, teal, curved line represents the overall costs of buying from a local store when transportation costs are low (value t_1). These costs increase quadratically with the distance to the next local store. As long as the teal curve is below the dotted line $A + p_{m,o}(t_1)$, consumers buy from the local stores. In the area in the middle, highlighted with gray on the axis of abscissae, consumers turn to the online channel. As the transportation costs increase to t_2, the online price p^o increases, while the curve representing the overall costs from buying in a local store shifts upwards. The demand addressed to the online channel augments by the two purple highlighted domains. These domains visualize the migration effect: these are the consumers shifting from the local to the online channel as the transportations costs parameter shifts from t_1 to t_2.

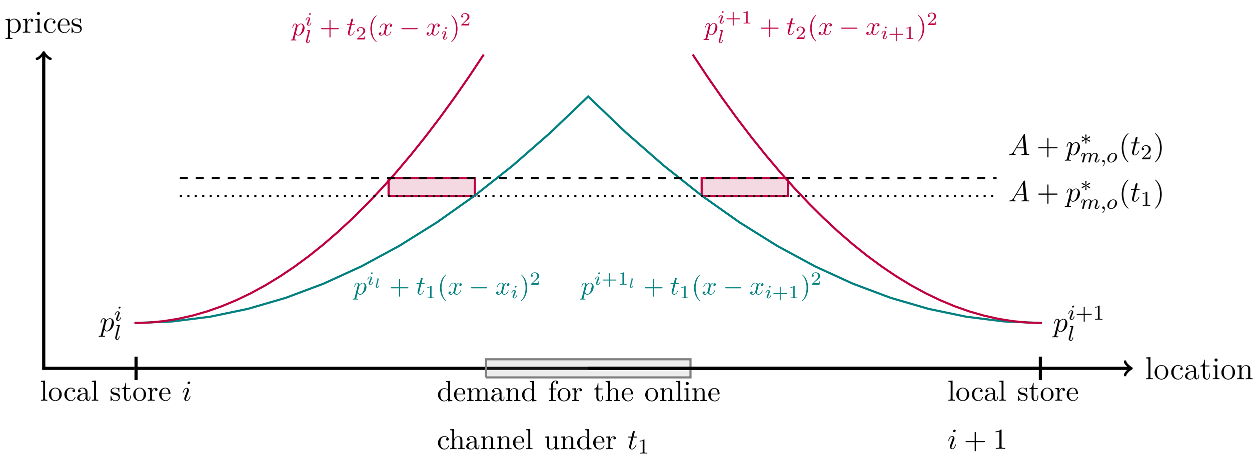


Figure 3.21: Migration effect

This migration effect is higher if, ceteris paribus, the horizontal line is higher, close to the peak of the teal curves. Figure 3.22 illustrates the maximum migration effect, corresponding to the limiting case in which the demand addressed to the online channel is initially converging to zero. The difference between the two curved lines is, indeed, increasing for higher horizontal levels of $A + p^o$. In the case of maximum migration effect, the overall price from buying online is equal to the price from buying in a local store of a multichannel for the consumer situated in the middle of two local firms. Remark that the price effect reduces the migration effect (if the price were constant at the level of the dotted line, the migration effect would be slightly higher), but cannot distort it.

Because of the shape of the transportation costs, the migration effect can offset the price effect in this initial domain, but declines rapidly so that the price effect mostly dominates.

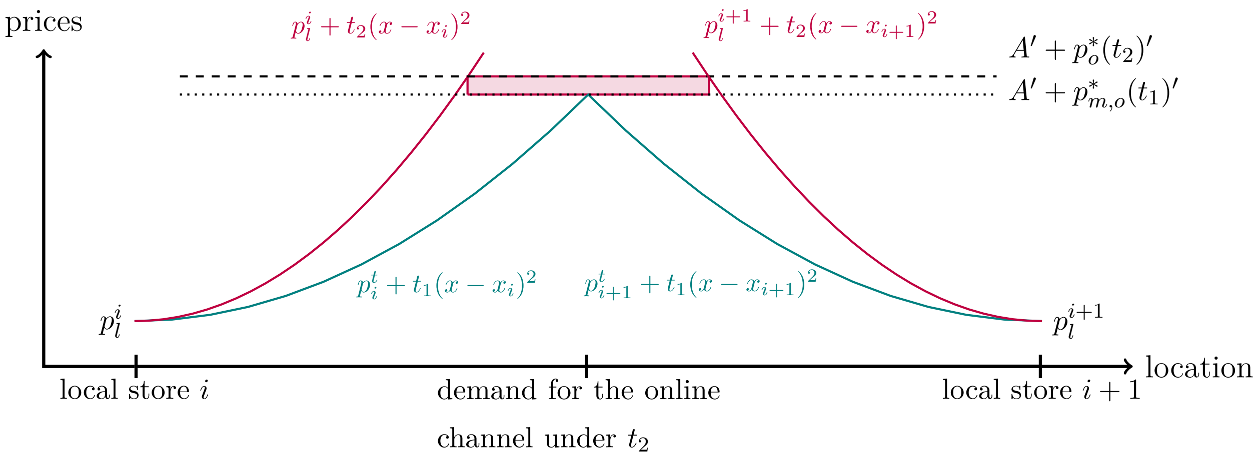


Figure 3.22: Maximum migration effect

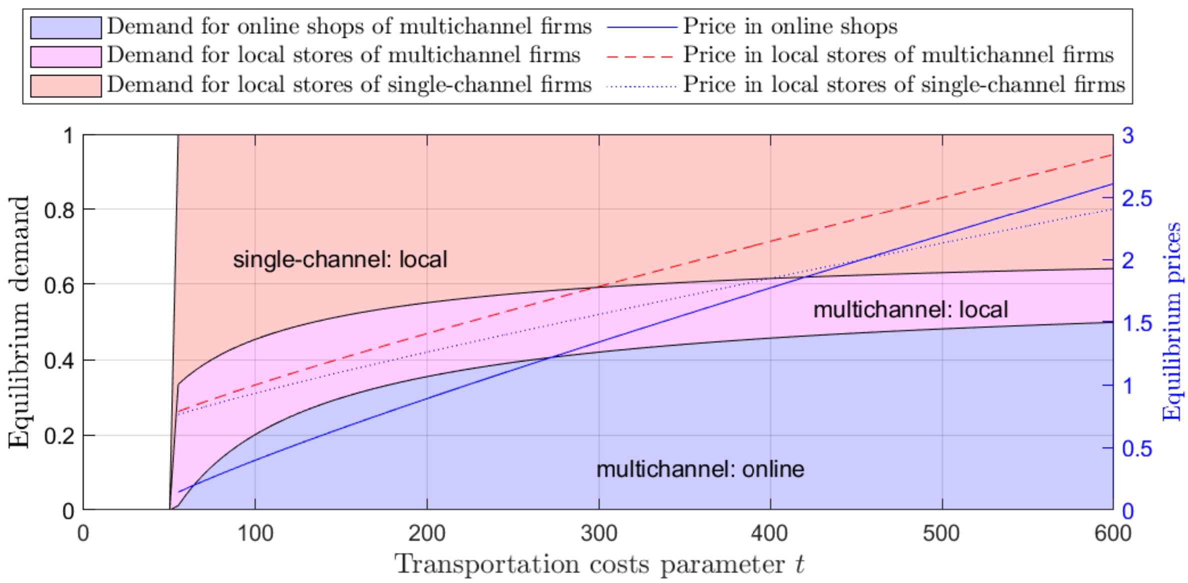


Figure 3.23: Evolution of price and demand as the transportation costs parameter t increases

Finally, let us have a look at the aggregated results. The steep individual migration effect in the online channel reflects in Figure 3.23: online demand increases at a decreasing rate. We observe that the demand addressed to the local stores of multichannel firms is modest, which is due as well to their small number in our numerical example, as to the concession effect: they set higher prices and risk a higher loss of demand than single-channel stores, because their associated online retailer inherits parts of this exited demand.

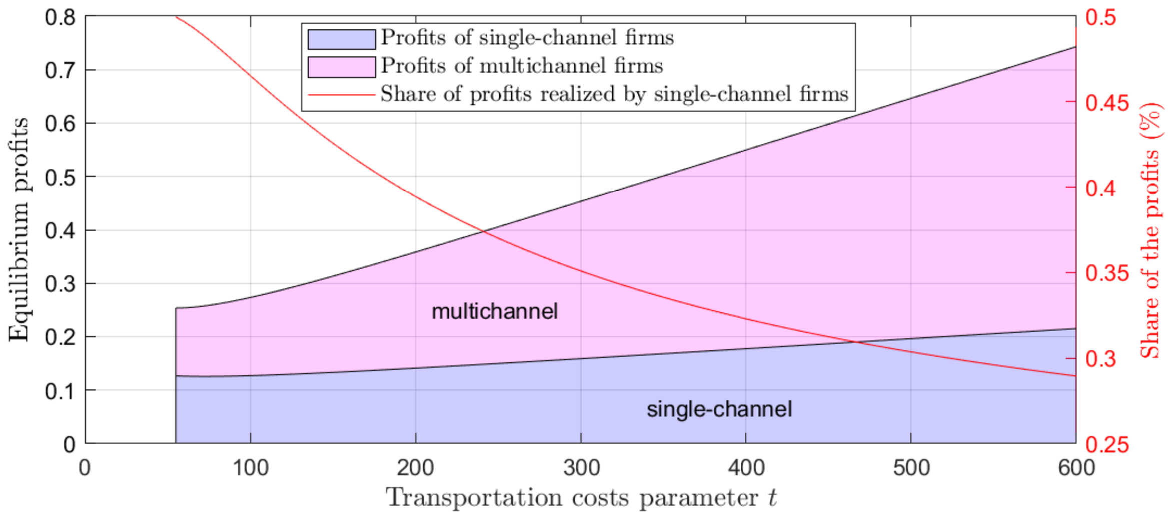


Figure 3.24: Evolution of firms' profits as the transportation costs parameter t increases

Figure 3.24 illustrates the evolution of the profits; the dominance of the price effect over the migration effect is visible in this graph over a broad range of values. We can observe here the success of the multichannel strategy: the share of profits realized by single-channel firms drops quickly as their competitiveness suffers under the increasing transportation costs.

Before we turn to the summary of this section, let us remark that both parameters A and t represent additional costs for consumers in the respective channels. It is therefore not surprising that their impact is quite comparable: if they decrease, they both lead to a constriction of profits, but confer a relative advantage to the channel they apply upon.

The main conclusion of this analysis of the dynamics of the transitional state are as follow.

The first result is that all firms should run an online retailer. A quick glance at the profit function of online retailers in equilibrium (3.17) confirms that these profits are indexed directly to the demand for the online channel. This demand increases as the reluctance parameter A decreases (see appendix, equation (D.13)). As soon as the level of the reluctance parameter A falls below the critical value for which some consumers - those located in the middle between two local stores - turn to the online channel, all firms opening an online retailer will gather positive profits. As their number n_o increases, the individual store's expected profits decrease, but it never turns negative. The asymmetric case is therefore, in essence, only a transitional state when market entry is free: as all n_l firms have an incentive to open an online retailer, there is no endogenous asymmetric structure of the market corresponding. If consumers' reluctance parameter A is low enough so that an expected demand for the online channel exists, all firms have an incentive to open an online retailer, and the endogenous structure will be symmetric.

Second, the online channel benefits from a comparative advantage especially when the transportation costs parameter is high and when the reluctance parameter is low - in this perspective, the lockdown 2020, as e-commerce was already well established, was a most improbable real-life experiment.[25] The asymmetry of the migration effect reinforces this phenomenon. The fact that this migration effect is at its peak exactly when the demand for the online channel appears, might well explain and justify the fears for a suffocation of the local channel expressed at the beginning of e-commerce (literature around 2000). The shape of the transportation costs yields a further phenomenon corroborated by literature: the fact that the price elasticity of demand is higher in the online channel.

We now turn to the specific case when all firms have converted to e-commerce. The comparative statics will confirm and precise many of the findings in this section, for the case of parity.

3.7 Dynamics of the Symmetry Case

3.7.1 Simplified Results

In the symmetry case when all firms are active in both the local and the online channel, the equality $n_l = n_o = n$ leads to a series of simplifications.[26] These simplifications yield explicit results for the equilibrium prices and expenditures. To distinguish the equilibrium results in the case of symmetry from those in the general case, we signal them with two

[25] In fact, Statista reports a considerable jump in the absolute volume of e-commerce retail in the year 2020 in Germany (Handelsverband Deutschland, 2021). This jump is clearly higher than the overall growth trend in the previous years and illustrates the result of our model: an increase in transportation costs (theoretically up to infinity in case of a lockdown) in an economy where e-commerce is well established (low reluctance against e-commerce) results in a major growth of the online channel. For the major economies worldwide, the United Nations Conference on Trade and Development (UNCTAD) reports an increase in e-commerce 2020 slightly above the trend in the previous years, contrasting with a contraction in overall retail volumes, meaning that the share of e-commerce increased noticeably (United Nations Conference on Trade and Development, 2021).

[26] The simplified results can be computed using the general results from above, or by solving the maximization problem specific to the case of parity, for any firm i:

$$\max_{p_l^i, p_o^i, w^i} \mathrm{E}\pi^i = p_l^i \cdot \mathrm{E}D_l^i(p_l^i, p_o^i, w^i) - f + p_o^i \cdot \mathrm{E}D_o^i(p_l^i, p_o^i, w^i) - w^i$$

stars as superscript. As all firms are multichannel, the index m becomes superfluous.

$$p_l^{**} = \frac{(2n+1)\left[t+\sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} + \frac{2nA}{(5n+1)} \tag{3.19}$$

$$p_o^{**} = \frac{3n\left[t+\sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} - \frac{2nA}{(5n+1)} \tag{3.20}$$

$$w^{**} = \left[\frac{3\left(t+\sqrt{\Delta}\right)}{2(5n+1)^2} - \frac{2A(n-1)}{(5n+1)}\right] \cdot \left[\frac{1}{n} - 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2 t}}\right] \tag{3.21}$$

$$\text{where} \quad \Delta = 4tA(n+1)(5n+1) + t^2$$

The equilibrium, individual expected demand functions simplify as:

$$\mathrm{E}D_l^{**} = 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2 t}} \tag{3.22}$$

$$\mathrm{E}D_o^{**} = \frac{1}{n} - 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2 t}} \tag{3.23}$$

We can check at one glance that their sum, that is the demand addressed to each firm via both channel, is $1/n$: expected demands spreads equally among the firms.

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with demand functions adjusted from (3.4) and (3.6) to the case of parity:

$$\mathrm{E}D_l^i(p_l^i, p_o^i, w^i) = \frac{1}{W} \cdot \sum_{j \neq i} \left(w^j \cdot 2\sqrt{\frac{A + p_o^j - p_l^i}{t}} \right) + \frac{w^i}{W} \cdot 2\sqrt{\frac{A + p_o^i - p_l^i}{t}}$$

$$\mathrm{E}D_o^i(p_l^i, p_o^i, w^i) = \frac{w^i}{W} \cdot \left[1 - 2\sum_{j \neq i} \sqrt{\frac{A + p_o^i - p_l^j}{t}} - 2\sqrt{\frac{A + p_o^i - p_l^i}{t}} \right] .$$

This maximization problem yields the first order conditions:

$$\frac{\partial \mathrm{E}\pi^i}{\partial p_l^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_l^{**} = \frac{2(A + p_o^{**})}{3} \frac{p_o^{**}}{3n}$$

$$\frac{\partial \mathrm{E}\pi^i}{\partial p_o^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_o^{**} = \frac{t + \sqrt{t^2 + 12nt\left[n(A - p_l^{**}) + p_l^{**}\right]}}{18n^2} - \frac{2(A - p_l^{**})}{3} + \frac{p_l^{**}}{3n}$$

$$\frac{\partial \mathrm{E}\pi^i}{\partial w^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad w^{**} = \frac{(n-1)}{n} \cdot p_o^{**} \cdot \left[\frac{1}{n} - 2\sqrt{\frac{A - p_l^{**} + p_l^{**}}{t}} \right] .$$

Because of the square root, the first order condition for the online price yields two solutions. The second order condition $-\frac{4An - 4np_l^{**} + 3np_o^{**} + p_l^{**}}{2n\sqrt{t}(A - p_l^{**} + p_o^{**})^{3/2}} < 0$ is fulfilled only for the upper root.

Finally, the equilibrium expected profit functions read:

$$\mathrm{E}\pi_m^{**} = \mathrm{E}\pi_l^{**}(n) + \mathrm{E}\pi_o^{**}(n) \qquad \text{where}$$

$$\mathrm{E}\pi_l^{**}(n) = \left[\frac{(2n+1)\left[t+\sqrt{\Delta}\right]}{(5n+1)^2(n-1)} + \frac{4nA}{(5n+1)}\right]\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}} - f \tag{3.24}$$

$$\mathrm{E}\pi_{m,o}^{*}(n) = \left[\frac{3\left(t+\sqrt{\Delta}\right)}{2(5n+1)^2(n-1)} - \frac{2A}{(5n+1)}\right]\cdot\left[\frac{1}{n} - 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}}\right] \tag{3.25}$$

As remarked in the general case, the expected profits in the online channel are as high as the visibility expenditures per competitor. Remark the strong similitude between the equilibrium price (3.20) and the price net of unit visibility costs, in the first square bracket of equation (3.25): the difference is only a factor n, their sign is the same.

The non-negativity conditions on prices and expected demand can be slightly simplified. In the local channel, the conditions $p_l^{**} \geq 0$ and $\mathrm{E}D_l^{**} \geq 0$ are obviously always fulfilled, as in the general case.

In the online channel, the non-negativity conditions are now symmetric, as all local stores are symmetric.[27] The non-negativity constraint on the equilibrium prices in the online channel $p_o^{**} \geq 0$ can be solved explicitly and yields the equivalent restrictions: [28]

$$t \geq \frac{4A(n-1)^2}{3} \quad\Leftrightarrow\quad A \leq \frac{3t}{4(n-1)^2} \quad\Leftrightarrow\quad n \leq 1 + \frac{\sqrt{3t}}{2\sqrt{A}}$$

The non-negativity constraint on the expected demand in the online channel can be solved explicitly with respect to A and t and yields more restrictive conditions:[29]

$$t \geq \frac{4A(n+1)n^2}{3n+1} \quad\Leftrightarrow\quad A \leq \frac{(3n+1)t}{4(n+1)n^2} \tag{3.26}$$

Because it is more restrictive,[30] this is the condition we will refer to in the following. Depending on the point of view, this condition can be formulated as a critical value with

[27] See discussion about local and global non-negativity conditions in appendix B, p. 153

[28] The non-negativity condition $p_o^{**} \geq 0$ can be reformulated as a quadratic, concave expression:

$$-(n-1)X^2 + 3\sqrt{t}(n+1)X + 2t(2n+1) \geq 0, \quad \text{where } X = \sqrt{4A(n+1)(5n+1)+t} \geq 0$$

This expression is positive between the roots. As $X \geq 0$, the solution resumes to:

$$\sqrt{4A(n+1)(5n+1)+t} \leq \frac{2(1+2n)}{(n-1)}\sqrt{t}$$

And can be transformed as condition on A, n or t.

[29] The non-negativity condition is cubic with respect to n and has no explicit solution.

[30] When the non-negativity condition on the expected demand 3.26 is fulfilled, the preceding non-negativity condition on the equilibrium prices always is. Using for example the formula for the maximal

respect to A, t or n. If is is not fulfilled, the online channel cannot come off and the market structure is the one in the initial benchmark.

We proceed with the analysis of this symmetry case. As for the transitional case, we will produce here only the numerical approximations; the analytical proofs referred to are gathered in appendix E, from p. 175 on. As the results are, of course, related to the general case, the analysis can be kept shorter. Especially, because all firms are multichannel and symmetric, there is no need to consider separately the market results at the firm level and at the channel level: the overall demands and profits are proportional to the firm level demands and profits in each channel. The approximations rely, unless otherwise declared, on the same set of assumptions as in the preceding section ($A = 0.01; t = 100; f = 0.0001; n = 6$).

3.7.2 Reluctance Parameter A

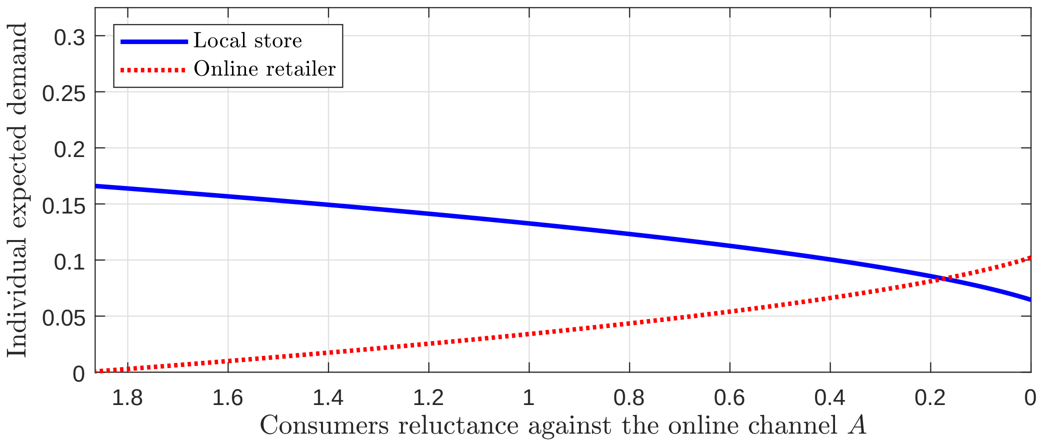

Figure 3.25: Migration of the expected demand as the reluctance parameter A decreases

As consumers' reluctance against the online channel decreases, we observe similar phenomena as in the general, asymmetric case.

First, the expected demand migrates from the local store to the online retailer (see Figure 3.25 and appendix, results (E.6) and (E.7). A slight difference is that the migration effect is here symmetric, as the number of local stores is equal to the number of online retailers: as consumers acceptance of the online channel increases, the expected demand in the on-

value for A:

$$\frac{3t}{4(n-1)^2} - \frac{(3n+1)t}{4(n+1)n^2} = \frac{(8n^2-n-1)t}{4(n-1)^2n^2(n+1)} > 0$$

line channel increases by the exact same value, as the expected demand in the local channel decreases. Their sum is constantly equal to $1/n$ (about 16,67% in our numerical example).

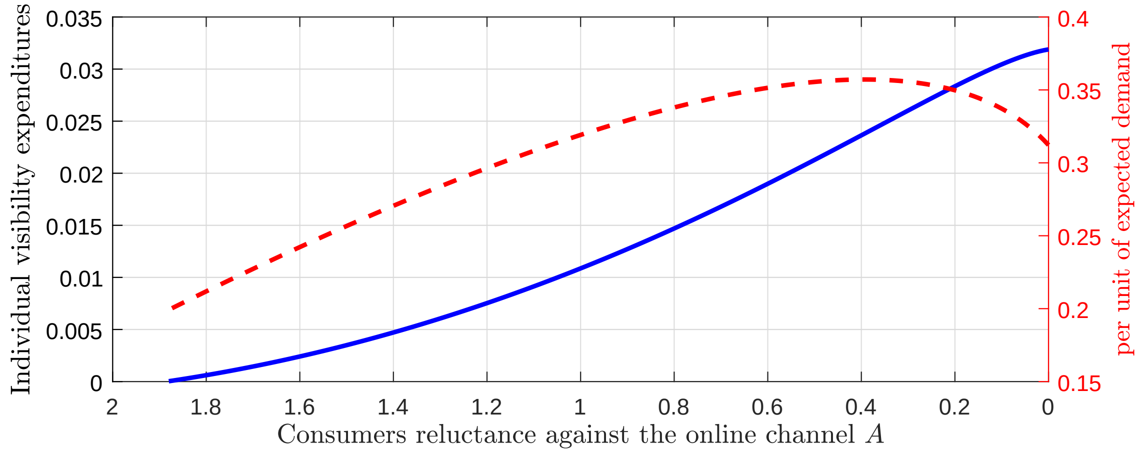

Figure 3.26: Variation of the visibility expenditures with the reluctance parameter A

As previously, we observe that a decrease in the reluctance parameter leads to increasing visibility expenditures (see Figure 3.26). A better acceptance leads to a migration of demand to the online channel, and therefore to an increased competition among online retailers (see appendix, (E.5)). The level of visibility expenditures per unit of expected demand is concave in A (see appendix, result (E.5); in the symmetric case, we can calculate the value at which it reaches its maximum (see appendix, (E.2)).

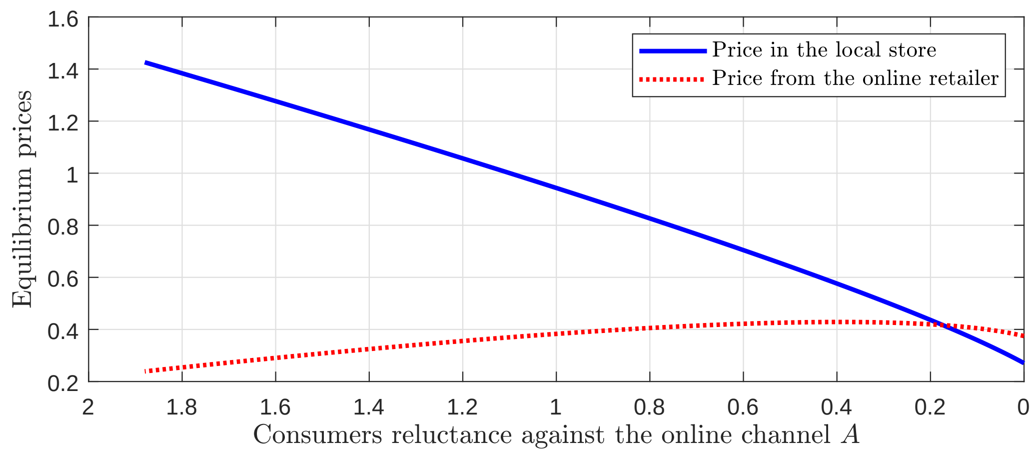

Figure 3.27: Variation of the equilibrium prices with the reluctance parameter A

As depicted in Figure 3.27, prices evolve as in the general, asymmetric case. As the reluctance against the online channel decreases, the expected demand migrates to the online channel and the competition between the channels increases. Both effects are negative for the local channel: local stores have an incentive to lower their price to limit the lost of expected demand (see appendix, results (E.1)). The two effects are, however, conflicting for the online channel (see appendix result (E.3)); the evolution of the online price reflects this conflict. Because of the increased price elasticity of demand for lower value of A, as discussed in the preceding section, the negative effect of increased competition dominates the positive effect of demand migration only for low values of A, and the online price curve is concave with respect to the reluctance parameter A. The critical value for which the online price reaches its maximum can be computed explicitly in the symmetric case; it is the same as for the visibility expenditures per unit of expected demand (see appendix, (E.2)). As in the general case, the variation in the local price is always steeper (see appendix, result (E.4)).

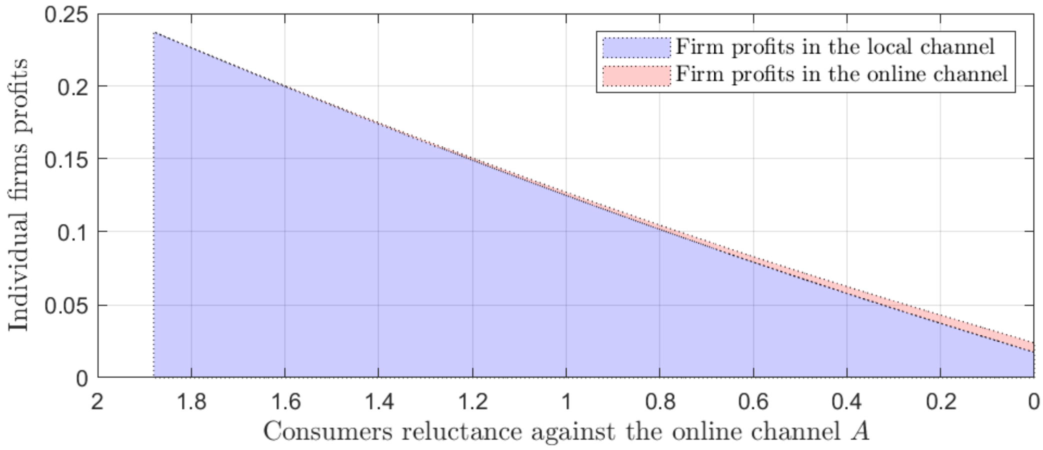

Figure 3.28: Variation of the equilibrium profits with the reluctance parameter A

The increased competition as well between the two channels as among the online retailers lead to a massive decrease in local and in overall profits (see appendix, results (E.8) and (E.10), and Figure 3.28). This does not imply that the absolute level of online retailers' profit always increases; but from a given threshold on, which is more plausible in the case of free market entry, it will increase (see discussion on the threshold $\underline{A}$ in the appendix, p. 175, and result (E.9)). The level of the online profits is here even lower than in the asymmetric case, precisely because of the nature of this asymmetry: the online retailers were less numerous, and therefore their share of the online profits higher.

3.7.3 Transportation Costs t

Higher transportation costs induce a migration of demand towards the online channel: as illustrated in Figure 3.29, in the symmetric case, this migration between the channels

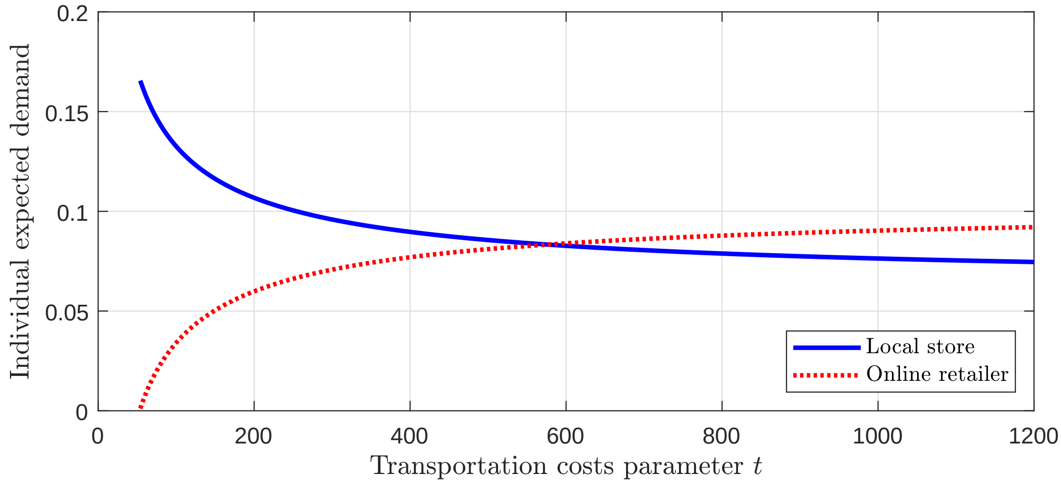

Figure 3.29: Migration of the demand as the transportation costs t increase

does not affect the distribution of the expected demand among the firms (see appendix, results (E.18) and (E.19)), as the expected demand lost in one channel is gained in the other.

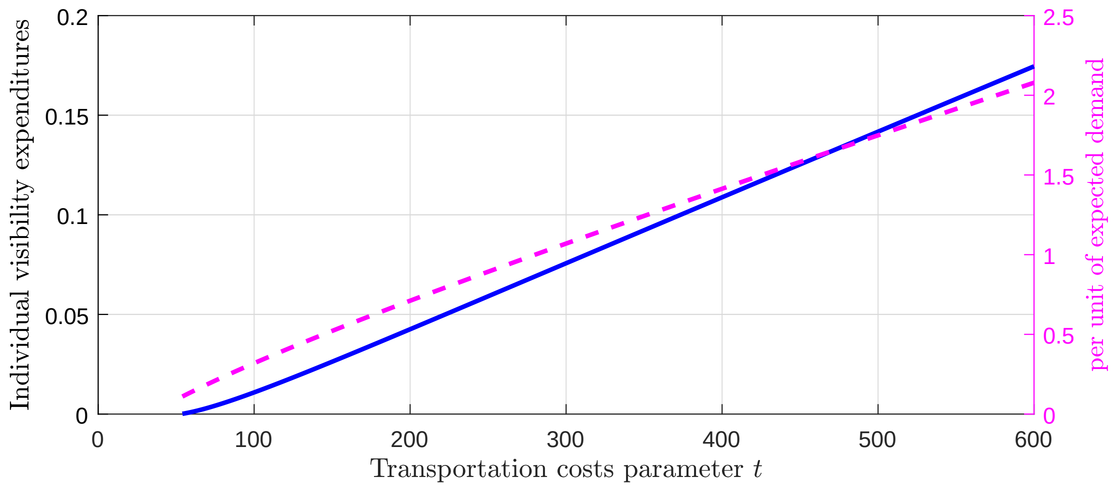

Figure 3.30: Increase of the visibility expenditures with the transportation costs t

As the competition among online firms gets fiercer, the overall expenditures for visibility increase as well absolutely as relatively per unit of expected demand (see appendix, (E.15) and (E.16))

Increasing transportation costs t affecting the local channel have the effect of relaxing competition: firms set higher prices in the local channel (see appendix, result (E.12). Here again, as in the asymmetric case, this relaxation extends to the online channel, as illustrated in Figure 3.31: the increase in the local prices allows for an even stronger increase in the online prices (see appendix, results (E.13) and (E.14)); so that even though the visibility expenditures increase, the online net prices still increase (appendix, result

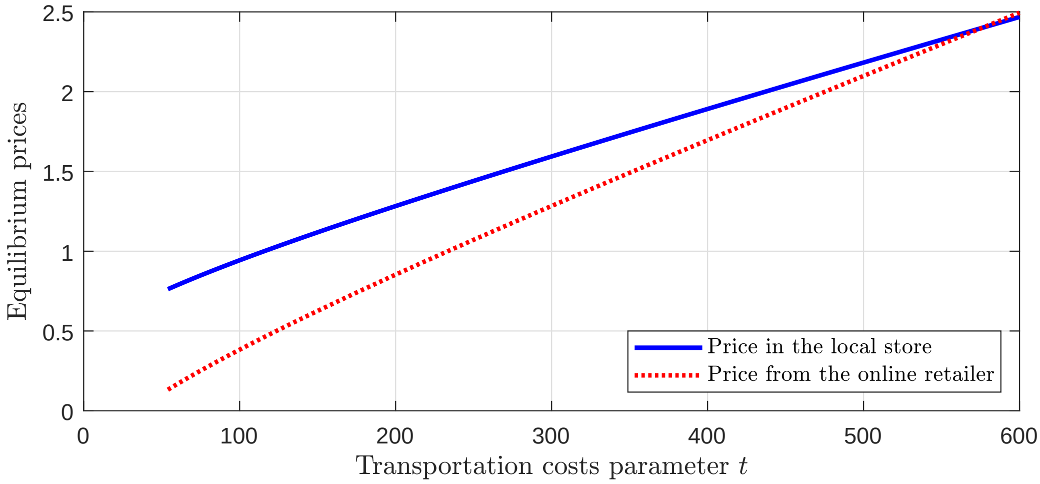

Figure 3.31: Increase in the prices with the transportation costs t

(E.17)).

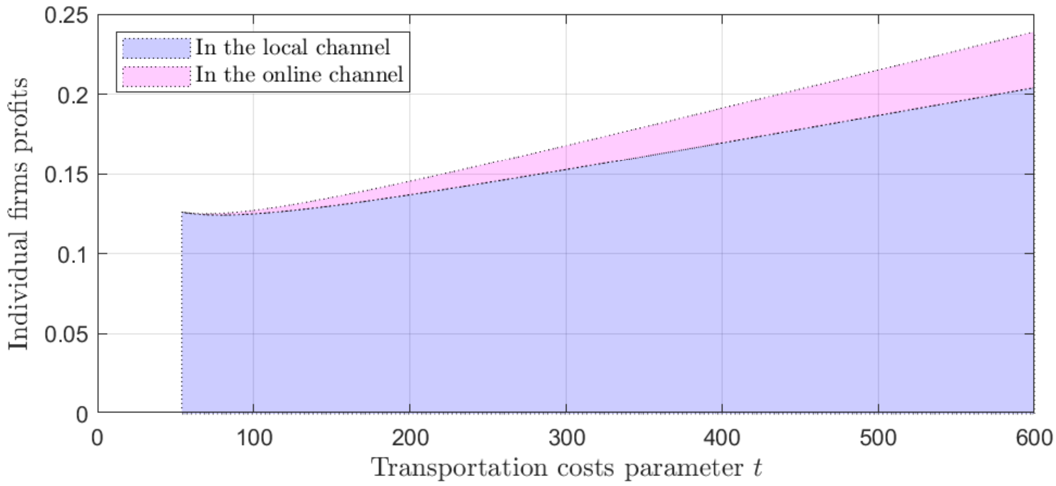

Figure 3.32: Evolution of profits as the transportation costs t increase

The relaxation of price competition finally reflects in increasing profits in both channels (see appendix, results (E.20) and (E.21), and Figure 3.32).

Remark that, inversely, very low transportation costs might eventually deprive the online channel from any demand and violate the non-negativity condition (3.26), which is the reason why the Figures in this subsection are truncated on the left hand-side (for values of the transportation costs below the critical level where the function is truncated, the only realizable market structure is the initial benchmark).

3.7.4 Number of Firms n

Finally, we turn to the effect of an increase in the number of firms, n, which represents a direct increase in competition.[31] As can be expected, increased competition goes along with higher prices respectively higher online prices net of visibility costs (see appendix, results (E.23), (E.24) respectively (E.26); and Figure 3.33) and with a reduction in the demand for the individual stores in both channels (see appendix, results (E.27) and (E.28)). As a result, firms profits decrease (see Figure 3.34). The relative profits in local channel are rather less affected: the increased competition is more harmful to the online profits, which share from the overall profits decreases (see Figure 3.34).

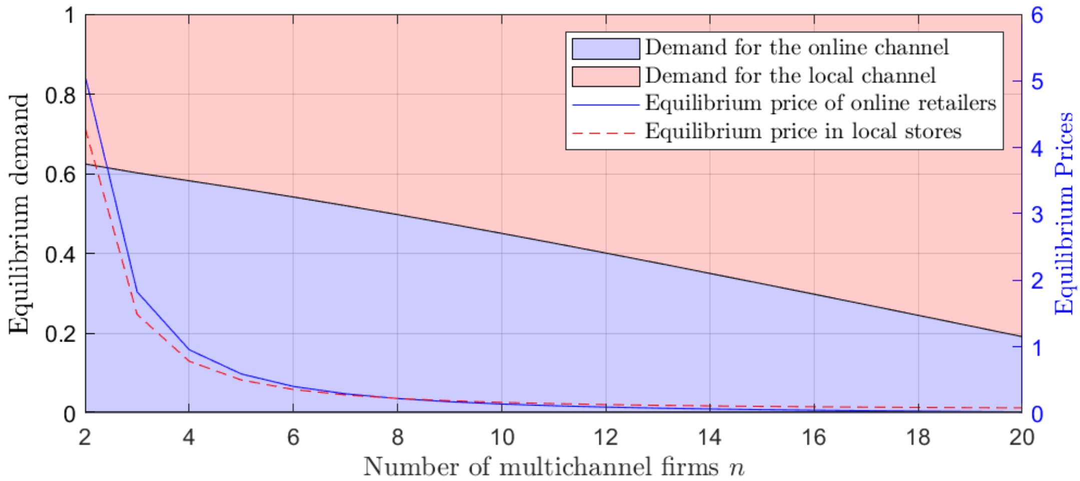

Figure 3.33: Evolution of price and demand as the number of firms n increases

[31] Here, to allow the consideration of a certain number of firms, we dropped the reluctance parameter A to 0.1, instead of 1 as in the basic example.

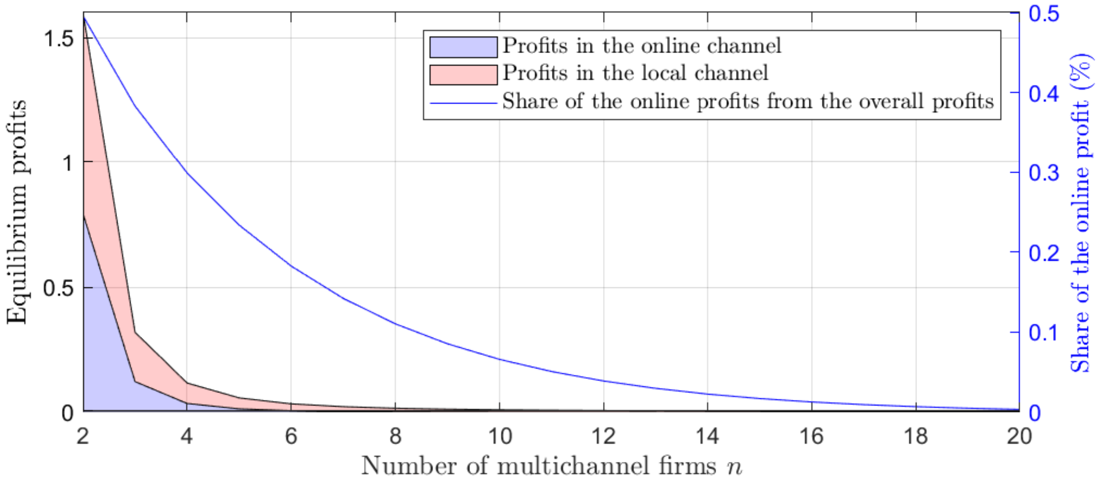

Figure 3.34: Evolution of firms' profits as the number of firms n increases

3.8 Welfare and Endogenous Market Structure

3.8.1 Welfare

Firms Profits

For a fix number of firms, the evolution of firms profits has been analyzed in the comparative statics above: the firms' overall profits, over both channels, decrease in the number of firms n (see result (E.29 and Figure 3.34); they increase with the transportation costs parameter t (see result (E.22 and Figure 3.32) and they decrease as the reluctance parameter A decreases (see result (E.10 and Figure 3.28). We refer to the preceding sections for the interpretation of these results.

Consumers' Surplus

In the case of parity, the surplus of an individual consumer depends its location: the location decides indeed on the channel chosen, local or online, and if relevant on the transportation costs. With $\overline{u}$ being the individual gross utility from consumption (or, equivalently, the utility of the mass of consumers, which is normalized to 1), we compute the surplus of all consumers as beeing the difference between this utility from consumption and the sum of the prices paid to the stores, transportation costs and reluctance costs:

$$CS^{**} = \overline{u} - n \cdot \mathbb{E}D_o^{**} \cdot p_o^* - n \cdot \mathbb{E}D_l^{**} \cdot p_l^{**} - 2n \cdot \int_0^{\sqrt{\frac{A+p_o^{**}-p_l^{**}}{t}}} t \cdot x_i^2 \, dx_i - n \cdot \mathbb{E}D_o^{**} \cdot A$$
$$= \overline{u} - \frac{A(3n+1)}{5n+1} - \frac{3n(\sqrt{\Delta}+t)}{2(n-1)(5n+1)^2} - \frac{2nt}{3} \cdot \left[\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{2(5n+1)^2 t}}\right]^3$$
$$\text{where} \quad \Delta = t(4A(n+1)(5n+1)+t) \tag{3.27}$$

The consumer surplus increases with the number of firms (see Figure 3.35), which is coherent with the expected result as competition among the firms gets fiercer. It declines as the level of transportation costs or the reluctance parameter increases (see, respectively, Figures 3.36 and 3.37): on the on hand, these parameters increase directly the respective overall costs for consumers; on the other hand, their increase relaxes competition among the firms and allows for higher firms prices. The derivatives are gathered in appendix F.1, p. 183f.

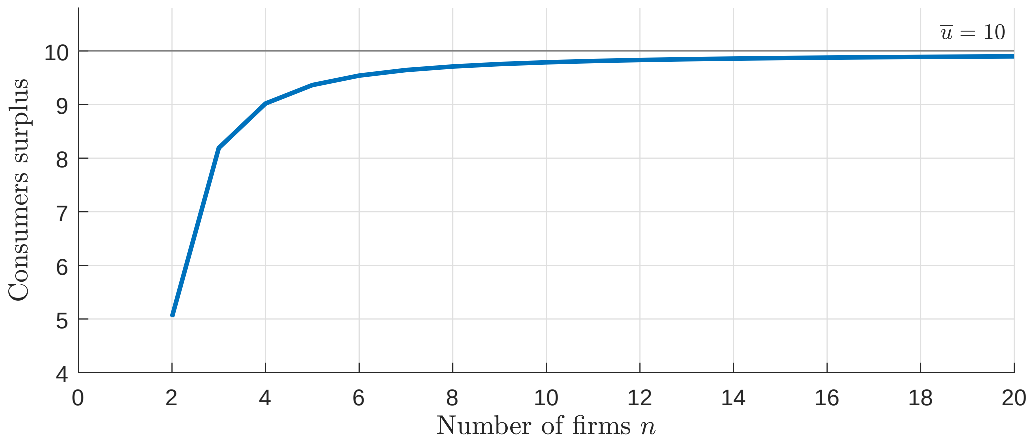

Figure 3.35: Evolution of the consumer surplus as the number of firms n increases

The conflict with the evolution of firms' profits is evident: these effects on the consumer surplus are exactly opposed to the effects on overall firms' profits. The analyze of the overall welfare gives us hints about which of the effects dominates.

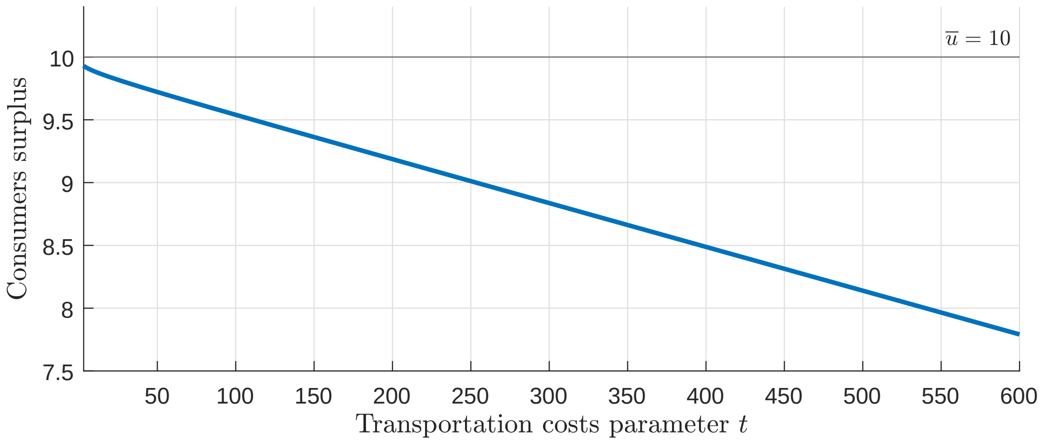

Figure 3.36: Evolution of the consumer surplus as the transportation costs t increase

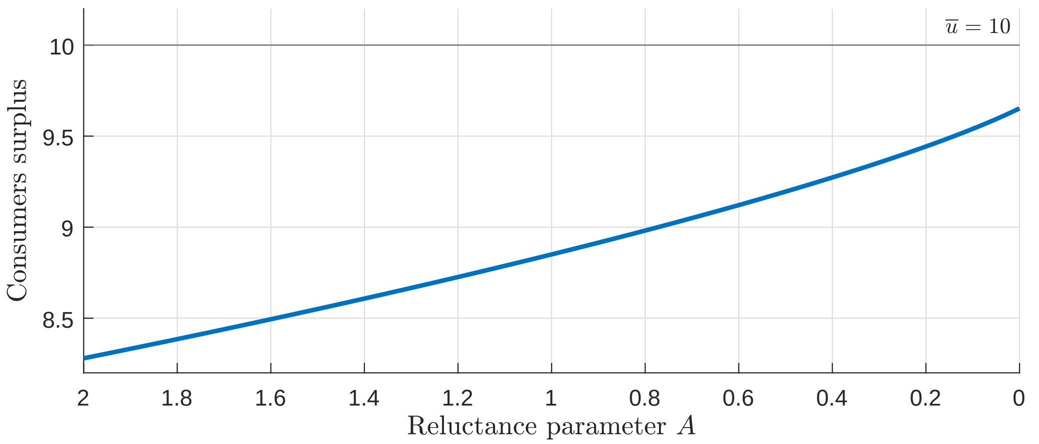

Figure 3.37: Evolution of the consumer surplus as the reluctance parameter A decreases

Overall Welfare

The evolution of consumers' surplus and firms' profits prove to be exactly opposite with respect to each of the parameters A, t and n. The absolute value of the variations in the consumers surplus proves to be mostly much higher than that of the variations in firms profits, so that consumers surplus drives the change in welfare. In case of an increase in the number of firms or in the level of transportation costs, the impact on welfare is determined by the trend in the consumer surplus. This phenomenon is illustrated in Figures 3.38 and 3.39, where the values of consumers surplus and firms profits are stacked, and their sum is the overall welfare. There is one exception: precisely the reluctance parameter A, which is at the core of our model, can have such a positive impact on prices in the online channel that over-compensates the low value of consumer surplus, and leads to an increase in welfare for high levels of A. This is illustrated in Figure 3.40.

The formula for overall welfare can be set up as follows:

$$\begin{aligned} W^{**} &= CS^{**} + n \cdot \pi^{**} = \overline{u} - n \cdot \mathbb{E}D_o^{**} \cdot p_o^{**} - n \cdot \mathbb{E}D_l^{**} \cdot p_l^{**} - 2n \cdot \int_0^{\sqrt{\frac{A+p_o^{**}-p_l^{**}}{t}}} t \cdot x_i^2 \, dx_i \\ &- n \cdot \mathbb{E}D_o^{**} \cdot A + n \cdot \mathbb{E}D_o^{**} \cdot p_o^{**} - n \cdot w^{**} + n \cdot \mathbb{E}D_l^{**} \cdot p_l^{**} - n \cdot f \\ &= \overline{u} - \frac{2nt}{3} \left[\sqrt{\frac{A + p_o^{**} - p_l^{**}}{t}} \right]^3 - n \cdot \mathbb{E}D_o^{**} \cdot A - n \cdot w^{**} - n \cdot f \\ &= \overline{u} - \frac{nt}{12} \left[\mathbb{E}D_l^{**} \right]^3 - \left[1 - n \cdot \mathbb{E}D_l^{**} \right] \cdot \left[A + \frac{(n-1)}{n} \cdot p_o^{**} \right] - n \cdot f \end{aligned} \tag{3.28}$$

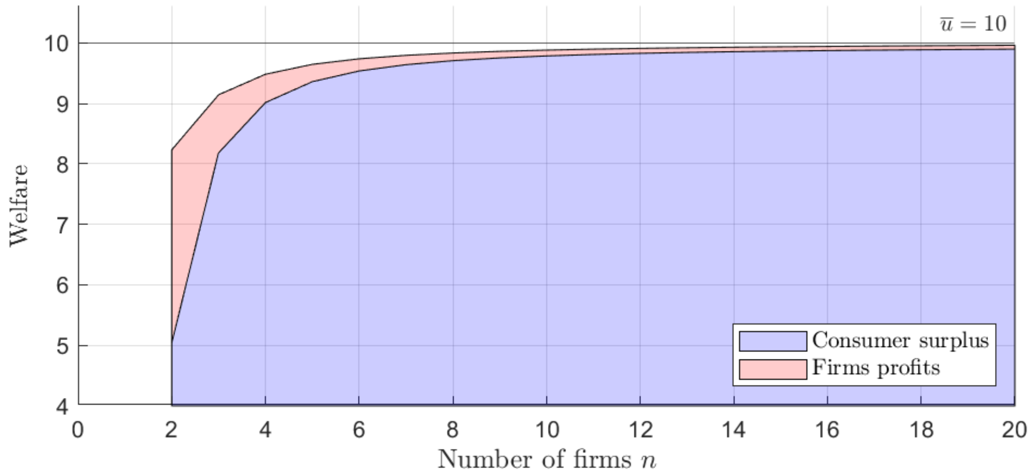

Figure 3.38: Evolution of welfare as the number of firms n increases

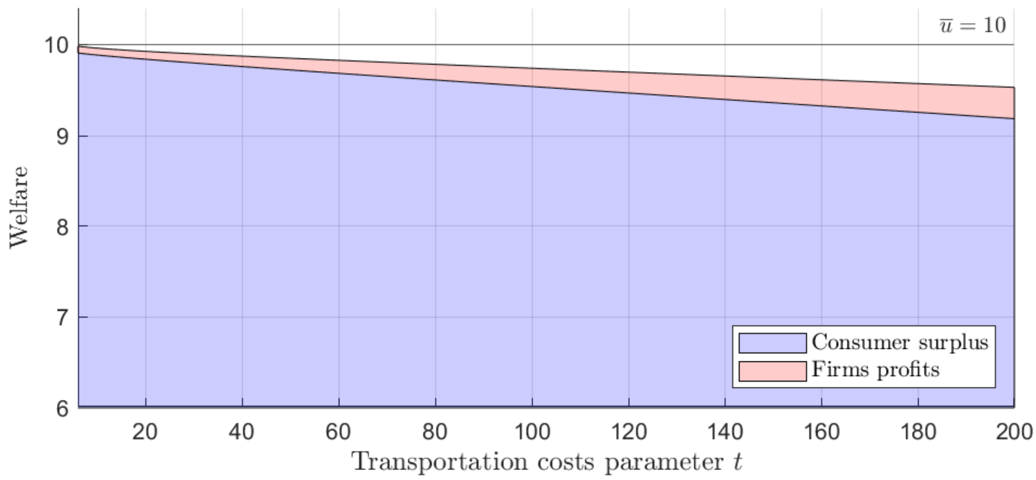

Figure 3.39: Evolution of welfare as the transportation costs t increase

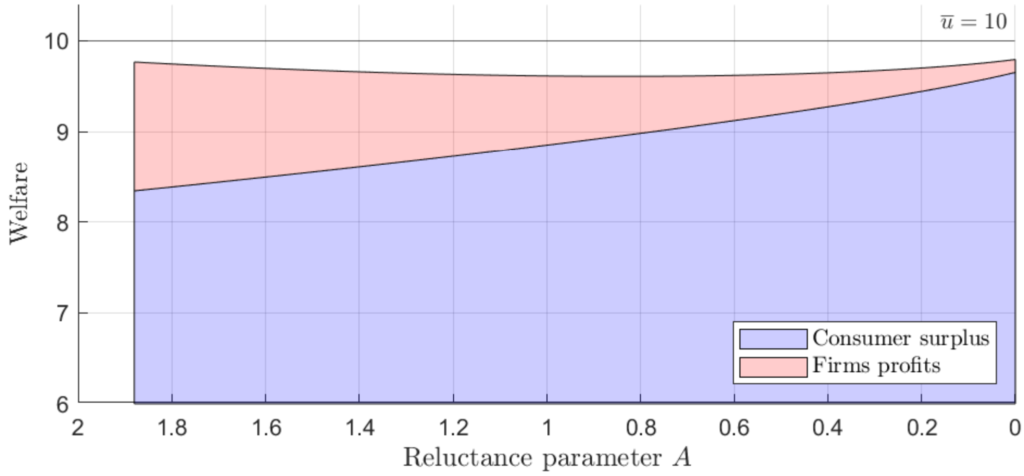

Figure 3.40: Evolution of welfare as the reluctance parameter A decreases

3.8.2 Endogenous Number of Firms

In case of free market entry, because all firms have an incentive to become active in the online channel, we expect a symmetric constellation of the market structure.

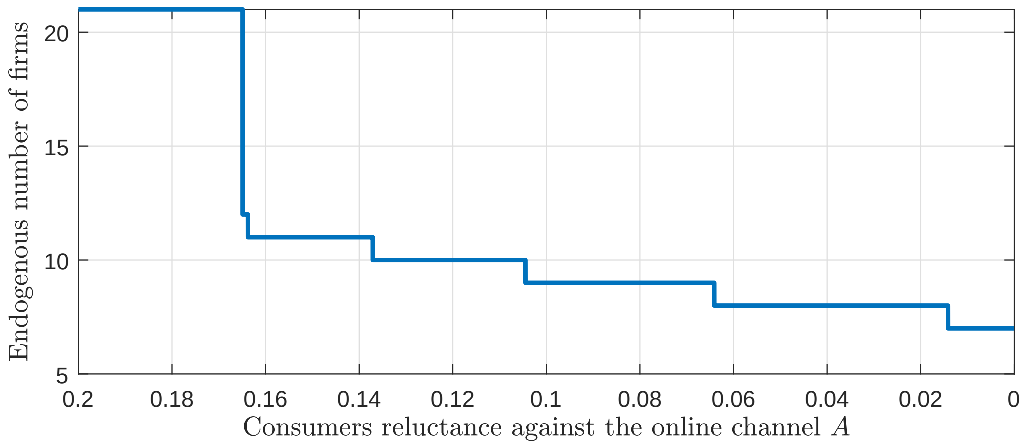

Figure 3.41: Endogenous number of firms as the reluctance A decreases

The market structure evolves depending on the reluctance parameter A, as illustrated in Figure 3.41.[32] The discontinuities in the different functions in this section are due to the fact that the number of firms is an integer: all functions related to the market structure are step functions. As the reluctance parameter A decreases from infinitely high value, there is a first phase in which e-commerce is always dominated by shopping in local stores. In this phase, the number of firms is constant and corresponds to the one defined in the benchmark, here 21 (see result (3.1) on p. 59). Then comes the critical point (here at

[32] As in our basic example, we assume a transportation costs parameter $t = 100$; we set the fix costs to $f = 0.01$ to expand the domain where the free market entry does not lead to the benchmark scenario.

about 0.165) when the demand for the online channel turns positive. This new source of competition leads to an abrupt drop in the number of firms. As consumers reluctance becomes lower, the endogenous number of firms decreases further on, but remains above the minimum number of two firms even when the reluctance parameter A is nil.

As illustrated in Figure 3.42, the rise of the online channel is concomitant with a decrease in the prices in local stores. Two effects drive the evolution of prices: the variation with respect to the continuous change in the reluctance parameter, and the discrete effect of the constriction of the market structure. The price in local stores is never, in the symmetry constellation, as high as in the benchmark. The parallel increase in prices and in expected demand for the online channel illustrates its increasing comparative advantage as the reluctance parameter declines.

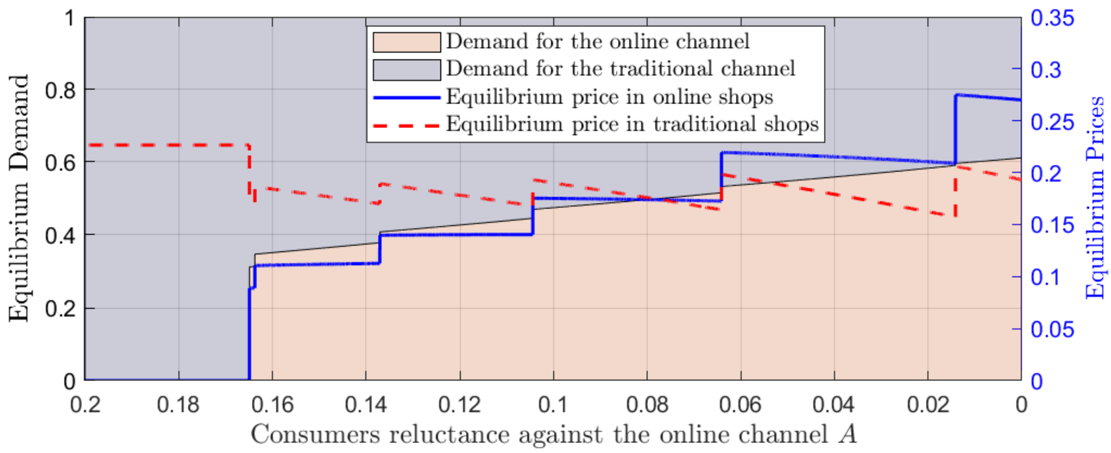

Figure 3.42: Demand shares and prices as reluctance A decreases

This comparative advantage is illustrated also in the relation between residuals profits in each channel, as depicted in Figure 3.43. Free market entry implies that if an additional firm were to enter the market, profits would turn negative; and even though their level is negligible, firms gather a certain amount of residual profits. These profits are the highest immediately after a reduction of the number of firms, i.e. after a point of discontinuity. Then, as the reluctance parameter A augments, online profits increase. This increase is partially hampered by the increasing visibility costs, which increase between any two jumps as well as with any jump. Meanwhile, between any two jumps, local profits decrease sharply: as well the prices (between any two jumps in the function) as the expected demand are decreasing in the local channel. In Figure 3.43, the profits of the two channel are stacked: when the area of the local channel infringes on those in the online channel, it means that they have turned negative and that firms finance their local store in deficit with the profits of the online channel. They go on with this cross-channel subvention until their overall profits are zero and the market structure is forced to a further constriction. The competitive advantage of the online channel leads to an atrophy of the local channel such that even a cross-channel subvention cannot prevent from an atrophy of the market structure.

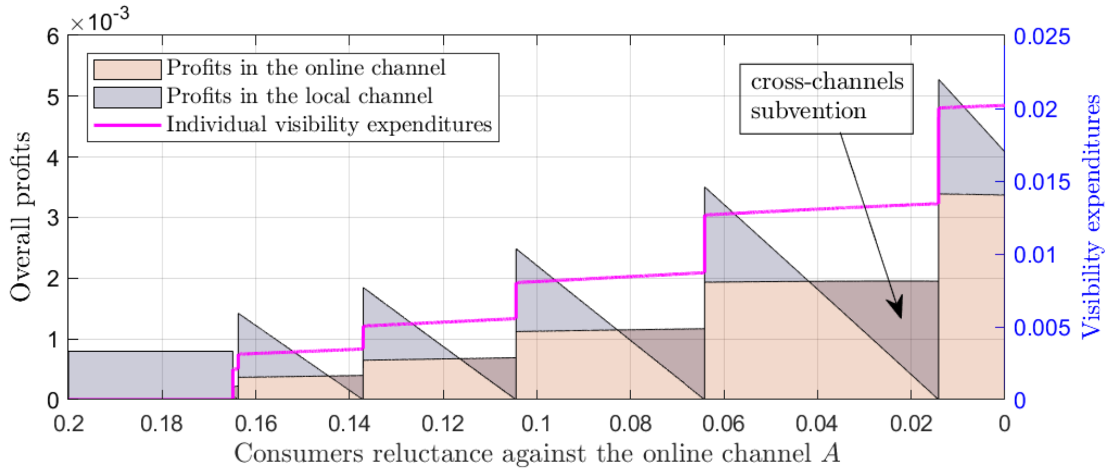

Figure 3.43: Profits and visibility expenditures as reluctance A decreases

The consequences for the consumers surplus are plotted in Figure 3.44. Consumers surplus underlies strong variations between any two jumps; it is at its highest, on each step of the function, just before a jump in the market structure - i.e. when firms profits are driven to exactly zero. The amplitude of these variations is so considerable that a conclusion on the effect of a reduction in the reluctance parameter is quite precarious. It is however noticeable that in most parameter constellations, the consumers surplus is not lesser than in the benchmark.

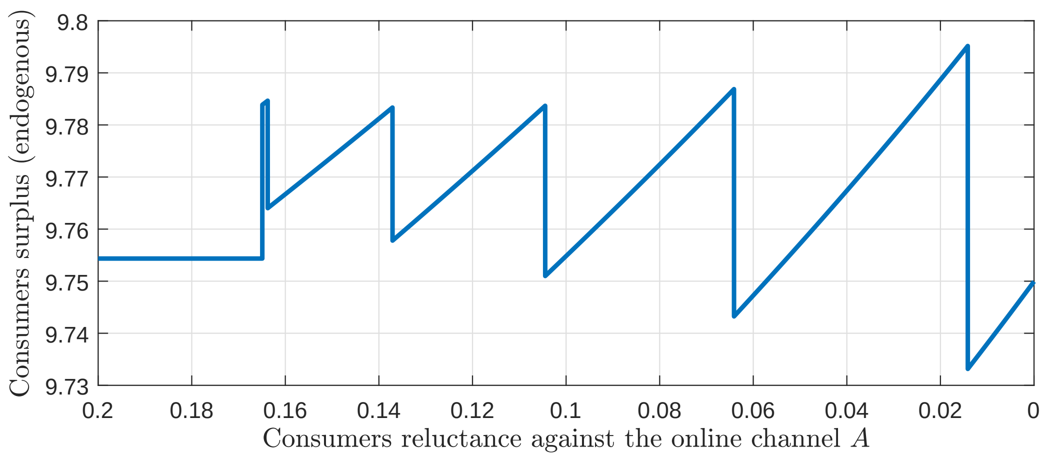

Figure 3.44: Consumers surplus in the endogenous market structure

Welfare, as sum of the consumers surplus and aggregated firms profits, is slightly less volatile than the consumers surplus (see Figure 3.45). Firms profits reach, even when aggregated over all firms, a lower absolute level, but this is in part due to our assumption on the reservation utility $\overline{u}$. As firms profits follow an exactly opposite evolution, the amplitude between any two jumps in the welfare function is lesser than in the consumers surplus function. The main finding is that generally, welfare is not lesser than in the benchmark situation.

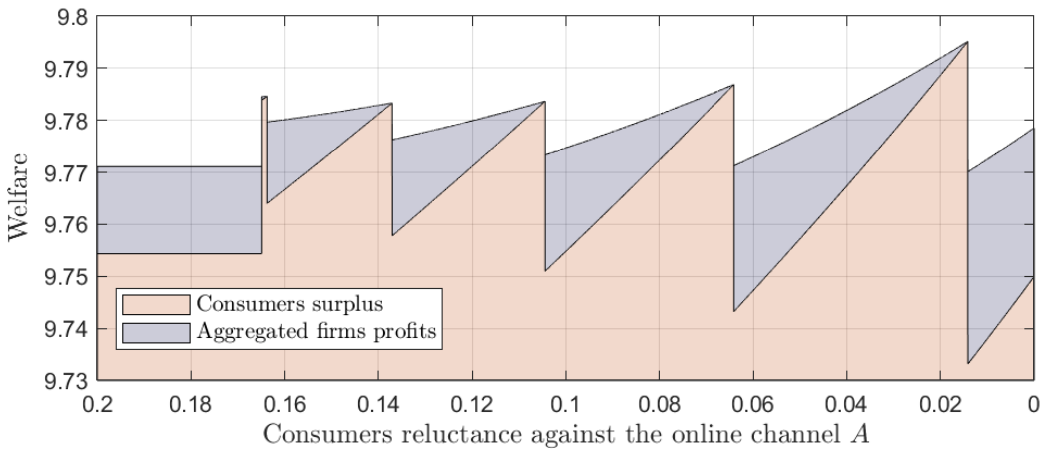

Figure 3.45: Welfare in the endogenous market structure

3.9 A Limiting Case

There is an interesting limiting case when the reluctance parameter tends to zero and fix costs are negligible. Due to these simplifications, this limiting case is characterized by aesthetical formulas and illustrations. Let us present it as a “trou normand”, this glass of calvados with sorbet served between two courses to aid digestion, before we turn to our core problematic, the question of whether firms behave in a cannibalistic way.

3.9.1 In the Transitional Case

When the reluctance parameter A as well as the fix costs f tend to zero, the equilibrium prices in the transitional case (equations (3.7), (3.11) and (3.12)) simplify as follows:

$$p^*_{s,\text{limit case}} = \frac{2n_o t}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^2}$$

$$p^*_{m,l,\text{limit case}} = \frac{(2n_o + 1)t}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^2}$$

$$p^*_{m,o,\text{limit case}} = \frac{3n_o t}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^2}$$

The common denominator makes the ranking of these prices evident: the single-channel firms set the lowest price, in their local stores; the multichannel firms have the possibility to set higher prices in their local stores, and the highest prices are set by the online retailers. An exemplary illustration can be found in Figure 3.46.[33]

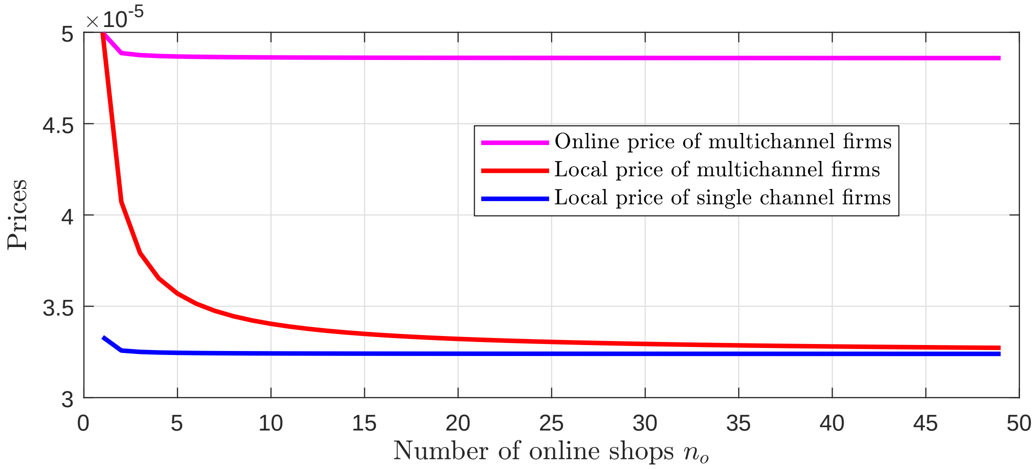

Figure 3.46: Prices in the transitional limit case

The visibility parameter in equilibrium from (3.13) amounts to:

$$w^*_{\text{limit case}} = \frac{3(n_o-1)t\cdot\left[3\sqrt{n_o}(n_l-n_o)+(3n_o+1)\sqrt{n_o-1}\right]}{n_o\left[5\sqrt{n_o}(n_l-n_o)+(5n_o+1)\sqrt{n_o-1}\right]^3}$$

Visibility expenditures reach their maximum in a small oligopoly, as illustrated in Figure 3.47: for a constant number of firms, visibility expenditures decrease as the share of multichannel firms augments (this illustration begins at $n_o = 2$, because there are no visibility expenditures in case online one firm acts as an online retailer).

It is however not linked to the fact that strategic interactions would be higher in small oligopolies, but to the simple fact that online retailers gather higher profits when they are less numerous, and can afford to bear higher expenditures (absolutely) for visibility. When putting these expenditures in relation to the profits they gather, we see that firms dedicate a higher share of their online profits to visibility expenditures as the number of online retailers increase (see Figure 3.48). This share becomes considerable: it tends to 50% as nearly all firms decide to run an online retailer. Seen as a share of overall profits,

[33]In all numerical examples in this section, the transportation costs parameter is set to $t = 1$. In all examples for the transitional state, we assume the existence of $n_l = 50$ firms and observe the effect of a successive conversion of these firms to the online online channel, i.e. we observe the impact of an increase of the number of multichannel firms n_o up to 49.

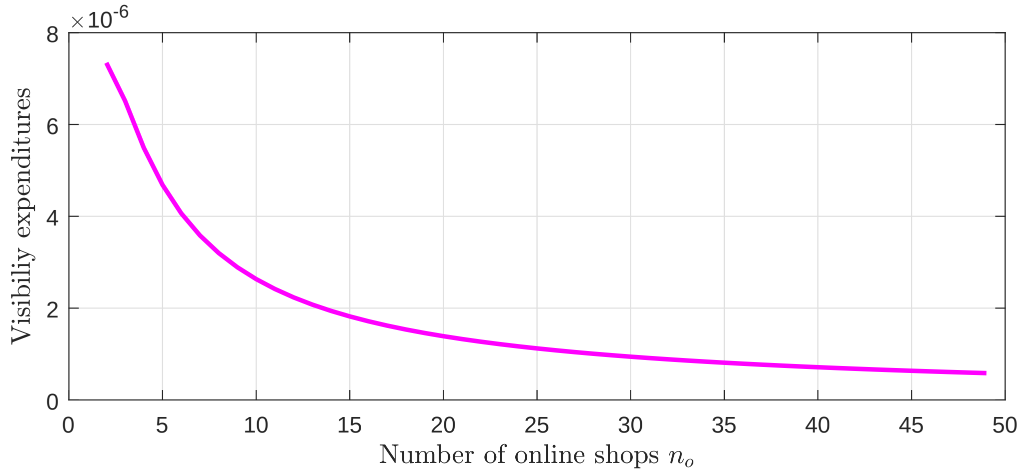

Figure 3.47: Visibility expenditures in the transitional limit case

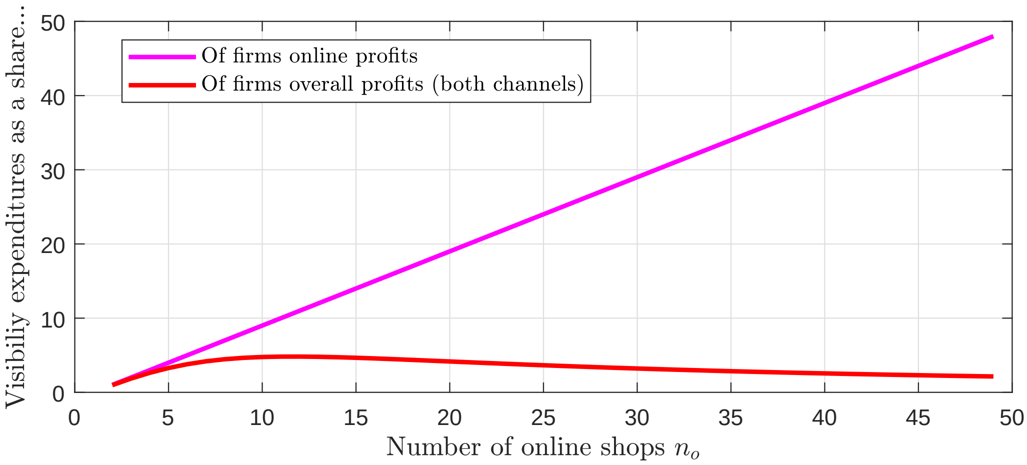

Figure 3.48: Relative level of visibility expenditures in the transitional limit case

the visibility expenditures converge to a modest 2.14%, which hints at the fact that local profits are not at all negligible when nearly all firms have converted to the digital world.

The distribution of the expected demand in equilibrium can be simplified from the results (3.8), (3.14) and (3.15):[34]

$$\mathrm{E}D_s^* = \frac{2\sqrt{n_o}}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]}$$

$$\mathrm{E}D_{m,l}^* = \frac{2\sqrt{n_o - 1}}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]}$$

$$\mathrm{E}D_{m,o}^* = \frac{3\sqrt{n_o}(n_l - n_o) + (3n_o + 1)\sqrt{n_o - 1}}{n_o \cdot \left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]}$$

[34] The sum of the expected demands $(n_l - n_o)\mathrm{E}D_s^* + n_o(\mathrm{E}D_{m,o}^* + \mathrm{E}D_{m,l}^{o*})$ is 1.

Here again, the similarities in the denominators make comparison easy: the individual expected demand is highest for online retailers, and lowest for online stores of multichannel firms (see illustration in Figure 3.49). Remark that these results are not dependent on t, which is due to the fact that they describe the distribution of a constant overall demand of mass 1.

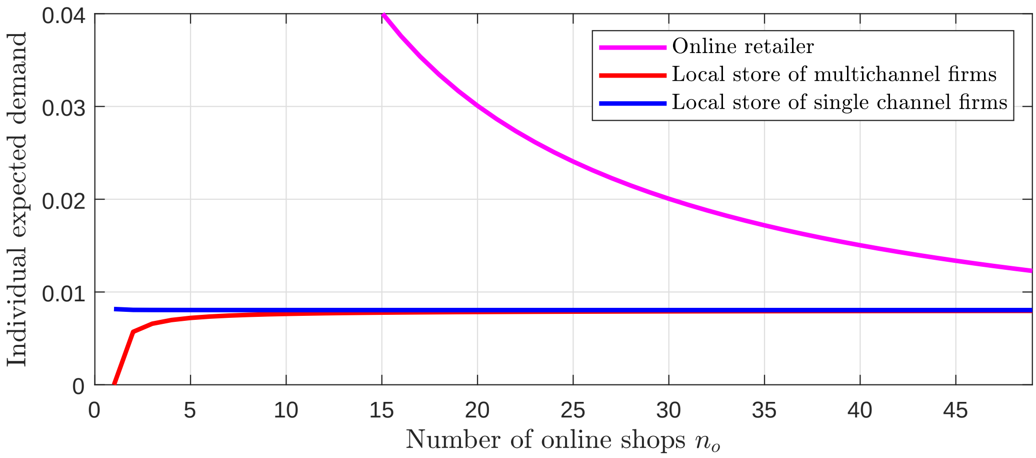

Figure 3.49: Individual expected demands in the transitional limit case

In equilibrium, single-channel firms attract a reduced, but still significant share of the demand:

$$(n_l - n_o)\mathrm{E}D_s^* = \frac{2\sqrt{n_o}(n_l - n_o)}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]} < \frac{2}{5}$$

This share will be decrease even further if the share of multichannel firms (and therefore of online retailers) is high (see Figure 3.50) - which is quite intuitive: the reduced number of single-channel firms attracts a reduced share of demand.

In a complementary way, the cumulative demand attracted by multichannel firms over both channels increases with n_o and reads:

$$n_o\mathrm{E}D_{m,l}^* + n_o\mathrm{E}D_{m,o}^* = \frac{3\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]} > \frac{3}{5}$$

We can also differentiate the distribution of demand with respect to the channels. The cumulative demand of the local channel is limited by the same upper boundary as the expected demand for single-channel firms: 2/5; but it decreases slower as the difference between the number of single and multichannel stores $n_l - n_o$ decreases (see Figure 3.51):

$$(n_l - n_o)\mathrm{E}D_s^* + n_o\mathrm{E}D_{m,l}^{t*} = \frac{2\sqrt{n_o}(n_l - n_o) + 2n_o\sqrt{n_o - 1}}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]} < \frac{2}{5}$$

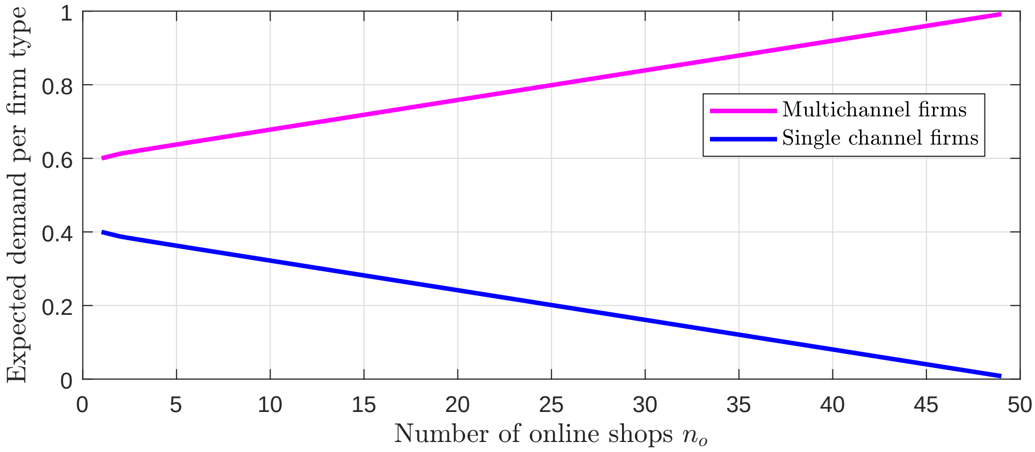

Figure 3.50: Expected demands per firm type in the transitional limit case

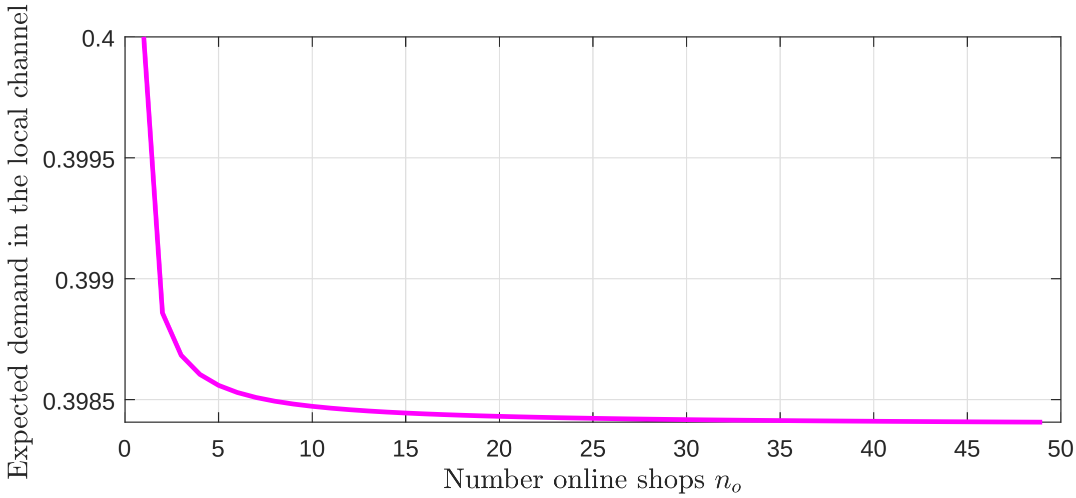

Figure 3.51: Expected demand for the local channel in the transitional limit case

The cumulative demand in the online channel reads:

$$n_o \mathrm{E}D^*_{m,o} = \frac{3\sqrt{n_o}(n_l - n_o) + (3n_o + 1)\sqrt{n_o - 1}}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]} > \frac{3}{5}$$

Because the variations in these curves are on a very tiny scale, we represent them in two separate graphs (in a common graph, they would look flat); the variations of the expected demand in the online channel are depicted in Figure 3.52.

In the transitional case, when the reluctance parameter gets negligible and consumers' acceptance of the online channel is maximal, the online channel attracts the majority of expected demand, but not the complete demand. Whether the local stores of single or multichannel firms are better off, in terms of expected demand, depends on their relative number.

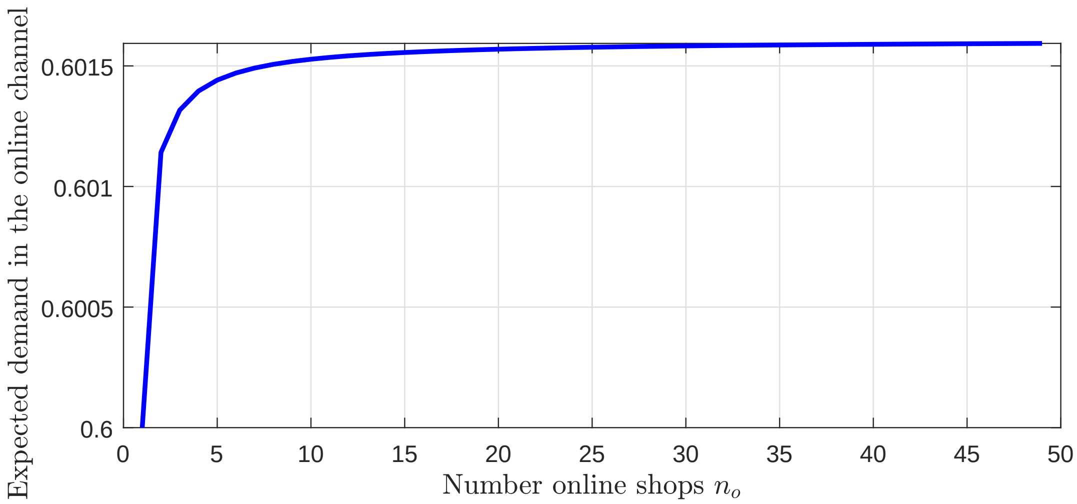

Figure 3.52: Expected demand for the online channel in the transitional limit case

Finally, the respective individual profit functions read:

$$\mathrm{E}\pi_s^* = \frac{4n_o^{3/2}t}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^3}$$

$$\mathrm{E}\pi_{m,l}^* = \frac{2(2n_o + 1)\sqrt{n_o - 1}t}{\left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^3}$$

$$\mathrm{E}\pi_{m,o}^* = \frac{3t \cdot \left[3\sqrt{n_o}(n_l - n_o) + (3n_o + 1)\sqrt{n_o - 1}\right]}{n_o \cdot \left[5\sqrt{n_o}(n_l - n_o) + (5n_o + 1)\sqrt{n_o - 1}\right]^3}$$

While the profits of local stores converge rapidly and are quite close to each other,[35] the online profits decline drastically as the number of competitors increase, due to the enhanced competition with respect to visibility and the corresponding expenditures. Remark that the profits of multichannel firms are always above those of single-channel firms: firms have, in this parameter constellation, a constant incentive to become active as an online retailer.

However, taken as a whole, the multichannel sector does not always perform better. The sum of profits of multichannel firms underlies two effects as the number of online retailers increase: a negative one, linked to the increasing competition which enhances the share

[35] A brief analysis using the same remarkable identity as in appendix C reveals that the single-channel local stores gather slightly more profits:

$$\begin{aligned}
\mathrm{E}\pi_s^* \geq \mathrm{E}\pi_{m,l}^* \quad & \Leftrightarrow 2n_o\sqrt{no} \geq (2n_o + 1)\sqrt{n_o - 1} \\
& \Leftrightarrow 2n_o\sqrt{no} - (2n_o + 1)\sqrt{n_o - 1} \geq 0 \\
& \Leftrightarrow (2n_o\sqrt{no} - (2n_o + 1)\sqrt{n_o - 1})(2n_o\sqrt{no} + (2n_o + 1)\sqrt{n_o - 1}) \geq 0 \\
& \Leftrightarrow 3n_o + 1 \geq 0 \quad q.e.d.
\end{aligned}$$

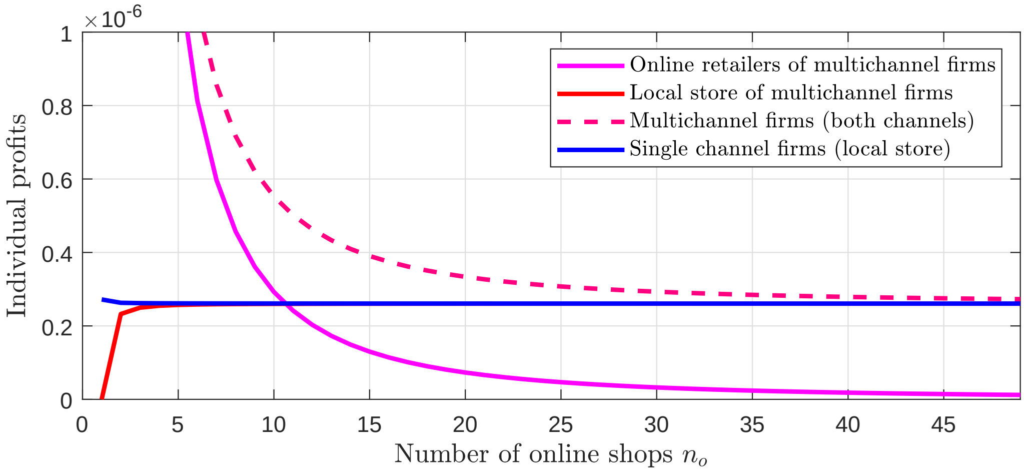

Figure 3.53: Expected individual profits in the transitional limit case

of online profits devoted to visibility expenditures and leads to lower online profits as the number of online retailers increase; and a positive one, which is simply that we add together the profits of more numerous multichannel firms. These two effects result in a u-shaped curve of the share of overall profits gathered by multichannel firms, with a complementary. inversed-u-shaped curve for the share gathered by single-channel firms (see Figure 3.54).

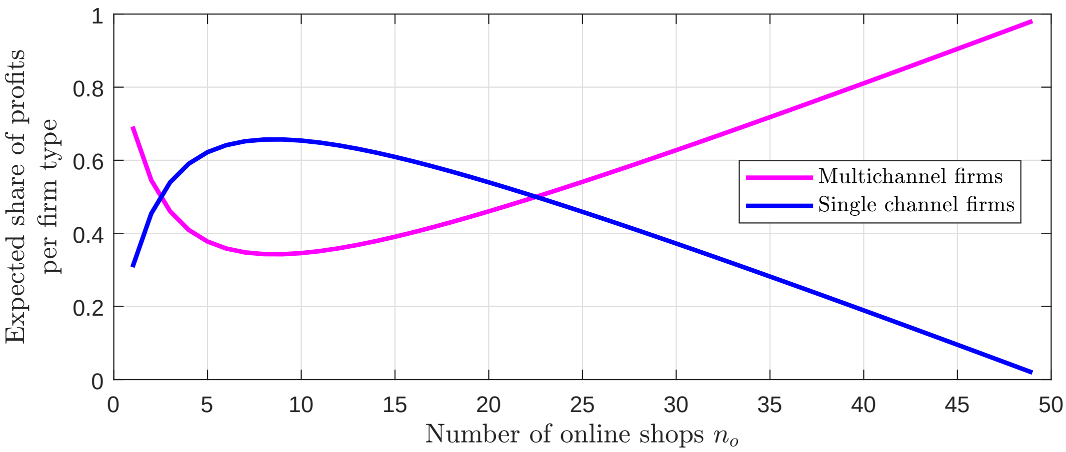

Figure 3.54: Expected share of profits per firm type in the transitional limit case

The effect of this enhanced competition among online retailers is visible also in the share of the profits realized in the online channel: notice the higher power of n_o in the denominator and the fact that this relative result does not depend any more on the transportation costs parameter t.

$$\frac{n_o \mathrm{E}\pi^*_{m,o}}{(n_l - n_o)\mathrm{E}\pi^*_s + n_o(\mathrm{E}\pi^*_{m,l} + \mathrm{E}\pi^*_{m,o})} = \frac{9\sqrt{n_o}(n_l - n_o) + (9n_o + 3)\sqrt{n_o - 1}}{(4n_o + 9)\sqrt{n_o}(n_l - n_o) + (4n_o^2 + 5n_o + 1)\sqrt{n_o - 1}}$$

The maximal share of the profits in the online channel is 9/13 (about 69%); it drops quickly as the number of online retailers increases (see Figure 3.55).

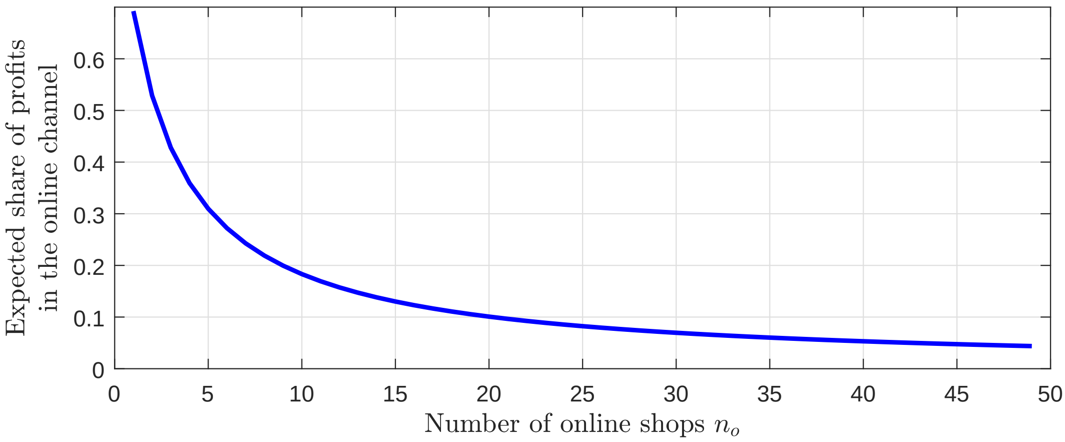

Figure 3.55: Expected profits in the online channel in the transitional limit case

As the overall local profits are converging quickly to a constant value when the number of online retailers augments, while the online profits decrease (even though at at decreasing rate), the share of profits gathered in the local channel proves to be increasing (see Figure 3.56): even and especially when nearly all firms are active as online retailers, the traditional, the local channel is of considerable importance.

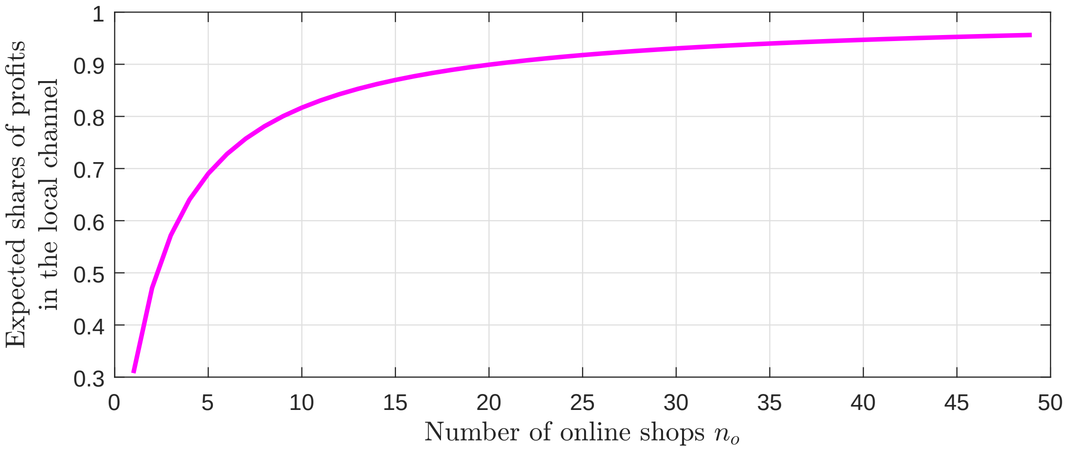

Figure 3.56: Expected share of profits in the local channel in the transitional limit case

3.9.2 In the Symmetric Case

In the case when all firms are active in the online channel, we can also analyze the impact of an increase in the overall number of firms n. The equilibrium prices from equations (3.19) and (3.20) simplify as follows:

$$p^{**}_{l,\text{limit case}} = \frac{(2n+1)\cdot t}{(5n+1)^2\cdot(n-1)} \qquad \text{and} \qquad p^{**}_{o,\text{limit case}} = \frac{3n\cdot t}{(5n+1)^2\cdot(n-1)}$$

As suggested by the higher power of n in the denominator and as illustrated in Figure 3.57, these prices are declining as the number of firms in the market, and therefore the level of competition augments.

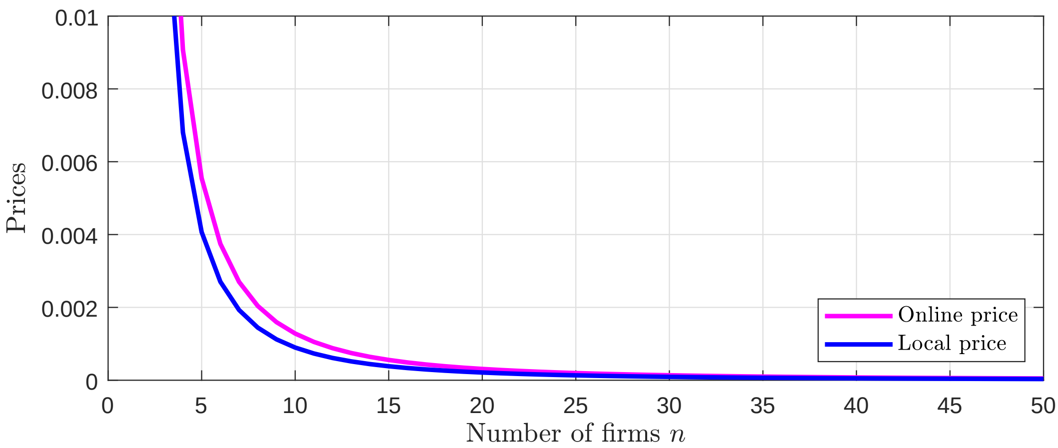

Figure 3.57: Prices in the symmetry limit case

The visibility parameter from (3.21) now reads:

$$w^{**}_{\text{limit case}} = \frac{3(3n+1)t}{n(5n+1)^3}$$

This expression is also evidently declining in n, which is illustrated in Figure 3.58.

As above, we put this decline in relationship to the level of profits: the share of online profits dedicated to visibility expenditure augments quickly; while the share of overall profits dedicated to these expenditures augments only slowly (see Figure 3.59).

The difference between the prices is:

$$p^{**}_{o,\text{limit case}} - p^{**}_{l,\text{limit case}} = \frac{t}{(5n+1)^2}$$

So that the demand functions from (3.22) and (3.23) turn into:

$$\text{E}D^{**}_o = \frac{3n+1}{n(5n+1)} \qquad \text{and} \qquad \text{E}D^{**}_l = \frac{2}{(5n+1)}$$

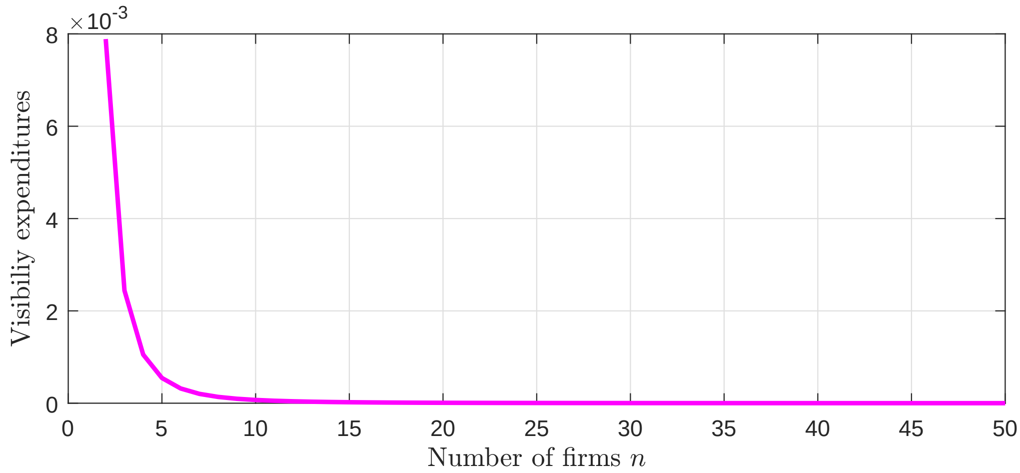

Figure 3.58: Visibility expenditures in the symmetry limit case

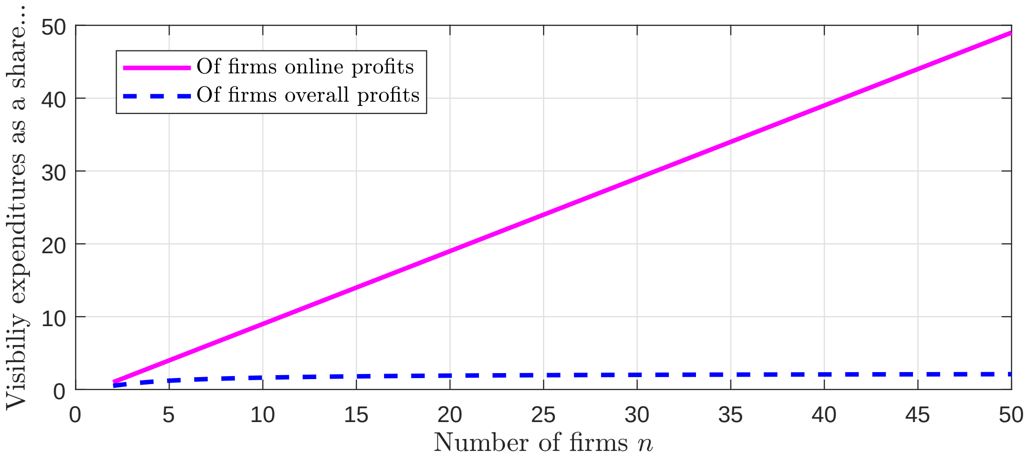

Figure 3.59: Visibility expenditures in the symmetry limit case

Again, we find that these individual levels of expected demand decline quickly as the number of firms augments (see Figure 3.60).

The distribution of the demand mass converges to a constant ratio when the number of firms increases to infinity:

$$\lim_{n \mapsto \infty} n\mathrm{E}D_o^{**} = \lim_{n \mapsto \infty} \frac{3n+1}{5n+1} = 60\% \qquad \text{and} \qquad \lim_{n \mapsto \infty} n\mathrm{E}D_l^{**} = \frac{2n}{5n+1} = 40\%$$

This convergence is already observable in Figure 3.61.

The respective expected profit functions of an online and a local store (3.24) and (3.25) turn into:

$$\mathrm{E}\pi_{o,\text{limit case}}^{**} = \frac{3t(3n+1)}{n(n-1)(5n+1)^3} \qquad \text{and} \qquad \mathrm{E}\pi_{l,\text{limit case}}^{**} = \frac{2t(2n+1)}{(n-1)(5n+1)^3}$$

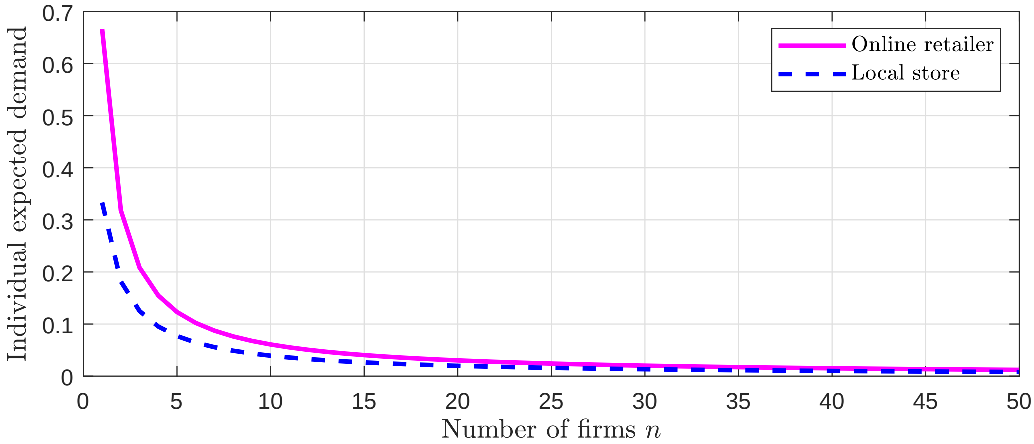

Figure 3.60: Expected demand in the symmetry limit case

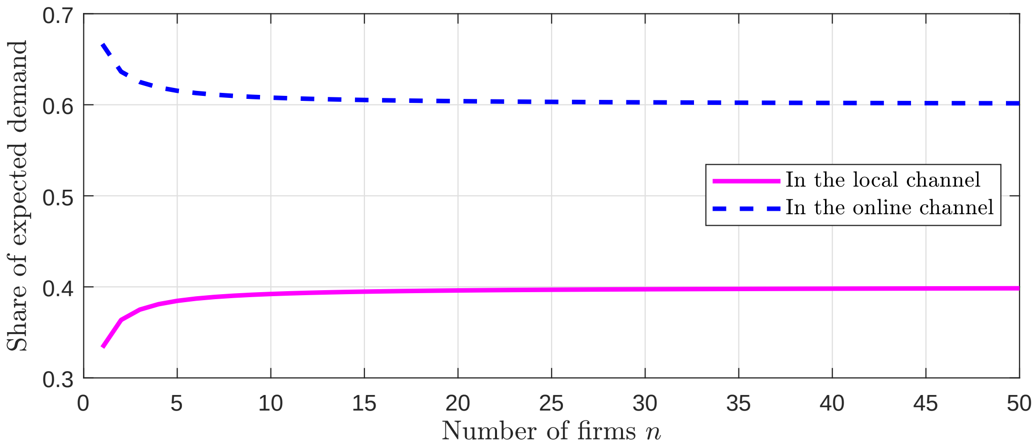

Figure 3.61: Shares of the expected demand in the symmetry limit case

Similarly to the prices and expected demands, these profits decline strongly with the number of firms (see Figure 3.62).

The ratio of the overall expected profits from the online channel out of the overall profits converges to a constant value when the number of firms increases to infinity:

$$\lim_{n\mapsto\infty}\frac{\mathrm{E}\pi^{**}_{o,\text{limit case}}}{\mathrm{E}\pi^{**}_{o,\text{limit case}}+\mathrm{E}\pi^{**}_{l,\text{limit case}}}=\lim_{n\mapsto\infty}\frac{3(3n+1)}{3(3n+1)+2n(2n+1)}=\frac{9}{4n}\approx 0$$

This convergence is illustrated in Figure 3.63.

In this specific situation, there is no phenomenon of desolation of the city center: the polypoly survives. In the equilibrium, the online channel is worse off: even though it attracts 60% of the demand, the profit it generates are negligible compared to the profit in the local channel.[36]

[36]The exact result depends of course strongly on the assumption on the shape of the visibility costs.

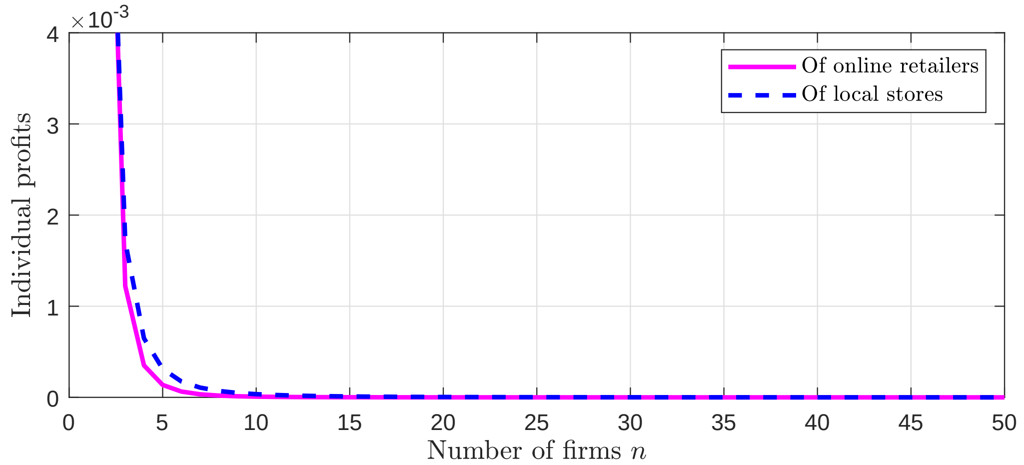

Figure 3.62: Expected profits in the symmetry limit case

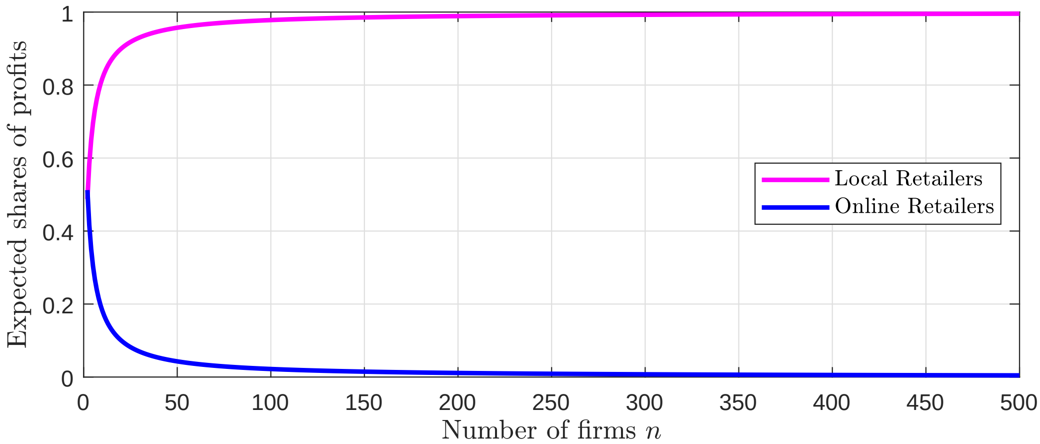

Figure 3.63: Shares of the expected profits in the symmetry limit case

The consumer surplus from equation (3.27) simplifies in this limiting case as:

$$CS^{**}_{\text{limit case}} = \overline{u} - \frac{n(41n+13)t}{3(n-1)(5n+1)^3}$$

and increases up to its maximal value $\overline{u}$ when the number of firms becomes large, especially in a polypoly (see Figure 3.64).

The linear assumption is quite severe; if we assume, instead, quadratic visibility costs $0 < w_i^2 < w_i < 1$, the online channel then gather 52.9412% of the overall profit, i.e. more than the half. But this affects only the distribution of profit; the distribution of demand is not affected by the shape of the visibility costs.

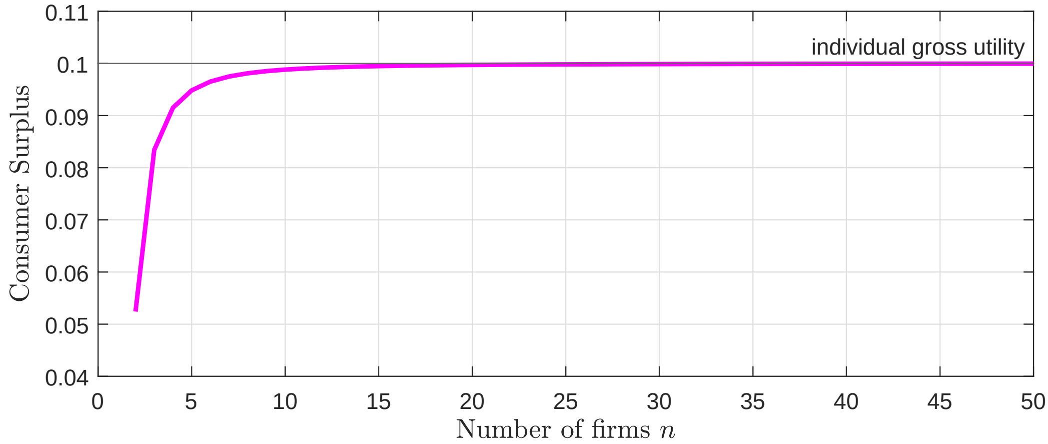

Figure 3.64: Consumers surplus in the symmetry limit case ($\overline{u} = 0.1$)

3.10 The Issue of Cannibalism

As mentioned in the literature overview, the danger of cannibalism is a topos in the literature on e-commerce and multichannel strategies. Under "cannibalism", we understand the threat that the competition among the different channels becomes a nuisance for firms performance.

In our model, a phenomenon of consumers' migration towards the online channel can be observed as the reluctance parameter A decreases. It would be, however, inappropriate to label this migration as a form of cannibalism: the shift between the channels accompanies a shift in the selling technologies. The fact that firms adapt to the technological evolution is, insofar, a form of creative destruction as defined by Schunpeter: a "process of industrial mutation that continuously revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one" (Schumpeter (1976), pp. 82–83). Even if the online channel supersedes the local one, the negative denotation of cannibalism is not justified.

Because of this denotation, we reserve the use of cannibalism to a situation when the coordination effort by multichannel firms were to become counterproductive, i.e. when their conjoint maximization over the two channels would lead to poorer results, than when stores were to set their respective prices in the two channels without any coordination. Can such a situation arise in our model? In a first step, we will answer this question analytically for the symmetric case with exogenous market structure. In a second step, we will turn to the case when the market structure is determined endogenously.

3.10.1 Symmetric case with exogenous number of firms n

We can isolate, in the system of solutions resulting from the first order conditions (3.19) to (3.21), the terms resulting specifically from the firms' coordination of the two channels, i.e. those who appear because the firms optimizes commonly over both channels and not only above the channel corresponding to the optimization variable. These terms are here highlighted in red:

$$\frac{\partial E\pi^i}{\partial p_l^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_l^{**} = \frac{2(A + p_o^{**})}{3} + \frac{p_o^{**}}{3n}$$
$$\frac{\partial E\pi^i}{\partial p_o^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad p_o^{**} = \frac{t + \sqrt{t^2 + 12nt\left[n(A - p_l^{**}) + p_l^{**}\right]}}{18n^2} - \frac{2(A - p_l^{**})}{3} + \frac{p_l^{**}}{3n}$$
$$\frac{\partial E\pi^i}{\partial w^i} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad w^{**} = \frac{(n-1)}{n} \cdot p_o^{**} \cdot \left[\frac{1}{n} - 2\sqrt{\frac{A - p_l^{**} + p_o^{**}}{t}}\right]$$

The highlighted summands are are positive: firms' coordination of the channels leads to a stronger positive interaction between the equilibrium prices. The optimal level of visibility expenditures depending on the prices is left unchanged by the consideration of this coordination. Similarly, we highlight the effect of coordination on the equilibrium prices and expenditures (see equations (3.19), (3.20) and (3.21)):

$$p_l^{**} = \frac{(2n+1)\left[t + \sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} + \frac{2nA}{(5n+1)}$$
$$p_o^{**} = \frac{3n\left[t + \sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} - \frac{2nA}{(5n+1)}$$
$$w^{**} = \left[\frac{3\left(t + \sqrt{\Delta}\right)}{2(5n+1)^2} - \frac{2A(n-1)}{(5n+1)}\right] \cdot \left[\frac{1}{n} - 2\sqrt{\frac{t + \sqrt{\Delta} + 2A(n+1)(5n+1)}{2(5n+1)^2 t}}\right]$$
$$\text{where} \quad \Delta = 4tA(n+1)(5n+1) + t^2$$

The net price in the online channel reads:

$$p_o^{**,net} = p_o^{**} - \frac{w^{**}}{\mathbb{E}D_o^{**}} = \frac{3\left[t + \sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} - \frac{2A}{(5n+1)} = \frac{1}{n} \cdot p_o^{**}$$

Firms' coordination leads to higher equilibrium prices as well in the online channel (this also applies to the net price) as in the local one.[37]

[37] See appendix G.1 on page 189.

The equilibrium expected demand functions (3.22) and (3.23) can be parsed as:

$$\mathrm{E}D_l^{**} = 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}}$$

$$\mathrm{E}D_o^{**} = \frac{1}{n} - 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}}$$

When firms coordinate the price and visibility decisions, the expected demand is higher in the local channel if $A > t/(16n^2)$:[38] if consumers' reluctance is high, the firm's coordination results in augmenting the demand addressed to the channel in which higher prices are possible, the local one. The critical value $A > t/(16n^2)$ is indeed the value from which on the equilibrium prices are higher in the local channel than in the online one, both with or without coordination.[39] The tables of variations in Figures 3.65 and 3.66 give an overview of the variations in prices and expected demands induced by firms' coordination, depending on the reluctance parameter A, in each channel respectively.

A	0		$\frac{t}{16n^2}$		$\frac{t(3n+1)}{4n^2(n+1)}$
δp_l^{**}		$+$		$+$	
$\delta \mathrm{E}D_l^{**}$		$-$	0	$+$	

Figure 3.65: Table of the variations in the local channel induced by firms' coordination

A	0		$\frac{t}{16n^2}$		$\frac{t(3n+1)}{4n^2(n+1)}$
δp_o^{**}		$+$		$+$	
$\delta \mathrm{E}D_o^{**}$		$+$	0	$-$	

Figure 3.66: Table of the variations in the online channel

Finally, the equilibrium expected profit functions (3.24) and 3.25 yield:

$$\mathrm{E}\pi_l^{**}(n) = \left[\frac{(2n+1)\left[t+\sqrt{\Delta}\right]}{(5n+1)^2(n-1)} + \frac{4nA}{(5n+1)}\right]\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}} - f$$

$$\mathrm{E}\pi_o^{**}(n) = \left[\frac{3\left(t+\sqrt{\Delta}\right)}{2(5n+1)^2(n-1)} - \frac{2A}{(5n+1)}\right]\cdot\left[\frac{1}{n} - 2\sqrt{\frac{t+\sqrt{\Delta}+2A(n+1)(5n+1)}{2(5n+1)^2t}}\right]$$

[38]See appendix G.2 on page 190. This critical value is compatible with the necessary non-negativity condition (3.26) on p. 88, as $t/(16n^2) < t(3n+1)/[4n^2(n+1)]$.

[39]See appendix G.3 on page 192.

Extensive computations are not necessary for our purpose. We can use the fact that the variations in expected demand are symmetric: the expected demand deserting one channel due to firms coordination is exactly the additional expected demand in the other channel. We introduce the superscripts nc for "no coordination" and c for "with coordination", we omit the $*$ in the superscript for simplification and denote the change in the expected demand in the local channel as δ, a positive or negative variable (the change in the online channel being $-\delta$). The change in expected profits reads:

$$\begin{aligned}
\mathrm{E}\pi^c - \mathrm{E}\pi^{nc} &= \left[\mathrm{E}D^{l,c}\cdot p^{l,c} - f + \mathrm{E}D^{o,c}\cdot p^{o,c,net}\right] - \left[\mathrm{E}D^{l,nc}\cdot p^{l,nc} - f + \mathrm{E}D^{o,nc}\cdot p^{o,nc,net}\right]\\
&= \left(\mathrm{E}D^{l,nc} + \delta\right)\cdot p^{l,c} + \left(\mathrm{E}D^{o,nc} - \delta\right)\cdot p^{o,c,net} - \mathrm{E}D^{l,nc}\cdot p^{l,nc} - \mathrm{E}D^{o,nc}\cdot p^{o,nc,net}\\
&= \mathrm{E}D^{l,nc}\underbrace{\left(p^{l,c} - p^{l,nc}\right)}_{>0} + \mathrm{E}D^{o,nc}\underbrace{\left(p^{o,c,net} - p^{o,nc,net}\right)}_{>0} + \delta\underbrace{\left(p^{l,c} - p^{o,c,net}\right)}_{>0}
\end{aligned}$$

In case expected demand is shifted by firms' coordination towards the local channel, i.e. in case reluctance is high: $A > t/(16n^2)$, the change δ is positive and the sign of the change in the overall profits $\mathrm{E}\pi^c - \mathrm{E}\pi^{nc}$ is unambiguously positive.

In case reluctance is low, i.e. $A < t/(16n^2)$, the variation δ is negative. Remark that, as the price difference $p^{l,c} - p^{o,c}$ is negative too in this case, the only factor hindering an increase of expected profits are the enhanced visibility expenditures:

$$\mathrm{E}\pi^c - \mathrm{E}\pi^{nc} = \mathrm{E}D^{l,nc}\underbrace{\left(p^{l,c} - p^{l,nc}\right)}_{>0} + \mathrm{E}D^{o,nc}\underbrace{\left(p^{o,c,net} - p^{o,nc,net}\right)}_{>0} + \underbrace{\delta\left(p^{l,c} - p^{o,c}\right)}_{>0} + \underbrace{\delta}_{<0}\cdot\frac{w^c}{\mathrm{E}D^{o,c}}$$

As prices as well as expected demand increase in the online channel, it is self-explaining that the profits in this channel increase too. Some additional computations prove that profits are increasing in the local channel too.[40] The negative impact of the visibility expenditure might dampen profits increase, but it does not prevent it: firms coordination is positive for both channel.

As a conclusion, firms coordination of the channels proves to augment the profits in both channels, not depending on the reluctance parameter i.e. on which channel is more attractive to firms. There is, when considering the exogenous case, no cannibalism, even though the establishment of the online channel implicates a radical transformation of the market structure.

This section proved that, ceteris paribus, expanding to the online channel does not lead to a cannibalistic behaviour of firms. A constant number of firms is a reasonable assumption

[40]See appendix G.4 on page 192.

in the short run[41] or when market entry cannot be considered free. Otherwise, we need to consider what happens when the market structure changes depending on the acceptance of e-commerce.

3.10.2 Endogenous market structure

When market entry is free, the equilibrium market structure depends on the maximal number of firms able to achieve positive profits, for a given value of the exogenous parameters. The profits gathered by firms oscillate around zero, depending on the jumps generated when the number of firms (a discrete variable) varies. Nevertheless, a look at the variations of market structure, consumers surplus, demand distribution and prices displays an interesting light on the fears of a cannibalistic behavior and of a desolated downtown. We comment here the findings of section 3.8.2 with respect to this topic.

On the first hand, the constriction observed in Figure 3.41 can indeed be looked upon as a downtown desolation: as the online channel becomes more attractive, the number of local stores is reduced. Our model does not yield a complete extinction of local stores, but the reduction of their number is indeed a by-product of the increased competition. Whether this evolution should be considered as "regular" in a Schumpetarian way or a societal decay is too normative a position to be decided upon in this model.

On the other hand, the impact of this constriction on welfare might well be positive. Figure 3.45 depicts the paradoxal results that, as the reluctance parameter A decreases, overall welfare underlies strong variations but mostly remains above its level in the benchmark. We therefore confirm our preceding statement that the negative denotation of "cannibalism" is not justified: even though some firms "die" in the process, the transformation induced by a stronger acceptance of the online channel is not malign. In an exogenous market structure, where we can observe the evolution of firms profits, the expansion to the online channel as well as the coordination of both channels are profit-augmenting for firms. In a case with free market entry, where he number of firms is endogenized, the effect on profits is negligible and disputable ; but the transformation of the market structure does not harm welfare.

[41] The question is whether considering a change in the exogenous parameters A and t is meaningful in the short run. The COVID-19 lockdown is an extreme case of a nearly instantaneous, major change in the transportation costs parameter t.

3.11 Conclusion

To conclude this extensive chapter, we will summarize its main findings, confront them to the results of existing literature and point at the limits of its scope.

Our model integrates a virtual space in the traditional frame of a Salop circular city: local stores are assumed to be evenly distributed on the circle, while online retailers gather in an e-commerce cloud accessible to all consumers. The model yields consistent results when considering different given types of market structure (an asymmetric one, when only some firms are active in the online channel; and a symmetric one). The analysis of these exogenous structures characterizes the dynamics of a transition from the benchmark case towards a bi-channel market structure. The model enables an endogenization of the market structure, under the assumption of free market entry, which is valuable especially for the analysis of an impact of a broader acceptance of the online channel on overall welfare.

The first main finding is that all firms have a strong incentive to participate in the online channel as soon as consumers' reluctance decreases below a certain critical value. This result stems from the analysis of the asymmetric, exogenous market structure. The asymmetry in the migration effect introduced on p. 73 further fuels this incentive: in an asymmetric market structure, a higher acceptance of the online channel leads to a migration of demand, whereby the expected demand drained off the numerous local stores is distributed among the less numerous online retailers: the individual change in expected demand is, absolutely, lesser for local stores than for online retailers. This incentive yields the simplifying results that in any endogenously defined structure of the market, all firms participate in the online channel.

Second, we observe that the rise of an additional channel enhances competition. Firms profits are, for each type of market structure (symmetric or asymmetric and exogenous; or endogenous) entailed as consumers reluctance against the online channel declines, while consumers surplus increases considerably, not last because of the reduction of transportation expenditures. This positive effect dominates, in our formulation of the model, the evolution of welfare; however, a calibration (e.g. by varying the size of the Salop circle) might influence the ratio of consumers surplus and firms profits.

We further proved that firms coordinate their decisions over the two channels to the benefit of the online channel. This is especially visible when comparing single-channel and multichannel firms: multichannel firms set, in their local stores, higher prices than their single-channel counterparts, taking into account a reduced expected demand, to enhance their results in the single channel. The possibility of such a comparison is an example of why the analysis of the asymmetric case is worthwhile, even though it is mathematically more complex and represents no eligible option when the market structure is considered endogenous. Firms coordination of the channels is determined by the migration effect: as the reluctance parameter declines, consumers gradually migrate from the local to the online channel. The first who migrate are the consumers located at the highest distance from local stores. The assumption that transportation costs increase quadratically with the distance to firms implies a convex shape of the expected demand functions, so that the migration effect is at its peak when the online channel only just arises. A strong migration effect leads to the paradox result that, as consumers acceptance of the online channel improves, online retailers can simultaneously set higher prices and gather more expected demand - this phenomenon always holds true when the market structure is endogenous. The fact that firms price decision over both channels is favorable to online activity can lead to a phenomenon of cross-channel subvention: under free market entry, the local channel might perform so poorly that firms have to take into account losses in their local stores to maximize their overall profits.

These results can be confronted to the expectations addressed at e-commerce and the results of the previous literature, as summarized in the introduction. As expected and as depicted by empirical and behavioral approaches, we find that the overall level of prices decreases when the online channel arises. Our model can explain a phenomenon mentioned in (Smith et al., 2001): sometimes, prices can be observed online that are higher than those in the local channel. We are not aware of a behavioral explanation for this phenomenon, but our model predicts such an outcome when the acceptance of the online channel is very high: when consumers have no reluctance against online shopping, they might prefer to order online at a higher price than buying from a lower-priced local store and bearing additionally transportation costs.

The expectation and empirical observation that the online channel enables a better level of information among the consumers is corroborated by our model, and - this is the novelty - not because searching costs are lower, but rather because firms engage in a fierce competition with respect to their visibility and *bring the information to the consumers*. The visibility expenditures are considerable in our model: each firms spends as much on visibility expenditures per online competitor, as it gathers in form of net profits.

Finally, the expectation of a higher price elasticity in the online channel is corroborated in our model. Because of this higher price elasticity, the amplitude of price variations is narrower in the online channel. This is related to the shape of the expected demand functions and holds true in any type of market structure.

The question of whether e-commerce compels firms to a cannibalistic behavior is made difficult by the normative implication that a constriction of the local market structure should be seen as negative. There is no "cannibalization" insofar as firms coordination of the two channels enhances profit: the different sales channels do not compete with each other in an unproductive way. Further on, the transformation of the market structure as consumers acceptance of the online channel improves leads generally to an increase of overall welfare. Nevertheless, when market structure is endogenized, it is true that the rise of the online channel leads to a drop in the number of local stores. Our model does not convey any assessment of whether such a transformation is undesirable.

Some further restrictions of the scope of this model need to be outlined. This model does not account for the existence of further sales channels (especially the long existing possibility of catalog selling). It does not take into account any question of capacities or scale issues; especially, we assume that the cost structure is unchanged for any number of firms (e.g. no economies of scale). It considers that the mass of consumers is constant and captive: further analysis concerning the role of the reservation price and the ease of substitution with further goods in each channel would be an interesting complement. Finally, the model does not consider the possible correlation of the cost structure (unit costs, fix costs) with variations in the exogenous parameters. Lower transportation costs might be reflected in lower unit costs for firms; a better acceptance of e-commerce can be assumed to go along with a better state of the art and, accordingly, with lower costs for online retailers.

Finally, a last point has to be stressed. Our formulation of the model is quite conservative - because of the formulation in terms of "reluctance against" the online channel rather than in terms of acceptance, as already mentioned; but also because we did not consider a third party who is an indisputable winner of the expansion of the online channel: the service industry linked with the technical implementation of the online presence and the visibility efforts of firms. The role of platforms generating no own content, but offering the possibility to publish and advertise some content is preponderant not only for music and arts, it is also central for e-commercial activity. Such platforms and services would build a market on its own and collect the visibility expenditures generated in our model. As we ignore this third parties for simplification, our estimation of the overall welfare is, actually, understated. Taking into account the profits of the third parties would lead to a more accentuated improvement of welfare as consumers acceptance of the online channel increases.

While in the first two chapter, either consumers or firms were purely passive, we consider in the last model of this work a strategic situation in which both parts play an active role, whereby the strategy of the firms relies on the searching activity of the consumers. Here again, time is central theme: the question is about the optimal timing of purchases, illustrated with the example of Black Friday.

Chapter 4

Searching for the Optimal Timing: Black Friday

The American sale discount tradition of the "Black Friday" is now well established. The connotation of this denomination is, initially, negative, as days to which the adjective "black" is applied upon usually refer to some historical disaster. The usage of the term "Black Friday" for the day following Thanksgiving seems to have been introduced by the police to describe the congestion in the streets, as people rushed in the shops for the beginning of Christmas shopping. Introduced in Europe by multinational enterprises in 2013, it turned the week following Thanksgiving into the top-selling one from 2018 on. Advertising opportunities, logistical challenges in the brick-and-mortar shops and, 2020, the wave of lockdown measures to combat coronavirus have enhanced corresponding online offers. "Cyber Monday", a similar discount day on the Monday following Thanksgiving (and Black Friday), was initially formed 2005 as a reply of online retail to the tradition of Black Friday in brick-and-mortar shops, relying on the observation that online sales were already high on that day. The lockdown 2020 has pushed the conjunction of these two marketing events into one online event, with an increased number of retailers turning it into a "Black Week", and might mark the durable consolidation of online and offline marketing, putting an end to their traditional rivalry.

This topic affiliates the thread of literature on intertemporal price discrimination relying on the seminal Coase (1972) conjecture. This conjecture states that a monopolist offering a durable good over an infinite time horizon to a constant population of consumers is reduced to offering the competitive price and loses its monopoly power. However, Coase (1972) neglects the cost of time, by assuming that in an efficient market, the allocation procedure would "take place in the twinkling of an eye". Stokey (1981) reformulates the problem analytically and embeds it in time, offering both a continuous, and a discrete time model where transactions are possible only at the beginning of some discrete time periods of variable length. The continuous time model verifies the Coase conjecture: the equilibrium strategy is a constant, competitive price. However, in the discrete case, the monopolist regains monopoly power as the periods length increases: as periods length increases to infinity, he skims the consumers' willingness to pay by setting the monopoly price in the (residual) demand in each period. This approach focuses on the monopolist's perspective and his effort to segregate the market; but it does not account for the consumers' perspective, who are reduced to a very passive role. The later literature filled this gap by modelling the behavior of so-called strategic consumers, who anticipate the monopolist's behaviour. Landsberger and Meilijson (1985) allow for a difference between consumers' time preference rate and the monopolist's interest rate. They show that intertemporal price discrimination, as a decreasing pricing path over time, is optimal for the seller if the consumers' time preference is higher than the seller's discount rate. Here again, only a constant consumers population is considered, so that the monopolist is confronted, as time goes by, only to a residual demand. Similarly to Conlisk et al. (1984), we expand the scope to a situation where new consumers enter the market at any point in time. Under the assumption of discrete time and of two groups of consumers differing in their valuation of the good, Conlisk et al. (1984) prove that a cycle of high pricing and sales is optimal for the monopolist: in this permanently rejuvenated market, the firm has an incentive to offer a type of price discrimination. The length of the cycle can be endogenously optimized; similarly, further models focusing on product life cycle (e.g. Aviv and Pazgal (2008) for seasonal products) or stochastic variations of consumers' valuations (as e.g. Garrett (2016)) can explain cyclical dynamic pricing but do not account for the existence of a calendar sales date. Nevertheless, the empirical evidence of season sales and Black Friday suggests that a simple calender cycle might be of some advantage, be it just because the mere simplicity and publicity of ritual calendar dates for sales reduces information costs. We therefore consider the length of the cycle, which turns out, in all models mentioned above, to be a parameter upon which the results depends very sensitively, as an exogenous calendar period length normalized to one. As remarked in Landsberger and Meilijson (1985), the inner relationship between time preference (respectively discount rate) and period length makes it superfluous to consider both parameters at the same time.[1]

The phenomenon of such a calendar discount tradition could also be explained as a price war between firms when facing massive demand in the Christmas time. Following Rotemberg and Saloner (1986) or Stadler (2015), firms are more likely to depart from collusive behavior at the beginning of a phase of boom. This price war is a strategical necessity, as the increased demand shakens the stability of the price collusion, and does not yield

[1] When computing our model with a period length generalized to T, we end up with similar optimality conditions: the only difference is that the consumers' time preference rate r is replaced by rT.

higher aggregated firms profits than in the benchmark of an (instable and therefore nor realisable) continued collusion.

Analysts[2] found evidence that not only seasonal Christmas shopping explains the high level of sales; a non negligible share of customers seem to postpone the acquisition of valuable goods to that day, expecting to benefit from an unusually high discount.[3] Black Friday is not only the strategical answer of firms to a peak in the trade cycles, it contributes to generate that peak.

The following model proposes an explanation for this phenomenon. We consider a time period, normalized to one, at the end of which the representative firm is expected to give a considerable discount. Depending on their time preference rate and valuation for the good, consumers might postpone consumption to that final point in time. The firm can strategically benefit from this discount tradition, as it segments the market and results in an intertemporal type of price discrimination. Therefore, the assumption of business cycles is not necessary to explain the success and attractiveness of the Black Friday tradition. This paper is structured as follows: the model's specifications are presented in the first section. The second section deals with consumer behaviour, the third with the firm's reaction to that behaviour. The features of the resulting market equilibrium are analysed in part four. A welfare analysis concludes the model.

4.1 The Model

We assume that the good is sold by a monopolist. The consideration of a monopolist aims at simplifying the model; the results would be left unchanged if we were to consider, instead, some or many firms setting a perfectly collusive price in a non-cooperative setting. In our model with rational, educated, and perfectly informed consumers able to postpone their purchase, there is no issue about the stability of such a tacit collusion. Should a firm deviate from the collusive price, then the consumers would anticipate the upcoming of a punishment phase, lean back and wait for the price war to rage. All consumers would lean back indeed, independently of their valuations and time preference, as the full price war is to be expected in the following, infinitesimal point in time. The deviating firm would benefit from no increased profits - and this absence of deviation profits saps any intention of deviating in the first place; it guarantees the stability of a non-cooperative

[2]See e.g. Handelsverband Deutschland (2020), last item: about 38% of the purchases made on Black Friday are not linked to Christmas shopping.

[3]Statista (2020), p. 28: about half of the customers interviewed expect discounts ranging from 10 to 50%.

collusion. The stability of a collusive price setting could also be justified in a different way: when assuming that consumers choose at random a firm where to buy among the many available firms, by the law of large numbers, the demand would then spread evenly over the firms. This is, for example, the assumption made in Hendershott and Zhang (2006). These two assumptions fit two lines of reasoning: the first one is at home in a world of information transparency where consumers and firms are perfectly informed about market prices; the second one would accommodate search-lazy consumers picking a firm at random. As the number of firms does not further impact the results, we stick to the simplifying assumption of a monopolistic, representative firm.

The considered product is typically bought once in the period, like for example appliances, perfume or clothing. It might be, but is not necessarily a durable good: to the difference of the models on durable goods, we do not focus on an infinite time horizon. Empirical evidence shows that the product categories in which consumers are most frequently postponing purchase until Black Friday are, in this order: (1) shoes and clothing; (2) consumer electronics; (3) media; (4) cosmetics and perfume; (5) appliances (see Statista (2020), p. 32). Production costs are simplified to zero, which is equivalent to considering prices as unit profits. We set aside any question about potential capacity limitations. The firm sells the considered product to the constant list price p over the complete period, excepted at the terminal point in time, normalized to 1, at which a discount rate b is offered. This unit period corresponds implicitely to one year, and the date one is the constant calendar date of Black Friday. Consumers do not need to rely on firms communication to be informed about b - they can forecast its equilibrium level. As Landsberger and Meilijson (1985), we assume that the firm commits itself to a list price $p \in [0, 1]$ (as 1 is the maximal consumer valuation) and a discount rate $b \in [0, 1]$ at the beginning of the period, and that these values are known with certainty to consumers.

Figure 4.1: Firms' price policy over the time period $[0; 1]$

We consider that consumers discount their utility with the instantaneous time preference rate $r > 0$: the higher r, the stronger their preference for immediate consumption.

Demand is generated by successive cohorts of consumers: at any point in time $\tau \in [0;1]$, a cohort of mass one appears in the market. Consumers valuations for the good, are independent identically, uniformly distributed between zero and one: for any consumer i, $v_i \sim \mathcal{U}(0,1)$. Scaling the mass of cohorts scales the results without changing the incentives neither for firms nor for consumers. We do not assume the existence of business cycles: the mass of each cohort of consumers, as well as the distribution of their valuation, are constant over time. Consumers buy maximally one unit of the good.

4.2 Consumers' Behavior

Any cohort addressing the market at time $\tau \in [0;1]$ can choose the exact time of consumption $t, t \in [\tau;1]$. Each consumer i from that cohort, having valuation v_i, can abstain from buying, which is associated tu zero utility, or buy and maximize the net utility from consumption:

$$\max_t u_\tau^i(t) = \begin{cases} e^{-r(t-\tau)}(v_i - p) & \text{if} \quad t < 1 \\ e^{-r(1-\tau)}\Big[v_i - (1-b)p\Big] & \text{if} \quad t = 1 \end{cases}$$

Postponing consumption to some later point in time anterior to the end of the period would only lead to a greater time discount without yielding any price advantage; so that the actual trade-off for consumers is whether to definitely abstain from consumption, or to consume immediately, or at the end of the period:

$$\max_t u_\tau^i(t) = \max\Big\{0, (v_i - p), e^{-r(1-\tau)}\Big[v_i - (1-b)p\Big]\Big\}$$

Consumers whose valuation is below the discount price $\underline{v} = (1-b)p$ do not buy, their utility is nil. Comparing the two further alternatives shows that consumers buy immediately if their valuations is above $\tilde{v}$, defined as:

$$\tilde{v}(\tau) = \min\left\{\frac{1 - e^{-r(1-\tau)}(1-b)}{1 - e^{-r(1-\tau)}} \cdot p \ ; \ 1\right\} \tag{4.1}$$

This critical value increases over time:

$$\frac{\partial \tilde{v}(\tau)}{\partial \tau} = \frac{rbe^{-r(1-\tau)}}{[1 - e^{-r(1-\tau)}]^2} \cdot p > 0$$

The initial level of the critical value is superior to the list price:

$$\tilde{v}(0) = p + \frac{e^{-r}}{1 - e^{-r}} \cdot bp > p$$

This means that there always are, even at the beginning of the period, some consumers deciding to buy immediately. As time goes by and Black Friday gets closer, an always higher share of each consumers cohort decides to postpone their purchase to Black Friday. This share increases until all consumers prefer to wait until Black Friday, i.e. there is no immediate consumption any more, and the critical valuation $\tilde{v}(\tau)$ reaches its upper boundary of one. Let the point in time at which this happens be called $\bar{t}$:

$$\bar{t} = 1 + \frac{1}{r} \ln \left[\frac{1-p}{1-(1-b)p}\right] \tag{4.2}$$

As the logarithm is negative, $\bar{t}$ always precedes the end of the period: $\bar{t} < 1$. This implies that there always is a phase preceding Black Friday during which all consumers will prefer to wait for Black Friday. Before $\bar{t}$, some consumers buy immediately, others postpone their purchase to Black Friday. We are only interested in the situation where at least some consumers decide to buy immediately, the situation when all consumers postpone their purchase is not interesting; i.e. we look at the cases when the critical value $\bar{t}$ is strictly positive. This translates into a condition on the time preference rate:

$$r > \ln \left[\frac{1-p+bp}{1-p}\right] \tag{4.3}$$

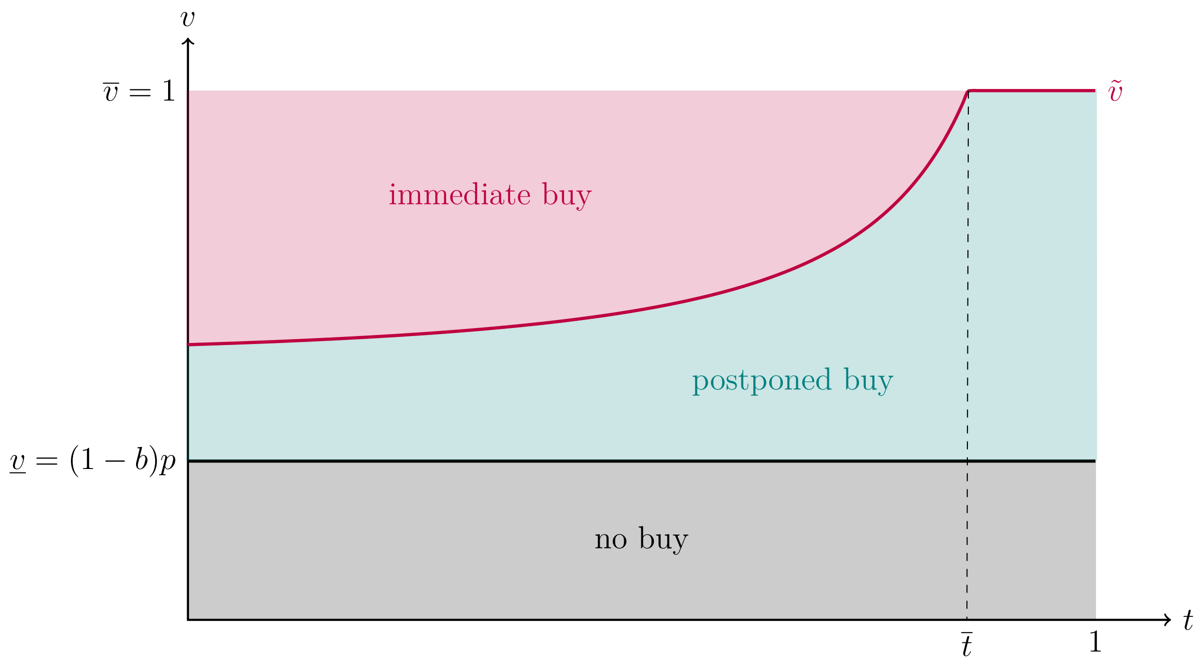

Figure 4.2: Buying behavior of consumers depending on their cohort and valuation

Until the critical date $\bar{t}$, only the consumers with a valuation above $\tilde{v}$ buys immediately, while consumers with intermediate preferences $\underline{v} < v_i < \tilde{v}$ postpone their acquisition to Black Friday. As Black Friday approaches, consumers' utility discounting is of lesser importance, and more consumers decide to postpone their purchase. The evolution of this trade-off in time is illustrated in Figure (4.2). Belief in the tradition of Black Friday therefore endogenously generates a trade cycle with an accelerating decrease of demand

and a sudden boom, as depicted in Figure (4.3).

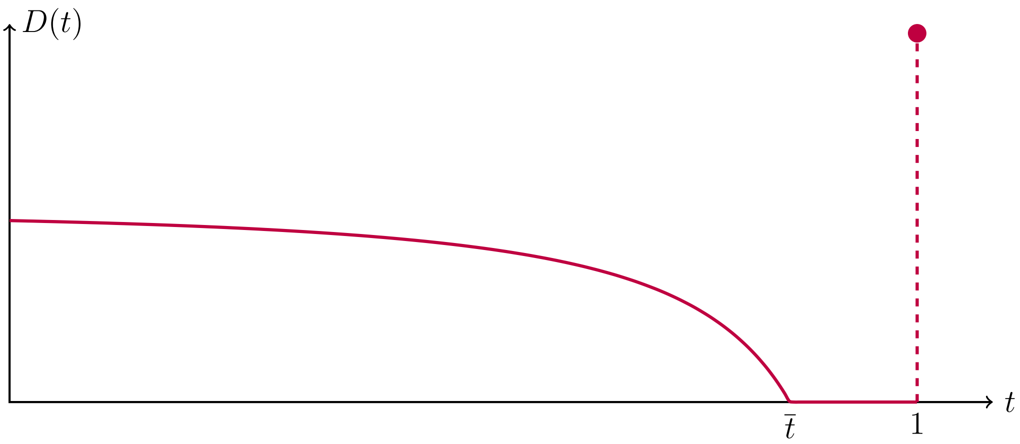

Figure 4.3: Endogenous business cycle

The market demand function is three-part. Between the beginning of the period and the critical time $\bar{t}$, the demand is equal to the number of consumers having a valuation superior to $\tilde{v}$ (see equation 4.1):

$$D(\tau) = 1 - p - \frac{b \cdot p \cdot e^{-r(1-\tau)}}{1 - e^{-r(1-\tau)}}$$

The aggregate demand for the full-price good amounts to:

$$\int_0^{\bar{t}} D(\tau)\ d\tau = (1-p)\bar{t} + \frac{bp}{r} \cdot \ln\left(\frac{1 - e^{-r(1-\bar{t})}}{1 - e^{-r}}\right)$$

We can check that between the critical time $\bar{t}$ and Black Friday, sales drop to zero:

$$\forall \tau \in [\bar{t}; 1], D(\tau) = 0$$

On Black Friday, all the consumers who delayed their purchase stream in the market:

$$D(1) = \int_0^1 [\tilde{v}(\tau) - \underline{v}]\, d\tau = \frac{(1-p)}{r} \ln\left[1 + \frac{bp}{1-p}\right] + bp\left[1 - \frac{1}{r}\ln\left(\frac{bp}{[1-(1-b)p][1-e^{-r}]}\right)\right]$$

It can be checked that the sum of $\int_0^{\bar{t}} D(\tau)\ d\tau$ and $D(1)$ is equal to the rectangular area corresponding to the actual buyers on Figure (4.2), which amounts to $1 - p + bp$.

4.3 Firm's Behavior

While time discounting is crucial for explaining consumers' behavior, its usage for firms is questionable in our model. Discounting the Black Friday sales, positioned at the end of the period, might strongly relativise the importance of these sales, which makes the model's results very sensitive to the interest rate respectively to the time preference rate. This problem is intrinsic to the family of models on intertemporal price discrimination and, as mentioned above, it has been elucidated in Landsberger and Meilijson (1985): intertemporal price discrimination can be explained only if the monopolist's interest rate is lower than consumer's time preference. Along with this technical justification, we consider that there is an intrinsic asymmetry between consumers and firm: while consumers are focused on a short term question (whether to buy one unit of the good now or later), the calendar year shapes the rhythm of firms' activity. Many a firm objective is related to non discounted, yearly aggregated sales and profit, not least for accounting and taxation purposes. It would not be adequate to assume that consumers' time preference rate and firm's interest rate are equal. For simplification, we assume that the opportunity costs from delayed purchases are negligible for the firm, and that its instantaneous discounting rate is zero.

Under these assumptions, the monopoly profit function reads:

$$\begin{aligned}\pi(p,b) &= \left[p\int_0^{\bar{t}} D(\tau)\ d\tau + (1-b)pD(1)\right]\\ &= p\Big[1-p+bp-b^2p\Big] + \frac{bp(1-p)}{r}\ln\left[\frac{1-p}{1-(1-b)p}\right] + \frac{b^2p^2}{r}\cdot\ln\left[\frac{bp}{[1-(1-b)p](1-e^{-r})}\right]\end{aligned} \tag{4.4}$$

4.3.1 Implicit Definition of the Equilibrium Values

The first order condition with respect to the price reads:

$$\frac{\partial\pi(p,b)}{\partial p} = \frac{r[1-2p+2bp-2b^2p] + b(1-2p)\ln\left[\frac{1-p}{1-(1-b)p}\right] + 2b^2p\cdot\ln\left[\frac{bp}{[1-(1-b)p](1-e^{-r})}\right]}{r} = 0$$

The first order condition with respect to the discount rate b reads:

$$\frac{\partial\pi(p,b)}{\partial b} = \frac{p}{r}\left[2bp\ln\left(\frac{bp}{[1-(1-b)p]\,(1-e^{-r})}\right) + (1-2b)pr + (1-p)\ln\left(\frac{1-p}{1-(1-b)p}\right)\right] = 0$$

Rearranging leads to the system of two conditions:

$$[1-2p(1-b+b^2)]r + b(1-2p)\ln\left[\frac{1-p}{1-p+bp}\right] + 2b^2p\ln\left[\frac{bp}{[1-p+bp](1-e^{-r})}\right] = 0 \tag{4.5}$$

$$(1-2b)pr + (1-p)\ln\left[\frac{1-p}{1-p+bp}\right] + 2bp\ln\left[\frac{bp}{[1-p+bp]\,(1-e^{-r})}\right] = 0 \tag{4.6}$$

These conditions implicitly determine the equilibrium values of the price and the discount rate. The presence of logarithms prevents a straightforward analytical solution and even restricts the choice of numerical approximation methods.

There is, however, a possibility to simplify the problem by first determining the pairs of values for the price and the discount rate eligible as solution pairs. Substituting the last logarithm from (4.5) into (4.6) leads to the condition:

$$r = \frac{bp}{1-2p+bp} \ln\left[\frac{1-p}{1-p+bp}\right] \tag{4.7}$$

As the time preference rate r is positive per assumption and the logarithm negative, the factor preceding it is negative, i.e. its denominator $1-2p+bp$ is negative and we can deduce an upper boundary for the price: $p > \frac{1}{2-b}$. This critical value amounts to $1/2$ (when $b = 0$) or more and increases with b: as the discount offered on Black Friday increases, the list price will be higher, i.e. the intertemporal price discrimination increases. Combining condition (4.3) and condition (4.7) from the first order conditions, yields an additional boundary for the price: $p < 1/[2(1-b)]$.

$$\frac{1}{2-b} < p < \frac{1}{2-2b} \tag{4.8}$$

By the squeeze theorem, the value for the price is unambiguous in case there is no discount on Black Friday: the price is then $1/2$. As the discount increases, the domain of eligible values for the prices increases and expands, as illustrated in Figure (4.4); in the case of a maximal discount $b = 1$, the price can only take value 1, implying a negligible demand.

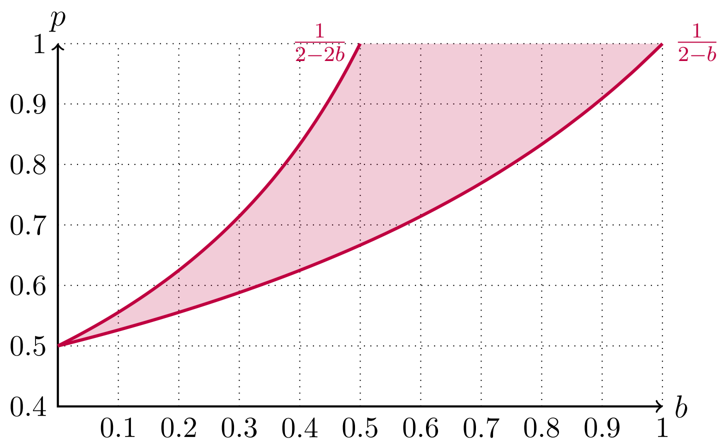

Figure 4.4: Positive relationship between list price and discount rate

The inequation (4.8) describes a positive relationship between price and discount rate illustrated in Figure (4.4): intertemporal price discrimination is enhanced as both list price and discount rate increase. This hints at the ambiguous effect of price discrimination on consumers welfare, as analysed below: while the discount rate on Black Friday is beneficial to consumers welfare, the accompanying increase in the list price is not.

The condition (4.7) defines, for each level of the time preference rate r, a set of admissible pairs of values for b and p. The right-hand size of equation (4.7) in the relevant domain can be plotted, as done in Figure (4.5), as a three dimensional graph. The resulting altitude is low for low values of the discount rate b, and rises to infinite height when price p and discount rate b are high. Condition (4.7) corresponds, geometrically speaking, to the intersection of this three-dimensional graph with a horizontal hyperplane of height r. This intersection is a two-dimensional function which is concave in the price p. It is represented for an exemplary level of r in Figure (4.6), computed here using a simple search method for numerical approximation.

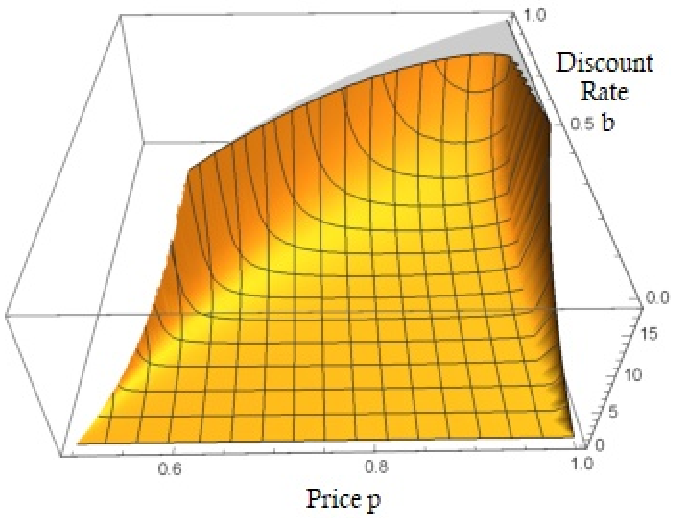

Figure 4.5: Condition on r: left-hand-side of (4.7)

Figure (4.6) represents all combinations of prices and discount rates fulfilling simultaneously the first order condition (4.7) and the restriction (4.8). This inequality does not depend on the time preference rate: the difference between the lowest and the highest possible prices, depending on any level of the discount rate b, i.e. the distance between the two dark lines on Figure (4.6), increases with b. The red curve, which corresponds to the pairs fulfilling the first-order condition (4.6), has a higher curvature for higher values of the time preference rate: the range of eligible values is broader when the time preference rate is higher. Figure (4.7) displays a exemplary comparison for the time preference rates 1 and 10. It might seem surprising to consider such high values of r, as it works, technically, similarly to an interest rate. Remember however that very high time preferences

Figure 4.6: Optimality Conditions 4.7 and 4.8 for a given value r

are not excluded; they signal an increasingly exclusive preference for the present moment.

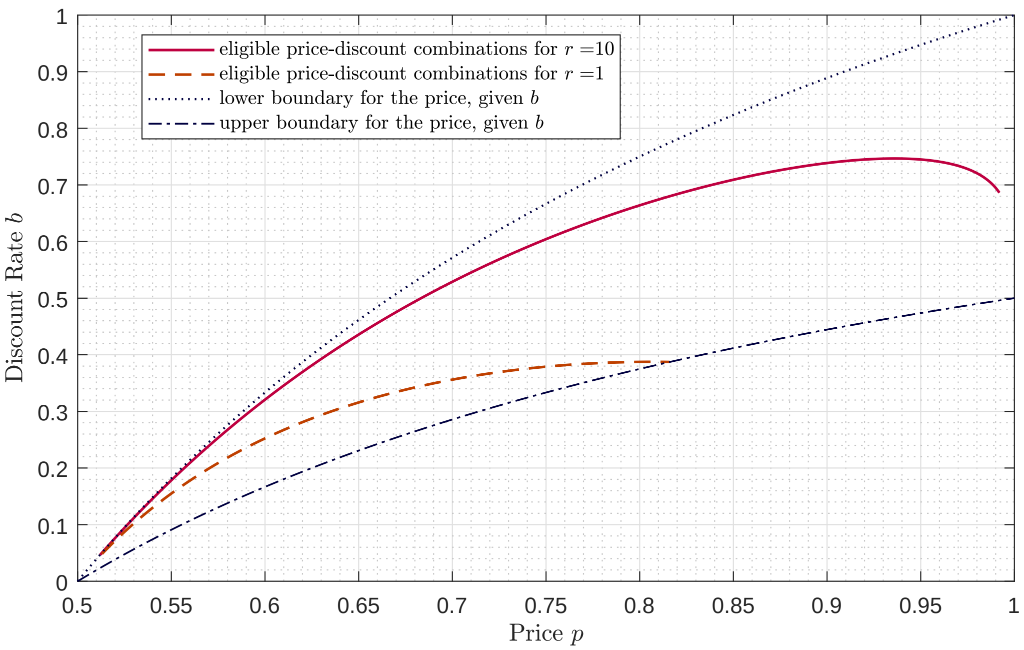

Figure 4.7: Eligible Combinations for Different Time Preference Rates

Figures (4.6) and (4.7) strengthen our assumption that consumers can assign a discount rate b unambiguously to any level of the price they might observe: they do not rely on uncertain expectations or firm's own proclamations.

In the following section, we will consider the impact of the decisive exogenous parameter, the time preference rate r.

4.4 Market Equilibrium

We analyse the influence of the time preference rate r^* on the equilibrium price p^*, discount b, discount price $(1-b^*)p^*$ and the firms profits. The mathematical analysis is completed by numerical computations relying on the same assumptions (especially of uniformly distributed preferences) and using Newton's method or, to circumvent the problem of poor initial estimates, a more exhaustive, iterative search for the optimal values. The numerical approximations account for the parameter restrictions mentioned until then:

1. $0 < b < 1$,
2. $\frac{1}{2-b} < p < \min\{1; \frac{1}{2-2b}\}$, and
3. $r > \ln\left[\frac{1-(1-b)p}{1-p}\right]$.

The illustrations result from the matlab files documented in appendix H.
For the mathematical analysis, we set up, on the basis of the condition (4.7), the following function F, which takes constantly value zero in equilibrium:

$$F(b^*, p^*, r) = \frac{b^* p^*}{1-2p^*+b^*p^*} \ln\left[\frac{1-p^*}{1-p^*+b^*p^*}\right] - r = 0 \tag{4.9}$$

4.4.1 Equilibrium Price

By the implicit function theorem, we get an expression for the partial derivative of the price p^* with respect to the time preference rate r^*:

$$\frac{\partial p^*}{\partial r} = -\frac{\partial F/\partial r}{\partial F/\partial p^*}$$

While the partial derivative in the numerator is trivial ($\partial F/\partial r = -1 < 0$), it proves more to be delicate to determine the sign of the partial derivative in the denominator.

$$\frac{\partial F}{\partial p^*} = \frac{b^*}{1-2p^*+b^*p^*} \cdot \left(\frac{1}{1-2p^*+b^*p^*} \ln\left[\frac{1-p^*}{1-p^*+b^*p^*}\right] - \frac{b^*p^*}{(1-p^*)(1-p^*+b^*p^*)}\right)$$

where $\frac{b^*}{1-2p^*+b^*p^*} < 0$. Be $G(b^*, p^*)$ defined as the expression in the round brackets:

$$G(b^*, p^*) = \frac{1}{1-2p^*+b^*p^*} \ln\left[\frac{1-p^*}{1-p^*+b^*p^*}\right] - \frac{b^*p^*}{(1-p^*)(1-p^*+b^*p^*)}$$

We can compute its partial derivative with respect to the price p^*:

$$\frac{\partial G(b^*, p^*)}{\partial p^*} = \frac{2-b^*}{(1-2p^*+b^*p^*)^2} \cdot \ln\left[\frac{1-p^*}{1-p^*+b^*p^*}\right] + \frac{b^*}{(1-p^*)(1-2p^*+b^*p^*)(1-p^*+b^*p^*)} + \\ + \frac{-b^*(1-p^{*2}+b^*p^{*2})}{(1-p^*)^2(1-p^*+b^*p^*)^2}$$

As each of the three summands is negative, the function $G(b^*, p^*)$ declines with increasing p^*; and as $G(b^*, 0)$ is nil, it will take negative values for any price above $1/2$. $G(p^*, b^*)$ is negative, as is the factor preceding it in $\partial F/\partial p^*$; and the overall sign of the partial derivative $\partial F/\partial p^*$ is positive. By the implicit function theorem, this implies that the sign of the partial derivative $\partial p^*/\partial r$ is positive: the optimal price increases in the time preference rate.

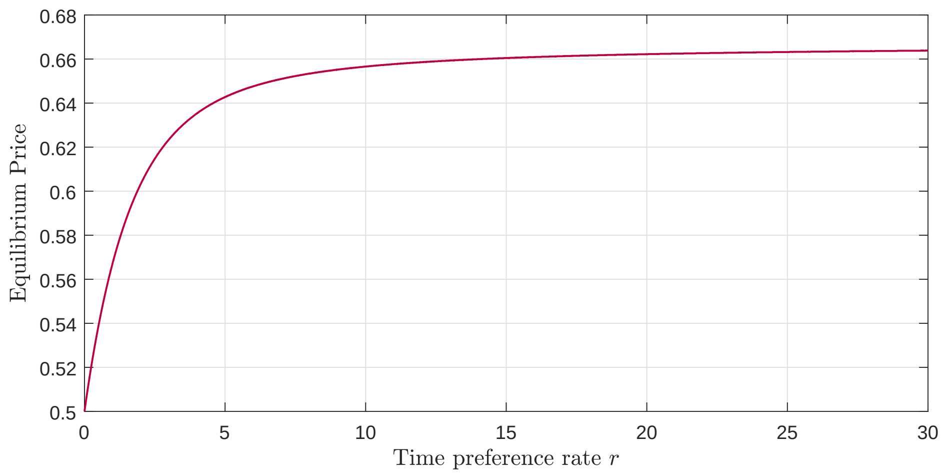

Figure 4.8: Impact of the time preference rate r on the optimal prices

4.4.2 Equilibrium Discount

We can proceed similarly to explore the relationship between the optimal discount b and the time preference rate r. Considering now the discount b^* as function of price p^* and time preference rate r, the implicit function theorem yields:

$$\frac{\partial b^*}{\partial r} = -\frac{\partial F/\partial r}{\partial F/\partial b^*}$$

Again, the sign of the denominator requires further computations.

$$\frac{\partial F}{\partial b^*} = \frac{p^*}{1 - 2p^* + b^*p^*} \cdot H(b^*, p^*) \qquad \text{where}$$

$$H(b^*, p^*) = \frac{1 - 2p^*}{1 - 2p^* + b^*p^*} \ln\left[\frac{1 - p^*}{1 - p^* + b^*p^*}\right] - \frac{b^*p^*}{1 - p^* + b^*p^*}$$

We can compute the partial derivative of the auxiliary function H with respect to the price p^*:

$$\frac{\partial H}{\partial b^*} = \frac{-p^*(1 - 2p^*)}{(1 - 2p^* + b^*p^*)^2} \ln\left[\frac{1 - p^*}{1 - p^* + b^*p^*}\right] - \frac{p^*(1 - 2p^*)}{(1 - 2p^* + b^*p^*)(1 - p^* + b^*p^*)} - \frac{p^*(1 - p^*)}{(1 - p^* + b^*p^*)^2}$$

$$\frac{\partial H}{\partial b^*} \leq 0 \qquad ; \quad H(0,p^*) = 0 \qquad ; \quad H(b^*,p^*) \leq 0 \quad \forall b^* \in [0,1]$$

As above, the sign of the auxiliary function is negative, the partial derivative in the denominator is positive; and the optimal discount factor b^* chosen by firms in equilibrium increases as the time preference rate increases.

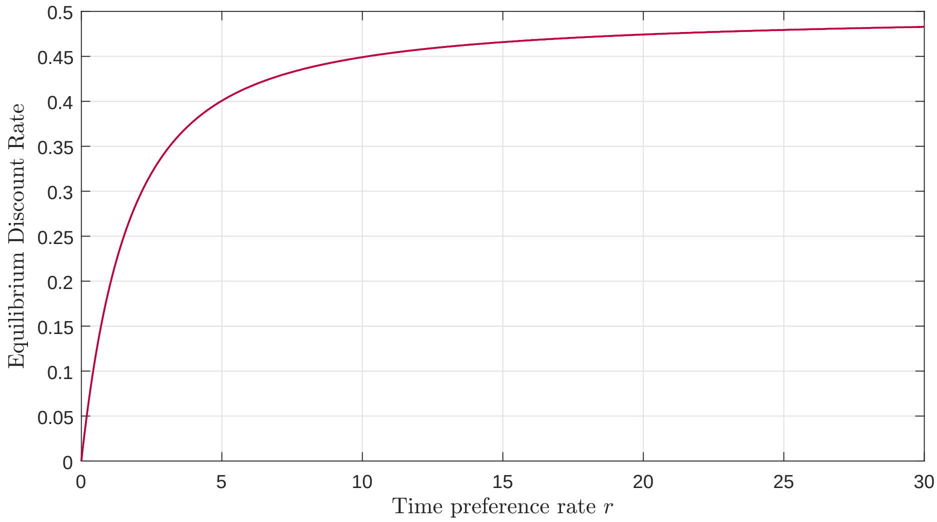

Figure 4.9: Impact of the time preference rate r on the optimal discount rate

4.4.3 Equilibrium Discount Price

We can also analyze the behaviour of the discount price. Let us call it, for this purpose, $d^* = p^*(1-b^*)$. The function $F(b^*,p^*,r)$ can be reformulated as a function of price, discount price and time preference rate:

$$F(d^*,p^*,r) = \frac{p^*-d^*}{1-d^*-p^*}\ln\left[\frac{1-p^*}{1-d^*}\right] - r = 0 \tag{4.10}$$

The implicit function theorem yields:[4]

$$\frac{\partial d^*}{\partial r} = -\frac{\partial F/\partial r}{\partial F/\partial d^*} = \frac{1}{\partial F/\partial d^*} < 0$$

The discount price decreases as the time preference rate increases: when consumers are more impatient, a stronger market segmentation is possible.

[4]This time, the sign of the denominator is straightforward: $\frac{\partial F}{\partial d^*} = \frac{2p^*-1}{(1-d^*-p^*)^2}\ln\left[\frac{1-p^*}{1-d^*}\right] + \frac{p^*-d^*}{(1-d^*-p^*)(1-d^*)} < 0$ as $\ln\left[\frac{1-p^*}{1-d^*}\right] < 0$, $\quad (1-d^*-p^*) < 0$, $\quad$ and the other factors are positive

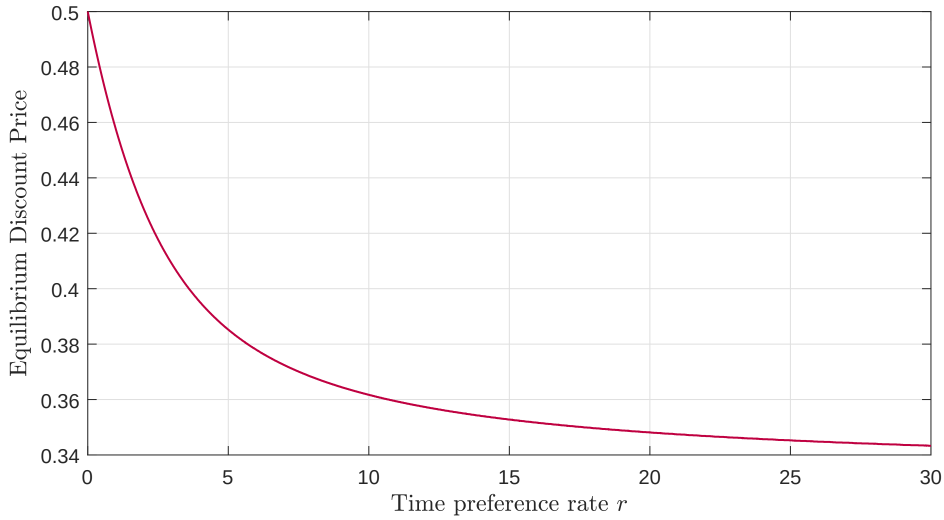

Figure 4.10: Impact of the time preference rate r on the discount price

4.4.4 Convergence Values for an Increasing Time Preference Rate

As mentioned above, the nature of r as time preference rates accommodates values above 1 or even higher.[5] The convergence values is not a core result; but it serves the analysis of the incentives at work. The influence of the time preference rate r on the optimal price and discount is always positive; but the impact of an additional increase in the time preference rate gets always more moderate as r increases.

As price p^* and discount b^* have an upper boundary, they will converge as the time preference rate r increases. This convergence can be captured mathematically with the use of either one of the first order conditions (4.5) or (4.6). We consider here the first order condition (4.6); a similar procedure starting from first order condition (4.5) would lead to the exact same result. We can divide equation (4.6) with the respective sides of the equality (4.7) as follows:

$$(1-2b^*)p^* + \frac{(1-p^*)(1-2p^*+b^*p^*)}{b^*p^*} + 2(1-2p^*+b^*p^*) \cdot \frac{\ln\left[\frac{b^*p^*}{[1-(1-b^*)p^*](1-e^{-r})}\right]}{\ln\left[\frac{1-p^*}{1-(1-b^*)p^*}\right]} = 0 \tag{4.11}$$

When the time preference rate r tends to infinity, the last quotient simplifies correspon-

[5] Remark additionally that when computing the model with a general period length T instead of the normalization to 1 (implicitly, one year), we have the exact same results, with factor rT instead of T. In that version of the model, the convergence of factor rT to infinity can be associated to a very high time preference and / or to an infinite period length.

dingly. We use the property that in equilibrium, this quotient converges to 1:

$$\lim_{r\mapsto\infty} \ln\left[\frac{b^*p^*}{[1-(1-b^*)p^*]}\right] \Big/ \ln\left[\frac{1-p^*}{1-(1-b^*)p^*}\right] = 1$$

Under this assumption and after some transformations, the equation (4.11) simplifies to

$$1 - 3p^* + 3b^*p^* + 2p^{*2} - 4b^*p^{*2} = 0\ ,$$

implying the convergence result for b when r increases to infinity:

$$b^* = \frac{1 - 3p^* + p^*2}{p^*(4p^* - 3)} \tag{4.12}$$

This new, explicit formula for b^* enables us to gain more information from the condition on r stated in equation (4.7). Using the fact that the discount rate b^* is at most 1, we can deduce from (4.12):

$$p^* < \sqrt{1/2}$$

As p^* is simultaneously above 1/2 and increasing with r, the numerator of the logarithm in equation (4.7) cannot converge close to neither 0 nor to 1.

Further on, in order to analyse the denominator of the same logarithm, we can deduce from (4.12) a formula for the discount price when r increases to infinity:

$$\frac{\partial(1-b^*)p^*}{\partial p^*} = \frac{4(2p^{*2} - 3p^* + 1)}{(4p^* - 3)^2} < 0 \quad \forall p^* \in (1/2, \sqrt{1/2})$$

As the discount price declines in r, the denominator of the logarithm in (4.7) cannot converge to zero. The logarithm cannot converge to infinity; the only possible driving element for the right hand side of (4.7) is therefore the numerator $1 - 2p^* + b^*p^*$: r increasing to infinity implies that this numerator is converging to zero and

$$b^* = \frac{2p^* - 1}{p^*} \tag{4.13}$$

Equations (4.12) and (4.13) yield a quadratic equation for p, with roots 1/2 and 2/3. The first root is not eligible, as the price is strictly above 1/2; the only analytical solution left is therefore a price converging to 2/3 for increasing values of r, implying a convergence of the optimal discount rate to 1/2, as confirmed by the numerical approach illustrated in graphs (4.8) and (4.9). As a consequence, the discount price $p^*(1-b^*)$ converges to 1/3 as the time preference rate approaches infinity. This is illustrated in Figure (4.10).

We can now verify the property used above:

$$\lim_{r\mapsto\infty} \ln\left[\frac{b^*p^*}{[1-(1-b^*)p^*]}\right] \Big/ \ln\left[\frac{1-p^*}{1-(1-b^*)p^*}\right] = \ln(1/2)/\ln(1/2) = 1$$

4.4.5 Convergence Values for Minimal Values of the Time Preference Rate

We consider the equilibrium values in the case when the time preference rate r is tending to its minimum, according to the condition on the lower boundary (crit:rT). The formulation of the limit values considered below, where we look at r tending to this lower boundary, includes this condition. Remark that in equilibrium, i.e. when considering the impact of the time preference rate on the equilibrium values for the list price p and the discount rate b, this quotient tends to $\ln 1 = 0$ when the time preference rate tends to zero.

This lower boundary yields a major simplification of the first order conditions (4.5) and (4.6):

$$\lim_{r \mapsto -\ln\left[\frac{1-p}{1-p+bp}\right]} \ln \frac{bp}{(1-p+bp)(1-e^{-r})} = 0$$

First order condition (4.6) yields, in the limit, $b = 1 - 1/(2p)$. First order condition (4.5) yields $-1 + 2p - b + 4bp + 2bp^2 = 0$. Combining these results, we deduce the limit equilibrium values:

$$\lim_{r \mapsto \ln\left[\frac{1-p+bp}{1-p}\right]} p = 1/2; \qquad \lim_{r \mapsto \ln\left[\frac{1-p+bp}{1-p}\right]} b = 0; \qquad \lim_{r \mapsto \ln\left[\frac{1-p+bp}{1-p}\right]} p(1-b) = 1/2;$$

$$\lim_{r \mapsto \ln\left[\frac{1-p+bp}{1-p}\right]} r = 0; \qquad \lim_{r \mapsto \ln\left[\frac{1-p+bp}{1-p}\right]} \pi = 0$$

The minimal equilibrium value for r tends to zero and implies a list price $p^* = 1/2$, no discount rate $b^* = 0$, and zero profits. It coincides with the benchmark results without Black Friday strategy, as described below in the section on welfare.

4.4.6 Equilibrium Profits

Profits can be expected to increase as the time preference rate r, and therefore the share of immediate purchases increases. We reformulate the profit function in equilibrium, using the condition on r in equilibrium (4.7):

$$\begin{aligned}\pi^*(p^*, b^*, r) =& p^* - p^{*2} + b^* p^{*2} - b^{*2} p^{*2} + \frac{1}{r} p^*(1-p^*) \cdot b^* \ln\left[\frac{1-p^*}{1-(1-b^*)p^*}\right] \\ &+ \frac{1}{r}\frac{b^* p^*}{2} \cdot 2b^* p^* \ln\left[\frac{b^* p^*}{[1-(1-b^*)p^*](1-e^{-r})}\right] \\ =& \frac{1-p^*+b^*p^*}{2}\end{aligned} \tag{4.14}$$

As the discount price $(1-b^*)p^*$ converges at a fast rate, over a small range of values, to $1/3$, the factor $(1-p^*+b^*p^*)/2$ converges to $1/3$ as r increases. In equilibrium, we have:

$$\lim_{r \mapsto \infty} \pi^*(p^*, b^*, r) = 1/3$$

The variation of the profits when the time preference rate increases can be deduced from the initial profits functions (4.4), using the properties (4.5), (4.6) and (4.7) in equilibrium, the convergence values in equilibrium and the inequality $e^{-rT}/(1-e^{-rT}) < (1-p^*)/b^*p^*$ derived from (4.3):

$$\frac{\partial \pi}{\partial r} > -\frac{b^*p^*}{2r^2} \ln\left[\frac{1-p^*}{1-(1-b^*)p^*}\right] \cdot \left[\frac{(1-p^*+b^*p^*)(1-2p^*+2b^*p^*)}{1-2p^*+b^*p^*}\right] > 0$$

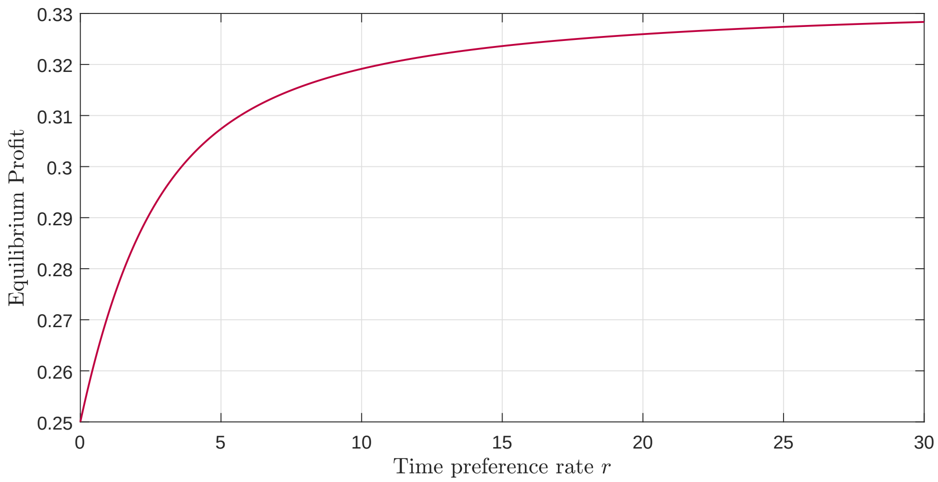

Figure 4.11: Impact of the time preference rate r on the equilibrium profits

For the minimal time preference rate $r \mapsto 0$, as computed above (see result 5), profits tend to zero as the time preference rate shrinks to zero: in this case, consumers are infinitely patient and incur no noticeable utility loss from postponing their purchase to Black Friday. Figure (4.11) illustrates the convergence of firm's profits.

4.4.7 Impact of the Time Preference Rate on the Market Equilibrium

When consumers' time preference is negligible, there are no costs of waiting for them: all consumers would wait for the Black Friday price. As a consequence, the firm has no incentive to pursue a Black Friday strategy any more and would set the monopoly price as list price, and offer no discount at Black Friday.

An increasing time preference rate enhances the possibility of differentiating the market. The higher the time preference rate is, the stronger consumers discount future net utility, and will prefer to buy immediately rather than to postpone their acquisition to Black Friday. As the time preference rate r increases, i.e. as the avidity of households to purchase the good immediately increases, the list price p^* increases: the firm can better skim the immediate willingness to pay (see figure 4.8). At the same time, the discount rate b^* increases and the discount price $(1 - b^*)p^*$ decreases (see figure 4.9): the firm can address a higher share of the market and more consumers with low valuations are able to buy at Black Friday. Overall, more impatient households foster a stronger possibility of market segmentation, which reflects in higher firms profits (see figure 4.11).

4.5 Welfare Analysis

We already know that more consumers are able to buy under a Black Friday strategy. We now turn to the overall welfare effect. This discussion does not aim at formulating policy recommendation, but rather at making out the overall effect of Black Friday on consumers welfare. This effect is not clear at first sight, as opposed effects influence it: on the one hand, the list price is higher than the benchmark price, so that consumers buying immediately see their utility decreased; on the other hand, consumers postponing their purchase have to bear the costs of waiting: the discount costs, but those among them with a valuation below the monopoly price are able to buy only under the Black Friday strategy.

4.5.1 Benchmark: Firm and Consumer Surpluses Without Black Friday Strategy

In the absence of any discount, consumers will not delay their purchase: all consumers with a valuation at least equal to the list price p buy immediately. The firm's profits function simplifies to the reference monopoly function:

$$\pi_{noBF}(p) = [1 - p] \cdot p$$

Firms optimal price strategy is to set the list price $p = 1/2$. The overall level of demand amounts to $1 - p = 1/2$. The resulting firm's surplus is:

$$FS_{noBF} = 1/4$$

In this benchmark case, consumer surplus is the aggregate value of the difference between preferences and price for each cohort, again aggregated over all periods:

$$CS_{noBF} = \int_0^1 \int_{1/2}^1 (v - 1/2) \ dv \ dt = 1/8$$

As consumers do not delay their purchase, their time preference rate does not influence this result.

There is a further perspective meeting this benchmark: the case where consumers time preference is zero leads to the same results. In that case, insofar as there is some discount

$b > 0$, all consumers will wait and buy at Black Friday. As demand concentrates on Black Friday, the optimization variable is now the discount price $\tilde{p} = (1 - b)p$; the exact levels of price p and discount b do not matter, any combination leading to the optimal discount price is optimal. The maximization problem is as stated above and the result is identical, with $\tilde{p}$ replacing p. Firms profits and consumers surplus are as above.

4.5.2 Firm Surplus Under Black Friday Strategy

The firm's profit proves to be higher under the Black Friday strategy. Using the formulation of the profits from result 6 in equation (4.14) and the right hand-side of the squeeze inequation (4.8), we get the strict inequality:

$$\forall r > 0, \quad FS_{BF} = \pi^* = \frac{1 - p + bp}{2} > \frac{1}{4}$$

The dominance of the Black Friday strategy compared to the benchmark is illustrated in Figure (4.12).

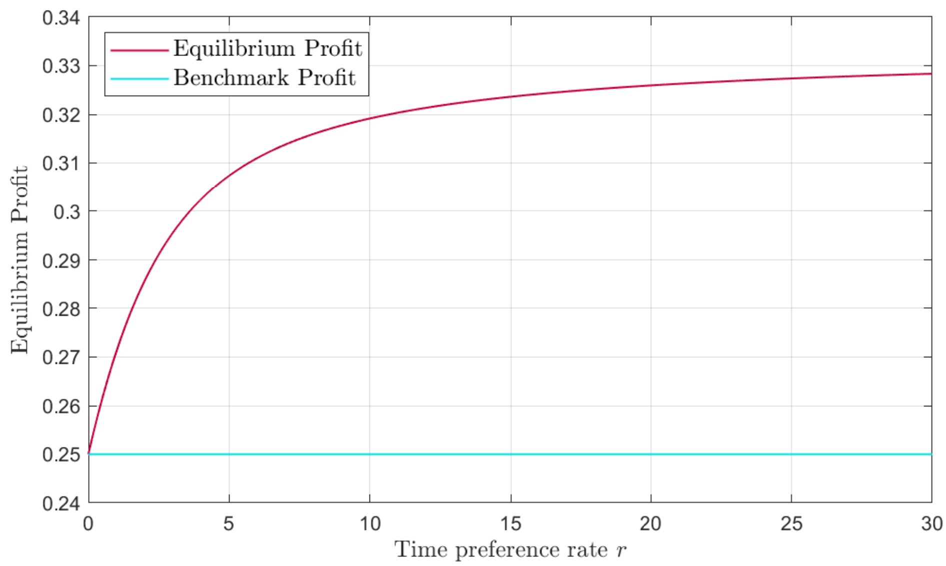

Figure 4.12: Black Friday and Benchmark Profits as Functions of the Time Preference Rate

4.5.3 Consumer Surplus Under Black Friday Strategy

Even though the computation of consumers' surplus follows the same logic as in the benchmark case, the varying share of consumers deciding to postpone their purchase to Black Friday, as well as the necessity to consider them discounting their utility, make the mathematics more complex. To keep the steps simple, we distinguish between consumers who buy immediately and consumers who buy on Black Friday.

For any cohort generated at some point in time between 0 and $\bar{t}$, all consumers with a valuation above the critical valuation $\tilde{v}(t)$ buy the good immediately and gather a level of utility equal to the difference between their valuation and the full list price p. We sum up these cohort utilities over time,

$$CS_{BF}^{immediate} = \int_0^{\bar{t}} \int_{\tilde{v}}^1 (v-p)\ dv\ dt = \frac{1-2p+p^2}{2} \cdot \bar{t} - \frac{b^2p^2}{2} \int_0^{\bar{t}} \frac{e^{-2r(1-\tau)}}{(1-e^{-r(1-\tau)})^2}\ d\tau$$

The second summand captures the "migration" of consumers attracted by the opportunity of a discount at Black Friday and postponing their purchase. It is easily verified that if the time preference rate becomes negligible, implying a price $p = 1/2$ and zero discount at Black Friday, the second summand drops to zero, while the first corresponds to the benchmark consumers' surplus, 1/8.

For the consumers buying on Black Friday, we additionally need to consider them discounting utility:

$$\begin{aligned} CS_{BF}^{postponed} &= \int_0^1 e^{-r(1-t)} \int_{(1-b)p}^{\tilde{v}} [v-(1-b)p]\ dv\ dt \\ &= \frac{(1-b)^2p^2}{2} \int_0^1 e^{-r(1-t)}\ dt + \frac{1-2p(1-b)}{2} \int_{\bar{t}}^1 e^{-r(1-t)}\ dt - \frac{p^2(1-2b)}{2} \int_0^{\bar{t}} e^{-r(1-t)}\ dt + \\ &+ b^2p^2 \int_0^{\bar{t}} \frac{e^{-2r(1-t)}}{1-e^{-r(1-t)}}\ dt + \frac{b^2p^2}{2} \int_0^{\bar{t}} \left[\frac{e^{-3r(1-t)}}{(1-e^{-r(1-t)})^2}\right]\ dt \end{aligned}$$

The sum results in the consumers' surplus under Black Friday strategy:[6]

$$CS_{BF} = \frac{(1-p)^2(1-2p)}{2bp} + \frac{5-(11-b)p+2(3-b^2)p^2}{4} + \frac{bp(1-p+bp)}{2r}$$

The time preference rate r has a negative impact on consumers' surplus. This is owing to two factors: on the one hand, a stronger time preference leads to a higher market segmentation; and on the other hand, the more impatient the consumers, the stronger their disutility from waiting until Black Friday. As illustrated in figure (4.13), consumers' surplus is always lower under the Black Friday strategy than in the benchmark.

[6] Computations are using: $\bar{t} = 1 + \frac{1}{r}\ln\left[\frac{1-p}{1-p+bp}\right]$, $\int_0^1 e^{-r(1-t)}\ dt = \frac{1}{r} - \frac{e^{-r}}{r}$, $\int_0^{\bar{t}} e^{-r(1-t)}\ dt = \frac{1-p}{r(1-p+bp)} - \frac{e^{-r}}{r}$, $\int_{\bar{t}}^1 e^{-r(1-t)}\ dt = \frac{bp}{r(1-p+bp)}$; partial integration; and transformation of the logarithms using respectively (4.5), (4.6) and (4.7).

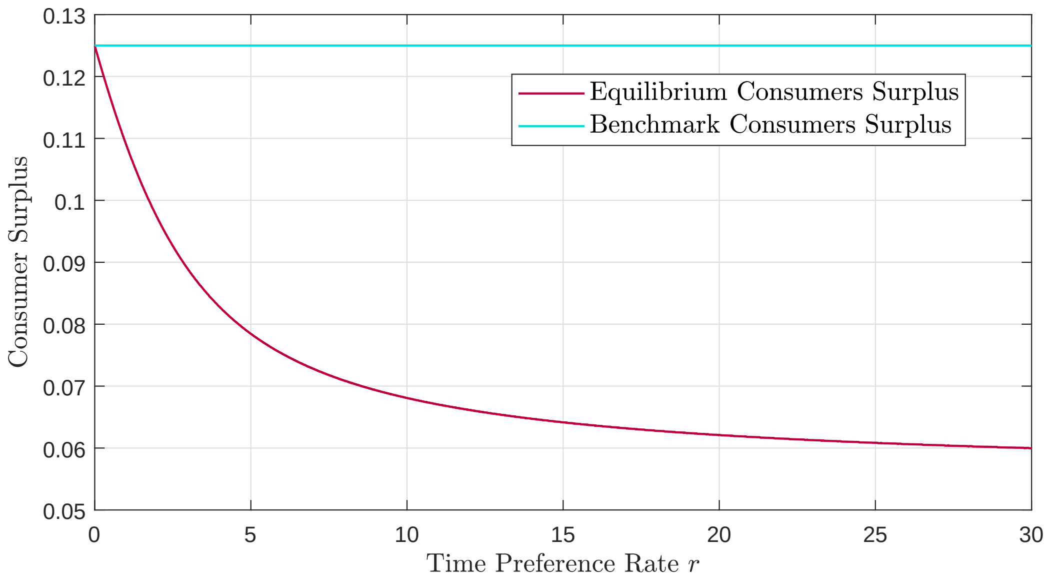

Figure 4.13: Consumers' Surplus With and Without Black Friday

Nevertheless, drawing the conclusion that consumers are better off without Black Friday would be inadequate. Consumers are heterogeneous; while the consumers with highest valuation obviously gather only a smaller surplus under the Black Friday strategy, consumers with a lower willingness to pay get the possibility to buy in the first place - possibly even with a small surplus - at the Black Friday discount price. Figure (4.14) illustrates such a case.

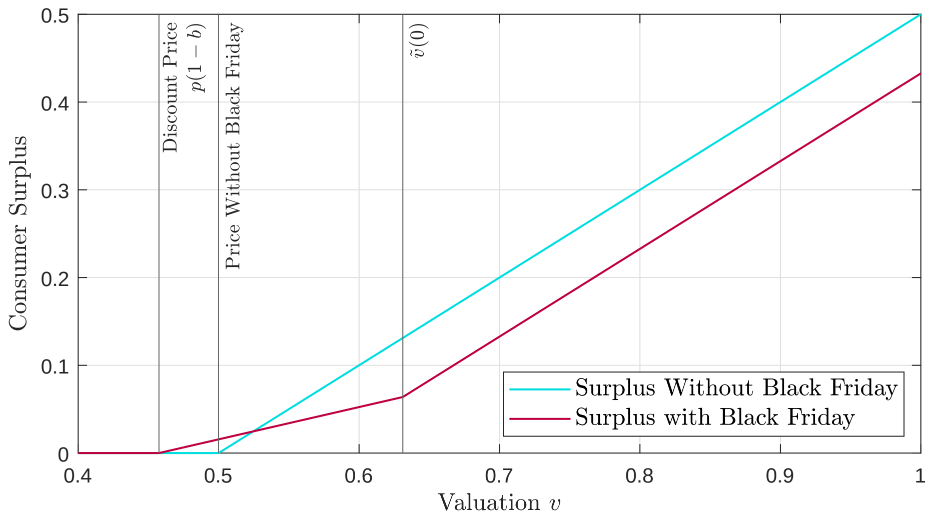

Figure 4.14: Consumers' Surplus of Initial Cohort, for $r = 1$

Black Friday enables few consumers with a valuation lower than the list price without Black Friday, $p = 0.5$, to buy; and some of the consumers with a valuation just above 1/2 are better off when benefiting from the low price at Black Friday. However, most consumers gather a lower surplus, either because they postpone their purchase and have to bear the costs of waiting, or, for those with a higher valuation, who buy immediately, because the list price is higher under the Black Friday strategy.

4.5.4 Overall Welfare

The overall welfare is the sum of firm and consumer surplus:

$$W = \frac{1-p^*+b^*p^*}{2} + \frac{(1-p^*)^2(1-2p^*)}{2b^*p^*} + \frac{5-(11-b^*)p^*+2(3-b^{*2})p^{*2}}{4} + \frac{b^*p^*(1-p^*+b^*p^*)}{2r}$$

Welfare increases in the time preference r from the benchmark value of $3/8$ and to the convergence value of $7/18$, as illustrated in figure (4.15).

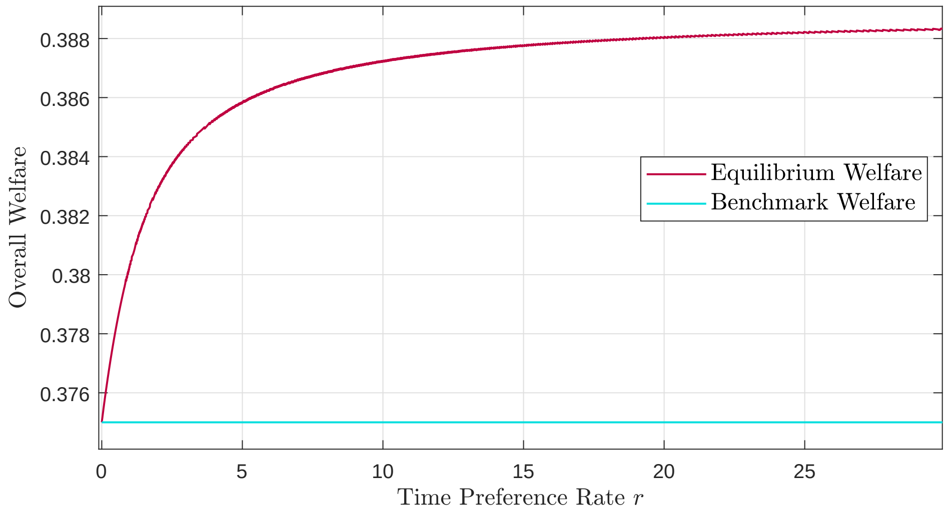

Figure 4.15: Welfare

4.5.5 Final Remark

The welfare results are, at first glance, mixed. In the benchmark equilibrium, the optimal price is lower than under the Black Friday strategy $p > 1/2$, but higher than the discount price $(1-b)p < 1/2$. Overall demand, as well as firms' profits, are lower in the benchmark than under the Black Friday strategy. In sum, these positive effects overweight the combined negative effect of the higher list price and of the utility costs of waiting; and welfare is increased under the Black Friday strategy, as depicted in figure (4.15).

4.6 Conclusion

The modeling of Black Friday presents a most interesting case of intertemporal price discrimination. The problem underlying the Coase conjecture is the difficulty in accounting for the evolution of the willingness to pay (our reservation price) of strategic consumers over time. We build upon selected contributions of the previous literature, by allowing for a difference between firms' discount rate and consumers' time preference rate; and by considering not only an initial cohort of consumers followed by a residual demand in later periods, but rather a steadily renewed pool of strategic consumers. These assumptions account for the existence of cycles of high and low prices. Different possibilities have been suggested in the previous literature for explaining the length of such cycles; we show that the mere arbitrary calendar tradition of Black Friday is enough to explain the performance of such a price discount strategy. Our model does not explain the origin of Black Friday, which is more likely to be explained as a price war at the beginning of the seasonal Christmas shopping. But it explains its success and perpetuation: since the publicity of Black Friday is such that consumers can safely assume its repetition after the upcoming Thanksgiving, the intrinsic dynamic of the price discrimination strategy is sufficient to explain its perpetuation and solve the paradox of the Coase conjecture.

The impact of this self fulfilling Black Friday strategy on welfare is mixed. Overall welfare is higher under the existence of Black Friday, which is owing to the fact that firms benefit strongly from the price discrimination. This positive effect compensates a decline in overall consumers welfare. The decrease in consumers welfare results itself from mixed effects: while some consumers with a low valuation for the good get access to the purchase or to a higher surplus owing to the low discount price offered on Black Friday, many consumers see their utility from consumption decrease, as the regular list price is higher than it would be without the Black Friday strategy: their willingness to pay gets more efficiently skimmed.

Our model neglects the seasonal effect of Christmas shopping. This might seem surprising, and the impact of Christmas shopping cannot be denied, but strategic postponing of purchases appears to be the main driver of the Black Friday sales: Statista (2020) found that nearly half of the purchases on Black Friday were planned in advance (ibid., p. 12), while only a quarter of the interviewed consider Black Friday as a good opportunity for Christmas shopping (ibid, p. 13). Even though the existence of a seasonal effect is evident, the postponing of purchases by strategic consumers, as modelled here, explains an important share of Black Friday purchases.

Chapter 5

Time. Money. Is there a Dark Side of E-commerce?

The different chapters of this work have approached the topic of e-commerce under very different angles, while following the central themes of search mechanisms and of whether e-commerce poses a threat to the traditional channel. A specificity is that this work is purely theoretical, while the literature on the topic is mainly empirical.

In the first chapter, we have extended the existing framework of a classical search model to describe in a new way the expansion by many decentralized firms to e-commerce. The momentum for this model was the COVID-19 pandemic, when many firms had to shift to the online channel at a very short notice. This model proves to be consistent with many empirical findings: the persisting heterogeneity in prices, the lower level of unit costs in the online channel, the stability of high-priced segments in the traditional channel. This first model outlines the "democratic" dimension of e-commerce, as it enables many consumers with lower reservation prices to participate at all in the market. We observe a shift of welfare from the traditional to the online channel, but there is no existential threat looming upon the traditional channel.

The second chapter is in many a way a contrast to the first one: it does not directly rely on an existing framework but elaborates a new one, a combination of a Salop Circle and of an omnipresent cloud above it. This fitting of the online channel as a virtual space into a location model is, as far as we are aware of, a novelty. A further difference to the first model is that here, consumers are passive (they display no searching behavior), but instead the retailers are active. In this framework, we have analyzed the transition from a purely traditional structure to a mixed structure with traditional and online channel. A main finding, which is at the same time a difference to the first model, is that here, all firms have a strong incentive to participate in the market as soon as the reluctance of consumers decrease below a critical level. The reason for this difference is that we considered heterogeneous unit costs in the first, and uniform unit costs in the second model. As in the first model, we find here again that the introduction of the online channel leads to a redistribution of welfare from the firms to the consumers, by enhancing the competition among firms. This second model offers the possibility to analyze how firms coordinate their decisions in the two channels, which prove to be beneficial to the online channel: we observe a phenomenon of cross-channel subventions inside firms. This model too accounts for the overall lower level of prices when an online channel exists, as well as for the possibility that some prices in the online channel are above the lowest prices in the traditional channel. In this model, the online channel leads to higher level of information, even though consumers are passive: firms bring information to the consumers. Finally, in this model, we dedicated an detailed analysis to the question of whether e-commerce represents a threat to traditional commerce. Here again, the answer is negative: there is no cannibalization insofar as firms coordination of the two channel enhances welfare, and that the transformation of the market is linked to a technological standard with a higher level of acceptance among consumers. However, insofar as a constriction of the traditional market structure is seen as negative and not only as a new standard, then yes, there is a threat for the traditional channel. The danger is here that the assessment of such a change becomes normative.

The last model in this work considers a case when both consumers and firms are active. With the example of Black Friday, we consider the question of the optimal timing of purchases, when firms strategically anticipate the searching and planning behavior of consumers. In this model, the question of the transition to the online channel does not arise, because the optimal conditions for such strategies implicate the possibility of a good information of consumers and virtually implicates the existence of e-commerce. The question of the channel is, however, not determinant. An interesting result of this model is that few assumptions (a steadily renewed pool of consumers; a difference between consumers' time preference and firms' discount rate) are sufficient to account for the existence of business cycles, with phases of high and low prices. The model does not explain the apparition of Black Friday, but it explains its perpetuation. The effect on welfare are mixed: while some consumers gain access to the market thanks to the discounted prices at Black Friday, many consumers see their surplus decrease and overall consumers' surplus declines. This decline is compensated by a strong increase in firms surpluses.

These three model have in common the attempt to model search behavior. Their guiding themes are time and money, and the overall tenor is that the idea of a dark side of the online channel is, mainly, a fear for novelty, as represented in the wording of the economic literature itself - where it is question of consumers' reluctance against, rather than of their acceptance of the online channel. There is a central topic missing in this work: the question of market power, which seems to be a very decisive one in the online channel. This might actually be the dark side of e-commerce.

Appendix A

Heaviside Function

The simplification of the expected demand function for a local store i using the Heaviside functions on p. 64 relies on the following computations:

$$\begin{aligned}
\mathrm{E}D_l^i(p_l^i, p_1^o, p_2^o, ..., p_{n_o}^o) &= \int_{x_{i,i-1}^0}^{x_{i,i+1}^0} \left[1 - \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \Big(w_\lambda \cdot F[p_l^i + t(x_i - x)^2 - A]\Big)\right] dx \\
&= \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \int_{x_{i,i-1}^0}^{x_{i,i+1}^0} \Big[1 - F[p_l^i + t(x_i - x)^2 - A]\Big]\, dx\right) \\
&= \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \int_{x_{i,i-1}^0}^{x_i - \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}} \Big[1 - F[p_l^i + t(x_i - x)^2 - A]\Big]\, dx\right) + \\
&\quad \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \int_{x_i - \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}^{x_i + \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}} \Big[1 - F[p_l^i + t(x_i - x)^2 - A]\Big]\, dx\right) + \\
&\quad \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \int_{x_i + \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}^{x_{i,i+1}^0} \Big[1 - F[p_l^i + t(x_i - x)^2 - A]\Big]\, dx\right) \\
&= \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \left[\int_{x_{i,i-1}^0}^{x_i - \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}} [1 - 1] dx + \right.\right. \\
&\quad \left.\left. \int_{x_i - \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}^{x_i + \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}} [1 - 0]\, dx + \int_{x_i + \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}^{x_{i,i+1}^0} [1 - 1] dx\right]\right) \\
&= \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot \Big[x\Big]_{x_i - \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}^{x_i + \sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}}\right) \\
&= \frac{1}{W} \cdot \sum_{\lambda=1}^{n_o} \left(w_\lambda \cdot 2\sqrt{\frac{A + p_\lambda^o - p_l^i}{t}}\right)
\end{aligned}$$

Appendix B

Issue of the Overlapping of Demand Areas

As consumers opt for a channel and, in case they choose the local channel, simultaneously opt for a given store, different indifference conditions constrain the distribution of demand among the channels and the firms. In equilibrium, the preferences between neighboring local stores and online retailers have to be transitive. This is the case when there is no overlapping between the different demand areas. More specifically, we consider here the conditions for transitivity of the preferences and for the existence of e-commerce (the expected demand for the online channel is positive).

Technically, we consider the different demand areas between any two neighbouring stores i and $i+1$ on the Salop circle. They are delimited by the points of indifference depicted in Figure B.1. There is no problematic overlapping if these points of indifference are ordered as in the graph, because then, preferences are transitive. Between store i and $x^o_{i,onl}$, the position of indifference between i and the online channel, consumers buy from i;[1] between the two points of indifference between local and online channels, $x^o_{i,onl}$ and $x^o_{i+1,onl}$, consumers buy online[2]; and finally, between $x^o_{i+1,onl}$ and the store $i+1$, consumers prefer to buy from store $i+1$.[3] For completeness, let us mention that the situation when e-commerce is non-existing arises when the red consumer indifferent between store i and the online channel is located further away from shop i, than the consumer being indifferent between store i and its local neighbour (teal). Such an indifferent consumer "in exile" would be indifferent between i and the online channel, but would actually prefer buying from store $i+1$. Such a situation might arise especially when consumers reluctance against the online channel is high, and results in the benchmark situation treated in section 3.3.1.

[1] In this first area, the order of preferences is $i \succ \textit{online} \succ i+1$.

[2] In this area, the order of preferences is $\textit{online} \succ i \succ i+1$ on the left-hand side of $x^o_{i,i+1}$ and $\textit{online} \succ i+1 \succ i$ on the right-hand side.

[3] In this last area, the order of preferences is $i+1 \succ \textit{online} \succ i$.

In the following, we verify that when the expected demand for the online channel is positive, preferences are transitive in the transitional, asymmetric case as well as in the equilibrium, parity case.

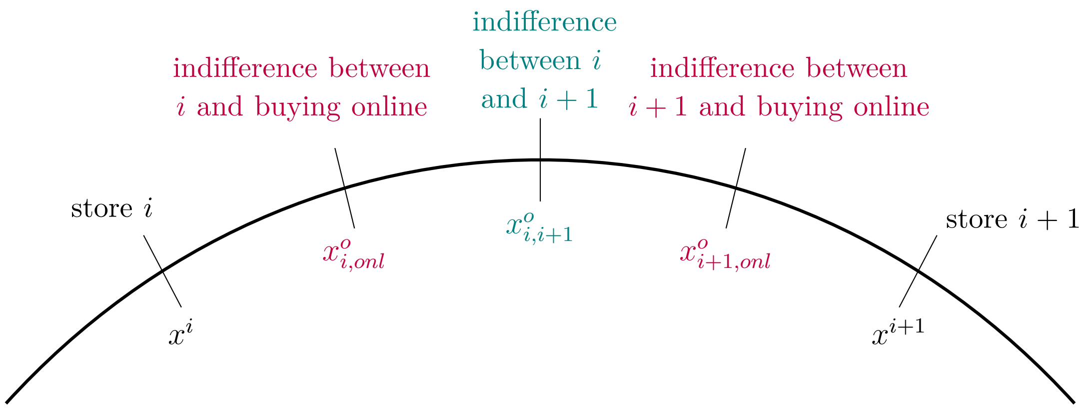

Figure B.1: Transitivity of Preferences as Defined by Indifference Conditions

B.1 In the Transitional State

The specificity of the transitional state if that the different kinds of firms, mutichannel and single-channel, follow different price strategies. Basically, three situations can arise (see Figure B.2). As mentioned above, we put aside the case when e-commerce is inexistent.

On the Salop circle, two neighboring stores might be symmetric, i.e. be both multichannel (case 1 in Figure B.2) or both single-channel (case 2 in the same figure). In such a situation, the two stores have identical price strategies. A noticeable consequence is that the consumer indifferent between them is located in their middle. As per assumption, the distance between two local stores is $1/n_l$,[4], the distance between this indifferent consumer and one of these stores is $1/(2n)$.

Multichannel firms set a slightly higher price in their local store than their single-channel couterparts (see results (3.7) and (3.11)):

$$\frac{2(A+p^*_{m,o})}{3}+\frac{p^*_{m,o}}{3n_o} \geq \frac{2(A+p^*_{m,o})}{3}$$

Correspondingly, on the Salop circle, consumers shift away more easily from the multichannel stores towards online offers than in the case of a single-channel store. The location of the indifferent consumer, $x^o_{i,onl}$, is closer to store i when i belongs to a multichannel

[4] This is, in the parity equilibrium, the optimal distribution of shops.

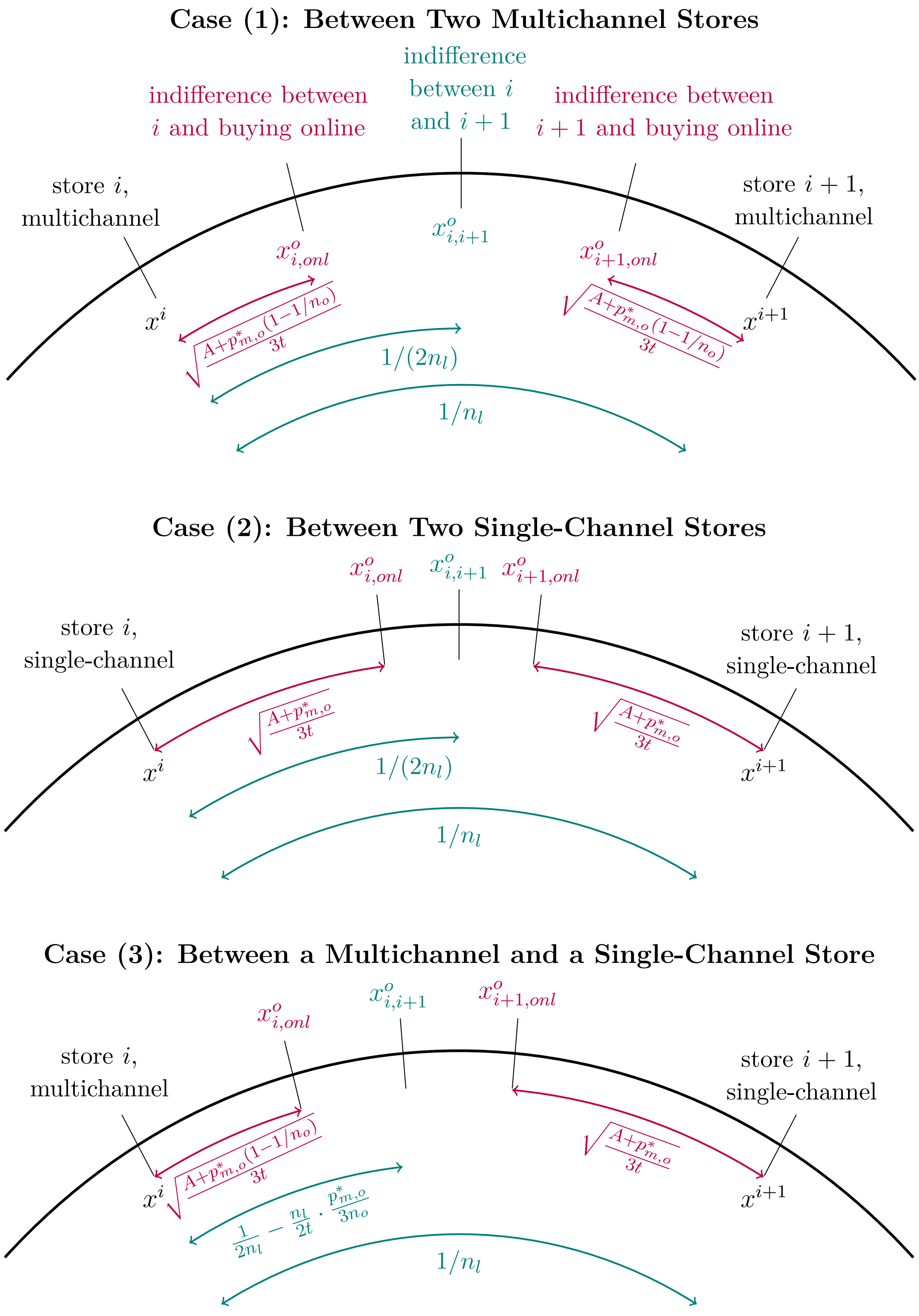

Figure B.2: Transitivity of Preferences in the Transitional State

firm. The indifference condition in the case of a store belonging to a multichannel firm reads (when looking at the right-hand side of firm i):

$$\frac{2(A+p^*_{m,o})}{3}+\frac{p^*_{m,o}}{3n_o}+t(x^o_{i,onl}-x_i)^2=A+p^*_{m,o} \;\Leftrightarrow\; x^o_{i,onl}=x_i+\sqrt{\frac{A+p^*_{m,o}(1-1/n_o)}{3t}}$$

The indifference condition in the case of a store belonging to a single-channel firm reads (looking again at the right-hand side):

$$\frac{2(A+p^*_{m,o})}{3}+t(x^o_{i,onl}-x_i)^2=A+p^*_{m,o} \;\Leftrightarrow\; x^o_{i,onl}=x_i+\sqrt{\frac{A+p^*_{m,o}}{3t}}$$

These values are entered in purple on the graphs in Figure B.2. The condition for the transitivity of preferences in these two symmetric cases (1) and (2) is that the location of indifference with the online channel does not infringe on the position of indifference between the two local stores. This condition is more restrictive when looking at two single-channel stores:

$$\sqrt{\frac{A+p^*_{m,o}}{3t}}\le\frac{1}{2n_l} \quad\Leftrightarrow\quad p^*_{m,o}\le\frac{3}{4n_l^2}-A \tag{B.1}$$

than when looking at two multichannel stores, as their areas of demand are smaller:

$$\sqrt{\frac{A+p^*_{m,o}(1-1/n_o)}{3t}}\le\frac{1}{2n_l} \quad\Leftrightarrow\quad p^*_{m,o}\le\left[\frac{3}{4n_l^2}-A\right]\cdot\frac{n_o}{n_o-1} \tag{B.2}$$

Case (3), in which we consider the distribution of demand between a multichannel and a single-channel store, is slightly different. Because of the different prices set by the two stores, the consumer indifferent between them is not located in the middle. Considering that the single-channel firms owns the store located to the right, and the multichannel firm the one located to the left:

$$\frac{2(A+p^*_{m,o})}{3}+\frac{p^*_{m,o}}{3n_o}+t(x^o_{i,i+1}-x_i)^2=\frac{2(A+p^*_{m,o})}{3}+t(x^o_{i,i+1}-x_i-1/n_l)^2$$
$$\Leftrightarrow\quad x^o_{i,i+1}=x_i+\frac{1}{2n_l}-\frac{n_l}{2t}\cdot\frac{p^*_{m,o}}{3n_o}$$

At the right-hand side, where the single-channel firm is located, the condition for the demand area of the single-channel firm not to infringe on the indifference condition between the two stores reads:

$$\sqrt{\frac{A+p^*_{m,o}}{3t}}\le\frac{1}{2n_l}+\frac{n_l}{2t}\cdot\frac{p^*_{m,o}}{3n_o}$$

which is obviously less restrictive than the condition (B.2) above.

At the left-hand side, the corresponding transitivity condition reads:

$$\sqrt{\frac{A+p^*_{m,o}(1-1/n_o)}{3t}}\le\frac{1}{2n_l}-\frac{n_l}{2t}\cdot\frac{p^*_{m,o}}{3n_o}$$
$$\Leftrightarrow\quad p^*_{m,o}\le\frac{6n_o^2t}{n_l^2}-\frac{2\sqrt{3}n_o\sqrt{An_l^2t+3t^2(n_o^2-1)}}{n_l^2} \tag{B.3}$$

The conditions (B.1), (B.2) and (B.3) are considered in the appropriate places, especially in the comparative statics.

As a final caveat, remark that not all three cases are necessarily always realized, depending on the distribution of the local stores. When there is only one multichannel firm, or when maximally half of the firms are multichannel and each of them is caught in sandwich between two single-channel firms, case (1) and condition (B.2) do not apply, which is of no direct consequence as (B.1) is stricter than (B.2). Similarly, with few single-channel firms caught in sandwich, case (2) does not apply, and neither does condition (B.1). In a situation with as many single-channel as multichannel firms alternating regularly, only case (3) applies, and therefore only condition (B.3). We say, therefore, that (B.3) is a necessary condition, and that (B.1) and (B.2) are configuration-specific conditions.

B.2 In the Parity Constellation

In the parity constellation, with all firms being multichannel firms, the explicit results (3.19) and (3.20) lead, here again, to substantial simplifications. The graphical representation in Figure B.3 is quite similar to the case (1) presented above.

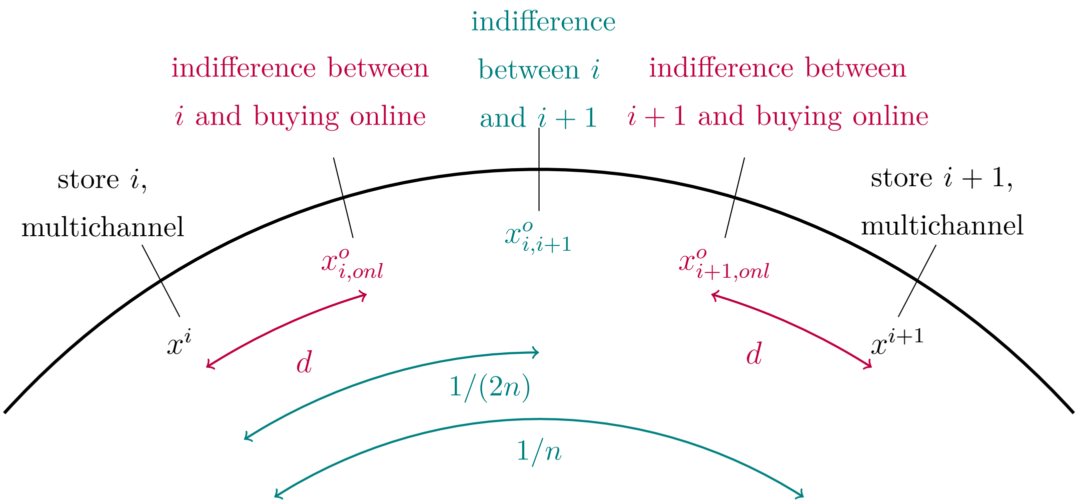

Figure B.3: Transitivity of Preferences in the Parity State

The consumer indifferent between two local stores is located in their middle, as they are all symmetric. The condition of indifference between the two channels can be solved explicitly:

$$p^*_{m,l} + t(x^o_{i,onl} - x_i)^2 = A + p^*_{m,o} \quad \text{where}$$
$$p^*_{m,l} = \frac{(2n+1)\left[t+\sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} + \frac{2nA}{(5n+1)}$$
$$p^*_{m,o} = \frac{3n\left[t+\sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} - \frac{2nA}{(5n+1)}$$
$$\text{with} \quad \Delta = 4tA(n+1)(5n+1) + t^2$$

yielding the solution:

$$x^o_{i,onl} = x_i + \frac{1}{2(5n+1)} + \frac{1}{2}\sqrt{\frac{4A(n+1)(5n+1)+t}{(5n+1)^2 t}}$$

For brevity, we define the help variable d corresponding to the distance between the store's location and the consumer indifferent between two channels:

$$d = \frac{1}{2(5n+1)} + \frac{1}{2}\sqrt{\frac{4A(n+1)(5n+1)+t}{(5n+1)^2 t}} \ .$$

The transitivity condition

$$d \leq 1/(2n)$$

is fulfilled for:

$$A < \frac{(3n+1)t}{4n^2(n+1)}$$

which corresponds to the non-negativity condition on the online demand as formulated in (3.26): if the demand area of a store (d) were to reach the position of indifference to the neighboring store ($1/(2n)$), local online demand would drop to zero. This local condition is the same for each side of any local store, so that it coincides with the global condition (3.26) as computed in the main part.

Appendix C

Comparison of the Profits in Local Stores

We compare here the expected profits realized by single-channel and multichannel firms in their local stores, as formulated in equations (3.9) and (3.16). Be Δ the difference between them:

$$\Delta = \mathrm{E}\pi_s^*(p_{m,o}^*) - \mathrm{E}\pi_{m,t}^*(p_{m,o}^*)$$
$$= \underbrace{\frac{2}{3\sqrt{t}}}_{>0} \cdot \left(\underbrace{2(A + p_{m,o}^*) \cdot \sqrt{\frac{A + p_{m,o}^*}{3}}}_{:=\Delta_1 > 0} - \underbrace{\left[2(A + p_{m,o}^*) + \frac{p_{m,o}^*}{n_o} \right] \cdot \sqrt{\frac{A + p_{m,o}^*}{3} - \frac{p_{m,o}^*}{3n_o}}}_{:=\Delta_2 > 0} \right)$$

Applying a remarkable identity, we multiply the difference $(\Delta_1 - \Delta_2)$ with $(\Delta_1 + \Delta_2) > 0$. The result Δ' will have the same sign as $\Delta_1 - \Delta_2$, and allows for many simplifications:

$$\begin{aligned}
\Delta' =& \Delta_1^2 - \Delta_2^2 \\
=& 4(A + p_{m,o}^*)^2 \cdot \frac{A + p_{m,o}^*}{3} - \left[4(A + p_{m,o}^*)^2 + 4(A + p_{m,o}^*) \cdot \frac{p_{m,o}^*}{n_o} + \frac{(p_{m,o}^*)^2}{n_o^2} \right] \cdot \frac{A + p_{m,o}^*}{3} \\
&+ \left[4(A + p_{m,o}^*)^2 + 4(A + p_{m,o}^*) \cdot \frac{p_{m,o}^*}{n_o} + \frac{(p_{m,o}^*)^2}{n_o^2} \right] \cdot \frac{p_{m,o}^*}{3n_o} \\
=& \frac{(p_{m,o}^*)^3}{3n_o^3} > 0 \\
\Delta =& \frac{2}{3\sqrt{t}} \cdot (\Delta_1 - \Delta_2) > 0
\end{aligned}$$

We can conclude that in the local channel, in the transitional case, single-channel firms always expect higher profits than multichannel ones:

$$\mathrm{E}\pi_s^*(p_{m,o}^*) > \mathrm{E}\pi_{m,t}^*(p_{m,o}^*)$$

Beware the scope of this result: it concerns only the local channel. Multichannel firms expect additional profits from the online channel, and it is well possible that their overall expected profits are higher than those of single-channel firms.

The first interesting conclusion from this result is that the profits from the online channel are crucial to explain the apparition of multichannel firms: if these profits were negligible, multichannel firms would earn less than single-channel firms, which questions the reason for their existence and the stability of such a configuration.

The second, central conclusion is that the sheer existence of the additional online channel has a negative impact on the results of multichannel firms in the local channel, which indeed makes the threat of cannibalization plausible - when looking only at the performance in the local store. The overall impact of getting active as an online retailer will be extensively analyzed in appendix G.

Appendix D

Comparative Statics in the Transisitional Case

D.1 Reluctance Parameter

D.1.1 Derivative of the equilibrium online price with respect to A

The derivative of the equilibrium online price can be obtained by implicitly derivating the first order condition (3.12):

$$\frac{\partial p^*_{m,o}}{\partial A} = \frac{\frac{\left[-(2A-p^*_{m,o})(An_o-p^*_{m,o}+n_o p^*_{m,o})\right](n_l-n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{\left[-(A+p^*_{m,o})(2An_o-5p^*_{m,o}-n_o p^*_{m,o})\right](n_o-1)}{\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}}{\frac{\left[(8A+5p^*_{m,o})(An_o-p^*_{m,o}+n_o p^*_{m,o})\right](n_l-n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{\left[(A+p^*_{m,o})(4An_o(1+2n_o)+(n_o-1)(5n_o+1)p^*_{m,o})\right](n_o-1)}{n_o\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}} \tag{D.1}$$

As $n_o \geq 1$, the two quotients in the denominator are positive. In the numerator, the sign of the two quotients is determined by their own numerators. Let us call the expressions in square brackets in the numerator P_1 and P_2. They are quadratic and convex in the price $p^*_{m,o}$, and quadratic and concave in the reluctance parameter A:

$$P_1 = \left[-(2A - p^*_{m,o})(An_o - p^*_{m,o} + n_o p^*_{m,o})\right]$$
$$P_2 = \left[-(A + p^*_{m,o})(2An_o - 5p^*_{m,o} - n_o p^*_{m,o})\right]$$

These polynomials, weighted respectively with the coefficients $(n_l - n_o)/\sqrt{A + p^*_{m,o}}$ and $(n_o - n_1)/\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}$, determine the sign of the derivative, as depicted in Figure D.1.[1]

[1] An analysis of the two polynomials leads the following results: The derivative $P_1'(A) = -4An_o + (2 - n_o)p^*_{m,o}$ is decreasing in A and reaches, at $A = 0$, the positive value $P_1(0) = (n_o - 1)(p^*_{m,o})^2$. $P_1(A)$ reaches

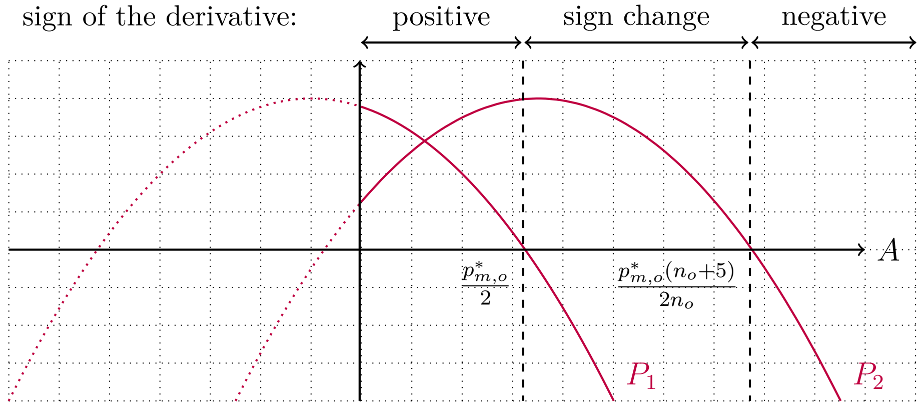

Figure D.1: Decomposition of the sign of the derivative $\partial p^*_{m,o}/\partial A$

This is consistent with the numerical approximation in Figure 3.10. For higher levels of reluctance, we can expect a negative impact of a further increase on the equilibrium price (negative derivative); for low values of A, a positive one. The exact value of A for which the sign changes cannot be formulated as a simple expression; it is situated between the two roots. We can however formulate simple sufficient, but not necessary conditions on A: for values of A below $p^*_{m,o}/2$, the derivative $\partial p^*_{m,o}/\partial A$ is positive. For values above $p^*_{m,o}/2 + (5p^*_{m,o})/(2n_o)$, this derivative is negative. The gap between these values is reduced as the number of online retailers increases.

Not any parameter constellation must necessary cover this complete evolution. When the transportation costs are high (higher equilibrium online price, see below (D.19)) or when the fix costs f are high (very low levels of the reluctance parameter A are not admissible, as the profits in the local channel turn negative), the derivative might be only positive. Inversely, for low fix costs and transportation costs, i.e. when the local channel still has certain competitive advantages, it might be only negative. It is in any case an interesting result that online prices do not increase indefinitely when the acceptance of the online channel is enhanced.

In any parameter constellation, it is however, certain that the non-negativity condition

its peak at $A = (2 - n_o)p^*_{m,o}/(4n_o)$; whether this peak lies within the relevant region of positive values for A or not depends on the number of online retailers n_o. In the illustration, we assume $2 < n_o < 5$, and the peak lies in the negative domain. P_1 has a non-relevant root at a negative value for A (defined by the second factor $(An_o - p^*_{m,o} + n_o p^*_{m,o})$) and a relevant one defined by the first factor, at $A = p^*_{m,o}/2$. The derivative $P_2'(A) = -4An_o + (5 - n_o)p^*_{m,o}$ is decreasing in A and reaches, at $A = 0$, the positive value $P_2(0) = (5 + n_o)(p^*_{m,o})^2$, higher than $P_1(0)$. $P_2(A)$ reaches its peak at $A = (5 - n_o)p^*_{m,o}/(4n_o)$, later than $P_1(A)$; whether this peak is reached for a positive value for A also depends on the number of online retailers n_o. P_2 has a non-relevant root at a negative value for A (defined by the first factor $(A + p^*_{m,o})$) and a relevant one defined by the second factor, at $A = p^*_{m,o}/2 + (5p^*_{m,o})/2n_o$). This root is above the relevant root of $P_1(A)$.

on the equilibrium online price becomes binding only for high values of the reluctance parameter. At first glance, as we expect an inverted-U-relationship between this price and A, we have to check the non-negativity conditions at the origin, for $A = 0$, and when A increases to high values. For $A = 0$, the implicit definition for the price in (3.12) can be solved explicitly:

$$p^*_{m,o} = \left[\frac{\sqrt{t}}{\frac{5(n_l - n_o)}{\sqrt{3}} + \frac{(5+1/n_o)\cdot(n_o-1)}{\sqrt{3}\sqrt{1-1/n_o}}} \right]^2 > 0 \Rightarrow \lim_{A \mapsto 0} p^*_{m,o} > 0$$

The non-negativity on the equilibrium online price is therefore binding only for high values of A.

A first explanatory factor for this inverted-U-relationship between the online price and the reluctance parameter is the contribution of the local channel. In the equation below, we highlight the parts stemming from the impact of the local channel:

$$\frac{\partial p^*_{m,o}}{\partial A} = \frac{\frac{\left[-(2A-p^*_{m,o})(An_o - p^*_{m,o} + n_o p^*_{m,o})\right](n_l - n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{\left[(A+p^*_{m,o})(-2An_o + 2p^*_{m,o} +3p^*_{m,o} - 2n_o p^*_{m,o} +3n_o p^*_{m,o})\right](n_o -1)}{\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}}}{\frac{\left[(8A+5p^*_{m,o})(An_o - p^*_{m,o} + n_o p^*_{m,o})\right](n_l - n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{\left[(A+p^*_{m,o})(2An_o(-1 +3 + 4n_o) + (n_o-1)(5n_o - 2 +3)p^*_{m,o})\right](n_o -1)}{n_o\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}}} \tag{D.2}$$

The highlighted terms prove to have a positive impact on the overall partial derivative. However, the contribution of the online channel is still mixed and varies with A.

The second factor, which explains this mixed contribution, is linked to the shape of the demand function for online retailers. It is decreasing and convex in the price. A consequence is that a decrease in the online price leads to a higher increase in the demand when the overall consumers' price $A + p^*_{m,o}$ is low: a low A, i.e. a high acceptance of the online channel, leads to a stronger migration effect. For low values of the parameter A, a price decrease becomes more attractive as it leads to a higher increase in demand.

D.1.2 Preliminary for the following steps of the comparative statics: some properties

The value of the derivative $\partial p^*_{m,o}/\partial A$ determines many results which are crucial in the following steps. We will therefore compute here the sign of different expressions useful later on.

First, we can prove that this derivative is always above -1. Let us call the quotients in the numerator N_1 and N_2 and those in the denominator $D_1 > 0$ and $D_2 > 0$, in order of

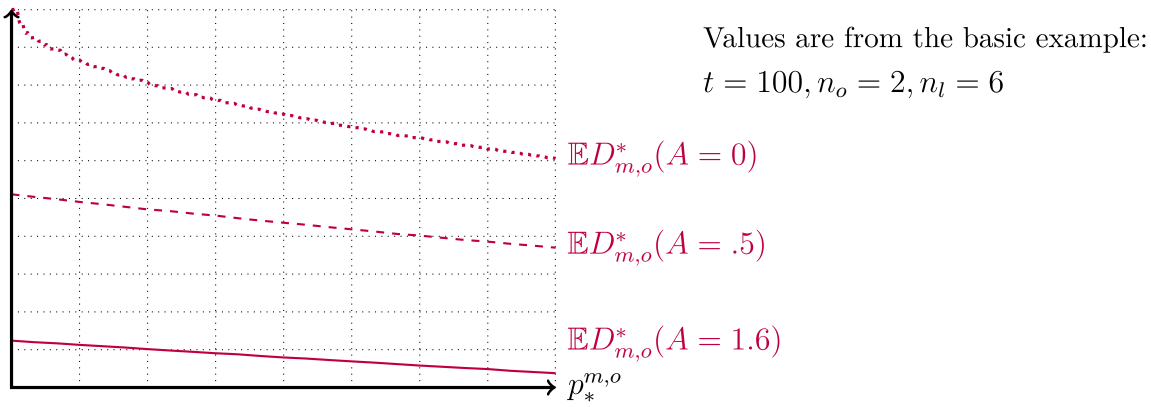

Figure D.2: Convexity of the demand for different levels of A

appearance.

$$\frac{\partial p^*_{m,o}}{\partial A} > -1 \quad \Leftrightarrow \quad \frac{N_1 + N_2}{D_1 + D_2} > -1 \quad \Leftrightarrow \quad N_1 + D_1 + N_2 + D_2 > 0$$

This sum is clearly positive, as all terms are positive:

$$\frac{\left[6(n_o-1)(p^*_{m,o})^2+A(12n_o-6)p^*_{m,o}+6A^2n_o\right](n_l-n_o)}{\sqrt{A+p^*_{m,o}}}+\frac{\left[\left(6n_o^2+n_o-1\right)(p^*_{m,o})^2+A\left(12n_o^2+5n_o-1\right)p^*_{m,o}+2A^2n_o(3n_o+2)\right](n_o-1)}{n_o\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}>0$$

And therefore we conclude that the derivative is above -1:

$$\frac{\partial p^*_{m,o}}{\partial A} > -1 \tag{D.3}$$

Second, for the following steps, we also need to prove that the derivative $\partial p^*_{m,o}/\partial A$ is greater than $-2n_o/(2n_o + 1)$.

$$\frac{\partial p^*_{m,o}}{\partial A} > -2n_o/(2n_o+1) \quad \Leftrightarrow \quad 2n_o(N_1 + D_1 + N_2 + D_2) + N_1 + N_2 > 0$$

Again, each and any factor in the numerators of this last sum is positive:

$$\frac{\left[(12n_o^2 - 11n_o + 1)(p^*_{m,o})^2 + A(24n_o^2 - 13n_o + 2)p^*_{m,o} + A^2(12n_o^2 - 2n_o)\right](n_l - n_o)}{\sqrt{A + p^*_{m,o}}} +$$
$$\frac{\left[(12n_o^3 + 3n_o^2 + 3n_o)(p^*_{m,o})^2 + A(24n_o^3 + 9n_o^2 + 3n_o)p^*_{m,o} + 2A^2 n_o(6n_o^2 + 3n_o)\right](n_o - 1)}{n_o\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} > 0$$

We conclude with the following property:

$$\frac{\partial p^*_{m,o}}{\partial A} > -2n_o/(2n_o+1) \quad \Leftrightarrow \quad 2n_o + \left(2n_o+1\right)\frac{\partial p^*_{m,o}}{\partial A} > 0 \tag{D.4}$$

Third, we will need the sign of the expression $1+\partial p^*_{m,o}/\partial A \cdot (n_o - 1)/n_o$. We can put aside the trivial case $n_o = 1$, in which the sign is obviously positive. For $n_o \geq 2$, we consider equivalently:

$$\frac{\partial p^*_{m,o}}{\partial A} > -n_o/(n_o-1) \quad \Leftrightarrow \quad n_o(N_1 + D_1 + N_2 + D_2) - N_1 - N_2 > 0$$

Here again, in this case for $n_o \geq 2$, this sum is positive:

$$\frac{\left[(6n_o^2 - 7n_o + 1)(p^*_{m,o})^2 + A(12n_o^2 - 5n_o - 2)p^*_{m,o} + 2A^2 n_o(3n_o - 1)\right](n_l - n_o)}{\sqrt{A + p^*_{m,o}}} + \frac{6\left[n_o\left(n_o^2 - 1\right)(p^*_{m,o})^2 + An_o\left(2n_o^2 + n_o - 1\right)p^*_{m,o} + A^2 n_o^2(n_o + 1)\right](n_o - 1)}{n_o\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} > 0$$

And our third property reads:

$$\frac{\partial p^*_{m,o}}{\partial A} > -n_o/(n_o - 1) \quad \Leftrightarrow \quad n_o + \left(n_o - 1\right)\frac{\partial p^*_{m,o}}{\partial A} > 0 \tag{D.5}$$

Fourth , we show that the partial derivative is always below 2:

$$2 - \frac{\partial p^*_{m,o}}{\partial A} = \frac{\frac{(n_o-1)(A+p^*_{m,o})\left[2An_o(9n_o+4)+\left(9n_o^2-13n_o-2\right)p^*_{m,o}\right]}{\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}} + \frac{9n_o(2A+p^*_{m,o})(n_t-n_o)(An_o+(n_o-1)p^*_{m,o})}{\sqrt{A+p^*_{m,o}}}}{\frac{\left[(8A+5p^*_{m,o})(An_o-p^*_{m,o}+n_o p^*_{m,o})\right](n_l-n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{\left[(A+p^*_{m,o})(4An_o(1+2n_o)+(n_o-1)(5n_o+1)p^*_{m,o})\right](n_o-1)}{n_o\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}}$$

All the terms are obviously positive for $n_o \geq 2$; and for $n_o = 1$, the first quotient in the numerator, in which the sign of the square bracket gets problematic, simply disappears due to the leading factor $(n_o - 1)$. As a conclusion, for any number of online retailers $n_o \geq 1$, the following property is verified:

$$\frac{\partial p^*_{m,o}}{\partial A} < 2 \tag{D.6}$$

Fifth and last, we compare the partial derivative to $p^*_{m,o}/A$:

$$\frac{\partial p^*_{m,o}}{\partial A} - \frac{p^*_{m,o}}{A} = -\frac{(A + p^*_{m,o})(An_o + (n_o - 1)p^*_{m,o})}{A} \cdot \frac{\left(\frac{n_o(2A+5p^*_{m,o})(n_l-n_o)}{\sqrt{A+p^*_{m,o}}} + \frac{(n_o-1)(2An_o+(5n_o+1)p^*_{m,o})}{\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}\right)}{\left(\frac{n_o(8A+5p^*_{m,o})(n_l-n_o)(An_o+n_o p^*_{m,o}-p^*_{m,o})}{\sqrt{A+p^*_{m,o}}} + \frac{(n_o-1)(A+p^*_{m,o})(4An_o(2n_o+1)+(n_o-1)(5n_o+1)p^*_{m,o})}{\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}\right)}$$

This expression is long, but all terms are positive, so that the overall sign is determined by the leading minus:

$$\frac{\partial p^*_{m,o}}{\partial A} - \frac{p^*_{m,o}}{A} < 0 \tag{D.7}$$

D.1.3 Derivative of the further prices with respect to A

Having computed these preliminary properties, we can go on with the comparative statics. From the equilibrium prices (3.7) and (3.11), we deduce, using the properties (D.3) and

(D.4):

$$\frac{\partial p_s^*}{\partial A} = \frac{2}{3}\left[1 + \frac{\partial p_{m,o}^*}{\partial A}\right] > 0 \tag{D.8}$$

$$\frac{\partial p_{m,l}^*}{\partial A} = \frac{1}{3n_o}\left[2n_o + \left(2n_o + 1\right)\frac{\partial p_{m,o}^*}{\partial A}\right] > 0 \tag{D.9}$$

The prices in the local channel unambiguously increase with the level of the reluctance parameter, as well in multichannel, as in single-channel firms.
Using property D.6, we prove that the variation of these prices is greater than the variation of the price in the online channel:

$$\frac{\partial p_s^*}{\partial A} - \frac{\partial p_{m,o}^*}{\partial A} = \frac{1}{3}\left[2 - \frac{\partial p_{m,o}^*}{\partial A}\right] > 0 \tag{D.10}$$

$$\frac{\partial p_{m,l}^*}{\partial A} - \frac{\partial p_{m,o}^*}{\partial A} = \frac{1}{3}\left[2 - \frac{\partial p_{m,o}^*}{\partial A}\right] + \frac{1}{3n_o}\frac{\partial p_{m,o}^*}{\partial A} > 0 \tag{D.11}$$

The equilibrium prices as formulated in (3.7) and (3.11) depend on A and on the equilibrium online price; the two results above prove that the effect of A dominates.

D.1.4 Derivative of the demand functions with respect to A

We turn now to the impact on the distribution of demand between the different channels and firms. From the equilibrium demand functions of multichannel firms (3.14) and (3.15), using the properties (D.3) and (D.5), we deduce:

$$\frac{\partial \mathrm{E}D_{m,l}^*}{\partial A} = \frac{1}{\sqrt{\frac{A+p_{m,o}^*}{3t} - \frac{p_{m,o}^*}{3n_o t}}} \cdot \frac{1}{3t}\left[1 + \frac{\partial p_{m,o}^*}{\partial A} \cdot \frac{n_o - 1}{n_o}\right] > 0 \tag{D.12}$$

$$\frac{\partial \mathrm{E}D_{m,o}^*}{\partial A} = -\frac{(n_l - n_o)}{n_o} \cdot \frac{\left[1 + \frac{\partial p_{m,o}^*}{\partial A}\right]}{\sqrt{3t}\sqrt{A + p_{m,o}^*}} - \frac{\left[n_o + (n_o - 1)\frac{\partial p_{m,o}^*}{\partial A}\right]}{\sqrt{3n_o t}\sqrt{n_o A + (n_o - 1)p_{m,o}^*}} < 0 \tag{D.13}$$

From the equilibrium demand of single-channel firms (3.8), using property (D.3), we get:

$$\frac{\partial \mathrm{E}D_s^*}{\partial A} = \frac{1}{\sqrt{3t}\sqrt{A + p_{m,o}^*}} \cdot \left[1 + \frac{\partial p_{m,o}^*}{\partial A}\right] > 0 \tag{D.14}$$

As in the main part, the interpretation focuses at a decrease of the reluctance parameter A, i.e. an improvement of consumers' acceptance of the online channel. A decrease in the reluctance parameter has a negative impact on the demand for the local channel, both for single-channel and multichannel firms; and a positive impact on the demand for the online channel: in the transitional case, a better acceptance of the online channel shifts indeed the demand from the local to the online channel.

D.1.5 Derivative of the visibility expenditures with respect to A

From the equilibrium visibility expenditure (3.13), we deduce the variation of the visibility expenditure per unit:

$$\frac{\partial(w^*/\mathrm{E}D^*_{m,o})}{\partial A} = \frac{n_o - 1}{n_o} \cdot \frac{\partial p^*_{m,o}}{\partial A}$$

We refer to the discussion on the sign of the derivative $\partial p^*_{m,o}/\partial A$ in section D.1.1. The variation of the overall visibility costs links the variations of the visibility costs per unit to the variation of the expected demand:

$$\frac{\partial(w^*)}{\partial A} = \frac{n_o - 1}{n_o} \cdot \left[p^*_{m,o} \cdot \frac{\partial \mathrm{E}D^*_{m,o}}{\partial A} + \frac{\partial p^*_{m,o}}{\partial A} \cdot \mathrm{E}D^*_{m,o} \right]$$

As this last factor shows up again later on in the analysis of the profits, we analyse it into more details and transform it using the first order condition (3.12) to replace the transportation costs t:

$$\begin{aligned} p^*_{m,o} \cdot \frac{\partial \mathrm{E}D^*_{m,o}}{\partial A} + \frac{\partial p^*_{m,o}}{\partial A} \cdot \mathrm{E}D^*_{m,o} \\ = \frac{ - \frac{p^*_{m,o} n_o (n_l - n_o)\left(1 - 2\frac{\partial p^*_{m,o}}{\partial A}\right)}{\sqrt{A + p^*_{m,o}}} - \frac{(2An_o + (n_o+1)p^*_{m,o})\frac{\partial p^*_{m,o}}{\partial A} + n_o^2 p^*_{m,o}\left(1 - 2\frac{\partial p^*_{m,o}}{\partial A}\right)}{\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} }{ \frac{n_o(n_l - n_o)(2A + 5p^*_{m,o})}{\sqrt{A + p^*_{m,o}}} + \frac{(n_o - 1)(2An_o + (5n_o + 1)p^*_{m,o}}{\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} } \end{aligned} \tag{D.15}$$

This expression (D.15) is increasing in the derivative $\partial p^*_{m,o}/\partial A$. An obvious result is the case when the derivative $\partial p^*_{m,o}/\partial A$ is below 1/2; then our expression is negative. The criterion $\partial p^*_{m,o}/\partial A < 1/2$ is mostly, but not always met; especially, it is met for $n_o = 1$ and $n_o \geq 5$.[2] For the values $2 \leq n_o \leq 4$, we can, however, describe the values of the

[2]The result for $1/2 - \partial p^*_{m,o}/\partial A$ reads:

$$\frac{1}{2} - \frac{\partial p^*_{m,o}}{\partial A} = \frac{ \frac{(n_o-1)(A+p^*_{m,o})\left(4An_o(3n_o+1) + \left(3n_o^2 - 14n_o - 1\right)p^*_{m,o}\right)}{\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} + \frac{3n_o(4A + p^*_{m,o})(n_l - n_o)(An_o + (n_o-1)p^*_{m,o})}{\sqrt{A + p^*_{m,o}}} }{ \frac{2\left[(8A + 5p^*_{m,o})(An_o - p^*_{m,o} + n_o p^*_{m,o})\right](n_l - n_o)}{\sqrt{A + p^*_{m,o}}} + \frac{2\left[(A + p^*_{m,o})(4An_o(1+2n_o) + (n_o-1)(5n_o+1)p^*_{m,o})\right](n_o - 1)}{n_o\sqrt{A - \frac{p^*_{m,o}}{n_o} + p^*_{m,o}}} }$$

Observe the first quotient in the numerator: is is zero for $n_o = 1$ and positive for $n_o \geq 5$. All other quotients are positive. We can put aside the case $n_o = 1$, as there is no need for visibility expenditure in case there is only one online retailer. A sufficient condition for the difference to be positive is therefore that $n_o \geq 5$. For intermediary values, the sign of this first quotient depends on the value of A and the price and cannot be determined at one sight.

derivative for which the expression (D.15) is negative:

$$\frac{\partial p^*_{m,o}}{\partial A} \leq b^*$$

$$\begin{aligned} b^* =& \frac{n_o^2 p^*_{m,o}\sqrt{A+p^*_{m,o}} + n_o p^*_{m,o}(n_l-n_o)\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}{\left[-2An_o + p^*_{m,o}(2n_o^2-n_o-1)\right]\sqrt{A+p^*_{m,o}} + 2n_o p^*_{m,o}(n_l-n_o)\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}} \\ =& \frac{1}{2} + \frac{\left[An_o + n_o p^*_{m,o}/2 + p^*_{m,o}/2)\right]\sqrt{A+p^*_{m,o}}}{\left[-2An_o + p^*_{m,o}(2n_o^2-n_o-1)\right]\sqrt{A+p^*_{m,o}} + 2n_o p^*_{m,o}(n_l-n_o)\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}} \end{aligned}$$

This boundary value b^* is discontinuous in A, but increasing in A on each side of the point of discontinuity, as illustrated in Figure D.3 (plotted with the values of the basic example).[3] For values of A above the point of discontinuity, the hurdle for consumers to choose the online channel is infinitely high; firms cannot charge any positive price and the online channel cannot survive. To make out the sign of expression (D.15) and of the

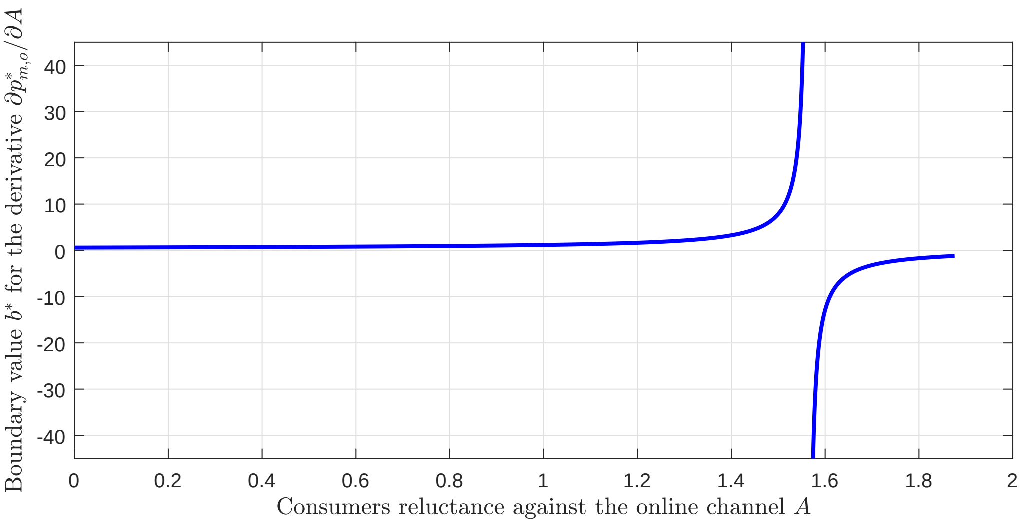

Figure D.3: Evolution of the boundary value b^*

derivative of the overall visibility expenditure as A varies, we have to check whether the

[3] The partial derivative with respect to A reads:

$$\frac{\left(p^*_{m,o} - A\frac{dp^*_{m,o}}{dA}\right)\left[4n_o^3(A+p^*_{m,o}) + \frac{(n_l-n_o)\left(2An_o(2An_o+(4n_o-3)p^*_{m,o})+\left(4n_o^2-5n_o-1\right)(p^*_{m,o})^2\right)}{\sqrt{A+p^*_{m,o}}\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}}\right]}{2\left(\sqrt{A+p^*_{m,o}}(2An_o-(n_o-1)(2n_o+1)p^*_{m,o}) - 2n_o p^*_{m,o}(n_l-n_o)\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}\right)^2} > 0$$

The sign of the factor $\left(p^*_{m,o} - A\frac{\partial p^*_{m,o}}{\partial A}\right)$ is determined in property (D.7). The boundary $b^*(A)$ is however not continuous; it is undetermined when the denominator converges to zero. The value of the denominator declines with increasing A and drops from positive to negative values. The boundary value is positive below the point of essential discontinuity and negative above it. The left-hand side limit at this point of discontinuity is plus infinity; the right-hand side limit is minus infinity. The convergence value when A tends to infinity is 0^-.

derivative $\partial p^*_{m,o}/\partial A$ is below the boundary b^* for the three values for n_o not yet solved for: $n_o \in [2,4]$. The formula (D.1) implies that $\partial p^*_{m,o}/\partial A - b^*$ is quadratic and concave in n_l, so that this expression is negative outside the roots. The initial expression in (D.15) is negative, respectively:

- for $n_o = 2$ outside the roots

$$n^{0,2}_{t;1,2} = \frac{\sqrt{2}\left(16A^3-140A^2p^*_{m,o}-115A(p^*_{m,o})^2+2(p^*_{m,o})^3\right)\sqrt{A+p^*_{m,o}}\sqrt{2A+p^*_{m,o}}+24p^*_{m,o}(4A+p^*_{m,o})(2A+p^*_{m,o})^2}{12p^*_{m,o}(2A+p^*_{m,o})^2(4A+p^*_{m,o})}$$
$$\pm\frac{\sqrt{2(A+p^*_{m,o})(2A+p^*_{m,o})\left(256A^6-1408A^5p^*_{m,o}-3280A^4(p^*_{m,o})^2-5560A^3(p^*_{m,o})^3-4087A^2(p^*_{m,o})^4-1348A(p^*_{m,o})^5+316(p^*_{m,o})^6\right)}}{12p^*_{m,o}(2A+p^*_{m,o})^2(4A+p^*_{m,o})}$$

  In case $A = 0$, the roots read $\frac{1}{6}\left(\sqrt{2}+12\pm\sqrt{158}\right)$ and this condition simplifies to $n_l \leq 5$.

- for $n_o = 3$ outside the roots

$$n^{0,3}_{t;1,2} = -\frac{-\sqrt{3}\left(12A^3-178A^2p^*_{m,o}-171A(p^*_{m,o})^2-10(p^*_{m,o})^3\right)\sqrt{A+p^*_{m,o}}\sqrt{3A+2p^*_{m,o}}-18p^*_{m,o}(4A+p^*_{m,o})(3A+2p^*_{m,o})^2}{6p^*_{m,o}(4A+p^*_{m,o})(3A+2p^*_{m,o})^2}$$
$$\pm\frac{\sqrt{3(A+p^*_{m,o})(3A+2p^*_{m,o})\left(144A^6-816A^5p^*_{m,o}-2660A^4(p^*_{m,o})^2-4740A^3(p^*_{m,o})^3-3615A^2(p^*_{m,o})^4-1060A(p^*_{m,o})^5+356(p^*_{m,o})^6\right)}}{6p^*_{m,o}(4A+p^*_{m,o})(3A+2p^*_{m,o})^2}$$

  In case $A = 0$, this condition is always fulfilled, as both roots $(3-\frac{5}{2\sqrt{6}}\pm\frac{\sqrt{89}}{2\sqrt{6}})$ are below the lowest acceptable value for n_l: 4.

- for $n_o = 4$ outside the roots

$$n^{0,4}_{t;1,2} = \frac{\left(64A^3-1336A^2p^*_{m,o}-1379A(p^*_{m,o})^2-132(p^*_{m,o})^3\right)\sqrt{A+p^*_{m,o}}\sqrt{4A+3p^*_{m,o}}+48p^*_{m,o}(4A+p^*_{m,o})(4A+3p^*_{m,o})^2}{12p^*_{m,o}(4A+p^*_{m,o})(4A+3p^*_{m,o})^2}$$
$$\pm\frac{\sqrt{\left(4096A^6-23552A^5p^*_{m,o}-87360A^4(p^*_{m,o})^2-159152A^3(p^*_{m,o})^3-123383A^2(p^*_{m,o})^4-33816A(p^*_{m,o})^5+13536(p^*_{m,o})^6\right)}}{12p^*_{m,o}(4A+p^*_{m,o})(4A+3p^*_{m,o})^2}$$
$$\cdot\sqrt{(A+p^*_{m,o})(4A+3p^*_{m,o})}$$

  In case $A = 0$, again, this condition is always fulfilled, as both roots $(\frac{1}{9}\left(36-11\sqrt{3}\pm\sqrt{282}\right))$are below the lowest acceptable value for n_l: 5.

As a result, we can conclude that the initial expression in (D.15), $p^*_{m,o}\cdot\frac{\partial \mathrm{E}D^*_{m,o}}{\partial A}+\frac{\partial p^*_{m,o}}{\partial A}\cdot \mathrm{E}D^*_{m,o}$ is negative excepted in some few cases, when the number of both online and local stores is very low. This limitation is however not relevant any more in the case of free market entry; because then, all firms have an incentive to open an online retailer and in the limiting case when they do so, the expression (D.15) is unambiguously negative.

D.1.6 Derivative of the profit functions with respect to A

The impact of the reluctance parameter on the profit functions in equilibrium (3.9), (3.16) and (3.17) can be deduced from the previous results, under consideration of the non-negativity condition defined in (B.3):

$$\frac{\partial \mathrm{E}\pi_s^*}{\partial A} = p_s^* \cdot \frac{\partial \mathrm{E}D_s^*}{\partial A} + \frac{\partial p_s^*}{\partial A} \cdot \mathrm{E}D_s^* > 0 \tag{D.16}$$

$$\frac{\partial \mathrm{E}\pi_{m,l}^*}{\partial A} = p_{m,l}^* \cdot \frac{\partial \mathrm{E}D_{m,l}^*}{\partial A} + \frac{\partial p_{m,l}^*}{\partial A} \cdot \mathrm{E}D_{m,l}^* > 0 \tag{D.17}$$

$$\frac{\partial \mathrm{E}\pi_{m,o}^*}{\partial A} = \frac{1}{n_o} \cdot \left[p_{m,o}^* \cdot \frac{\partial \mathrm{E}D_{m,o}^*}{\partial A} + \frac{\partial p_{m,o}^*}{\partial A} \cdot \mathrm{E}D_{m,o}^*\right] \gtreqless 0 \tag{D.18}$$

For the interpretation of this last result, we refer to the discussion above, on the overall visibility costs (p. 167).

D.2 Transportation Costs

Derivative of the equilibrium online price with respect to t

Here again, the derivative of the equilibrium online price results from an implicit derivation of the first order condition (3.12):

$$\frac{\partial p_{m,o}^*}{\partial t} = \frac{\frac{\sqrt{A+p_{m,o}^*}[An_o+(n_o-1)p_{m,o}^*](2A+5p_{m,o}^*)}{n_o-1} + \frac{(A+p_{m,o}^*)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}(2An_o+5n_op_{m,o}^*+p_{m,o}^*)}{n_l-n_o}}{\frac{t(A+p_{m,o}^*)[4An_o(2n_o+1)+(n_o-1)(5n_o+1)p_{m,o}^*]}{n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}} + \frac{t[An_o+(n_o-1)p_{m,o}^*](8A+5p_{m,o}^*)}{(n_o-1)\sqrt{A+p_{m,o}^*}}}$$

The quotients in the numerator as well as in the denominator are obviously positive for any number of online retailers ($n_o \geq 1$). Despite the complexity of this partial derivative, its sign is therefore unambiguously positive. As the transportation costs increase, price competition is relaxed for the online channel, and the equilibrium price increases:

$$\frac{\partial p_{m,o}^*}{\partial t} > 0 \tag{D.19}$$

The non-negativity condition on the equilibrium online price $p_{m,o}^*$ might therefore be binding only for low levels of the transportation costs parameter t.

Preliminary for the following steps of the comparative statics: a property

Only one property implying this derivative will be needed later on. It is the sign of the difference:

$$\frac{\partial p_{m,o}^*}{\partial t} - \frac{(A+p_{m,o}^*)}{t} = \frac{-\frac{6A\sqrt{A+p_{m,o}^*}(An_o+(n_o-1)p_{m,o}^*)}{n_o-1} - \frac{A(A+p_{m,o}^*)\left(2An_o(3n_o+2)+(3n_o-1)(2n_o+1)p_{m,o}^*\right)}{n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}}}{\frac{t(A+p_{m,o}^*)[4An_o(2n_o+1)+(n_o-1)(5n_o+1)p_{m,o}^*]}{n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}} + \frac{t[An_o+(n_o-1)p_{m,o}^*](8A+5p_{m,o}^*)}{(n_o-1)\sqrt{A+p_{m,o}^*}}}$$

For any number of multichannel firms $n_o \geq 1$, the sign of the quotients in the numerator is unambiguous and the whole quotient is negative, which leads to the property:

$$\frac{\partial p_{m,o}^*}{\partial t} - \frac{(A+p_{m,o}^*)}{t} < 0 \tag{D.20}$$

Derivative of the further prices with respect to t

From the equilibrium prices (3.7) and (3.11), we deduce:

$$\frac{\partial p_s^*}{\partial t} = \frac{2}{3} \cdot \frac{\partial p_{m,o}^*}{\partial t} > 0 \tag{D.21}$$
$$\frac{\partial p_{m,l}^*}{\partial t} = \frac{2n_o + 1}{3n_o} \cdot \frac{\partial p_{m,o}^*}{\partial t} > 0 \tag{D.22}$$

The sign of these derivatives follows directly from (D.19). The equilibrium prices increase with the level of the transportation costs, in both channels and for both types of firms: the well-known relaxing effect of the transportation parameter on price competition[4] is reflected in our model too, and as in the transitional case, it expands to the online channel.

Remark that the variation of $p_{m,o}^*$ is the steepest, and that of p_s^* the most moderate:

$$\underbrace{\frac{2}{3} \cdot \frac{\partial p_{m,o}^*}{\partial t}}_{\dfrac{\partial p_s^*}{\partial t}} < \underbrace{\frac{2n_o + 1}{3n_o} \cdot \frac{\partial p_{m,o}^*}{\partial t}}_{\dfrac{\partial p_{m,l}^*}{\partial t}} \leq \frac{\partial p_{m,o}^*}{\partial t} \tag{D.23}$$

In the case of symmetry, we will observe a similar phenomenon: an increase in the transportation costs has a stronger positive impact on equilibrium prices in online retailers than in local stores. Here, we observe that the multichannel firms can, in such a situation, increase their equilibrium price in the local channel stronger than single-channel local stores: they benefit from the channel diversification.

Derivative of the visibility expenditures with respect to t

From the equilibrium visibility expenditure (3.13), we deduce the variation of the visibility expenditure per unit of expected demand:

$$\frac{\partial (w^*/\mathrm{E}D_{m,o}^*)}{\partial t} = \frac{n_o - 1}{n_o} \cdot \frac{\partial p_{m,o}^*}{\partial t} > 0 \tag{D.24}$$

The overall visibility expenditure are a share of the online profits and follow the same evolution: they increase with the transportation costs (see below, D.31).
Under increasing transportation costs, both the online price and the unitary visibility expenditure increase. In case $n_o = 1$, there is only one online retailer, and this sole store does not need to invest in visibility expenditures, regardless of the level of transportation costs. The online price net of unitary visibility expenditure, denoted with p_1^{o*net} increases at a lower pace for $n_o > 2$:

$$\frac{\partial p_{m,o}^{*net}}{\partial t} = \frac{1}{n_o} \cdot \frac{\partial p_{m,o}^*}{\partial t} > 0 \tag{D.25}$$

[4]See e.g. Salop (1979), pp. 148 sqq.

Derivative of the demand functions with respect to t

From the equilibrium demands (3.14), (3.15) and (3.8) and using property (D.20), we deduce:

$$\frac{\partial \mathrm{E}D_s^*}{\partial t} = \frac{1}{\sqrt{\frac{A+p_{m,o}^*}{3t}}} \cdot \frac{1}{3t} \cdot \left[\frac{\partial p_{m,o}^*}{\partial t} - \frac{(A+p_{m,o}^*)}{t}\right] < 0 \tag{D.26}$$

$$\frac{\partial \mathrm{E}D_{m,l}^*}{\partial t} = \frac{1}{\sqrt{\frac{A+p_{m,o}^*}{3t} - \frac{p_{m,o}^*}{3n_o t}}} \cdot \frac{1}{3n_o t} \cdot \left[(n_o-1)\left(\frac{\partial p_{m,o}^*}{\partial t} - \frac{A+p_{m,o}^*}{t}\right) - \frac{A}{t}\right] < 0 \tag{D.27}$$

$$\frac{\partial \mathrm{E}D_{m,o}^*}{\partial t} = \frac{(n_l-n_o)\left[\frac{(A+p_{m,o}^*)}{t} - \frac{\partial p_{m,o}^*}{\partial t}\right]}{3t \cdot n_o \cdot \sqrt{\frac{A+p_{m,o}^*}{3t}}} + \frac{(n_o-1)\left[\frac{A+p_{m,o}^*}{t} - \frac{\partial p_{m,o}^*}{\partial t}\right] + \frac{A}{t}}{3t \cdot n_o \cdot \sqrt{\frac{A+p_{m,o}^*}{3t} - \frac{p_{m,o}^*}{3n_o t}}} > 0 \tag{D.28}$$

Increasing transportation costs make the local channel less attractive, and foster a shift of the demand to the online channel.

Derivative of the profit functions with respect to t

From the profit function in equilibrium (3.9), we deduce:

$$\frac{\partial \mathrm{E}\pi_s^*(p_{m,o}^*)}{\partial t} = 2 \cdot \sqrt{\frac{A+p_{m,o}^*}{3t}} \cdot \left[\frac{\partial p_{m,o}^*}{\partial t} - \frac{(A+p_{m,o}^*)}{3t}\right]$$

$$\frac{\partial \mathrm{E}\pi_s^*(p_{m,o}^*)}{\partial t} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\partial p_{m,o}^*}{\partial t} \gtreqless \frac{(A+p_{m,o}^*)}{3t} \tag{D.29}$$

The sign of the corresponding difference depends on the factor $(A - 5p_{m,o}^*)$: the denominator is positive, and the two expressions in square brackets are positive:

$$\frac{\partial p_{m,o}^*}{\partial t} - \frac{(A+p_{m,o}^*)}{3t} =$$

$$\frac{-(A-5p_{m,o}^*)\left[\frac{2\sqrt{A+p_{m,o}^*}(An_o+(n_o-1)p_{m,o}^*)}{3(n_o-1)} + \frac{2n_o^2(A+p_{m,o}^*)^2}{3n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}}\right] - \left[\frac{(A+p_{m,o}^*)\left(4A^2n_o+A(3n_o-1)p_{m,o}^*+2(4n_o+1)(p_{m,o}^*)^2\right)}{3n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}}\right]}{\frac{t(A+p_{m,o}^*)[4An_o(2n_o+1)+(n_o-1)(5n_o+1)p_{m,o}^*]}{n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}} + \frac{t[An_o+(n_o-1)p_{m,o}^*](8A+5p_{m,o}^*)}{(n_o-1)\sqrt{A+p_{m,o}^*}}}$$

Even though higher transportation costs relax the price competition among neighbouring local stores, this increase in prices might not be high enough to compensate the migration of consumers to the online channel. Both sides of equation D.29 are decreasing in t, but not at the same rate: the right hand-side is steeper. A short way to prove this property is to transform the equation (D.29), using the property that the denominator is positive, into:

$$n_o(n_l-n_o)\sqrt{A-\frac{p_{m,o}^*}{n_o}+p_{m,o}^*}((10p_{m,o}^*-2A)(An_o+(n_o-1)p_{m,o}^*)) \gtreqless$$
$$(n_o-1)\sqrt{A+p_{m,o}^*}\left(2A^2n_o(n_o+2) - A\left(8n_o^2-3n_o+1\right)p_{m,o}^* + 2\left(-5n_o^2+4n_o+1\right)(p_{m,o}^*)^2\right)$$

The right-hand side is decreasing in t; while the left-hand side is increasing in t.[5] As t increases, it becomes more probable that the left-hand side is greater than the right-hand side. As a consequence, we expect that equation (D.29) implies decreasing profits in the single channel only for low values of t, if any.

The increase in the demand for the online channel is indeed the highest at its origin, i.e. as online sales emerge. When looking at the numerical simulations, this is, actually, the rather less intuitive realization of (D.29). As t increases, the trend flips and the price effect dominates the migration effect. A graphical interpretation of the migration effect is given in the main part of this work (see p. 83).

From the profit functions in equilibrium (3.16), (3.17), (3.18) and the property (D.20), we deduce:

$$\frac{\partial \mathrm{E}\pi^*_{m,l}}{\partial t} = \frac{\left(2An_o^2 + (n_o-1)(2n_o+1)p^*_{m,o}\right)\left[\frac{\partial p^*_{m,o}}{\partial t} - \frac{An_o+(n_o-1)p^*_{m,o}}{3n_o t}\right] - (2n_o+1)\frac{\partial p^*_{m,o}}{\partial t}}{3n_o^2 t\sqrt{\frac{An_o+(n_o-1)p^*_{m,o}}{3n_o t}}}$$

$$\frac{\partial \mathrm{E}\pi^*_{m,l}}{\partial t} \gtreqless 0 \quad \Leftrightarrow \quad \frac{\partial p^*_{m,o}}{\partial t} \gtreqless \frac{\left[2n_o(A+p^*_{m,o}) + p^*_{m,o}\right]\cdot\left[An_o + (n_o-1)p^*_{m,o}\right]}{3t\left[2An_o^2 + (2n_o+1)(n_o-1)p^*_{m,o}\right]} \tag{D.30}$$

$$\frac{\partial \mathrm{E}\pi^*_{m,o}}{\partial t} = \frac{1}{n_o}\cdot\underbrace{\frac{\partial p^*_{m,o}}{\partial t}}_{>0}\cdot \mathrm{E}D^*_{m,o} + \frac{1}{n_o}\cdot p^*_{m,o}\cdot\underbrace{\frac{\partial \mathrm{E}D^*_{m,o}}{\partial t}}_{>0} > 0 \tag{D.31}$$

Equation (D.30) can be analyzed as (D.29), with qualitatively similar results: for the lowest possible levels of the transportations costs t, the migration effect dominates. As t increases, the price effect dominates and the local channel benefits from increasing profits.

While the online channel of multichannel firms always enjoys higher profits when the transportation costs increase, the local channel might suffer from it in a specific case: when the online channel is emerging and growing rapidly. This might be also an explanation why such fears of a cannibalistic behavior of online retailers were quite justified exactly at the beginning of e-commerce.

[5]The respective derivatives are: for the right-hand side:

$$-\frac{(n_o-1)\left(2A^2\left(7n_o^2-5n_o+1\right)+A\left(64n_o^2-41n_o-5\right)p^*_{m,o}+10\left(5n_o^2-4n_o-1\right)(p^*_{m,o})^2\right)\frac{\partial p^*_{m,o}}{\partial t}}{2\sqrt{A+p^*_{m,o}}} < 0$$

and for the left-hand side:

$$n_o(n_l-n_o)(A(7n_o+3)+25(n_o-1)p^*_{m,o})\sqrt{A-\frac{p^*_{m,o}}{n_o}+p^*_{m,o}}\cdot\frac{\partial p^*_{m,o}}{\partial t} > 0$$

Appendix E

Comparative Statics in the Symmetry Case

E.1 Reluctance Parameter

Remark that we focus at a decrease of the reluctance parameter, corresponding to an improvement of consumers' acceptance of the online channel. The interpretation of the following results rather refers to a decrease in A, and therefore to an impact with the opposite sign.

E.1.1 Derivative of the equilibrium prices with respect to A

From the equilibrium prices (3.19) and (3.20), we deduce:

$$\frac{\partial p^{**}_{m,l}}{\partial A} = \frac{(2n+1)(n+1)t + 2(n-1)n\sqrt{t\left(4A(5n+1)(n+1)+t\right)}}{(n-1)(5n+1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}} > 0 \tag{E.1}$$

$$\frac{\partial p^{**}_{m,o}}{\partial A} = \frac{n\left(3(n+1)t - 2(n-1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}\right)}{(n-1)(5n+1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}}$$

The sign of the second derivative might vary. It is positive for low values of A and negative from $\overline{A}$ on:

$$\overline{A} = \frac{(n+2)(3n-1)\left(3n^2+n+2\right)t}{16(n-1)^2(n+1)(5n+1)} \tag{E.2}$$

This value for A might not, however, be eligible insofar as it does not fulfill the non-negativity condition (3.26) ($\overline{A} > \frac{t+3nt}{4n^2(n+1)}$ for $n \geq 2, t > 0$). The derivative is therefore concave in A.

$$\frac{\partial p^{**}_{m,o}}{\partial A} = \frac{n\left(3(n+1)t - 2(n-1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}\right)}{(n-1)(5n+1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}} \lesseqgtr 0 \tag{E.3}$$

Remark that the variation of $p^{**}_{m,l}$ is steeper:

$$\frac{\partial p^{**}_{m,l}}{\partial A} - \frac{\partial p^{**}_{m,o}}{\partial A} = \frac{4n\sqrt{\Delta} - (n+1)t}{(5n+1)\sqrt{\Delta}} \geq \frac{4nt - (n+1)t}{(5n+1)\sqrt{\Delta}} > 0 \tag{E.4}$$

E.1.2 Derivative of the visibility expenditure with respect to A

From the equilibrium visibility expenditure (3.21), we deduce the variation of the visibility expenditure per unit:

$$\frac{\partial(w^* * /\mathrm{E}D^{**}_o)}{\partial A} = \frac{3(n+1)t - 2(n-1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}}{(5n+1)\sqrt{t\left(4A(5n+1)(n+1)+t\right)}} \lesseqqgtr 0 \tag{E.5}$$

The same reasoning concerning the sign of this derivative apply as in the preceding case: it is concave in A.

In a scenario where A decreases, both the online price and the unitary visibility expenditure decrease, but not at the same pace. The online price net of unitary visibility expenditure, denoted here with p_2^{o*net}, might therefore increase or decrease, depending on the level of A:

$$\frac{\partial p^{**net}_o}{\partial A} > 0 \quad \Leftrightarrow \quad A \leq \frac{(n+5)t}{16(n-1)^2(n+1)}$$

E.1.3 Derivative of the demand functions with respect to A

From the equilibrium demands (3.22) and (3.23), we deduce:

$$\frac{\partial \mathrm{E}D^{**}_l}{\partial A} = \frac{\sqrt{2}(n+1)\left(\sqrt{t\left(4A(5n+1)(n+1)+t\right)}+t\right)}{t\sqrt{t\left(4A(5n+1)(n+1)+t\right)}\sqrt{\frac{\sqrt{t(4A(5n+1)(n+1)+t)}+2A(5n+1)(n+1)+t}{t}}} > 0 \tag{E.6}$$

$$\frac{\partial \mathrm{E}D^{**}_o}{\partial A} = \frac{-\sqrt{2}(n+1)\left(\sqrt{t\left(4A(5n+1)(n+1)+t\right)}+t\right)}{t\sqrt{t\left(4A(5n+1)(n+1)+t\right)}\sqrt{\frac{\sqrt{t(4A(5n+1)(n+1)+t)}+2A(5n+1)(n+1)+t}{t}}} < 0 \tag{E.7}$$

As the overall demand is constantly 1, these results also illustrate the impact on the respective shares of the demand addressed to each channel; and the results are of the same absolute value, with opposite signs. A decrease in the reluctance parameter, meaning an enhanced acceptance of the online channel, shifts the demand from the local to the online channel.

E.1.4 Derivative of the profit functions with respect to A

From the profit functions in equilibrium (3.24) and (3.25), we deduce:

$$\frac{\partial \mathrm{E}\pi_l^{**}}{\partial A} = \frac{2\sqrt{2}(n+1)(2n+1)t\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}}{(n-1)(5n+1)^2\sqrt{\Delta}} > 0 \tag{E.8}$$

$$\begin{aligned}\frac{\partial \mathrm{E}\pi_o^{**}}{\partial A} =& \frac{2(5n+1)\frac{1}{\sqrt{\Delta}}\left[3n(n+1)t-2(n-1)\sqrt{\Delta}\right]\left(\frac{1}{n}-\sqrt{2}\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{(5n+1)^2 t}}\right)}{2(n-1)(5n+1)^2} \\ &- \frac{(n+1)\left(\frac{t}{\sqrt{\Delta}}+1\right)\left[3\left(n\sqrt{\Delta}+t\right)-4A(5n+1)(n-1)\right]}{\sqrt{2}(n-1)(5n+1)^2 t\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}} \\ \text{where} \quad & \Delta = t(4A(n+1)(5n+1)+t)\end{aligned}$$

The expression in the first pair of square brackets is similar to the denominator of $\partial p_{m,o}^{**}/\partial A$, and therefore positive. The expression in the second pair of square brackets: $\left[3\left(n\sqrt{\Delta}+t\right)-4A(5n+1)(n-1)\right]$ corresponds to the denominator of the online price net of unit visibility costs and is positive. Which effect dominates, depends on the relative level of A and n. For $A \leq \underline{A}$:

$$\underline{A}=\frac{\left(9n^6+24n^5+28n^4-24n^3-8n^2+6n+1+\left(-3n^3-7n^2+3n+1\right)\sqrt{9n^6+6n^5+34n^4-18n^3-2n^2+6n+1}\right)t}{18(n-1)^2n^2(n+1)(5n+1)}$$

the derivative is positive. This value is eligible, as it is below the boundary defined by the non-negativity condition (3.26). For values above this threshold $\underline{A}$ and below the non-negativity boundary, it is negative. This second case is the one interesting to us, as we consider, later on, a scenario with free market entry, where the number of firms n is as high as possible for a given level of the reluctance parameter. But in a first step, before the endogenization of n, the online profits are concave in A:

$$\frac{\partial \mathrm{E}\pi_o^{**}}{\partial A} \lesseqqgtr 0 \tag{E.9}$$

A higher value of the reluctance parameter reflects indeed in higher profits for the local channel, and lower profits for the online channel.

Not depending on the threshold $\underline{A}$, the sum of the individual profits in both channels (firm profits) increases wit A:

$$\frac{\partial \mathrm{E}\pi^{**}}{\partial A} = \frac{\partial \mathrm{E}\pi_l^{**}}{\partial A} + \frac{\partial \mathrm{E}\pi_o^{**}}{\partial A} > 0 \tag{E.10}$$

More specifically, the impact on the share of the profits collected in the online channel is positive:[1]

$$\frac{\partial\left[\frac{\mathrm{E}\pi_o^{**}}{\mathrm{E}\pi_o^{**}+\mathrm{E}\pi_l^{**}}\right]}{\partial A} = \frac{\partial \mathrm{E}\pi_o^{**}}{\partial A}\cdot \mathrm{E}\pi_l^{**} - \frac{\partial \mathrm{E}\pi_l^{**}}{\partial A}\cdot \mathrm{E}\pi_o^{**} < 0 \tag{E.11}$$

[1] As $\frac{\partial \mathrm{E}\pi_o^{**}}{\partial A} < 0$, $\frac{\partial \mathrm{E}\pi_l^{**}}{\partial A} > 0$, $\mathrm{E}\pi_o^{**} \geq 0$ and $\mathrm{E}\pi_l^{**} \geq 0$, no further computations are needed.

E.2 Transportation Costs

E.2.1 Derivative of the equilibrium prices with respect to t

From the equilibrium prices (3.19) and (3.20), we deduce:

$$\frac{\partial p^{**}_{m,l}}{\partial t} = \frac{(2n+1)\left(\sqrt{\Delta} + 2A\,(5n^2+6n+1) + t\right)}{2(n-1)(5n+1)^2\sqrt{\Delta}} > 0 \tag{E.12}$$

$$\frac{\partial p^{**}_{m,o}}{\partial t} = \frac{3n}{2(n-1)(5n+1)^2} \cdot \left(\frac{2A\,(5n^2+6n+1)+t}{\sqrt{\Delta}} + 1\right) > 0 \tag{E.13}$$

The relaxing effect of the transportation costs on competition becomes here apparent; interestingly enough, it affects, indeed, the prices in the online channel too.

Remark that the variation of $p^{**}_{m,o}$ is steeper:

$$\frac{\partial p^{**}_{m,l}}{\partial t} - \frac{\partial p^{**}_{m,o}}{\partial t} = \frac{-\sqrt{\Delta} - 2A\,(5n^2+6n+1) - t}{2(5n+1)^2\sqrt{\Delta}} < 0 \tag{E.14}$$

E.2.2 Derivative of the visibility expenditures with respect to t

From the equilibrium visibility expenditure (3.21), we deduce the variation of the visibility expenditure per unit:

$$\frac{\partial (w^{**}/\mathrm{E}D^{**}_o)}{\partial t} = \frac{3}{2(n-1)(5n+1)^2} \cdot \left(\frac{2A\,(5n^2+6n+1)+t}{\sqrt{\Delta}} + 1\right) > 0 \tag{E.15}$$

As the visibility expenditures per unit and the expected demand in the online channel both augment with the transportation costs parameter t, the overall visibility expenditure increase too:

$$\frac{\partial w^{**}}{\partial t} > 0 \tag{E.16}$$

As the visibility expenditures represent a share of the online profits, they increase in the same way when the transportation costs increase (see below, result E.21).

In a scenario where t increases, for example as experienced lately with the COVID-19 health restrictions, both the online price and the unitary visibility expenditure decrease. The online price net of unitary visibility expenditure, denoted with p_2^{o*net} varies at the exact same pace:

$$\frac{\partial p^{**net}_o}{\partial t} = \frac{3}{2(n-1)(5n+1)^2} \cdot \left(\frac{2A\,(5n^2+6n+1)+t}{\sqrt{\Delta}} + 1\right) > 0 \tag{E.17}$$

E.2.3 Derivative of the demand functions with respect to t

From the equilibrium demands (3.22) and (3.23), we deduce:

$$\frac{\partial \mathrm{E}D_l^{**}}{\partial t} = \frac{-\left[\sqrt{t^2+t4A(5n^2+6n+1)}-t\right]\sqrt{\frac{\sqrt{\Delta}+2A(5n^2+6n+1)+t}{t}}}{\sqrt{2}(5nt+t)\sqrt{\Delta}} < 0 \tag{E.18}$$

$$\frac{\partial \mathrm{E}D_o^{**}}{\partial t} = \frac{\left[\sqrt{t^2+t4A(5n^2+6n+1)}-t\right]\sqrt{\frac{\sqrt{\Delta}+2A(5n^2+6n+1)+t}{t}}}{\sqrt{2}(5nt+t)\sqrt{\Delta}} > 0 \tag{E.19}$$

$$\text{where} \quad \Delta = t(4A(n+1)(5n+1)+t)$$

Increasing transportation costs make the local channel less attractive, so that, other things being equal, demand shifts to the online channel.

E.2.4 Derivative of the profit functions with respect to t

From the local profit functions in equilibrium (3.24), we deduce:

$$\frac{\partial \mathrm{E}\pi_l^{**}}{\partial t} = \frac{(2n+1)\left(\sqrt{\Delta}+t\right)\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}}{\sqrt{2}(n-1)(5n+1)^3\sqrt{\Delta}} > 0 \tag{E.20}$$

With higher transportation costs relaxing the price competition among neighboring local stores, the increase in prices is high enough to compensate the migration of consumers to the online channel and firms realize higher profits in the local channel. The online profit functions in equilibrium (3.25) yield:

$$\frac{\partial \mathrm{E}\pi_o^{**}}{\partial t} = \sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}\left[\frac{3t\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}}{2(n-1)n(5n+1)^2\sqrt{\Delta}}+\right.$$
$$\left.\frac{-3t\left(\sqrt{\Delta}+t\right)-2A(n-1)(5n+1)\left(\sqrt{\Delta}-t\right)}{\sqrt{2}(n-1)(5n+1)^3t\sqrt{\Delta}}\right]$$
$$\text{where} \quad \Delta = t(4A(n+1)(5n+1)+t)$$

This expression is positive if:

$$t > \frac{4A(n-1)n\left(3n+3-2\sqrt{3}\sqrt{\frac{n(n^2-1)}{3n+1}}\right)}{3(n+3)}$$

which is always fulfilled insofar as t fulfills the non-negativity condition (3.26). The impact of t on the profits in the online channel is always positive:

$$\frac{\partial \mathrm{E}\pi_o^{**}}{\partial t} > 0 \tag{E.21}$$

Higher transportation costs benefit both the online and the local channel and the sum of the individual profits in both channels (firm profits) increases with t:

$$\frac{\partial \mathrm{E}\pi^{**}}{\partial t} = \frac{\partial \mathrm{E}\pi_l^{**}}{\partial t} + \frac{\partial \mathrm{E}\pi_o^{**}}{\partial t} > 0 \tag{E.22}$$

E.3 Number of Firms

E.3.1 Derivative of the equilibrium prices with respect to n

From the equilibrium prices (3.19) and (3.20), we deduce, under consideration of the non-negativity constraint (3.26):

$$\frac{\partial p_l^{**}}{\partial n} = -\frac{\left(20n^2+5n-7\right)t\left(\sqrt{\Delta}+t\right)-4A(5n+1)\left((n-1)^2\sqrt{\Delta}-2\left(5n^3+12n^2+3n-2\right)t\right)}{2(n-1)^2(5n+1)^3\sqrt{\Delta}} < 0 \tag{E.23}$$

$$\frac{\partial p_o^{**}}{\partial n} = -\frac{3\left(10n^2-5n+1\right)t\left(\sqrt{\Delta}+t\right)+4A(5n+1)\left((n-1)^2\sqrt{\Delta}+3\left(5n^3+7n^2-n+1\right)t\right)}{2(n-1)^2(5n+1)^3\sqrt{\Delta}} < 0 \tag{E.24}$$

$$\text{where} \quad \Delta = t(4A(n+1)(5n+1)+t)$$

The fiercer competition linked to a higher number of market participants leads to lower prices in both channels. The difference is steepness is positive:

$$\frac{\partial p_l^{**}}{\partial n} - \frac{\partial p_o^{*}}{\partial n} = \frac{5t\left(\sqrt{\Delta}+t\right)+2A(5n+1)\left(2\sqrt{\Delta}+5nt+7t\right)}{(5n+1)^3\sqrt{\Delta}} > 0 \tag{E.25}$$

This means that the online price decreases faster than the local one as the number of firms increases.

The online price net of unitary visibility expenditure, denoted with p_2^{o*net}, also decreases as competition gets more intensive:

$$\frac{\partial p_o^{**net}}{\partial n} = -\frac{3\left(5t\left(\sqrt{\Delta}+t\right)+2A(5n+1)\left(2\sqrt{\Delta}+5nt+7t\right)\right)}{(5n+1)^3\sqrt{\Delta}} < 0 \tag{E.26}$$

E.3.2 Derivative of the demand and profit functions with respect to n

From the equilibrium demands (3.22) and (3.23), we deduce:

$$\frac{\partial \mathrm{E}D_l^{**}}{\partial n} = -\frac{\sqrt{2}\left(5t\left(\sqrt{\Delta}+t\right)+2A(5n+1)\left(2\sqrt{\Delta}+5nt+7t\right)\right)}{(5n+1)^2t\sqrt{\Delta}\sqrt{\frac{\sqrt{\Delta}+2A(5n^2+6n+1)+t}{t}}} < 0 \tag{E.27}$$

$$\frac{\partial \mathrm{E}D_o^{**}}{\partial n} = \frac{(5n+3)t\sqrt{2\Delta}\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}}{(n+1)(5n+1)^2\Delta} + \frac{2\sqrt{2}\sqrt{\frac{\sqrt{\Delta}+2A(n+1)(5n+1)+t}{t}}}{(n+1)(5n+1)^2} - \frac{1}{n^2} < 0 \tag{E.28}$$

$$\text{where} \quad \Delta = t(4A(n+1)(5n+1)+t)$$

This result, too, is intuitive: as the number of firms increases, the demand in each channel for each store decreases.

As both prices and expected demands decline in the number of firms, the channel specific store profits as well as the individual firms profits over both channels decline as the number of competitors increase:

$$\frac{\partial \mathbb{E}\pi_o^{**}}{\partial n} < 0 \; ; \qquad \frac{\partial \mathbb{E}\pi_t^{**}}{\partial n} < 0 \; ; \qquad \frac{\partial \mathbb{E}\pi^{**}}{\partial n} < 0 \tag{E.29}$$

Appendix F

Analysis of Consumer Surplus and Welfare in the Parity Equilibrium

F.1 Consumers' Surplus

The derivative of the consumers' surplus in (3.27) with respect to the number of firms is:

$$\begin{aligned}\frac{\partial CS^{**}}{\partial n} =& \Big[8\sqrt{2}A(5n+1)(n-1)^2\Big(A(n^2-1)(5n-4)(5n+1)t-3(5n^2+6n-1)t^2+[A(n+1)(5n+1)(5n^2+1)\\ &-(n+2)(5n-1)t]\sqrt{t(4A(n+1)(5n+1)+t)}\Big)+4\sqrt{2}(1-10n)(n-1)^2t^2\Big(\sqrt{t(4A(n+1)(5n+1)+t)}+t\Big)\\ &+\Big[3(5n+1)t\big(4A(5n+1)(n-1)^2+3\big(10n^2-5n+1\big)t\big)\sqrt{t(4A(n+1)(5n+1)+t)}\\ &+9(5n+1)t^2\big(4A(5n+1)\big(5n^3+7n^2-n+1\big)+\big(10n^2-5n+1\big)t\big)\Big]\sqrt{\frac{\sqrt{t(4A(n+1)(5n+1)+t)}+2A(n+1)(5n+1)+t}{t}}\Big]\Big/\\ &\Big[6(n-1)^2(5n+1)^4t\sqrt{t(4A(n+1)(5n+1)+t)}\sqrt{\frac{\sqrt{t(4A(n+1)(5n+1)+t)}+2A(n+1)(5n+1)+t}{t}}\Big]\end{aligned}$$

which is always positive for any realizable value of the reluctance parameter $A \leq (3nt + t)/(4n^2(n+1))$ (see condition (3.26)), any positive t and any number of firms $n > 1$.

The derivative of consumers' surpluses with respect to the transportation costs is:

$$\frac{\partial CS^{**}}{\partial t} = \frac{n\left(\sqrt{t\,(20An^2+24An+4A+t)}+10An^2+12An+2A+t\right)}{6(n-1)(5n+1)^3t^2\sqrt{t\,(20An^2+24An+4A+t)}\sqrt{\frac{\sqrt{t(20An^2+24An+4A+t)}+10An^2+12An+2A+t}{t}}}\cdot$$

$$\begin{aligned}&\Big[-\frac{45n+9}{\sqrt{2}}\cdot t\cdot\sqrt{2t\sqrt{t\,(20An^2+24An+4A+t)}+t(20An^2+24An+4A+2t)}\\ &+\frac{4n-4}{\sqrt{2}}\cdot t\cdot\left(t+\sqrt{t\,(20An^2+24An+4A+t)}\right)\\ &-2\sqrt{2}(n-1)\left(5An^2+6An+A\right)\left(\sqrt{t\,(20An^2+24An+4A+t)}-t\right)\Big]\end{aligned}$$

where the quotient is always positive while the subsequent expression in square brackets is always negative for any positive values of A and t and any number of firms $n > 1$.

The derivative of consumers' surpluses with respect to the reluctance parameter A is:

$$\frac{\partial CS^{**}}{\partial A} = \left[2\sqrt{2}(n-1)n(n+1)\left(\frac{\sqrt{t(4A(n+1)(5n+1)+t)}(A(n+1)(5n+1)+t)}{t}+3A(n+1)(5n+1)+t\right) - (5n+1)\cdot\left((n-1)(3n+1)\sqrt{t(4A(n+1)(5n+1)+t)}+3n(n+1)t\right)\cdot\sqrt{\frac{\sqrt{t(4A(n+1)(5n+1)+t)}+2A(n+1)(5n+1)+t}{t}}\right] \Big/ \left[(n-1)(5n+1)^2\sqrt{t(20An^2+24An+4A+t)}\sqrt{\frac{\sqrt{t(20An^2+24An+4A+t)}+10An^2+12An+2A+t}{t}}\right]$$

where the denominator is positive while the numerator is negative for any realizable value of the reluctance parameter $A \leq (3nt+t)/(4n^2(n+1))$, any positive t and any number of firms $n > 1$.

F.2 Overall Welfare

F.2.1 Variation with respect to the market structure

Using formula 3.28, we deduce:

$$\frac{\partial W^{**}}{\partial n} = -\frac{t}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + 3n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right] + \left[\mathbb{E}D_l^{**} + n\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right]\cdot\left[A + \frac{(n-1)}{n}\cdot p_o^{**}\right] - \left[1 - n\cdot\mathbb{E}D_l^{**}\right]\cdot\left[\frac{1}{n^2}\cdot p_o^{**} + \frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial n}\right] - f$$

We transform it for simplification of the next computation steps:

$$\frac{\partial W^{**}}{\partial n} = -\frac{t}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + 4n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right] + \left[\mathbb{E}D_l^{**} + n\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right]\cdot\left[A + \frac{(n-1)}{n}\cdot p_o^{**}\right] - \left(\left[1 - n\cdot\mathbb{E}D_l^{**}\right]\cdot\left[\frac{1}{n^2}\cdot p_o^{**} + \frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial n}\right] - \frac{nt}{12}\left[\mathbb{E}D_l^{**}\right]^2\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right) - f$$

Following the properties (E.24) and (E.27), both the price and demand derivatives $\partial p_o^{**}/\partial n$ and $\partial \mathbb{E}D_l^{**}/\partial n$ are negative. We consider the crucial parts of this formula and deduce, under consideration of the non-negativity condition (3.26):

$$\mathbb{E}D_l^{**} + 4n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n} = \frac{\sqrt{2}\left(2A(5n+1)\left(\left(5n^2-2n+1\right)\sqrt{\Delta}-2\left(5n^2+8n-1\right)t\right)-(15n-1)t\left(\sqrt{\Delta}+t\right)\right)}{(5n+1)^2 t\sqrt{\left(4A\left(5n^2+6n+1\right)+t\right)\left(\sqrt{\Delta}+2A\left(5n^2+6n+1\right)+t\right)}} < 0$$

so that the first summand is positive:

$$-\frac{t}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + 4n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right] > 0\ .$$

The following summand is positive too:

$$\mathbb{E}D_l^{**} + n \cdot \frac{\partial \mathbb{E}D_l^{**}}{\partial n} = \frac{t\left(\sqrt{\Delta}+t\right) + 2A(5n+1)\left((5n^2+4n+1)\sqrt{\Delta} + (5n^2+5n+2)\,t\right)}{(5n+1)^3 t\sqrt{\Delta}} > 0$$

The second factor in the expression in the rounded brackets is positive:

$$\frac{1}{n^2}\cdot p_o^{**} + \frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial n} = \frac{-3\left(10n^2+5n+1\right)t\left(\sqrt{\Delta}+t\right) - 4A(5n+1)\left(\left(n^2+4n+1\right)\sqrt{\Delta}+3\left(5n^3+12n^2+6n+1\right)t\right)}{2n^2(5n+1)^3\sqrt{\Delta}} < 0$$

The expression in the rounded brackets is negative when the non-negativity condition (3.26) is fulfilled:

$$\left([1-n\cdot\mathbb{E}D_l^{**}]\cdot\left[\frac{1}{n^2}\cdot p_o^{**} + \frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial n}\right] - \frac{nt}{12}\left[\mathbb{E}D_l^{**}\right]^2\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial n}\right) =$$
$$\frac{\left(18(5n+1)t - 19\sqrt{2}n\sqrt{t\left(\sqrt{\Delta}+2A(5n^2+6n+1)+t\right)}\right)\left(5t\left(\sqrt{\Delta}+t\right)+2A(5n+1)\left(2\sqrt{\Delta}+5nt+7t\right)\right)}{6(5n+1)^4 t\sqrt{\Delta}} < 0$$

The sum of the three first summands (the explicit formula is not reproduced here because of its length) is positive, but reduced by f. The level of f is, however, restricted by the non-negativity condition on the overall profits:

$$\mathrm{E}\pi^{**} = \mathrm{E}\pi_l^{**}(n) + \mathrm{E}\pi_o^{**}(n) \geq 0$$
$$.,\Leftrightarrow f \leq \left[\frac{(2n+1)\left[t+\sqrt{\Delta}\right]}{2(5n+1)^2(n-1)} + \frac{2nA}{(5n+1)}\right]\mathbb{E}D_l^{**} + \left[\frac{3\left(t+\sqrt{\Delta}\right)}{2(5n+1)^2(n-1)} - \frac{2A}{(5n+1)}\right]\cdot\left[\frac{1}{n} - \mathbb{E}D_l^{**}\right]$$

This restriction is sufficient to ensure that the overall expression is positive:

$$\frac{\partial W^{**}}{\partial n} > 0$$

F.2.2 Variation with respect to the transportation costs

The derivative of welfare with respect to the transportation costs reads:

$$\frac{\partial W^{**}}{\partial t} = -\frac{n}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + 3t\frac{\partial \mathbb{E}D_l^{**}}{\partial t}\right] + n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial t}\cdot\left[A + \frac{(n-1)}{n}\cdot p_o^{**}\right]$$
$$-\left[1-n\cdot\mathbb{E}D_l^{**}\right]\cdot\left[\frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial t}\right]$$

We reformulate it for convenience as:

$$\frac{\partial W^{**}}{\partial t} = -\frac{n}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + \frac{5t}{2}\frac{\partial \mathbb{E}D_l^{**}}{\partial t}\right] + n\cdot\frac{\partial \mathbb{E}D_l^{**}}{\partial t}\cdot\left[A + \frac{(n-1)}{n}\cdot p_o^{**} - \frac{t}{24}\left[\mathbb{E}D_l^{**}\right]^2\right]$$
$$-\left[1-n\cdot\mathbb{E}D_l^{**}\right]\cdot\left[\frac{(n-1)}{n}\cdot\frac{\partial p_o^{**}}{\partial t}\right]$$

Following the property (E.12), the price derivative $\partial p_o^{**}/\partial t$ is positive; and following (E.18), the derivative of the demand $\partial \mathbb{E}D_l^{**}/\partial n$ is negative. No further information is needed to make out the sign of the last summand:

$$-\left[1 - n \cdot \mathbb{E}D_l^{**}\right] \cdot \left[\frac{(n-1)}{n} \cdot \frac{\partial p_o^{**}}{\partial t}\right] < 0$$

We check the sign of the last-but-one summand:

$$A + \frac{(n-1)}{n} \cdot p_o^{**} - \frac{t}{24}\left[\mathbb{E}D_l^{**}\right]^2 = \frac{17\left(\sqrt{\Delta} + 2A\left(5n^2+6n+1\right)+t\right)}{12(5n+1)^2} > 0$$

implies, due to the negative demand derivative, that the last-but-one summand is negative:

$$n \cdot \frac{\partial \mathbb{E}D_l^{**}}{\partial t} \cdot \left[A + \frac{(n-1)}{n} \cdot p_o^{**} - \frac{t}{24}\left[\mathbb{E}D_l^{**}\right]^2\right] < 0\ .$$

And finally, we check the sign of the first summand:

$$\mathbb{E}D_l^{**} + \frac{5t}{2}\frac{\partial \mathbb{E}D_l^{**}}{\partial t} = \frac{\left(5t - \sqrt{\Delta}\right)\sqrt{\frac{\sqrt{\Delta}+2A(5n^2+6n+1)+t}{4A(5n^2+6n+1)+t}}}{2\sqrt{2}(5nt+t)}$$

The sign depends on the first factor in the numerator, which is declining in A. Considering the upper boundary for A as defined by the non-negativity condition (3.26), we state that the sign is always positive:

$$5t - \sqrt{\Delta} \geq 5t - \sqrt{t\left(4A\left(5n^2+6n+1\right)+t\right)} \geq \frac{(n-1)t}{n} > 0$$

so that the first summand is negative:

$$-\frac{n}{12}\left[\mathbb{E}D_l^{**}\right]^2\left[\mathbb{E}D_l^{**} + \frac{5t}{2}\frac{\partial \mathbb{E}D_l^{**}}{\partial t}\right] < 0$$

As all summands are negative, the sign of the overall derivative with respect to the transportation costs is clear:

$$\frac{\partial W^{**}}{\partial t} < 0$$

F.2.3 Variation with respect to the reluctance parameter

The derivative of welfare with respect to the reluctance parameter reads:

$$\frac{\partial W^{**}}{\partial A} = -\frac{nt}{4}\left[\mathbb{E}D_l^{**}\right]^2\frac{\partial \mathbb{E}D_l^{**}}{\partial A} + n \cdot \frac{\partial \mathbb{E}D_l^{**}}{\partial A} \cdot \left[A + \frac{(n-1)}{n} \cdot p_o^{**}\right]$$
$$- \left[1 - n \cdot \mathbb{E}D_l^{**}\right] \cdot \left[1 + \frac{(n-1)}{n} \cdot \frac{\partial p_o^{**}}{\partial A}\right]$$

This derivative is, itself, increasing from negative values to positive ones;

$$
\begin{aligned}
\left.\frac{\partial W^{**}}{\partial A}\right|_{A=0} &= -\frac{2(n+1)(7n+3)}{(5n+1)^2} \\
\left.\frac{\partial W^{**}}{\partial A}\right|_{A=\frac{(3n+1)t}{4n^2(n+1)}} &= \frac{n+1}{4n+1} \\
\left.\frac{\partial^2 W^{**}}{\partial A^2}\right|_{A\leq\frac{(3n+1)t}{4n^2(n+1)}} &\geq 0
\end{aligned}
$$

so that welfare has a U-shaped relationship to the reluctance parameter.

Appendix G

Issue of Cannibalism

G.1 Positive impact of firms' coordination on prices

The difference between the equilibrium price in the online channel with and without firms' coordination of the channels is positive if :

$$\frac{3n\sqrt{t(4A(n+1)(5n+1)+t)}}{2(n-1)(5n+1)^2} - \frac{3\sqrt{t\left(20An^2+t\right)}}{50n^2} + \frac{2A}{5(5n+1)} + \frac{3t(15n^2+9n+1)}{50(n-1)n^2(5n+1)^2} \geq 0$$
$$\Leftrightarrow \quad 75n^3\sqrt{t(4A(n+1)(5n+1)+t)} - 3(n-1)(5n+1)^2\sqrt{t\left(20An^2+t\right)}$$
$$\geq -\Big(20A(n-1)n^2(5n+1) + 3t(15n^2+9n+1)\Big)$$

where

$$\sqrt{t(4A(n+1)(5n+1)+t)} > \sqrt{t\left(20An^2+t\right)}$$

We deduce a lower boundary for the left hand side of our expression:

$$75n^3\sqrt{t(4A(n+1)(5n+1)+t)} - \Big(3(n-1)(5n+1)^2\Big)\sqrt{t\left(20An^2+t\right)}$$
$$> \Big(75n^3 - 3(n-1)(5n+1)^2\Big)\sqrt{t\left(20An^2+t\right)}$$

This lower boundary is positive:

$$\Big(75n^3 - 3(n-1)(5n+1)^2\Big)\sqrt{t\left(20An^2+t\right)} = (45n^2+27n+3)\sqrt{t\left(20An^2+t\right)} > 0$$

And therefore above the negative right-hand side:

$$\Leftrightarrow \quad 75n^3\sqrt{t(4A(n+1)(5n+1)+t)} - 3(n-1)(5n+1)^2\sqrt{t\left(20An^2+t\right)}$$
$$\geq 0$$
$$\geq -\Big(20A(n-1)n^2(5n+1) + 3t(15n^2+9n+1)\Big)$$

For the net price, the same proof applies, with all values divided by n.

The difference between the equilibrium price in the local channel with and without firms' coordination of the channels is equal to:

$$\frac{(2n+1)\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}}{2(n-1)(5n+1)^2} - \frac{\sqrt{t}\sqrt{20An^2+t}}{25n^2} + \frac{(55n^2+18n+2)\,t}{50(n-1)n^2(5n+1)^2} - \frac{2A}{5(5n+1)}$$

As above, we use the property $\sqrt{4A(n+1)(5n+1)+t} > \sqrt{20An^2+t}$ for a series of inequalities and prove that, under the non-negativity restriction (3.26), this difference is positive:

$$\begin{aligned}
&\frac{(2n+1)\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}}{2(n-1)(5n+1)^2} - \frac{\sqrt{t}\sqrt{20An^2+t}}{25n^2} + \frac{(55n^2+18n+2)\,t}{50(n-1)n^2(5n+1)^2} - \frac{2A}{5(5n+1)}\\
&\geq \frac{(2n+1)\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}}{2(n-1)(5n+1)^2} - \frac{\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}}{25n^2}\\
&\quad + \frac{(55n^2+18n+2)\,t}{50(n-1)n^2(5n+1)^2} - \frac{2\frac{(3n+1)t}{4n^2(n+1)}}{5(5n+1)}\\
&\geq \frac{(55n^2+18n+2)\sqrt{t}\sqrt{4A\,(5n^2+6n+1)+t}}{50(n-1)n^2(5n+1)^2} + \frac{(55n^2+18n+2)\,t}{50(n-1)n^2(5n+1)^2}\\
&\quad - \frac{(3n+1)t}{10n^2(5n+1)(n+1)}\\
&\geq \frac{(55n^2+18n+2)\,t}{25(n-1)n^2(5n+1)^2} - \frac{(3n+1)t}{10n^2(5n+1)(n+1)}\\
&\geq \frac{(35n^3+181n^2+75n+9)\,t}{50n^2(5n+1)^2\,(n^2-1)} > 0 \qquad \text{q.e.d.}
\end{aligned}$$

G.2 Impact of firms' coordination on expected demand

As the overall demand for a firm is, in this parity case, symmetric over the firms and always equal to $1/n$, firms' coordination of the channels impacts only the distribution of this expected demand: it shifts a certain amount of the expected demand from one channel to the other, so that the absolute values of the variations of demand in each channel are the same, only the sign is opposite. Whether we start with the expected demand in the local or in the online channel, we get the same condition: the expected demand in the local channel is increased by coordination, respectively the expected demand in the online channel decreased by coordination, if:

$$\sqrt{\frac{\sqrt{4A(n+1)(5n+1)t+t^2}+2A(n+1)(5n+1)+t}{2(5n+1)^2t}} > \sqrt{\frac{\sqrt{4An(5n)t+t^2}+2A(5n)n+t}{2(5n)^2t}}$$

$$\frac{\sqrt{4A(n+1)(5n+1)t+t^2}+2A(n+1)(5n+1)+t}{(5n+1)^2} > \frac{\sqrt{4An(5n)t+t^2}+2A(5n)n+t}{25n^2}$$

$$25n^2\sqrt{4A(n+1)(5n+1)t+t^2}+40A(5n+1)n^2 > (5n+1)^2\sqrt{4An(5n)t+t^2}+(10n+1)t$$

We isolate one square root and take the square of the two positive sides:

$$\frac{625n^4\left(4A(n+1)(5n+1)t+t^2\right)+50n^2\left(40An^2(5n+1)-(10n+1)t\right)\sqrt{4A(n+1)(5n+1)t+t^2}}{(5n+1)^4}$$
$$+\frac{\left(40An^2(5n+1)-(10n+1)t\right)^2}{(5n+1)^4} > 4An(5n)t+t^2$$

After some transformations, when isolating the remaining square root, we are confronted to a sign problem:

$$\begin{aligned} I: \quad & \left(32n^2(5An+A)^2+2A\left(50n^3-45n^2-16n-1\right)t-(10n+1)t^2\right) \\ & > -\left(40An^2(5n+1)-(10n+1)t\right)\sqrt{4A(n+1)(5n+1)t+t^2} \end{aligned}$$

Over the range of admissible values $A \in [0; t/(16n^2)]$, the two expressions $P_1 = 32n^2(5An+A)^2+2A\left(50n^3-45n^2-16n-1\right)t-(10n+1)t^2$ and $P_2 = -\left(40An^2(5n+1)-(10n+1)t\right)$ change sign. The sign table in Figure G.1 gives an overview of the different constellations.

A	0		$\frac{10nt+t}{200n^3+40n^2}$		r		$\frac{t(3n+1)}{4n^2(n+1)}$
P_1		$-$		$-$	0	$+$	
P_2		$+$	0	$-$		$-$	
I fulfilled?		*never*		*possibly*		*always*	

where $r = \frac{\left(-10n^2+\sqrt{100n^4+100n^3+133n^2+22n+1}+11n+1\right)t}{32n^2(5n+1)}$

Figure G.1: Sign table for the variations in expected demand induced by firms' coordination

We focus on the case when both P_1 and P_2 are negative, i.e. for $A \in [\frac{10nt+t}{200n^3+40n^2}; r]$ and transform I to end up with the equivalent condition:

$$256A^2n^4-32A\left(10n^2+6n+1\right)n^2t+\left(20n^2+12n+1\right)t^2<0$$

This is a simple quadratic equation with a positive leading term. It is negative between the two roots:

$$A_{1,2} = \frac{(10n^2 + 6n + 1)\,t \pm 2n(5n+3)t}{16n^2}$$

The lower root is $A_2 = t/(16n^2)$. The upper root appears to be above the maximal value admissible for A: $A_1 = \frac{(20n^2+12n+1)t}{16n^2} > t(3n+1)/(4n^2(n+1)$. The result is illustrated in Figure G.2 : for high levels of the reluctance parameter, $A > t/(16n^2)$, firms' coordination leads to a shift of the expected demand from the online to the local channel. For a lower reluctance, demand is shifted inversely from the local to the online channel.

A	0	$\frac{t}{16n^2}$	$\frac{t(3n+1)}{4n^2(n+1)}$
reluctance	low	high	
I fulfilled	no	yes	
variation in $\mathbb{E}D_2^{t*}$	decreases	increases	
variation in $\mathbb{E}D_2^{o*}$	increases	decreases	

Figure G.2: Variations in expected demand induced by firms' coordination

G.3 Comparison of the prices

Under firms' coordination of the channels, the equilibrium prices in the local channel are higher than those in the online one if:

$$\begin{aligned} p^*_{m,l} > p^*_{m,o} \quad &\Leftrightarrow \quad \frac{(-n+1)\left[t + \sqrt{4tA(n+1)(5n+1)+t^2}\right]}{2(5n+1)^2(n-1)} + \frac{4nA}{5n+1} > 0 \\ &\Leftrightarrow \quad 8nA(5n+1) - t > \sqrt{4tA(n+1)(5n+1)+t^2} \\ &\Leftrightarrow \quad 16n^2A - t > 0 \end{aligned}$$

In the hypothetical case without firms' coordination, the equilibrium prices in the local channel are higher than those in the online one if:

$$\begin{aligned} p^*_{m,l} > p^*_{m,o} \quad &\Leftrightarrow \quad \frac{-n\left[t + \sqrt{20n^2tA+t^2}\right]}{2n(5n)^2} + \frac{4nA}{5n} > 0 \\ &\Leftrightarrow \quad 40n^2A - t > \sqrt{20n^2tA+t^2} \\ &\Leftrightarrow \quad 16n^2A - t > 0 \end{aligned}$$

G.4 Comparison of the profits in the local channel

For simplification, we focus here at the case into question, when demand is shifted by firms' coordination from the local to the online channel, i.e. when $A < t/(16n^2)$, as it is clear that profits in the local channel increase with firms coordination in the complementary case, when $t/(16n^2) < A < (t(3n+1)/(4n^2(n+1))$. The extensive formula for the difference profits in the local channel with or without firms coordination reads, after only straightforward transformations:

$$\mathbb{E}\pi_{2t}^{*,c} - \mathbb{E}\pi_{2t}^{*,nc} = \left(\frac{(2n+1)\left(\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}+t\right)}{(5n+1)^2(n-1)} + \frac{4An}{5n+1} \right) \cdot \frac{1}{(5n+1)} \sqrt{\frac{\sqrt{t(4A(n+1)(5n+1)+t)} + 2A(n+1)(5n+1)+t}{2t}} - \left(\frac{2\left(\sqrt{t}\sqrt{20An^2+t}+t\right)}{(5n)^2} + \frac{4A}{5} \right) \frac{1}{5n} \sqrt{\frac{\sqrt{t(20n^2A+t)}+10An^2+t}{2t}}$$

The main difficulty in the simplification of this formula are, obviously, the different square roots. Remark that the square roots weighted negatively (last line) are smaller than their respective counterpart in the positive summand:

$$\sqrt{4A(n+1)(5n+1)+t} > \sqrt{20An^2+t}$$

$$\sqrt{\frac{\sqrt{t(4A(n+1)(5n+1)+t)} + 2A(n+1)(5n+1)+t}{2t}} > \sqrt{\frac{\sqrt{t(20n^2A+t)}+10An^2+t}{2t}}$$

As we are here interested in the sign of this difference in profits, we can lower the positive part by replacing the square roots and go on with the inequality:

$$\mathbb{E}\pi_{2t}^{*,c} - \mathbb{E}\pi_{2t}^{*,nc} \geq \left(\frac{(2n+1)\left(\sqrt{t}\sqrt{20An^2+t}+t\right)}{(5n+1)^2(n-1)} + \frac{4An}{5n+1} \right) \frac{1}{(5n+1)} \sqrt{\frac{\sqrt{t(20n^2A+t)}+10An^2+t}{2t}} - \left(\frac{2\left(\sqrt{t}\sqrt{20An^2+t}+t\right)}{(5n)^2} + \frac{4A}{5} \right) \frac{1}{5n} \sqrt{\frac{\sqrt{t(20n^2A+t)}+10An^2+t}{2t}}$$

Putting the initial brackets on a common denominator yields:

$$\mathbb{E}\pi_{2t}^{*,c} - \mathbb{E}\pi_{2t}^{*,nc} \geq \left[\frac{(225n^3+120n^2+28n+2)\left(\sqrt{t}\sqrt{4A(n+1)(5n+1)+t}+t\right)}{25n^2(5n+1)^2(n-1)} - \frac{4A(10n+1)}{5(5n+1)} \right] \cdot \underbrace{\frac{1}{5n(5n+1)} \sqrt{\frac{\sqrt{t(20n^2A+t)}+10An^2+t}{2t}}}_{>0} \tag{G.1}$$

Let us consider the square bracket as a function of A, $\phi(A)$. Its derivative is positive for $A < t/(16n^2)$:

$$\phi'(A) = \frac{2\left(\frac{(n+1)\left(225n^3+120n^2+28n+2\right)\sqrt{t}}{(n-1)n^2\sqrt{4A(5n^2+6n+1)+t}} - 10(10n+1) \right)}{25(5n+1)} > \frac{4\left(15n^3+83n^2+25n+2\right)}{25(n-1)n(3n+1)} > 0$$

The initial value $\phi(0)$ is positive:

$$\phi(0) = \frac{(225n^3 + 120n^2 + 28n + 2)\, 2t}{25n^2(5n+1)^2(n-1)} > 0$$

The square bracket is increasing in A on the relevant domain and positive for $A = 0$, so that we can conclude that this bracket and, as a consequence, the complete expression in the right-hand side of the inequality (G.1) are positive. Our simplification of the square roots, as simple as it is, is sufficient to prove that the difference in expected profits induced by firms' coordination of channels is positive: even in the case when firms' coordination leads to a reduced expected demand in the local channel, the expected profits in this channel increase. The reason is that the increased prices overcompensate the decrease in expected demand.

Appendix H

Matlab Code for Section 4.4

```
%% arrays
rr=0:0.01:30;
pi = zeros(length(rr),1);
P = NaN(length(rr),1);
B = NaN(length(rr),1);
D = NaN(length(rr),1);
CSbf = NaN(length(rr),1);
PSbf = NaN(length(rr),1);
W = NaN(length(rr),1);
tcrit = NaN(length(rr),1);
pprevious=0.5;
bprevious=0;

%% convergence values r mapsto 0
P(1)=0.5;
B(1)=0;
pi(1)=.25;
D(1)=.5;
CSbf(1)=.125;
PSbf(1) =.25;
W(1)=.375;
tcrit(1)=NaN;

%% approximation
for k=2:length(rr)
    r=rr(k);
    pmin=pprevious-0.001;
```

```
    pmin=max([0.5 pmin]);
    pmax=pprevious+ 0.001;
    pmax=min([pmax 1]);
    pp = [pmin:0.000001:pmax]';
    bmin=bprevious- 0.01;
    bmin=max([0 bmin]);
    bmax=bprevious+ 0.01;
    bmax=min([bmax 0.5]);
    bb= [bmin:0.000001:bmax]';
    R = NaN(length(pp),1);
    for mm = 1:length(pp)
        Abs=1;
        for nn=1:length(bb)
            if pp(mm)>= 1/(2-bb(nn))
                crit=abs(FOCr(r,bb(nn),pp(mm)));
                if crit<Abs
                    Abs=crit;
                    R(mm)=bb(nn);
                end
            end
        end
        top=Profits(r,R(mm),pp(mm));
        if top>pi(k)
            pi(k)=top;
            pos=mm;
        end
    end
    P(k)=pp(pos);
    pprevious=P(k);
    B(k)=R(pos);
    bprevious=B(k);
    D(k)=P(k)*(1-B(k));
    CSbf(k)=CS(B(k),P(k),r);
    PSbf(k) = (1-P(k)+B(k)*P(k))/2;
    W(k)=CSbf(k)+PSbf(k);
    tcrit(k)=1+log((1-P(k))/(1-P(k)+B(k)*P(k)))/r;
end
```

```
function [crit] = FOCr(r,b,p)
```

```
% Condition on b, p depending on r
%   See script, equation FOCr
if (b>=0) &&(b <= 1) && (1/(2-b) <= p) && (p <= 1) && (r >= -log((1-p)/(1-p+b*p))  )
    crit = r - b*p/(1-2*p+b*p)*log((1-p)/(1-p+b*p));
else
    crit=1;
end
end
```

```
function [pi] = Profits(r,b,p)
% Profit function in collusion
%   As stated in paper
pi = p*(1-p+b*p-b^2*p) +b*p*(1-p)*log((1-p)/(1-p+b*p))/(r) + ...
    b^2*p^2*log(b*p/((1-p+b*p)*(1-exp(-r))))/(r);
end
```

Appendix I

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